Mathematics

Textbook for Class VII

0756

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

ISBN 81-7450-669-1

First Edition
February 2007 Phalguna 1928

Reprinted
October 2007 Kartika 1929
January 2009 Pausa 1930
December 2009 Pausa 1931
November 2010 Kartika 1932
January 2012 Magha 1933
November 2012 Kartika 1934
October 2013 Asvina 1935
November 2014 Agrahayana 1936
December 2015 Agrahayana 1937
December 2016 Pausa 1938
November 2017 Agrahayana 1939
January 2019 Pausha 1940
August 2019 Bhadrapada 1941

PD 1000T RPS

₹ 65.00

Printed on 80 GSM paper with NCERT watermark

Published at the Publication Division by the Secretary, National Council of Educational Research and Training, Sri Aurobindo Marg, New Delhi 110 016 and printed at Anand Publications, 106/1/A, Nusali Fata, N.H. 06, Duwe Road, Tal. Dharangaon, District Jalgaon - 425 103

OFFICES OF THE PUBLICATION DIVISION, NCERT

NCERT Campus
Sri Aurobindo Marg
New Delhi 110 016 **Phone : 011-26562708**

108, 100 Feet Road
Hosdakere Halli Extension
Banashankari III Stage
Bengaluru 560 085 **Phone : 080-26725740**

Navjivan Trust Building
P.O.Navjivan
Ahmedabad 380 014 **Phone : 079-27541446**

CWC Campus
Opp. Dhankal Bus Stop
Panihati
Kolkata 700 114 **Phone : 033-25530454**

CWC Complex
Maligaon
Guwahati 781 021 **Phone : 0361-2674869**

Publication Team

Head, Publication Division : *M. Siraj Anwar*

Chief Editor : *Shveta Uppal*

Chief Production Officer : *Arun Chitkara*

Chief Business Manager : *Bibash Kumar Das*

Editor : *Bijnan Sutar*

Production Assistant : *Om Prakash*

Cover *Shweta Rao*

Illustrations *Prashant Soni*

Foreword

The National Curriculum Framework (NCF), 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that, given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

The National Council of Educational Research and Training (NCERT) appreciates the hard work done by the textbook development committee responsible for this book. We wish to thank the Chairperson of the advisory group in science and mathematics, Professor J.V. Narlikar and the Chief Advisor for this book, Dr H.K. Dewan for guiding the work of this committee. Several teachers contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organisations which have generously permitted us to draw upon their resources, material and personnel. We are especially grateful to the members of the National Monitoring Committee, appointed by the Department of Secondary and Higher Education, Ministry of Human Resource Development under the Chairpersonship of Professor Mrinal Miri and Professor G.P. Deshpande, for their valuable time anc contribution. As an organisation committed to systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

Director
National Council of Educational
Research and Training

New Delhi
20 November 2006

Preface

The National Curriculum Framework (NCF), 2005 suggests the need for developing the ability for mathematisation in the child. It points out that the aim of learning mathematics is not merely being able to do quantitative calculations but also to develop abilities in the child that would enable her/him to redefine her/his relationship with the World. The NCF-2005 also lays emphasis on development in the children logical abilities as well as abilities to comprehend space, spatial transformations and develop the ability to visualise both these. It recommends that mathematics needs to slowly move towards abstraction even though it starts from concrete experiences and models. The ability to generalise and perceive patterns is an important step in being able to relate to the abstract and logic governed nature of the subject.

We also know that most children in upper primary and secondary classes develop a fear of mathematics and it is one of the reasons for students not being able to continue in schools. NCF-2005 has also mentioned this problem and has therefore emphasised the need to develop a programme which is relevant and meaningful. The need for conceptualising mathematics teaching allows children to explore concepts as well as develop their own ways of solving problems. This also forms corner-stone of the principles highlighted in the NCF-2005.

In Class VI we have begun the process of developing a programme which would help children understand the abstract nature of mathematics while developing in them the ability to construct their own concepts. As suggested by NCF-2005, an attempt has been made to allow multiple ways of solving problems and encouraging children to develop strategies different from each other. There is an emphasis on working with basic principles rather than on memorisation of algorithms and short-cuts.

The Class VII textbook has continued that spirit and has attempted to use language which the children can read and understand themselves. This reading can be in groups or individual and at some places require help and support by the teacher. We also tried to include a variety of examples and opportunities for children to set problems. The appearance of the book has sought to be made pleasant by including many illustrations. The book attempts to engage the mind of the child actively and provides opportunities to use concepts and develop her/his own structures rather than struggling with unnecessarily complicated terms and numbers.

We hope that this book would help all children in their attempt to learn mathematics and would build in them the ability to appreciate its power and beauty. We also hope that this would enable to revisit and consolidate concepts and skills that they have learnt in the primary school. We hope to strengthen the foundation of mathematics, on which further engagement with studies as well as her daily life would become possible in an enriched manner.

The team in developing the textbook consists of many teachers who are experienced and brought to the team the view point of the child and the school. We also had people who have done research in learning of mathematics and those who have been writing textbooks for mathematics for many years. The team has tried to make an effort to remove fear of mathematics from the minds of children and make it a part of their daily routine even outside the school. We had many discussions and a review process with some other teachers of schools across the country. The effort by the team has been to accommodate all the comments.

In the end, I would like to place on record our gratefulness to Prof Krishna Kumar, Director, NCERT, Prof G. Ravindra, Joint Director, NCERT and Prof Hukum Singh, Head, DESM, for giving opportunity to me and the team to work on this challenging task. I am also grateful to

Prof J.V. Narlikar, Chairperson of the Advisory Group in Science and Mathematics for his suggestions. I am also grateful for the support of all those who were part of this team including Prof S.K. Singh Gautam, Dr V.P. Singh and Dr Ashutosh K. Wazalwar from NCERT, who have worked very hard to make this possible. In the end I must thank the Publication Department of NCERT for its support and advice and those from Vidya Bhawan who helped produce the book.

The process of developing materials is a continuous one and we would hope to make this book better. Suggestions and comments on the book are most welcome.

Dr H.K. Dewan
Chief Advisor
Textbook Development Committee

A Note for the Teachers

This book is a continuation of the process and builds on what was initiated in Class VI. We had shared with you the main points reflected in NCF-2005. These include relating mathematics to a wider development of abilities in children, moving away from complex calculations and algorithms following, to understanding and constructing a framework of understanding. The mathematical ideas in the mind of the child grow neither by telling nor by merely giving explanations. For children to learn mathematics, to be confident in it and understand the foundational ideas, they need to develop their own framework of concepts. This would require a classroom where children discuss ideas, look for solutions of problems, set new problems and find not only their own ways of solving problems but also their own definitions with the language they can use and understand. These definitions need not be as general and complete as the standard ones.

In the mathematics class it is important to help children read with understanding the textbook and other references. The reading of materials is not normally considered to be related to learning of mathematics but learning mathematics any further would require the child to comprehend the text. The text in mathematics uses a language that has brevity. It requires the ability to deal with terseness and with symbols, to follow logical arguments and appreciate the need for keeping certain factors and constraints. Children need practice in translating mathematical statements into normal statements expressing ideas in words and vice-a-versa. We would require children to become confident of using language in words and also being able to communicate through mathematical statements.

Mathematics at the upper primary stage is a major challenge and has to perform the dual role of being both close to the experience and environment of the child and being abstract. Children often are not able to work in terms of ideas alone. They need the comfort of context and/or models linked to their experience to find meaning. This stage presents before us the challenge of engaging the children while using the contexts but gradually moving them away from such dependence. So while children should be able to identify the principles to be used in a contextual situation, they should not be dependent or be limited to contexts. As we progress further in the middle school there would be greater requirement from the child to be able to do this.

Learning mathematics is not about remembering solutions or methods but knowing how to solve problems. Problem-solving strategies give learners opportunities to think rationally, enabling them to understand and create methods as well as processes; they become active participants in the construction of new knowledge rather than being passive receivers. Learners need to identify and define a problem, select or design possible solutions and revise or redesign the steps, if required. The role of a teacher gets modified to that of a guide and facilitator. Students need to be provided with activities and challenging problems, along with sets of many problem-solving experiences.

On being presented a problem, children first need to decode it. They need to identify the knowledge required for attempting it and build a model for it. This model could be in the form of an illustration or a situation construct. We must remember that for generating proofs in geometry the figures constructed are also models of the ideal dimensionless figure. These diagrams are, however, more abstract than the concrete models required for attempting problems in arithmetic and algebra. Helping children to develop the ability to construct appropriate models by breaking up the problems and evolving their own strategies and analysis of problems is extremely important. This should replace prescriptive algorithms to solve problems.

Teachers are expected to encourage cooperative learning. Children learn a lot in purposeful conversation with each other. Our classrooms should develop in the students the desire and capacity to learn from each other rather than compete. Conversation is not noise and consultation is not cheating. It is a challenge to make possible classroom groups that benefit the most from being with each other and in which each child contributes to the learning of the group. Teachers must recognise that different children and different groups will use distinct strategies. Some of these strategies would appear to be more efficient and some not as efficient. They would reflect the modelling done by each group and would indicate the process of thinking used. It is inappropriate to identify the best strategy or pull down incorrect strategies. We need to record all strategies adopted and analyse them. During this, it is crucial to discuss why some of the strategies are unsuccessful. The class as a group can improve upon the ineffective and unsuccessful strategies and correct them. This implies that we need to complete each strategy rather than discard some as incorrect or inappropriate. Exposures to a variety of strategies would deepen mathematical understanding and ability to learn from others. This would also help them to understand the importance of being aware of what one is doing.

Enquiry to understand is one of the natural ways by which students acquire and construct knowledge. The process can even begin with casual observations and end in generation and acquisition of knowledge. This can be aided by providing examples for different forms of questioning-explorative, open-ended, contextual, error detection etc. Students need to get exposed to challenging investigations. For example in geometry there could be things like, experimenting with suitable nets for solids, visualising solids through shadow play, slicing and elevations etc. In arithmetic we can make them explore relationships among members, generalise the relationships, discover patterns and rules and then form algebraic relations etc.

Children need the opportunity to follow logical arguments and find loopholes in the arguments presented. This will lead them to understand the requirement of a proof.

At this stage topics like Geometry are poised to enter a formal stage. Provide activities that encourage students to exercise creativity and imagination while discovering geometric vocabulary and relationships using simple geometric tools.

Mathematics has to emerge as a subject of exploration and creation rather than an exercise of finding answers to old and complicated problems. There is a need to encourage children to find many different ways to solve problems. They also need to appreciate the use of many alternative algorithms and strategies that may be adopted to solve a problem.

Topics like Integers, Fractions and Decimals, Symmetry have been presented here by linking them with their introductory parts studied in earlier classes. An attempt has been made to link chapters with each other and the ideas introduced in the initial chapters have been used to evolve concepts in the subsequent chapters. Please devote enough time to the ideas of negative integers, rational numbers, exploring statements in Geometry and visualising solids shapes.

We hope that the book will help children learn to enjoy mathematics and be confident in the concepts introduced. We want to recommend the creation of opportunity for thinking individually and collectively. Group discussions need to become a regular feature of mathematics classroom thereby making learners confident about mathematics and make the fear of mathematics a thing of past.

We look forward to your comments and suggestions regarding the book and hope that you will send interesting exercises, activities and tasks that you develop during the course of teaching, to be included in the future editions.

Textbook Development Committee

Chairperson, Advisory Group in Science and Mathematics

J.V. Narlikar, *Emeritus Professor*, Inter University Centre for Astronomy and Astrophysics (IUCCA), Ganeshkhind, Pune University, Pune, Maharashtra

Chief Advisor

H.K. Dewan, Vidya Bhawan Society, Udaipur, Rajasthan

Chief Coordinator

Hukum Singh, *Professor* and *Head (Retd.),* DESM, NCERT, New Delhi

Members

Anjali Gupte, *Teacher*, Vidya Bhawan Public School, Udaipur, Rajasthan

Avantika Dam, *TGT*, CIE Experimental Basic School, Department of Education, Delhi

H.C. Pradhan, *Professor*, Homi Bhabha Centre for Science Education, TIFR, Mumbai, Maharashtra

Mahendra Shankar, *Lecturer (S.G.) (Retd.),* NCERT, New Delhi

Meena Shrimali, *Teacher*, Vidya Bhawan Senior Secondary School, Udaipur, Rajasthan

R. Athmaraman, *Mathematics Education Consultant*, TI Matric Higher Secondary School and AMTI, Chennai, Tamil Nadu

S.K.S. Gautam, *Professor*, DESM, NCERT, New Delhi

Shradha Agarwal, *PGT*, Sir Padampat Singhania Education Centre, Kanpur (U.P.)

Srijata Das, *Senior Lecturer* in Mathematics, SCERT, New Delhi

V.P. Singh, *Reader (Retd.)*, DESM, NCERT, New Delhi

Member-coordinator

Ashutosh K. Wazalwar, *Professor,* DESM, NCERT, New Delhi

Acknowledgements

The Council gratefully acknowledges the valuable contributions of the following participants of the Textbook Review Workshop – Ms. Nirupma Sahni, *TGT,* Mahavir Digambar Jain Sr. Sec. School, Jaipur; Dr Roohi Fatima, *TGT,* Jamia Middle School, New Delhi; Ms. Deepti Mathur, *TGT,* Mother's International School, New Delhi; Shri K. Balaji, *TGT,* Kendriya Vidyalaya, Donimalai, Karnataka; Shri Amit Bajaj, *TGT,* CRPF Public School, Delhi; Ms. Omlata Singh, *TGT,* Presentation Convent Sr. Sec. School, Delhi; Shri Nagesh S. Mone, *TGT,* Dravid High School, Wai, Maharashtra; Shri Gorakh Nath Sharma, *PGT,* Jawahar Navodaya Vidyalaya, Mesra, Ranchi, Jharkhand; Shri Ajay Kumar Singh, *TGT,* Ramjas Sr. Sec. School, No.3, Delhi; Ms. Ragini Subramanian, *TGT,* SRDF Vivekananda Vidyalaya, Chennai, Tamil Nadu; Shri Rajkumar Dhawan, *PGT,* Geeta Sr. Sec. School No.2, Delhi; Dr Sanjay Mudgil, *Lecturer,* CIET, NCERT, New Delhi; Dr. Sushma Jaireth, *Reader,* DWS, NCERT, New Delhi; Dr Mona Yadav, *Lecturer,* DWS, NCERT, New Delhi.

The Council acknowledges the comments/suggestions given by Dr Ram Avtar (*Retd. Professor,* NCERT) *Consultant,* DESM, NCERT, New Delhi, Dr R.P. Maurya, *Reader,* DESM, NCERT, New Delhi and Shri Sanjay Bolia, *Senior Teacher,* Vidya Bhawan Basic Secondary School, Udaipur, Rajasthan for the improvement of the content.

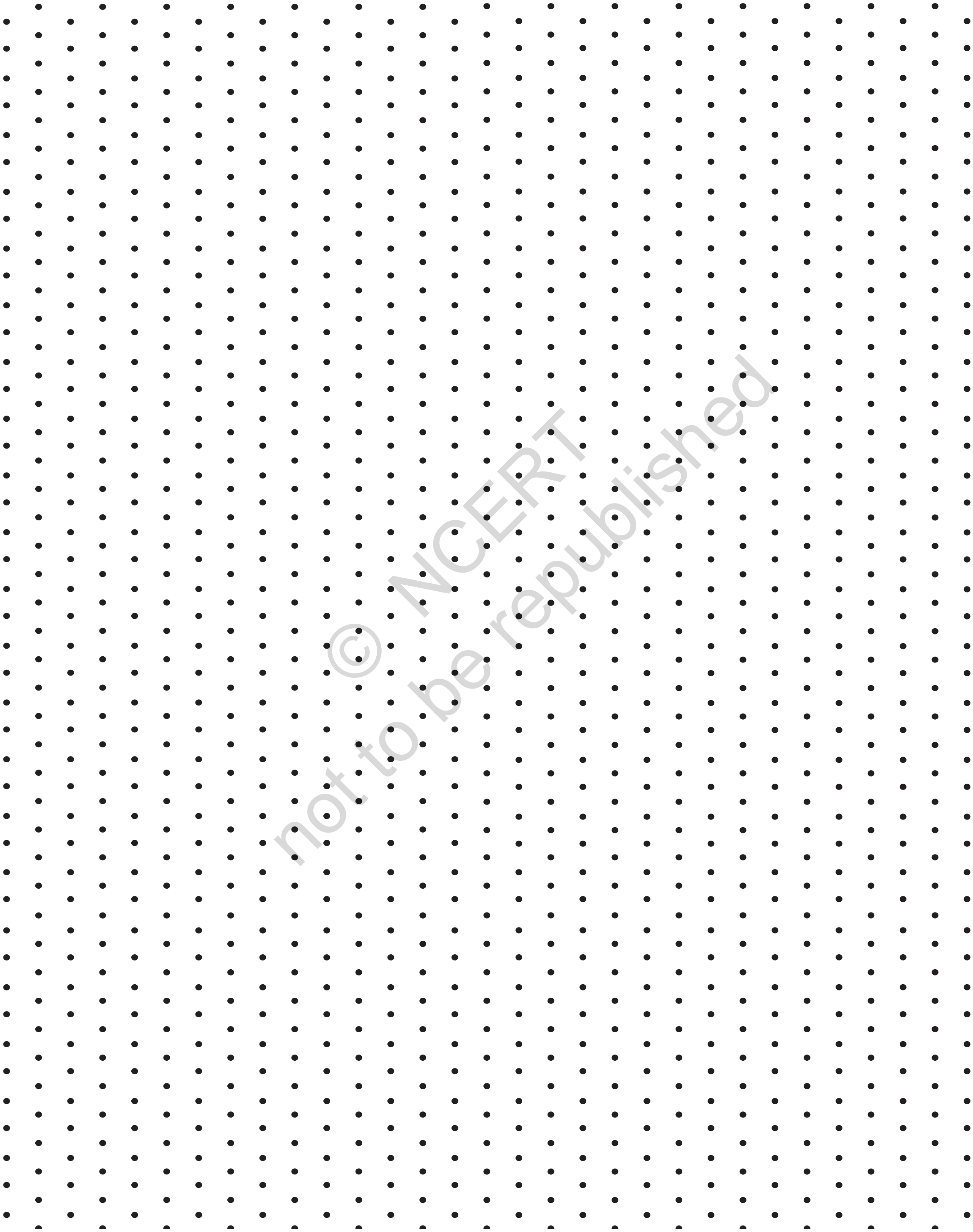
The Council acknowledges the support and facilities provided by Vidya Bhawan Society and its staff, Udaipur, for conducting workshops of the development committee at Udaipur, and to the Director, Centre for Science Education and Communication (CSEC), Delhi University for providing library help.

The Council acknowledges the academic and administrative support of Professor Hukum Singh, *Head,* DESM, NCERT, New Delhi.

The Council also acknowledges the efforts of S.M. Ikram, *DTP Operator,* Vidya Bhawan Society Udaipur; Sajjad Haider Ansari, Rakesh Kumar and Neelam Walecha, *DTP Operators,* Kanwar Singh, *Copy Editor,* NCERT; Abhimanu Mohanty, *Proof Reader,* NCERT; Deepak Kapoor, *Computer Station Incharge,* DESM, NCERT for technical assistance, APC-office and the Administrative Staff, DESM, NCERT; and the Publication Department of the NCERT.

Contents

Isometric Dot Sheet



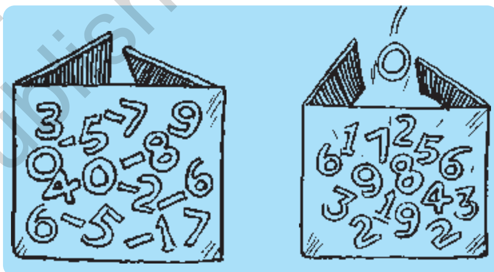
Chapter 1

Integers

0756CH01

1.1 Introduction

We have learnt about whole numbers and integers in Class VI. We know that integers form a *bigger* collection of numbers which contains whole numbers and negative numbers. What other differences do you find between whole numbers and integers? In this chapter, we will study more about integers, their properties and operations. First of all, we will review and revise what we have done about integers in our previous class.

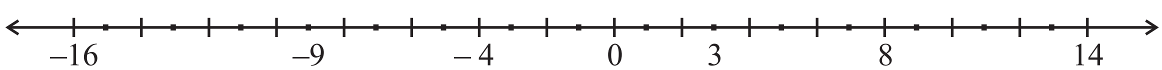


1.2 Recall

We know how to represent integers on a number line. Some integers are marked on the number line given below.

Can you write these marked integers in ascending order? The ascending order of these numbers is – 5, – 1, 3. Why did we choose – 5 as the smallest number?

Some points are marked with integers on the following number line. Write these integers in descending order.

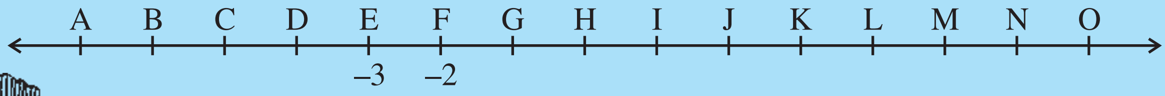


The descending order of these integers is 14, 8, 3, ...

The above number line has only a few integers filled. Write appropriate numbers at each dot.

TRY THESE

1. A number line representing integers is given below



–3 and –2 are marked by E and F respectively. Which integers are marked by B, D, H, J, M and O?

2. Arrange 7, –5, 4, 0 and – 4 in ascending order and then mark them on a number line to check your answer.

We have done addition and subtraction of integers in our previous class. Read the following statements.

On a number line when we

(i) add a positive integer, we move to the right.
(ii) add a negative integer, we move to the left.
(iii) subtract a positive integer, we move to the left.
(iv) subtract a negative integer, we move to the right.

State whether the following statements are correct or incorrect. Correct those which are wrong:

(i) When two positive integers are added we get a positive integer.
(ii) When two negative integers are added we get a positive integer.
(iii) When a positive integer and a negative integer are added, we always get a negative integer.
(iv) Additive inverse of an integer 8 is (– 8) and additive inverse of (– 8) is 8.
(v) For subtraction, we add the additive inverse of the integer that is being subtracted, to the other integer.
(vi) $(-10) + 3 = 10 - 3$
(vii) $8 + (-7) - (-4) = 8 + 7 - 4$

Compare your answers with the answers given below:

(i) Correct. For example:

(a) $56 + 73 = 129$ (b) $113 + 82 = 195$ etc.

Construct five more examples in support of this statement.

(ii) Incorrect, since $(-6) + (-7) = -13$, which is not a positive integer. The correct statement is: When two negative integers are added we get a negative integer.

For example,

(a) $(-56) + (-73) = -129$ (b) $(-113) + (-82) = -195$, etc.

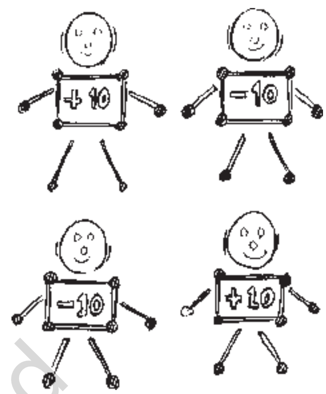
Construct five more examples on your own to verify this statement.

(iii) Incorrect, since $-9+16=7$, which is not a negative integer. The correct statement is : When one positive and one negative integers are added, we take their difference and place the sign of the bigger integer. The bigger integer is decided by ignoring the signs of both the integers. For example:

(a) $(-56)+(73)=17$ (b) $(-113)+82=-31$

(c) $16+(-23)=-7$ (d) $125+(-101)=24$

Construct five more examples for verifying this statement.



(iv) Correct. Some other examples of additive inverse are as given below:

Integer	Additive inverse
10	–10
–10	10
76	–76
–76	76

Thus, the additive inverse of any integer a is $-a$ and additive inverse of $(-a)$ is a.

(v) Correct. Subtraction is opposite of addition and therefore, we add the additive inverse of the integer that is being subtracted, to the other integer. For example:

(a) $56-73=56$ + additive inverse of $73=56+(-73)=-17$

(b) $56-(-73)=56$ + additive inverse of $(-73)=56+73=129$

(c) $(-79)-45=(-79)+(-45)=-124$

(d) $(-100)-(-172)=-100+172=72$ etc.

Write atleast five such examples to verify this statement.

Thus, we find that for any two integers a and b,

$a-b=a$ + additive inverse of $b=a+(-b)$

and $a-(-b)=a$ + additive inverse of $(-b)=a+b$

(vi) Incorrect, since $(-10)+3=-7$ and $10-3=7$

therefore, $(-10)+3\neq 10-3$

(vii) Incorrect, since, $8+(-7)-(-4)=8+(-7)+4=1+4=5$

and $8+7-4=15-4=11$

However, $8+(-7)-(-4)=8-7+4$

TRY THESE

We have done various patterns with numbers in our previous class.

Can you find a pattern for each of the following? If yes, complete them:

(a) 7, 3, – 1, – 5, ______, ______, ______.

(b) – 2, – 4, – 6, – 8, ______, ______, ______.

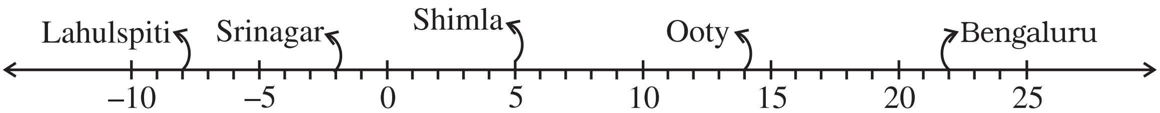
(c) 15, 10, 5, 0, ______, ______, ______.

(d) – 11, – 8, – 5, – 2, ______, ______, ______.

Make some more such patterns and ask your friends to complete them.

EXERCISE 1.1

1. Following number line shows the temperature in degree celsius (°C) at different places on a particular day.



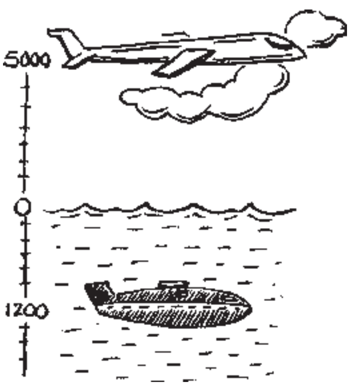
(a) Observe this number line and write the temperature of the places marked on it.

(b) What is the temperature difference between the hottest and the coldest places among the above?

(c) What is the temperature difference between Lahulspiti and Srinagar?

(d) Can we say temperature of Srinagar and Shimla taken together is less than the temperature at Shimla? Is it also less than the temperature at Srinagar?

2. In a quiz, positive marks are given for correct answers and negative marks are given for incorrect answers. If Jack's scores in five successive rounds were 25, – 5, – 10, 15 and 10, what was his total at the end?

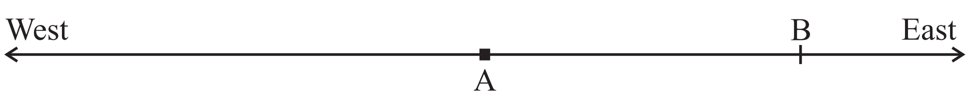


3. At Srinagar temperature was – 5°C on Monday and then it dropped by 2°C on Tuesday. What was the temperature of Srinagar on Tuesday? On Wednesday, it rose by 4°C. What was the temperature on this day?

4. A plane is flying at the height of 5000 m above the sea level. At a particular point, it is exactly above a submarine floating 1200 m below the sea level. What is the vertical distance between them?

5. Mohan deposits ₹ 2,000 in his bank account and withdraws ₹ 1,642 from it, the next day. If withdrawal of amount from the account is represented by a negative integer, then how will you represent the amount deposited? Find the balance in Mohan's account after the withdrawal.

6. Rita goes 20 km towards east from a point A to the point B. From B, she moves 30 km towards west along the same road. If the distance towards east is represented by a positive integer then, how will you represent the distance travelled towards west? By which integer will you represent her final position from A?



7. In a magic square each row, column and diagonal have the same sum. Check which of the following is a magic square.

5	–1	– 4
–5	–2	7
0	3	–3

(i)

1	–10	0
– 4	–3	–2
– 6	4	–7

(ii)

8. Verify $a - (-b) = a + b$ for the following values of a and b.

(i) $a = 21$, $b = 18$ (ii) $a = 118$, b = 125

(iii) $a = 75$, $b = 84$ (iv) $a = 28$, $b = 11$

9. Use the sign of >, < or = in the box to make the statements true.

(a) (– 8) + (– 4) $\square$ (–8) – (– 4)

(b) (– 3) + 7 – (19) $\square$ 15 – 8 + (– 9)

(c) 23 – 41 + 11 $\square$ 23 – 41 – 11

(d) 39 + (– 24) – (15) $\square$ 36 + (– 52) – (– 36)

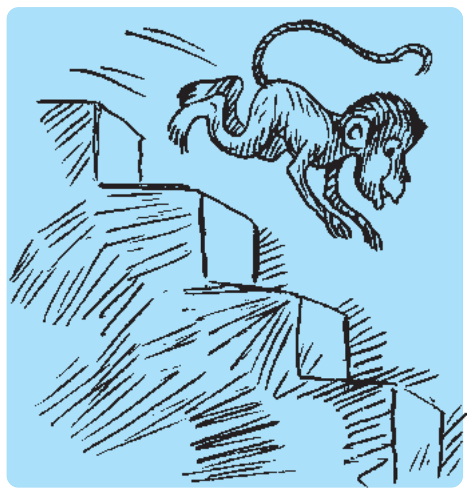
(e) – 231 + 79 + 51 $\square$ –399 + 159 + 81

10. A water tank has steps inside it. A monkey is sitting on the topmost step (i.e., the first step). The water level is at the ninth step.

(i) He jumps 3 steps down and then jumps back 2 steps up. In how many jumps will he reach the water level?

(ii) After drinking water, he wants to go back. For this, he jumps 4 steps up and then jumps back 2 steps down in every move. In how many jumps will he reach back the top step?

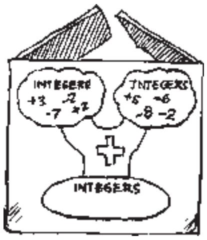
(iii) If the number of steps moved down is represented by negative integers and the number of steps moved up by positive integers, represent his moves in part (i) and (ii) by completing the following; (a) – 3 + 2 – ... = – 8 (b) 4 – 2 + ... = 8. In (a) the sum (– 8) represents going down by eight steps. So, what will the sum 8 in (b) represent?



1.3 Properties of Addition and Subtraction of Integers

1.3.1 Closure under Addition

We have learnt that sum of two whole numbers is again a whole number. For example, 17 + 24 = 41 which is again a whole number. We know that, this property is known as the closure property for addition of the whole numbers.



Let us see whether this property is true for integers or not.

Following are some pairs of integers. Observe the following table and complete it.

Statement	Observation
(i) 17 + 23 = 40	Result is an integer
(ii) (–10) + 3 = _____	_______________
(iii) (– 75) + 18 = _____	_______________
(iv) 19 + (– 25) = – 6	Result is an integer
(v) 27 + (– 27) = _____	_______________
(vi) (– 20) + 0 = _____	_______________
(vii) (– 35) + (– 10) = _____	_______________

What do you observe? Is the sum of two integers always an integer?

Did you find a pair of integers whose sum is not an integer?

Since addition of integers gives integers, we say **integers are closed under addition**.

In general, **for any two integers *a* and *b*, *a* + *b* is an integer.**

1.3.2 Closure under Subtraction

What happens when we subtract an integer from another integer? Can we say that their difference is also an integer?

Observe the following table and complete it:

Statement	Observation
(i) 7 – 9 = – 2	Result is an integer
(ii) 17 – (– 21) = _______	_______________
(iii) (– 8) – (–14) = 6	Result is an integer
(iv) (– 21) – (– 10) = _______	_______________
(v) 32 – (–17) = _______	_______________
(vi) (– 18) – (– 18) = _______	_______________
(vii) (– 29) – 0 = _______	_______________

What do you observe? Is there any pair of integers whose difference is not an integer? Can we say integers are closed under subtraction? Yes, we can see that *integers are closed under subtraction*.

Thus, *if a and b are two integers then a – b is also an intger*. Do the whole numbers satisfy this property?

1.3.3 Commutative Property

We know that $3 + 5 = 5 + 3 = 8$, that is, the whole numbers can be added in any order. In other words, addition is commutative for whole numbers.

Can we say the same for integers also?

We have $5 + (-6) = -1$ and $(-6) + 5 = -1$

So, $5 + (-6) = (-6) + 5$

Are the following equal?

(i) $(-8) + (-9)$ and $(-9) + (-8)$

(ii) $(-23) + 32$ and $32 + (-23)$

(iii) $(-45) + 0$ and $0 + (-45)$

Try this with five other pairs of integers. Do you find any pair of integers for which the sums are different when the order is changed? Certainly not. We say that *addition is commutative for integers.*

In general, for any two integers a and b, we can say

$$a + b = b + a$$

- We know that subtraction is not commutative for whole numbers. Is it commutative for integers?

  Consider the integers 5 and (–3).

  Is $5 - (-3)$ the same as $(-3) - 5$? No, because $5 - (-3) = 5 + 3 = 8$, and $(-3) - 5 = -3 - 5 = -8$.

  Take atleast five different pairs of integers and check this.

  We conclude that subtraction is not commutative for integers.

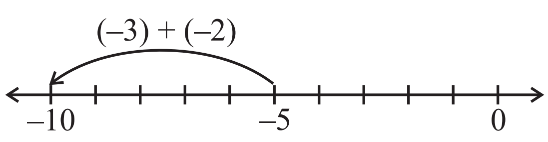
1.3.4 Associative Property

Observe the following examples:

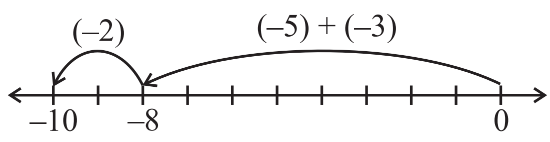
Consider the integers –3, –2 and –5.

Look at $(-5) + [(-3) + (-2)]$ and $[(-5) + (-3)] + (-2)$.

In the first sum (–3) and (–2) are grouped together and in the second (–5) and (–3) are grouped together. We will check whether we get different results.



$(-5) + [(-3) + (-2)]$



$[(-5) + (-3)] + (-2)$

In both the cases, we get –10.

i.e., $(-5) + [(-3) + (-2)] = [(-5) + (-2)] + (-3)$

Similarly consider –3 , 1 and –7.

$(-3) + [1 + (-7)] = -3 +$ __________ = __________

$[(-3) + 1] + (-7) = -2 +$ __________ = __________

Is $(-3) + [1 + (-7)]$ same as $[(-3) + 1] + (-7)$?

Take five more such examples. You will not find any example for which the sums are different. *Addition is associative for integers*.

In general for any integers *a*, *b* and *c*, we can say

$$a + (b + c) = (a + b) + c$$

1.3.5 Additive Identity

When we add zero to any whole number, we get the same whole number. Zero is an additive identity for whole numbers. Is it an additive identity again for integers also?

Observe the following and fill in the blanks:

(i) $(-8) + 0 = -8$ (ii) $0 + (-8) = -8$

(iii) $(-23) + 0 =$ _____ (iv) $0 + (-37) = -37$

(v) $0 + (-59) =$ _____ (vi) $0 +$ _____ $= -43$

(vii) $-61 +$ _____ $= -61$ (viii) _____ $+ 0 =$ _____

The above examples show that zero is an additive identity for integers.

You can verify it by adding zero to any other five integers.

In general, for any integer *a*

$$a + 0 = a = 0 + a$$

TRY THESE

1. Write a pair of integers whose sum gives

(a) a negative integer (b) zero

(c) an integer smaller than both the integers. (d) an integer smaller than only one of the integers.

(e) an integer greater than both the integers.

2. Write a pair of integers whose difference gives

(a) a negative integer. (b) zero.

(c) an integer smaller than both the integers. (d) an integer greater than only one of the integers.

(e) an integer greater than both the integers.

EXAMPLE 1 Write down a pair of integers whose

(a) sum is –3 (b) difference is –5

(c) difference is 2 (d) sum is 0

SOLUTION

(a) $(-1) + (-2) = -3$ or $(-5) + 2 = -3$

(b) $(-9) - (-4) = -5$ or $(-2) - 3 = -5$

(c) $(-7) - (-9) = 2$ or $1 - (-1) = 2$

(d) $(-10) + 10 = 0$ or $5 + (-5) = 0$

Can you write more pairs in these examples?

EXERCISE 1.2

1. Write down a pair of integers whose:

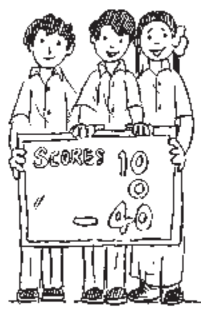
   (a) sum is –7 (b) difference is –10 (c) sum is 0

2. (a) Write a pair of negative integers whose difference gives 8.

   (b) Write a negative integer and a positive integer whose sum is –5.

   (c) Write a negative integer and a positive integer whose difference is –3.

3. In a quiz, team A scored – 40, 10, 0 and team B scored 10, 0, – 40 in three successive rounds. Which team scored more? Can we say that we can add integers in any order?



4. Fill in the blanks to make the following statements true:

   (i) $(-5) + (-8) = (-8) + (\text{............})$

   (ii) $-53 + \text{............} = -53$

   (iii) $17 + \text{............} = 0$

   (iv) $[13 + (-12)] + (\text{............}) = 13 + [(-12) + (-7)]$

   (v) $(-4) + [15 + (-3)] = [-4 + 15] + \text{............}$

1.4 MULTIPLICATION OF INTEGERS

We can add and subtract integers. Let us now learn how to multiply integers.

1.4.1 Multiplication of a Positive and a Negative Integer

We know that multiplication of whole numbers is repeated addition. For example,

$$5 + 5 + 5 = 3 \times 5 = 15$$

Can you represent addition of integers in the same way?

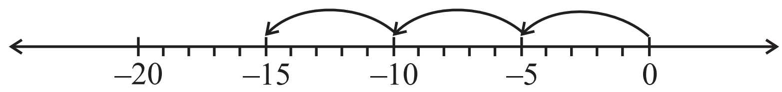
TRY THESE

Find:

$4 \times (-8)$,
$8 \times (-2)$,
$3 \times (-7)$,
$10 \times (-1)$

using number line.

We have from the following number line, $(-5) + (-5) + (-5) = -15$

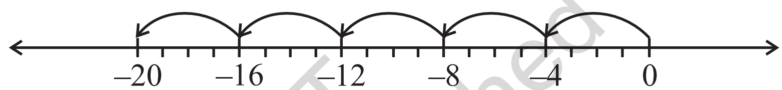


But we can also write

$$(-5) + (-5) + (-5) = 3 \times (-5)$$

Therefore, $\quad 3 \times (-5) = -15$

Similarly $\quad (-4) + (-4) + (-4) + (-4) + (-4) = 5 \times (-4) = -20$



And $\quad (-3) + (-3) + (-3) + (-3) = \_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_$

Also, $\quad (-7) + (-7) + (-7) = \_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_$

Let us see how to find the product of a positive integer and a negative integer without using number line.

Let us find $3 \times (-5)$ in a different way. First find 3×5 and then put minus sign (–) before the product obtained. You get –15. That is we find $-(3 \times 5)$ to get –15.

Similarly, $\quad 5 \times (-4) = -(5 \times 4) = -20$.

Find in a similar way,

$4 \times (-8) = \_\_\_\_ = \_\_\_\_$, $\quad 3 \times (-7) = \_\_\_\_ = \_\_\_\_$

$6 \times (-5) = \_\_\_\_ = \_\_\_\_$, $\quad 2 \times (-9) = \_\_\_\_ = \_\_\_\_$

TRY THESE

Find:

(i) $6 \times (-19)$
(ii) $12 \times (-32)$
(iii) $7 \times (-22)$

Using this method we thus have,

$$10 \times (-43) = \_\_\_\_ - (10 \times 43) = -430$$

Till now we multiplied integers as (positive integer) × (negative integer).

Let us now multiply them as (negative integer) × (positive integer).

We first find -3×5.

To find this, observe the following pattern:

We have, $\quad 3 \times 5 = 15$

$2 \times 5 = 10 = 15 - 5$

$1 \times 5 = 5 = 10 - 5$

$0 \times 5 = 0 = 5 - 5$

So, $\quad -1 \times 5 = 0 - 5 = -5$

$-2 \times 5 = -5 - 5 = -10$

$-3 \times 5 = -10 - 5 = -15$

We already have $3 \times (-5) = -15$

So we get $(-3) \times 5 = -15 = 3 \times (-5)$

Using such patterns, we also get $(-5) \times 4 = -20 = 5 \times (-4)$

Using patterns, find $(-4) \times 8$, $(-3) \times 7$, $(-6) \times 5$ and $(-2) \times 9$

Check whether, $(-4) \times 8 = 4 \times (-8)$, $(-3) \times 7 = 3 \times (-7)$, $(-6) \times 5 = 6 \times (-5)$

and $(-2) \times 9 = 2 \times (-9)$

Using this we get, $(-33) \times 5 = 33 \times (-5) = -165$

We thus find that while *multiplying a positive integer and a negative integer, we multiply them as whole numbers and put a minus sign (–) before the product. We thus get a negative integer.*

TRY THESE

1. Find: (a) $15 \times (-16)$ (b) $21 \times (-32)$
 (c) $(-42) \times 12$ (d) -55×15
2. Check if (a) $25 \times (-21) = (-25) \times 21$ (b) $(-23) \times 20 = 23 \times (-20)$
 Write five more such examples.

In general, for any two positive integers a and b we can say

$$a \times (-b) = (-a) \times b = -(a \times b)$$

1.4.2 Multiplication of two Negative Integers

Can you find the product $(-3) \times (-2)$?

Observe the following:

$-3 \times 4 = -12$

$-3 \times 3 = -9 = -12 - (-3)$

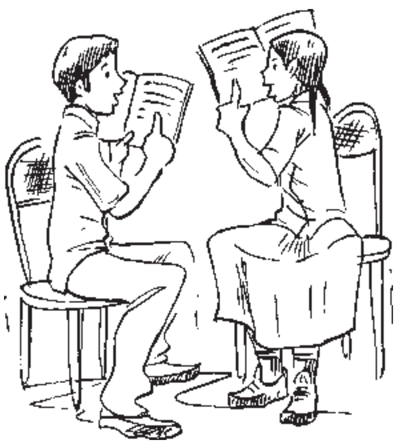
$-3 \times 2 = -6 = -9 - (-3)$

$-3 \times 1 = -3 = -6 - (-3)$

$-3 \times 0 = 0 = -3 - (-3)$

$-3 \times -1 = 0 - (-3) = 0 + 3 = 3$

$-3 \times -2 = 3 - (-3) = 3 + 3 = 6$



Do you see any pattern? Observe how the products change.

Based on this observation, complete the following:

$-3 \times -3 =$ _____ $-3 \times -4 =$ _____

Now observe these products and fill in the blanks:

$-4 \times 4 = -16$

$-4 \times 3 = -12 = -16 + 4$

$-4 \times 2 =$ _____ $= -12 + 4$

$-4 \times 1 =$ _____

$-4 \times 0 =$ _____

$-4 \times (-1) =$ _____

$-4 \times (-2) =$ _____

$-4 \times (-3) =$ _____

TRY THESE

(i) Starting from $(-5) \times 4$, find $(-5) \times (-6)$

(ii) Starting from $(-6) \times 3$, find $(-6) \times (-7)$

From these patterns we observe that,

$(-3) \times (-1) = 3 = 3 \times 1$

$(-3) \times (-2) = 6 = 3 \times 2$

$(-3) \times (-3) = 9 = 3 \times 3$

and $(-4) \times (-1) = 4 = 4 \times 1$

So, $(-4) \times (-2) = 4 \times 2 =$ _____

$(-4) \times (-3) =$ _____ $=$ _____

So observing these products we can say that the *product of two negative integers is a positive integer. We multiply the two negative integers as whole numbers and put the positive sign before the product.*

Thus, we have $(-10) \times (-12) = +120 = 120$

Similarly $(-15) \times (-6) = +90 = 90$

In general, for any two positive integers a and b,

$$(-a) \times (-b) = a \times b$$

TRY THESE

Find: $(-31) \times (-100)$, $(-25) \times (-72)$, $(-83) \times (-28)$

Game 1

(i) Take a board marked from –104 to 104 as shown in the figure.

(ii) Take a bag containing two blue and two red dice. Number of dots on the blue dice indicate positive integers and number of dots on the red dice indicate negative integers.

(iii) Every player will place his/her counter at zero.

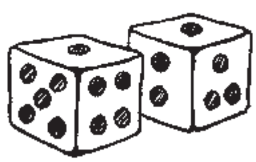
(iv) Each player will take out two dice at a time from the bag and throw them.

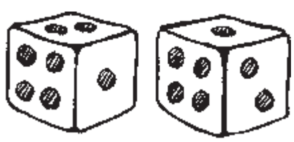
104	103	102	101	100	99	98	97	96	95	94
83	84	85	86	87	88	89	90	91	92	93
82	81	80	79	78	77	76	75	74	73	72
61	62	63	64	65	66	67	68	69	70	71
60	59	58	57	56	55	54	53	52	51	50
39	40	41	42	43	44	45	46	47	48	49
38	37	36	35	34	33	32	31	30	29	28
17	18	19	20	21	22	23	24	25	26	27
16	15	14	13	12	11	10	9	8	7	6
–5	– 4	–3	–2	–1	0	1	2	3	4	5
– 6	–7	– 8	–9	–10	–11	–12	–13	–14	–15	–16
–27	–26	–25	–24	–23	–22	–21	–20	–19	–18	–17
–28	–29	–30	–31	–32	–33	–34	–35	–36	–37	–38
– 49	–48	–47	–46	–45	–44	–43	–42	–41	–40	–39
– 50	–51	–52	–53	–54	–55	–56	–57	–58	–59	– 60
–71	–70	–69	–68	–67	–66	–65	–64	–63	–62	–61
–72	–73	–74	–75	–76	–77	–78	–79	–80	–81	–82
–93	–92	–91	–90	–89	–88	–87	–86	–85	–84	–83
–94	–95	–96	–97	–98	–99	–100	–101	–102	–103	–104

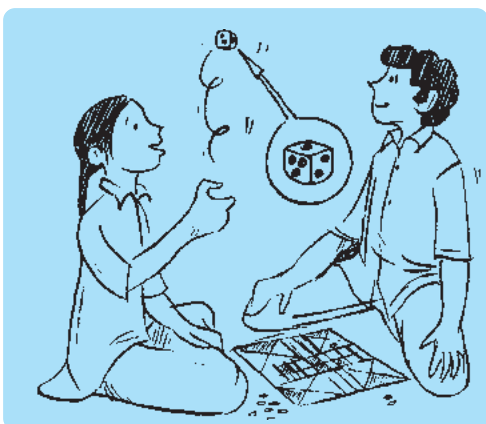
(v) After every throw, the player has to multiply the numbers marked on the dice.

(vi) If the product is a positive integer then the player will move his counter towards 104; if the product is a negative integer then the player will move his counter towards –104.

(vii) The player who reaches either -104 or 104 first is the winner.







1.4.3 Product of three or more Negative Integers

Euler in his book Ankitung zur Algebra(1770), was one of the first mathematicians to attempt to prove $(-1) \times (-1) = 1$

We observed that the product of two negative integers is a positive integer. What will be the product of three negative integers? Four negative integers? Let us observe the following examples:

(a) $(-4) \times (-3) = 12$

(b) $(-4) \times (-3) \times (-2) = [(-4) \times (-3)] \times (-2) = 12 \times (-2) = -24$

(c) $(-4) \times (-3) \times (-2) \times (-1) = [(-4) \times (-3) \times (-2)] \times (-1) = (-24) \times (-1)$

(d) $(-5) \times [(-4) \times (-3) \times (-2) \times (-1)] = (-5) \times 24 = -120$

From the above products we observe that

(a) the product of two negative integers is a positive integer;

(b) the product of three negative integers is a negative integer.

(c) product of four negative integers is a positive integer.

What is the product of five negative integers in (d)?

So what will be the product of six negative integers?

We further see that in (a) and (c) above, the number of negative integers that are multiplied are even [two and four respectively] and the product obtained in (a) and (c) are positive integers. The number of negative integers that are multiplied in (b) and (d) is odd and the products obtained in (b) and (d) are negative integers.

A Special Case

Consider the following statements and the resultant products:

$(-1) \times (-1) = +1$

$(-1) \times (-1) \times (-1) = -1$

$(-1) \times (-1) \times (-1) \times (-1) = +1$

$(-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$

This means that if the integer (-1) is multiplied even number of times, the product is $+1$ and if the integer (-1) is multiplied odd number of times, the product is -1. You can check this by making pairs of (-1) in the statement. This is useful in working out products of integers.

We find that *if the number of negative integers in a product is even, then the product is a positive integer; if the number of negative integers in a product is odd, then the product is a negative integer.*

Justify it by taking five more examples of each kind.

THINK, DISCUSS AND WRITE

(i) The product $(-9) \times (-5) \times (-6) \times (-3)$ is positive whereas the product $(-9) \times (-5) \times 6 \times (-3)$ is negative. Why?

(ii) What will be the sign of the product if we multiply together:

(a) 8 negative integers and 3 positive integers?

(b) 5 negative integers and 4 positive integers?

(c) (–1), twelve times?

(d) (–1), $2m$ times, m is a natural number?

1.5 Properties of Multiplication of Integers

1.5.1 Closure under Multiplication

1. Observe the following table and complete it:

Statement	Inference
(–20) × (–5) = 100	Product is an integer
(–15) × 17 = – 255	Product is an integer
(–30) × 12 = ______	
(–15) × (–23) = ______	
(–14) × (–13) = ______	
12 × (–30) = ______	

What do you observe? Can you find a pair of integers whose product is not an integer? No. This gives us an idea that the product of two integers is again an integer. So we can say that *integers are closed under multiplication*.

In general,

$a \times b$ is an integer, for all integers a and b.

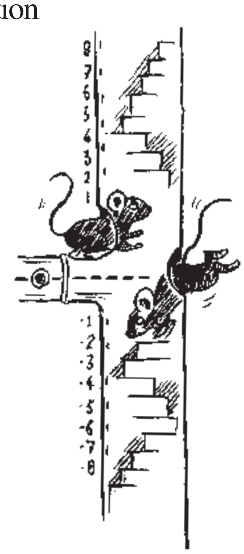
Find the product of five more pairs of integers and verify the above statement.

1.5.2 Commutativity of Multiplication

We know that multiplication is commutative for whole numbers. Can we say, multiplication is also commutative for integers?

Observe the following table and complete it:

Statement 1	Statement 2	Inference
3 × (– 4) = –12	(– 4) × 3 = –12	3 × (– 4) = (– 4) × 3
(–30) × 12 = ______	12 × (–30) = ______	
(–15) × (–10) = 150	(–10) × (–15) = 150	
(–35) × (–12) = ______	(–12) × (–35) =	
(–17) × 0 = ______		
___________ = ______	(–1) × (–15) =	



What are your observations? The above examples suggest *multiplication is commutative for integers*. Write five more such examples and verify.

In general, for any two integers a and b,

$$a \times b = b \times a$$

1.5.3 Multiplication by Zero

We know that any whole number when multiplied by zero gives zero. Observe the following products of negative integers and zero. These are obtained from the patterns done earlier.

$(-3) \times 0 = 0$

$0 \times (-4) = 0$

$-5 \times 0 =$ _____

$0 \times (-6) =$ _____

This shows that the product of a negative integer and zero is zero.

In general, for any integer a,

$$a \times 0 = 0 \times a = 0$$

1.5.4 Multiplicative Identity

We know that 1 is the multiplicative identity for whole numbers.

Check that 1 is the multiplicative identity for integers as well. Observe the following products of integers with 1.

$(-3) \times 1 = -3$	$1 \times 5 = 5$
$(-4) \times 1 =$ _____	$1 \times 8 =$ _____
$1 \times (-5) =$ _____	$3 \times 1 =$ _____
$1 \times (-6) =$ _____	$7 \times 1 =$ _____

This shows that 1 is the multiplicative identity for integers also.

In general, for any integer a we have,

$$a \times 1 = 1 \times a = a$$

What happens when we multiply any integer with -1? Complete the following:

$(-3) \times (-1) = 3$

$3 \times (-1) = -3$

$(-6) \times (-1) =$ _____

$(-1) \times 13 =$ _____

$(-1) \times (-25) =$ _____

$18 \times (-1) =$ _____

0 is the additive identity whereas 1 is the multiplicative identity for integers. We get additive inverse of an integer a when we multiply (–1) to a, i.e. $a \times (-1) = (-1) \times a = -a$

What do you observe?

Can we say -1 is a multiplicative identity of integers? No.

1.5.5 Associativity for Multiplication

Consider –3, –2 and 5.

Look at $[(-3) \times (-2)] \times 5$ and $(-3) \times [(-2) \times 5]$.

In the first case (–3) and (–2) are grouped together and in the second (–2) and 5 are grouped together.

We see that $[(-3) \times (-2)] \times 5 = 6 \times 5 = 30$

and $(-3) \times [(-2) \times 5] = (-3) \times (-10) = 30$

So, we get the same answer in both the cases.

Thus, $[(-3) \times (-2)] \times 5 = (-3) \times [(-2) \times 5]$

Look at this and complete the products:

$[(7) \times (-6)] \times 4 =$ __________ $\times 4 =$ __________

$7 \times [(-6) \times 4] = 7 \times$ __________ $=$ __________

Is $[7 \times (-6)] \times 4 = 7 \times [(-6) \times 4]$?

Does the grouping of integers affect the product of integers? No.

In general, for any three integers *a*, *b* and *c*

$$(a \times b) \times c = a \times (b \times c)$$

Take any five values for *a*, *b* and *c* each and verify this property.

Thus, like whole numbers, *the product of three integers does not depend upon the grouping of integers and this is called the associative property for multiplication of integers.*

1.5.6 Distributive Property

We know

$16 \times (10 + 2) = (16 \times 10) + (16 \times 2)$ [Distributivity of multiplication over addition]

Let us check if this is true for integers also.

Observe the following:

(a) $(-2) \times (3 + 5) = -2 \times 8 = -16$

and $[(-2) \times 3] + [(-2) \times 5] = (-6) + (-10) = -16$

So, $(-2) \times (3 + 5) = [(-2) \times 3] + [(-2) \times 5]$

(b) $(-4) \times [(-2) + 7] = (-4) \times 5 = -20$

and $[(-4) \times (-2)] + [(-4) \times 7] = 8 + (-28) = -20$

So, $(-4) \times [(-2) + 7] = [(-4) \times (-2)] + [(-4) \times 7]$

(c) $(-8) \times [(-2) + (-1)] = (-8) \times (-3) = 24$

and $[(-8) \times (-2)] + [(-8) \times (-1)] = 16 + 8 = 24$

So, $(-8) \times [(-2) + (-1)] = [(-8) \times (-2)] + [(-8) \times (-1)]$

Can we say that the distributivity of multiplication over addition is true for integers also? Yes.

In general, for any integers a, b and c,

$$a \times (b + c) = a \times b + a \times c$$

Take atleast five different values for each of a, b and c and verify the above Distributive property.

TRY THESE

(i) Is $10 \times [(6 + (-2)] = 10 \times 6 + 10 \times (-2)$?

(ii) Is $(-15) \times [(-7) + (-1)] = (-15) \times (-7) + (-15) \times (-1)$?

Now consider the following:

Can we say $4 \times (3 - 8) = 4 \times 3 - 4 \times 8$?

Let us check:

$4 \times (3 - 8) = 4 \times (-5) = -20$

$4 \times 3 - 4 \times 8 = 12 - 32 = -20$

So, $4 \times (3 - 8) = 4 \times 3 - 4 \times 8$.

Look at the following:

$(-5) \times [(-4) - (-6)] = (-5) \times 2 = -10$

$[(-5) \times (-4)] - [(-5) \times (-6)] = 20 - 30 = -10$

So, $(-5) \times [(-4) - (-6)] = [(-5) \times (-4)] - [(-5) \times (-6)]$

Check this for $(-9) \times [10 - (-3)]$ and $[(-9) \times 10] - [(-9) \times (-3)]$

You will find that these are also equal.

In general, for any three integers a, b and c,

$$a \times (b - c) = a \times b - a \times c$$

Take atleast five different values for each of a, b and c and verify this property.

TRY THESE

(i) Is $10 \times (6 - (-2)] = 10 \times 6 - 10 \times (-2)$?

(ii) Is $(-15) \times [(-7) - (-1)] = (-15) \times (-7) - (-15) \times (-1)$?

1.5.7 Making Multiplication Easier

Consider the following:

(i) We can find $(-25) \times 37 \times 4$ as

$[(-25) \times 37] \times 4 = (-925) \times 4 = -3700$

Or, we can do it this way,

$(-25) \times 37 \times 4 = (-25) \times 4 \times 37 = [(-25) \times 4] \times 37 = (-100) \times 37 = -3700$

Which is the easier way?

Obviously the second way is easier because multiplication of (-25) and 4 gives -100 which is easier to multiply with 37. Note that the second way involves commutativity and associativity of integers.

So, we find that the commutativity, associativity and distributivity of integers help to make our calculations simpler. Let us further see how calculations can be made easier using these properties.

(ii) Find 16×12

16×12 can be written as $16 \times (10 + 2)$.

$16 \times 12 = 16 \times (10 + 2) = 16 \times 10 + 16 \times 2 = 160 + 32 = 192$

(iii) $(-23) \times 48 = (-23) \times [50 - 2] = (-23) \times 50 - (-23) \times 2 = (-1150) - (-46)$

$= -1104$

(iv) $(-35) \times (-98) = (-35) \times [(-100) + 2] = (-35) \times (-100) + (-35) \times 2$

$= 3500 + (-70) = 3430$

(v) $52 \times (-8) + (-52) \times 2$

$(-52) \times 2$ can also be written as $52 \times (-2)$.

Therefore, $52 \times (-8) + (-52) \times 2 = 52 \times (-8) + 52 \times (-2)$

$= 52 \times [(-8) + (-2)] = 52 \times [(-10)] = -520$

TRY THESE

Find $(-49) \times 18$; $(-25) \times (-31)$; $70 \times (-19) + (-1) \times 70$ using distributive property.

EXAMPLE 2 Find each of the following products:

(i) $(-18) \times (-10) \times 9$ (ii) $(-20) \times (-2) \times (-5) \times 7$

(iii) $(-1) \times (-5) \times (-4) \times (-6)$

SOLUTION

(i) $(-18) \times (-10) \times 9 = [(-18) \times (-10)] \times 9 = 180 \times 9 = 1620$

(ii) $(-20) \times (-2) \times (-5) \times 7 = -20 \times (-2 \times -5) \times 7 = [-20 \times 10] \times 7 = -1400$

(iii) $(-1) \times (-5) \times (-4) \times (-6) = [(-1) \times (-5)] \times [(-4) \times (-6)] = 5 \times 24 = 120$

EXAMPLE 3 Verify $(-30) \times [13 + (-3)] = [(-30) \times 13] + [(-30) \times (-3)]$

SOLUTION $(-30) \times [13 + (-3)] = (-30) \times 10 = -300$

$[(-30) \times 13] + [(-30) \times (-3)] = -390 + 90 = -300$

So, $(-30) \times [13 + (-3)] = [(-30) \times 13] + [(-30) \times (-3)]$

Example 4 In a class test containing 15 questions, 4 marks are given for every correct answer and (–2) marks are given for every incorrect answer. (i) Gurpreet attempts all questions but only 9 of her answers are correct. What is her total score? (ii) One of her friends gets only 5 answers correct. What will be her score?

Solution

(i) Marks given for one correct answer = 4

So, marks given for 9 correct answers = $4 \times 9 = 36$

Marks given for one incorrect answer = – 2

So, marks given for 6 = (15 – 9) incorrect answers = $(-2) \times 6 = -12$

Therefore, Gurpreet's total score = $36 + (-12) = 24$

(ii) Marks given for one correct answer = 4

So, marks given for 5 correct answers = $4 \times 5 = 20$

Marks given for one incorrect answer = (–2)

So, marks given for 10 (=15 – 5) incorrect answers = $(-2) \times 10 = -20$

Therefore, her friend's total score = $20 + (-20) = 0$

Example 5 Suppose we represent the distance above the ground by a positive integer and that below the ground by a negative integer, then answer the following:

(i) An elevator descends into a mine shaft at the rate of 5 metre per minute. What will be its position after one hour?

(ii) If it begins to descend from 15 m above the ground, what will be its position after 45 minutes?

Solution

(i) Since the elevator is going down, so the distance covered by it will be represented by a negative integer.

Change in position of the elevator in one minute = – 5 m

Position of the elevator after 60 minutes = $(-5) \times 60 = -300$ m, i.e., 300 m below down from the starting position of elevator.

(ii) Change in position of the elevator in 45 minutes = $(-5) \times 45 = -225$ m, i.e., 225 m below ground level.

So, the final position of the elevator = $-225 + 15 = -210$ m, i.e., 210 m below ground level.

Exercise 1.3

1. Find each of the following products:

(a) $3 \times (-1)$ (b) $(-1) \times 225$

(c) $(-21) \times (-30)$ (d) $(-316) \times (-1)$

(e) $(-15) \times 0 \times (-18)$ (f) $(-12) \times (-11) \times (10)$

(g) $9 \times (-3) \times (-6)$ (h) $(-18) \times (-5) \times (-4)$

(i) $(-1) \times (-2) \times (-3) \times 4$ (j) $(-3) \times (-6) \times (-2) \times (-1)$

2. Verify the following:

(a) $18 \times [7 + (-3)] = [18 \times 7] + [18 \times (-3)]$

(b) $(-21) \times [(-4) + (-6)] = [(-21) \times (-4)] + [(-21) \times (-6)]$

3. (i) For any integer a, what is $(-1) \times a$ equal to?

(ii) Determine the integer whose product with (-1) is

(a) -22 (b) 37 (c) 0

4. Starting from $(-1) \times 5$, write various products showing some pattern to show $(-1) \times (-1) = 1$.

5. Find the product, using suitable properties:

(a) $26 \times (-48) + (-48) \times (-36)$ (b) $8 \times 53 \times (-125)$

(c) $15 \times (-25) \times (-4) \times (-10)$ (d) $(-41) \times 102$

(e) $625 \times (-35) + (-625) \times 65$ (f) $7 \times (50 - 2)$

(g) $(-17) \times (-29)$ (h) $(-57) \times (-19) + 57$

6. A certain freezing process requires that room temperature be lowered from 40°C at the rate of 5°C every hour. What will be the room temperature 10 hours after the process begins?

7. In a class test containing 10 questions, 5 marks are awarded for every correct answer and (–2) marks are awarded for every incorrect answer and 0 for questions not attempted.

(i) Mohan gets four correct and six incorrect answers. What is his score?

(ii) Reshma gets five correct answers and five incorrect answers, what is her score?

(iii) Heena gets two correct and five incorrect answers out of seven questions she attempts. What is her score?

8. A cement company earns a profit of ₹ 8 per bag of white cement sold and a loss of ₹ 5 per bag of grey cement sold.

(a) The company sells 3,000 bags of white cement and 5,000 bags of grey cement in a month. What is its profit or loss?

(b) What is the number of white cement bags it must sell to have neither profit nor loss, if the number of grey bags sold is 6,400 bags.

9. Replace the blank with an integer to make it a true statement.

(a) (–3) × _____ = 27 (b) 5 × _____ = –35

(c) _____ × (– 8) = –56 (d) _____ × (–12) = 132

1.6 Division of Integers

We know that division is the inverse operation of multiplication. Let us see an example for whole numbers.

Since 3 × 5 = 15

So 15 ÷ 5 = 3 and 15 ÷ 3 = 5

Similarly, 4 × 3 = 12 gives 12 ÷ 4 = 3 and 12 ÷ 3 = 4

We can say for each multiplication statement of whole numbers there are two division statements.

Can you write multiplication statement and its corresponding divison statements for integers?

- Observe the following and complete it.

Multiplication Statement	Corresponding Division Statements	
2 × (– 6) = (–12)	(–12) ÷ (– 6) = 2 ,	(–12) ÷ 2 = (– 6)
(– 4) × 5 = (–20)	(–20) ÷ 5 = (– 4) ,	(–20) ÷ (– 4) = 5
(– 8) × (–9) = 72	72 ÷ _____ = _____ ,	72 ÷ _____ = _____
(–3) × (–7) = _____	_____ ÷ (–3) = _____ ,	_______________
(– 8) × 4 = _____	_______________ ,	_______________
5 × (– 9) = _____	_______________ ,	_______________
(–10) × (–5) =	_______________ ,	_______________

From the above we observe that :

(–12) ÷ 2 = (– 6)

(–20) ÷ 5 = (– 4)

(–32) ÷ 4 = (– 8)

(– 45) ÷ 5 = (– 9)

Try These

Find:

(a) (–100) ÷ 5 (b) (–81) ÷ 9

(c) (–75) ÷ 5 (d) (–32) ÷ 2

We observe that *when we divide a negative integer by a positive integer, we divide them as whole numbers and then put a minus sign* (–) *before the quotient.*

- We also observe that:

  $72 \div (-8) = -9$ and $50 \div (-10) = -5$

  $72 \div (-9) = -8$ $\quad$ $50 \div (-5) = -10$

So we can say that *when we divide a positive integer by a negative integer, we first divide them as whole numbers and then put a minus sign (–) before the quotient.*

In general, for any two positive integers a and b

$$a \div (-b) = (-a) \div b \qquad \text{where } b \neq 0$$

Can we say that

$(-48) \div 8 = 48 \div (-8)$?

Let us check. We know that

$(-48) \div 8 = -6$

and $48 \div (-8) = -6$

So $(-48) \div 8 = 48 \div (-8)$

Check this for

(i) $90 \div (-45)$ and $(-90) \div 45$

(ii) $(-136) \div 4$ and $136 \div (-4)$

TRY THESE

Find: (a) $125 \div (-25)$ (b) $80 \div (-5)$ (c) $64 \div (-16)$

- Lastly, we observe that

  $(-12) \div (-6) = 2; (-20) \div (-4) = 5; (-32) \div (-8) = 4; (-45) \div (-9) = 5$

So, we can say that when we divide a negative integer by a negative integer, we first divide them as whole numbers and then put a positive sign (+).

In general, for any two positive integers a and b

$$(-a) \div (-b) = a \div b \qquad \text{where } b \neq 0$$

TRY THESE

Find: (a) $(-36) \div (-4)$ (b) $(-201) \div (-3)$ (c) $(-325) \div (-13)$

1.7 PROPERTIES OF DIVISION OF INTEGERS

Observe the following table and complete it:

What do you observe? We observe that integers are not closed under division.

Statement	Inference	Statement	Inference
$(-8) \div (-4) = 2$	Result is an integer	$(-8) \div 3 = \dfrac{-8}{3}$	________________
$(-4) \div (-8) = \dfrac{-4}{-8}$	Result is not an integer	$3 \div (-8) = \dfrac{3}{-8}$	________________

Justify it by taking five more examples of your own.

- We know that division is not commutative for whole numbers. Let us check it for integers also.

You can see from the table that $(-8) \div (-4) \neq (-4) \div (-8)$.

Is $(-9) \div 3$ the same as $3 \div (-9)$?

Is $(-30) \div (-6)$ the same as $(-6) \div (-30)$?

Can we say that division is commutative for integers? No.

You can verify it by taking five more pairs of integers.

- Like whole numbers, any integer divided by zero is meaningless and zero divided by an integer other than zero is equal to zero i.e., *for any integer a,* $a \div 0$ *is not defined but* $0 \div a = 0$ *for* $a \neq 0$.

- When we divide a whole number by 1 it gives the same whole number. Let us check whether it is true for negative integers also.

  Observe the following :

$(-8) \div 1 = (-8)$	$(-11) \div 1 = -11$	$(-13) \div 1 = -13$
$(-25) \div 1 =$ ____	$(-37) \div 1 =$ ____	$(-48) \div 1 =$ ____

  This shows that negative integer divided by 1 gives the same negative integer. So, *any integer divided by 1 gives the same integer.*

  In general, for any integer a,

  $$a \div 1 = a$$

- What happens when we divide any integer by (-1)? Complete the following table

$(-8) \div (-1) = 8$	$11 \div (-1) = -11$	$13 \div (-1) =$ ____
$(-25) \div (-1) =$ ____	$(-37) \div (-1) =$ ____	$-48 \div (-1) =$ ____

  What do you observe?

  We can say that if any integer is divided by (-1) it does not give the same integer.

TRY THESE

Is (i) $1 \div a = 1$?

(ii) $a \div (-1) = -a$? for any integer a.

Take different values of a and check.

- Can we say $[(-16) \div 4] \div (-2)$ is the same as $(-16) \div [4 \div (-2)]$?

  We know that $[(-16) \div 4] \div (-2) = (-4) \div (-2) = 2$

  and $(-16) \div [4 \div (-2)] = (-16) \div (-2) = 8$

  So $[(-16) \div 4] \div (-2) \neq (-16) \div [4 \div (-2)]$

  Can you say that division is associative for integers? No.

  Verify it by taking five more examples of your own.

EXAMPLE 6 In a test (+5) marks are given for every correct answer and (–2) marks are given for every incorrect answer. (i) Radhika answered all the questions and scored 30 marks though she got 10 correct answers. (ii) Jay also answered all the questions and scored (–12) marks though he got 4 correct answers. How many incorrect answers had they attempted?

SOLUTION

(i) Marks given for one correct answer = 5

So, marks given for 10 correct answers = 5 × 10 = 50

Radhika's score = 30

Marks obtained for incorrect answers = 30 – 50 = – 20

Marks given for one incorrect answer = (–2)

Therefore, number of incorrect answers = (–20) ÷ (–2) = 10

(ii) Marks given for 4 correct answers = 5 × 4 = 20

Jay's score = –12

Marks obtained for incorrect answers = –12 – 20 = – 32

Marks given for one incorrect answer = (–2)

Therefore number of incorrect answers = (–32) ÷ (–2) = 16

EXAMPLE 7 A shopkeeper earns a profit of ₹ 1 by selling one pen and incurs a loss of 40 paise per pencil while selling pencils of her old stock.

(i) In a particular month she incurs a loss of ₹ 5. In this period, she sold 45 pens. How many pencils did she sell in this period?

(ii) In the next month she earns neither profit nor loss. If she sold 70 pens, how many pencils did she sell?

SOLUTION

(i) Profit earned by selling one pen = ₹ 1

Profit earned by selling 45 pens = ₹ 45, which we denote by + ₹ 45

Total loss given = ₹ 5, which we denote by – ₹ 5

Profit earned + Loss incurred = Total loss

Therefore, Loss incurred = Total Loss – Profit earned

= ₹ (– 5 – 45) = ₹ (–50) = –5000 paise

Loss incurred by selling one pencil = 40 paise which we write as –40 paise

So, number of pencils sold = (–5000) ÷ (– 40) = 125

(ii) In the next month there is neither profit nor loss.

So, Profit earned + Loss incurred = 0

i.e., Profit earned = – Loss incurred.

Now, profit earned by selling 70 pens = ₹ 70

Hence, loss incurred by selling pencils = ₹ 70 which we indicate by – ₹ 70 or – 7,000 paise.

Total number of pencils sold = (–7000) ÷ (– 40) = 175 pencils.

Exercise 1.4

1. Evaluate each of the following:

   (a) (–30) ÷ 10 (b) 50 ÷ (–5) (c) (–36) ÷ (–9)

   (d) (– 49) ÷ (49) (e) 13 ÷ [(–2) + 1] (f) 0 ÷ (–12)

   (g) (–31) ÷ [(–30) + (–1)]

   (h) [(–36) ÷ 12] ÷ 3 (i) [(– 6) + 5)] ÷ [(–2) + 1]

2. Verify that $a \div (b + c) \neq (a \div b) + (a \div c)$ for each of the following values of a, b and c.

   (a) $a = 12$, $b = -4$, $c = 2$ (b) $a = (-10)$, $b = 1$, $c = 1$

3. Fill in the blanks:

   (a) 369 ÷ _____ = 369 (b) (–75) ÷ _____ = –1

   (c) (–206) ÷ _____ = 1 (d) – 87 ÷ _____ = 87

   (e) _____ ÷ 1 = – 87 (f) _____ ÷ 48 = –1

   (g) 20 ÷ _____ = –2 (h) _____ ÷ (4) = –3

4. Write five pairs of integers (a, b) such that $a \div b = -3$. One such pair is (6, –2) because 6 ÷ (–2) = (–3).

5. The temperature at 12 noon was 10°C above zero. If it decreases at the rate of 2°C per hour until midnight, at what time would the temperature be 8°C below zero? What would be the temperature at mid-night?

6. In a class test (+ 3) marks are given for every correct answer and (–2) marks are given for every incorrect answer and no marks for not attempting any question. (i) Radhika scored 20 marks. If she has got 12 correct answers, how many questions has she attempted incorrectly? (ii) Mohini scores –5 marks in this test, though she has got 7 correct answers. How many questions has she attempted incorrectly?

7. An elevator descends into a mine shaft at the rate of 6 m/min. If the descent starts from 10 m above the ground level, how long will it take to reach – 350 m.

What have We Discussed?

1. Integers are a bigger collection of numbers which is formed by whole numbers and their negatives. These were introduced in Class VI.

2. You have studied in the earlier class, about the representation of integers on the number line and their addition and subtraction.

3. We now study the properties satisfied by addition and subtraction.

   (a) Integers are closed for addition and subtraction both. That is, $a + b$ and $a - b$ are again integers, where a and b are any integers.

   (b) Addition is commutative for integers, i.e., $a + b = b + a$ for all integers a and b.

   (c) Addition is associative for integers, i.e., $(a + b) + c = a + (b + c)$ for all integers a, b and c.

   (d) Integer 0 is the identity under addition. That is, $a + 0 = 0 + a = a$ for every integer a.

4. We studied, how integers could be multiplied, and found that product of a positive and a negative integer is a negative integer, whereas the product of two negative integers is a positive integer. For example, $-2 \times 7 = -14$ and $-3 \times -8 = 24$.

5. Product of even number of negative integers is positive, whereas the product of odd number of negative integers is negative.

6. Integers show some properties under multiplication.

   (a) Integers are closed under multiplication. That is, $a \times b$ is an integer for any two integers a and b.

   (b) Multiplication is commutative for integers. That is, $a \times b = b \times a$ for any integers a and b.

   (c) The integer 1 is the identity under multiplication, i.e., $1 \times a = a \times 1 = a$ for any integer a.

   (d) Multiplication is associative for integers, i.e., $(a \times b) \times c = a \times (b \times c)$ for any three integers a, b and c.

7. Under addition and multiplication, integers show a property called distributive property. That is, $a \times (b + c) = a \times b + a \times c$ for any three integers a, b and c.

8. The properties of commutativity, associativity under addition and multiplication, and the distributive property help us to make our calculations easier.

9. We also learnt how to divide integers. We found that,

(a) When a positive integer is divided by a negative integer, the quotient obtained is negative and vice-versa.

(b) Division of a negative integer by another negative integer gives positive as quotient.

10. For any integer a, we have

(a) $a \div 0$ is not defined

(b) $a \div 1 = a$

Chapter 2

Fractions and Decimals

0756CH02

2.1 Introduction

You have learnt fractions and decimals in earlier classes. The study of fractions included proper, improper and mixed fractions as well as their addition and subtraction. We also studied comparison of fractions, equivalent fractions, representation of fractions on the number line and ordering of fractions.

Our study of decimals included, their comparison, their representation on the number line and their addition and subtraction.

We shall now learn multiplication and division of fractions as well as of decimals.

2.2 How Well have You Learnt about Fractions?

A **proper fraction** is a fraction that represents a part of a whole. Is $\frac{7}{4}$ a proper fraction? Which is bigger, the numerator or the denominator?

An **improper fraction** is a combination of whole and a proper fraction. Is $\frac{7}{4}$ an improper fraction? Which is bigger here, the numerator or the denominator?

The improper fraction $\frac{7}{4}$ can be written as $1\frac{3}{4}$. This is a **mixed fraction**.

Can you write five examples each of proper, improper and mixed fractions?

Example 1 Write five equivalent fractions of $\frac{3}{5}$.

Solution One of the equivalent fractions of $\frac{3}{5}$ is

$\frac{3}{5} = \frac{3\times 2}{5\times 2} = \frac{6}{10}$. Find the other four.

EXAMPLE 2 Ramesh solved $\frac{2}{7}$ part of an exercise while Seema solved $\frac{4}{5}$ of it. Who solved lesser part?

SOLUTION In order to find who solved lesser part of the exercise, let us compare $\frac{2}{7}$ and $\frac{4}{5}$.

Converting them to like fractions we have, $\frac{2}{7}=\frac{10}{35}$, $\frac{4}{5}=\frac{28}{35}$.

Since $10 < 28$, so $\frac{10}{35} < \frac{28}{35}$.

Thus, $\frac{2}{7} < \frac{4}{5}$.

Ramesh solved lesser part than Seema.

EXAMPLE 3 Sameera purchased $3\frac{1}{2}$ kg apples and $4\frac{3}{4}$ kg oranges. What is the total weight of fruits purchased by her?

SOLUTION The total weight of the fruits $= \left(3\frac{1}{2}+4\frac{3}{4}\right)$ kg

$$= \left(\frac{7}{2}+\frac{19}{4}\right) \text{kg} = \left(\frac{14}{4}+\frac{19}{4}\right) \text{kg}$$

$$= \frac{33}{4} \text{kg} = 8\frac{1}{4} \text{kg}$$

EXAMPLE 4 Suman studies for $5\frac{2}{3}$ hours daily. She devotes $2\frac{4}{5}$ hours of her time for Science and Mathematics. How much time does she devote for other subjects?

SOLUTION Total time of Suman's study $= 5\frac{2}{3}$ h $= \frac{17}{3}$ h

Time devoted by her for Science and Mathematics $= 2\frac{4}{5} = \frac{14}{5}$ h

Thus, time devoted by her for other subjects $= \left(\frac{17}{3} - \frac{14}{5}\right)$ h

$$= \left(\frac{17 \times 5}{15} - \frac{14 \times 3}{15}\right) \text{h} = \left(\frac{85 - 42}{15}\right) \text{h}$$

$$= \frac{43}{15} \text{ h} = 2\frac{13}{15} \text{ h}$$

Exercise 2.1

1. Solve:

   (i) $2 - \frac{3}{5}$ (ii) $4 + \frac{7}{8}$ (iii) $\frac{3}{5} + \frac{2}{7}$ (iv) $\frac{9}{11} - \frac{4}{15}$

   (v) $\frac{7}{10} + \frac{2}{5} + \frac{3}{2}$ (vi) $2\frac{2}{3} + 3\frac{1}{2}$ (vii) $8\frac{1}{2} - 3\frac{5}{8}$

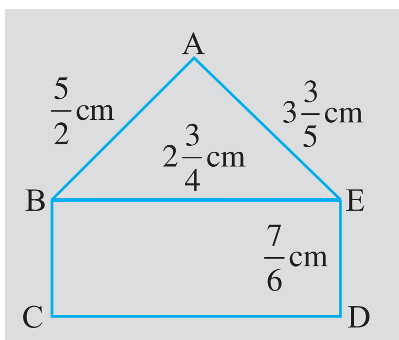
2. Arrange the following in descending order:

   (i) $\frac{2}{9}, \frac{2}{3}, \frac{8}{21}$ (ii) $\frac{1}{5}, \frac{3}{7}, \frac{7}{10}$.

3. In a "magic square", the sum of the numbers in each row, in each column and along the diagonals is the same. Is this a magic square?

$\frac{4}{11}$	$\frac{9}{11}$	$\frac{2}{11}$
$\frac{3}{11}$	$\frac{5}{11}$	$\frac{7}{11}$
$\frac{8}{11}$	$\frac{1}{11}$	$\frac{6}{11}$

(Along the first row $\frac{4}{11} + \frac{9}{11} + \frac{2}{11} = \frac{15}{11}$).



4. A rectangular sheet of paper is $12\frac{1}{2}$ cm long and $10\frac{2}{3}$ cm wide. Find its perimeter.

5. Find the perimeters of (i) ΔABE (ii) the rectangle BCDE in this figure. Whose perimeter is greater?

6. Salil wants to put a picture in a frame. The picture is $7\frac{3}{5}$ cm wide. To fit in the frame the picture cannot be more than $7\frac{3}{10}$ cm wide. How much should the picture be trimmed?

7. Ritu ate $\frac{3}{5}$ part of an apple and the remaining apple was eaten by her brother Somu. How much part of the apple did Somu eat? Who had the larger share? By how much?

8. Michael finished colouring a picture in $\frac{7}{12}$ hour. Vaibhav finished colouring the same picture in $\frac{3}{4}$ hour. Who worked longer? By what fraction was it longer?

2.3 Multiplication of Fractions

You know how to find the area of a rectangle. It is equal to length × breadth. If the length and breadth of a rectangle are 7 cm and 4 cm respectively, then what will be its area? Its area would be $7 \times 4 = 28$ cm^2.

What will be the area of the rectangle if its length and breadth are $7\frac{1}{2}$ cm and $3\frac{1}{2}$ cm respectively? You will say it will be $7\frac{1}{2} \times 3\frac{1}{2} = \frac{15}{2} \times \frac{7}{2}$ cm^2. The numbers $\frac{15}{2}$ and $\frac{7}{2}$ are fractions. To calculate the area of the given rectangle, we need to know how to multiply fractions. We shall learn that now.

2.3.1 Multiplication of a Fraction by a Whole Number

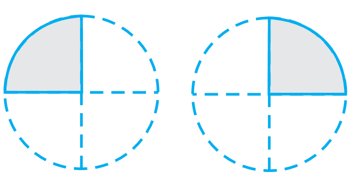


Fig 2.1

Observe the pictures at the left (Fig 2.1). Each shaded part is $\frac{1}{4}$ part of a circle. How much will the two shaded parts represent together? They will represent $\frac{1}{4}+\frac{1}{4} = 2\times\frac{1}{4}$.

Combining the two shaded parts, we get Fig 2.2 . What part of a circle does the shaded part in Fig 2.2 represent? It represents $\frac{2}{4}$ part of a circle .

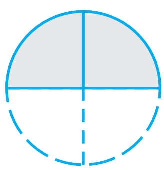


Fig 2.2

The shaded portions in Fig 2.1 taken together are the same as the shaded portion in Fig 2.2, i.e., we get Fig 2.3.

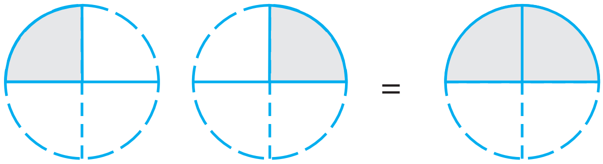


Fig 2.3

or $2\times\frac{1}{4} = \frac{2}{4}$.

Can you now tell what this picture will represent? (Fig 2.4)

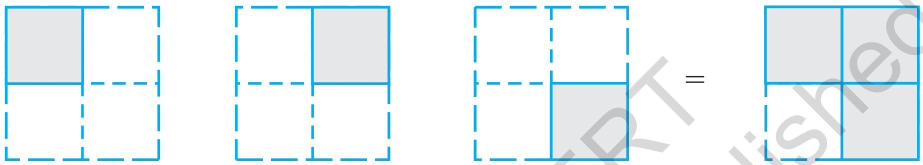


Fig 2.4

And this? (Fig 2.5)

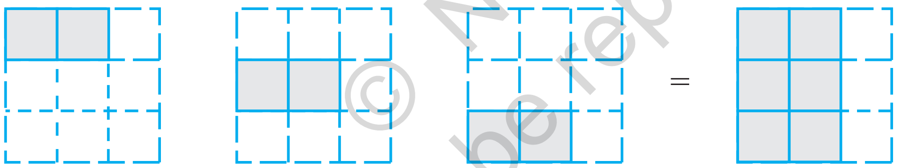


Fig 2.5

Let us now find $3\times\frac{1}{2}$.

We have $3\times\frac{1}{2} = \frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{3}{2}$

We also have $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{1+1+1}{2}=\frac{3\times1}{2}=\frac{3}{2}$

So $3\times\frac{1}{2} = \frac{3\times1}{2} = \frac{3}{2}$

Similarly $\frac{2}{3}\times5 = \frac{2\times5}{3} = ?$

Can you tell $3\times\frac{2}{7} = ?$ $4\times\frac{3}{5} = ?$

The fractions that we considered till now, i.e., $\frac{1}{2},\frac{2}{3},\frac{2}{7}$ and $\frac{3}{5}$ were proper fractions.

For improper fractions also we have,

$$2 \times \frac{5}{3} = \frac{2 \times 5}{3} = \frac{10}{3}$$

Try, $3 \times \frac{8}{7} = ?\quad 4 \times \frac{7}{5} = ?$

Thus, *to multiply a whole number with a proper or an improper fraction, we multiply the whole number with the numerator of the fraction, keeping the denominator same.*

Try These

1. Find: (a) $\frac{2}{7} \times 3$ (b) $\frac{9}{7} \times 6$ (c) $3 \times \frac{1}{8}$ (d) $\frac{13}{11} \times 6$

   If the product is an improper fraction express it as a mixed fraction.

2. Represent pictorially : $2 \times \frac{2}{5} = \frac{4}{5}$

Try These

Find: (i) $5 \times 2\frac{3}{7}$

(ii) $1\frac{4}{9} \times 6$

To multiply a mixed fraction to a whole number, first convert the mixed fraction to an improper fraction and then multiply.

Therefore, $3 \times 2\frac{5}{7} = 3 \times \frac{19}{7} = \frac{57}{7} = 8\frac{1}{7}$.

Similarly, $2 \times 4\frac{2}{5} = 2 \times \frac{22}{5} = ?$

Fraction as an operator 'of'

Observe these figures (Fig 2.6)

The two squares are exactly similar.

Each shaded portion represents $\frac{1}{2}$ of 1.

So, both the shaded portions together will represent $\frac{1}{2}$ of 2.

Combine the 2 shaded $\frac{1}{2}$ parts. It represents 1.

So, we say $\frac{1}{2}$ of 2 is 1. We can also get it as $\frac{1}{2} \times 2 = 1$.

Thus, $\frac{1}{2}$ of $2 = \frac{1}{2} \times 2 = 1$

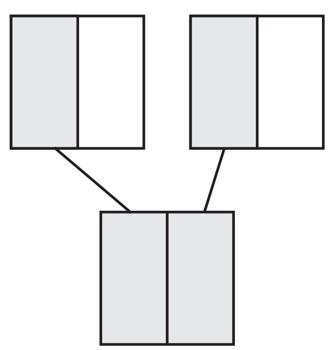


Fig 2.6

Also, look at these similar squares (Fig 2.7).

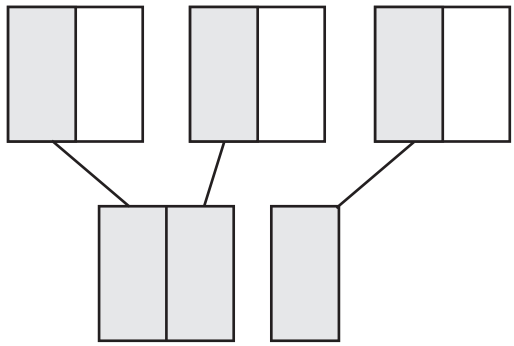


Fig 2.7

Each shaded portion represents $\frac{1}{2}$ of 1.

So, the three shaded portions represent $\frac{1}{2}$ of 3.

Combine the 3 shaded parts.

It represents $1\frac{1}{2}$ i.e., $\frac{3}{2}$.

So, $\frac{1}{2}$ of 3 is $\frac{3}{2}$. Also, $\frac{1}{2} \times 3 = \frac{3}{2}$.

Thus, $\frac{1}{2}$ of $3 = \frac{1}{2} \times 3 = \frac{3}{2}$.

So we see that **'of' represents multiplication.**

Farida has 20 marbles. Reshma has $\frac{1}{5}$th of the number of marbles what Farida has. How many marbles Reshma has? As, 'of' indicates multiplication, so, Reshma has $\frac{1}{5} \times 20 = 4$ marbles.

Similarly, we have $\frac{1}{2}$ of 16 is $\frac{1}{2} \times 16 = \frac{16}{2} = 8$.

Try These

Can you tell, what is (i) $\frac{1}{2}$ of 10?, (ii) $\frac{1}{4}$ of 16?, (iii) $\frac{2}{5}$ of 25?

Example 5 In a class of 40 students $\frac{1}{5}$ of the total number of studetns like to study English, $\frac{2}{5}$ of the total number like to study Mathematics and the remaining students like to study Science.

(i) How many students like to study English?

(ii) How many students like to study Mathematics?

(iii) What fraction of the total number of students like to study Science?

Solution Total number of students in the class = 40.

(i) Of these $\frac{1}{5}$ of the total number of students like to study English.

Thus, the number of students who like to study English = $\frac{1}{5}$ of $40 = \frac{1}{5} \times 40 = 8$.

(ii) Try yourself.

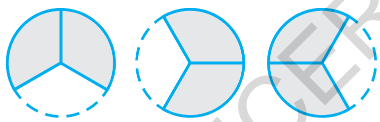
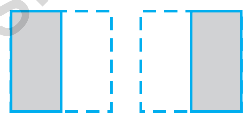
(iii) The number of students who like English and Mathematics = 8 + 16 = 24. Thus, the number of students who like Science = 40 – 24 = 16.

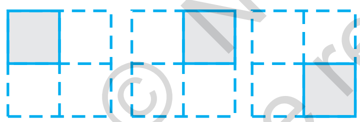
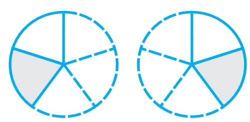
Thus, the required fraction is $\frac{16}{40}$.

Exercise 2.2

1. Which of the drawings (a) to (d) show :

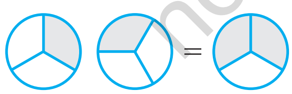
(i) $2 \times \frac{1}{5}$ (ii) $2 \times \frac{1}{2}$ (iii) $3 \times \frac{2}{3}$ (iv) $3 \times \frac{1}{4}$

(a)  (b) 

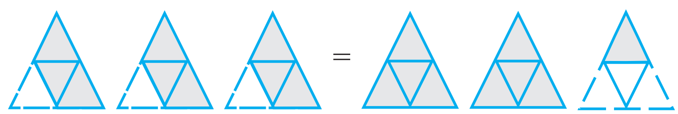
(c)  (d) 

2. Some pictures (a) to (c) are given below. Tell which of them show:

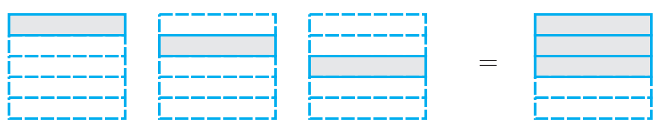
(i) $3 \times \frac{1}{5} = \frac{3}{5}$ (ii) $2 \times \frac{1}{3} = \frac{2}{3}$ (iii) $3 \times \frac{3}{4} = 2\frac{1}{4}$



(a)



(b)



(c)

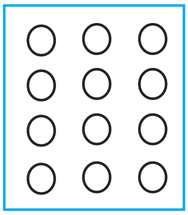
3. Multiply and reduce to lowest form and convert into a mixed fraction:

(i) $7 \times \frac{3}{5}$ (ii) $4 \times \frac{1}{3}$ (iii) $2 \times \frac{6}{7}$ (iv) $5 \times \frac{2}{9}$ (v) $\frac{2}{3} \times 4$

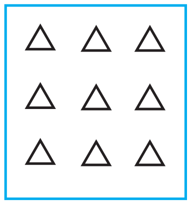
(vi) $\frac{5}{2} \times 6$ (vii) $11 \times \frac{4}{7}$ (viii) $20 \times \frac{4}{5}$ (ix) $13 \times \frac{1}{3}$ (x) $15 \times \frac{3}{5}$

4. Shade: (i) $\frac{1}{2}$ of the circles in box (a) (ii) $\frac{2}{3}$ of the triangles in box (b)

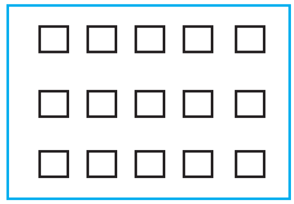
(iii) $\frac{3}{5}$ of the squares in box (c).



(a)



(b)



(c)

5. Find:

(a) $\frac{1}{2}$ of (i) 24 (ii) 46 (b) $\frac{2}{3}$ of (i) 18 (ii) 27

(c) $\frac{3}{4}$ of (i) 16 (ii) 36 (d) $\frac{4}{5}$ of (i) 20 (ii) 35

6. Multiply and express as a mixed fraction :

(a) $3\times5\frac{1}{5}$ (b) $5\times6\frac{3}{4}$ (c) $7\times2\frac{1}{4}$

(d) $4\times6\frac{1}{3}$ (e) $3\frac{1}{4}\times6$ (f) $3\frac{2}{5}\times8$

7. Find: (a) $\frac{1}{2}$ of (i) $2\frac{3}{4}$ (ii) $4\frac{2}{9}$ (b) $\frac{5}{8}$ of (i) $3\frac{5}{6}$ (ii) $9\frac{2}{3}$

8. Vidya and Pratap went for a picnic. Their mother gave them a water bottle that contained 5 litres of water. Vidya consumed $\frac{2}{5}$ of the water. Pratap consumed the remaining water.

(i) How much water did Vidya drink?

(ii) What fraction of the total quantity of water did Pratap drink?

2.3.2 Multiplication of a Fraction by a Fraction

Farida had a 9 cm long strip of ribbon. She cut this strip into four equal parts. How did she do it? She folded the strip twice. What fraction of the total length will each part represent?

Each part will be $\frac{9}{4}$ of the strip. She took one part and divided it in two equal parts by folding the part once. What will one of the pieces represent? It will represent $\frac{1}{2}$ of $\frac{9}{4}$ or $\frac{1}{2} \times \frac{9}{4}$.

Let us now see how to find the product of two fractions like $\frac{1}{2} \times \frac{9}{4}$.

To do this we first learn to find the products like $\frac{1}{2} \times \frac{1}{3}$.

Fig 2.8

(a) How do we find $\frac{1}{3}$ of a whole? We divide the whole in three equal parts. Each of the three parts represents $\frac{1}{3}$ of the whole. Take one part of these three parts, and shade it as shown in Fig 2.8.

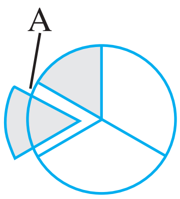


Fig 2.9

(b) How will you find $\frac{1}{2}$ of this shaded part? Divide this one-third ($\frac{1}{3}$) shaded part into two equal parts. Each of these two parts represents $\frac{1}{2}$ of $\frac{1}{3}$ i.e., $\frac{1}{2} \times \frac{1}{3}$ (Fig 2.9).

Take out 1 part of these two and name it 'A'. 'A' represents $\frac{1}{2} \times \frac{1}{3}$.

(c) What fraction is 'A' of the whole? For this, divide each of the remaining $\frac{1}{3}$ parts also in two equal parts. How many such equal parts do you have now?

There are six such equal parts. 'A' is one of these parts.

So, 'A' is $\frac{1}{6}$ of the whole. Thus, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

How did we decide that 'A' was $\frac{1}{6}$ of the whole? The whole was divided in $6 = 2 \times 3$ parts and $1 = 1 \times 1$ part was taken out of it.

Thus, $$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6} = \frac{1\times1}{2\times3}$$

or $$\frac{1}{2} \times \frac{1}{3} = \frac{1\times1}{2\times3}$$

The value of $\frac{1}{3}\times\frac{1}{2}$ can be found in a similar way. Divide the whole into two equal parts and then divide one of these parts in three equal parts. Take one of these parts. This will represent $\frac{1}{3}\times\frac{1}{2}$ i.e., $\frac{1}{6}$.

Therefore $\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}=\frac{1\times1}{3\times2}$ as discussed earlier.

Hence $\frac{1}{2}\times\frac{1}{3}=\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$

Find $\frac{1}{3}\times\frac{1}{4}$ and $\frac{1}{4}\times\frac{1}{3}$; $\frac{1}{2}\times\frac{1}{5}$ and $\frac{1}{5}\times\frac{1}{2}$ and check whether you get

$$\frac{1}{3}\times\frac{1}{4}=\frac{1}{4}\times\frac{1}{3};\ \frac{1}{2}\times\frac{1}{5}=\frac{1}{5}\times\frac{1}{2}$$

TRY THESE

Fill in these boxes:

(i) $\frac{1}{2}\times\frac{1}{7}=\frac{1\times1}{2\times7}=\boxed{}$

(ii) $\frac{1}{5}\times\frac{1}{7}=\boxed{}=\boxed{}$

(iii) $\frac{1}{7}\times\frac{1}{2}=\boxed{}=\boxed{}$

(iv) $\frac{1}{7}\times\frac{1}{5}=\boxed{}=\boxed{}$

EXAMPLE 6 Sushant reads $\frac{1}{3}$ part of a book in 1 hour. How much part of the book will he read in $2\frac{1}{5}$ hours?

SOLUTION The part of the book read by Sushant in 1 hour $=\frac{1}{3}$.

So, the part of the book read by him in $2\frac{1}{5}$ hours $=2\frac{1}{5}\times\frac{1}{3}$

$$=\frac{11}{5}\times\frac{1}{3}=\frac{11\times1}{5\times3}=\frac{11}{15}$$

Let us now find $\frac{1}{2}\times\frac{5}{3}$. We know that $\frac{5}{3}=\frac{1}{3}\times5$.

$$\text{So, } \frac{1}{2}\times\frac{5}{3}=\frac{1}{2}\times\frac{1}{3}\times5=\frac{1}{6}\quad5=\frac{5}{6}$$

Also, $\frac{5}{6} = \frac{1\times5}{2\times3}$. Thus, $\frac{1}{2}\times\frac{5}{3} = \frac{1\times5}{2\times3} = \frac{5}{6}$.

This is also shown by the figures drawn below. Each of these five equal shapes (Fig 2.10) are parts of five similar circles. Take one such shape. To obtain this shape we first divide a circle in three equal parts. Further divide each of these three parts in two equal parts. One part out of it is the shape we considered. What will it represent?

It will represent $\frac{1}{2}\times\frac{1}{3} = \frac{1}{6}$. The total of such parts would be $5\times\frac{1}{6} = \frac{5}{6}$.

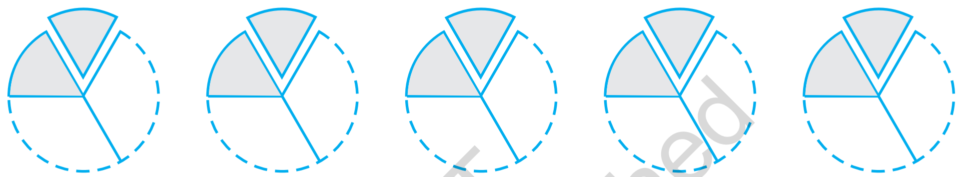


Fig 2.10

TRY THESE

Find: $\frac{1}{3}\times\frac{4}{5}$; $\frac{2}{3}\times\frac{1}{5}$

Similarly $\frac{3}{5}\times\frac{1}{7} = \frac{3\times1}{5\times7} = \frac{3}{35}$.

We can thus find $\frac{2}{3}\times\frac{7}{5}$ as $\frac{2}{3}\times\frac{7}{5} = \frac{2\times7}{3\times5} = \frac{14}{15}$.

So, we find that we multiply two fractions as $\frac{\text{Product of Numerators}}{\text{Product of Denominators}}$.

Value of the Products

TRY THESE

Find: $\frac{8}{3}\times\frac{4}{7}$; $\frac{3}{4}\times\frac{2}{3}$.

You have seen that the product of two whole numbers is bigger than each of the two whole numbers. For example, $3 \times 4 = 12$ and $12 > 4$, $12 > 3$. What happens to the value of the product when we multiply two fractions?

Let us first consider the product of two proper fractions.

We have,

$\frac{2}{3}\times\frac{4}{5}=\frac{8}{15}$	$\frac{8}{15}<\frac{2}{3},\frac{8}{15}<\frac{4}{5}$	Product is less than each of the fractions
$\frac{1}{5}\times\frac{2}{7}$ = ---------	--------,--------	-------------------------------------
$\frac{3}{5}\times\frac{\square}{8}$ =	--------,--------	-------------------------------------
$\frac{2}{\square}\times\frac{4}{9}=\frac{8}{45}$	--------,--------	-------------------------------------

You will find that *when two proper fractions are multiplied, the product is less than each of the fractions*. Or, *we say the value of the product of two proper fractions is smaller than each of the two fractions.*

Check this by constructing five more examples.

Let us now multiply two improper fractions.

$\frac{7}{3} \times \frac{5}{2} = \frac{35}{6}$	$\frac{35}{6} > \frac{7}{3}, \frac{35}{6} > \frac{5}{2}$	Product is greater than each of the fractions
$\frac{6}{5} \times \frac{\square}{3} = \frac{24}{15}$	--------,--------	--
$\frac{9}{2} \times \frac{7}{\square} = \frac{63}{8}$	--------,--------	--
$\frac{3}{\square} \times \frac{8}{7} = \frac{24}{14}$	--------,--------	--

We find that *the product of two improper fractions is greater than each of the two fractions.*

Or, *the value of the product of two improper fractions is more than each of the two fractions*.

Construct five more examples for yourself and verify the above statement.

Let us now multiply a proper and an improper fraction, say $\frac{2}{3}$ and $\frac{7}{5}$.

We have $\frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$. Here, $\frac{14}{15} < \frac{7}{5}$ and $\frac{14}{15} > \frac{2}{3}$

The product obtained is less than the improper fraction and greater than the proper fraction involved in the multiplication.

Check it for $\frac{6}{5} \times \frac{2}{8}$, $\frac{8}{3} \times \frac{4}{5}$.

EXERCISE 2.3

1. Find:

(i) $\frac{1}{4}$ of (a) $\frac{1}{4}$ (b) $\frac{3}{5}$ (c) $\frac{4}{3}$

(ii) $\frac{1}{7}$ of (a) $\frac{2}{9}$ (b) $\frac{6}{5}$ (c) $\frac{3}{10}$

2. Multiply and reduce to lowest form (if possible) :

(i) $\frac{2}{3} \times 2\frac{2}{3}$ (ii) $\frac{2}{7} \times \frac{7}{9}$ (iii) $\frac{3}{8} \times \frac{6}{4}$ (iv) $\frac{9}{5} \times \frac{3}{5}$

(v) $\frac{1}{3} \times \frac{15}{8}$ (vi) $\frac{11}{2} \times \frac{3}{10}$ (vii) $\frac{4}{5} \times \frac{12}{7}$

3. Multiply the following fractions:

(i) $\frac{2}{5} \times 5\frac{1}{4}$ (ii) $6\frac{2}{5} \times \frac{7}{9}$ (iii) $\frac{3}{2} \times 5\frac{1}{3}$ (iv) $\frac{5}{6} \times 2\frac{3}{7}$

(v) $3\frac{2}{5} \times \frac{4}{7}$ (vi) $2\frac{3}{5} \times 3$ (vii) $3\frac{4}{7} \times \frac{3}{5}$

4. Which is greater:

(i) $\frac{2}{7}$ of $\frac{3}{4}$ or $\frac{3}{5}$ of $\frac{5}{8}$ (ii) $\frac{1}{2}$ of $\frac{6}{7}$ or $\frac{2}{3}$ of $\frac{3}{7}$

5. Saili plants 4 saplings, in a row, in her garden. The distance between two adjacent saplings is $\frac{3}{4}$ m. Find the distance between the first and the last sapling.

6. Lipika reads a book for $1\frac{3}{4}$ hours everyday. She reads the entire book in 6 days. How many hours in all were required by her to read the book?

7. A car runs 16 km using 1 litre of petrol. How much distance will it cover using $2\frac{3}{4}$ litres of petrol.

8. (a) (i) Provide the number in the box $\square$, such that $\frac{2}{3} \times \square = \frac{10}{30}$.

(ii) The simplest form of the number obtained in $\square$ is _____.

(b) (i) Provide the number in the box $\square$, such that $\frac{3}{5} \times \square = \frac{24}{75}$.

(ii) The simplest form of the number obtained in $\square$ is _____.

2.4 Division of Fractions

John has a paper strip of length 6 cm. He cuts this strip in smaller strips of length 2 cm each. You know that he would get 6 ÷ 2 =3 strips.

John cuts another strip of length 6 cm into smaller strips of length $\frac{3}{2}$ cm each. How many strips will he get now? He will get $6 \div \frac{3}{2}$ strips.

A paper strip of length $\frac{15}{2}$ cm can be cut into smaller strips of length $\frac{3}{2}$ cm each to give $\frac{15}{2} \div \frac{3}{2}$ pieces.

So, we are required to divide a whole number by a fraction or a fraction by another fraction. Let us see how to do that.

2.4.1 Division of Whole Number by a Fraction

Let us find $1 \div \frac{1}{2}$.

We divide a whole into a number of equal parts such that each part is half of the whole. The number of such half ($\frac{1}{2}$) parts would be $1 \div \frac{1}{2}$. Observe the figure (Fig 2.11). How many half parts do you see?

There are two half parts.

So, $1 \div \frac{1}{2} = 2$. Also, $1 \times \frac{2}{1} = 1 \times 2 = 2$. Thus, $1 \div \frac{1}{2} = 1 \times \frac{2}{1}$

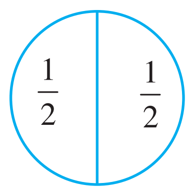


Fig 2.11

Similarly, $3 \div \frac{1}{4}$ = number of $\frac{1}{4}$ parts obtained when each of the 3 whole, are divided into $\frac{1}{4}$ equal parts = 12 (From Fig 2.12)

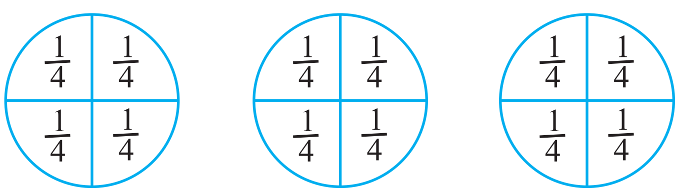


Fig 2.12

Observe also that, $3 \times \frac{4}{1} = 3 \times 4 = 12$. Thus, $3 \div \frac{1}{4} = 3 \times \frac{4}{1} = 12$.

Find in a similar way, $3 \div \frac{1}{2}$ and $3 \times \frac{2}{1}$.

Reciprocal of a fraction

The number $\frac{2}{1}$ can be obtained by interchanging the numerator and denominator of $\frac{1}{2}$ or by inverting $\frac{1}{2}$. Similarly, $\frac{3}{1}$ is obtained by inverting $\frac{1}{3}$.

Let us first see about the inverting of such numbers.

Observe these products and fill in the blanks :

$7 \times \frac{1}{7} = 1$	$\frac{5}{4} \times \frac{4}{5} = $ ---------
$\frac{1}{9} \times 9 = $ ------	$\frac{2}{7} \times$ ------- $= 1$
$\frac{2}{3} \times \frac{3}{2} = \frac{2 \times 3}{3 \times 2} = \frac{6}{6} = 1$	------ $\times \frac{5}{9} = 1$

Multiply five more such pairs.

The non-zero numbers whose product with each other is 1, are called the reciprocals of each other. So reciprocal of $\frac{5}{9}$ is $\frac{9}{5}$ and the reciprocal of $\frac{9}{5}$ is $\frac{5}{9}$. What is the receiprocal of $\frac{1}{9}$? $\frac{2}{7}$?

You will see that the reciprocal of $\frac{2}{3}$ is obtained by inverting it. You get $\frac{3}{2}$.

Think, Discuss and Write

(i) Will the reciprocal of a proper fraction be again a proper fraction?

(ii) Will the reciprocal of an improper fraction be again an improper fraction?

Therefore, we can say that

$$1 \div \frac{1}{2} = 1 \times \frac{2}{1} = 1 \times \text{ reciprocal of } \frac{1}{2}.$$

$$3 \div \frac{1}{4} = 3 \times \frac{4}{1} = 3 \times \text{ reciprocal of } \frac{1}{4}.$$

$$3 \div \frac{1}{2} = \text{------} = \text{----------------------}.$$

$$\text{So, } 2 \div \frac{3}{4} = 2 \times \text{ reciprocal of } \frac{3}{4} = 2 \times \frac{4}{3}.$$

$$5 \div \frac{2}{9} = 5 \times \text{ ------------------ } = 5 \times \text{ ------------}$$

Thus, *to divide a whole number by any fraction, multiply that whole number by the reciprocal of that fraction.*

TRY THESE

Find : (i) $7 \div \frac{2}{5}$ (ii) $6 \div \frac{4}{7}$ (iii) $2 \div \frac{8}{9}$

- *While dividing a whole number by a mixed fraction, first convert the mixed fraction into improper fraction and then solve it.*

Thus, $4 \div 2\frac{2}{5} = 4 \div \frac{12}{5} = ?$ Also, $5 \div 3\frac{1}{3} = 3 \div \frac{10}{3} = ?$

TRY THESE

Find: (i) $6 \div 5\frac{1}{3}$ (ii) $7 \div 2\frac{4}{7}$

2.4.2 Division of a Fraction by a Whole Number

- What will be $\frac{3}{4} \div 3$?

Based on our earlier observations we have: $\frac{3}{4} \div 3 = \frac{3}{4} \div \frac{3}{1} = \frac{3}{4} \times \frac{1}{3} = \frac{3}{12} = \frac{1}{4}$

So, $\frac{2}{3} \div 7 = \frac{2}{3} \times \frac{1}{7} = ?$ What is $\frac{5}{7} \div 6$, $\frac{2}{7} \div 8$?

- While dividing mixed fractions by whole numbers, convert the mixed fractions into improper fractions. That is,

$2\frac{2}{3} \div 5 = \frac{8}{3} \div 5 =$ ------ ; $4\frac{2}{5} \div 3 =$ ------ $=$ ------; $2\frac{3}{5} \div 2 =$ ------ $=$ ------

2.4.3 Division of a Fraction by Another Fraction

We can now find $\frac{1}{3} \div \frac{6}{5}$.

$\frac{1}{3} \div \frac{6}{5} = \frac{1}{3} \times$ reciprocal of $\frac{6}{5} = \frac{1}{3} \times \frac{5}{6} = \frac{5}{18}$.

Similarly, $\frac{8}{5} \div \frac{2}{3} = \frac{8}{5} \times$ reciprocal of $\frac{2}{3} = ?$ and, $\frac{1}{2} \div \frac{3}{4} = ?$

TRY THESE

Find: (i) $\frac{3}{5} \div \frac{1}{2}$ (ii) $\frac{1}{2} \div \frac{3}{5}$ (iii) $2\frac{1}{2} \div \frac{3}{5}$ (iv) $5\frac{1}{6} \div \frac{9}{2}$

Exercise 2.4

1. Find:

(i) $12 \div \frac{3}{4}$ (ii) $14 \div \frac{5}{6}$ (iii) $8 \div \frac{7}{3}$ (iv) $4 \div \frac{8}{3}$

(v) $3 \div 2\frac{1}{3}$ (vi) $5 \div 3\frac{4}{7}$

2. Find the reciprocal of each of the following fractions. Classify the reciprocals as proper fractions, improper fractions and whole numbers.

(i) $\frac{3}{7}$ (ii) $\frac{5}{8}$ (iii) $\frac{9}{7}$ (iv) $\frac{6}{5}$

(v) $\frac{12}{7}$ (vi) $\frac{1}{8}$ (vii) $\frac{1}{11}$

3. Find:

(i) $\frac{7}{3} \div 2$ (ii) $\frac{4}{9} \div 5$ (iii) $\frac{6}{13} \div 7$ (iv) $4\frac{1}{3} \div 3$

(v) $3\frac{1}{2} \div 4$ (vi) $4\frac{3}{7} \div 7$

4. Find:

(i) $\frac{2}{5} \div \frac{1}{2}$ (ii) $\frac{4}{9} \div \frac{2}{3}$ (iii) $\frac{3}{7} \div \frac{8}{7}$ (iv) $2\frac{1}{3} \div \frac{3}{5}$ (v) $3\frac{1}{2} \div \frac{8}{3}$

(vi) $\frac{2}{5} \div 1\frac{1}{2}$ (vii) $3\frac{1}{5} \div 1\frac{2}{3}$ (viii) $2\frac{1}{5} \div 1\frac{1}{5}$

2.5 How Well have you Learnt about Decimal Numbers

You have learnt about decimal numbers in the earlier classes. Let us briefly recall them here. Look at the following table and fill up the blank spaces.

Hundreds (100)	Tens (10)	Ones (1)	Tenths $\left(\frac{1}{10}\right)$	Hundredths $\left(\frac{1}{100}\right)$	Thousandths $\left(\frac{1}{1000}\right)$	Number
2	5	3	1	4	7	253.147
6	2	9	3	2	1	....
0	4	3	1	9	2	....
....	1	4	2	5	1	514.251
2	....	6	5	1	2	236.512
....	2	....	5	....	3	724.503
6	....	4	....	2	....	614.326
0	1	0	5	3	0	....

In the table, you wrote the decimal number, given its place-value expansion. You can do the reverse, too. That is, given the number you can write its expanded form. For example, $253.417 = 2 \times 100 + 5 \times 10 + 3 \times 1 + 4 \times \left(\frac{1}{10}\right) + 1 \times \left(\frac{1}{100}\right) + 7 \times \left(\frac{1}{1000}\right)$.

John has ₹ 15.50 and Salma has ₹ 15.75. Who has more money? To find this we need to compare the decimal numbers 15.50 and 15.75. To do this, we first compare the digits on the left of the decimal point, starting from the leftmost digit. Here both the digits 1 and 5, to the left of the decimal point, are same. So we compare the digits on the right of the decimal point starting from the tenths place. We find that $5 < 7$, so we say $15.50 < 15.75$. Thus, Salma has more money than John.

If the digits at the tenths place are also same then compare the digits at the hundredths place and so on.

Now compare quickly, 35.63 and 35.67; 20.1 and 20.01; 19.36 and 29.36.

While converting lower units of money, length and weight, to their higher units, we are required to use decimals. For example, 3 paise = ₹ $\frac{3}{100}$ = ₹ 0.03, 5g = $\frac{5}{1000}$ kg = 0.005 kg, 7 cm = 0.07 m.

Write 75 paise = ₹ ______, 250 g = ______ kg, 85 cm = ______m.

We also know how to add and subtract decimals. Thus, 21.36 + 37.35 is

$$\begin{array}{r} 21.36 \\ +\ 37.35 \\ \hline 58.71 \end{array}$$

What is the value of 0.19 + 2.3 ?

The difference 29.35 – 4.56 is

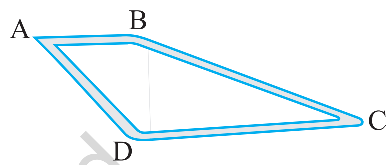
$$\begin{array}{r} 29.35 \\ -\ 04.56 \\ \hline 24.79 \end{array}$$

Tell the value of 39.87 – 21.98.

EXERCISE 2.5

1. Which is greater?
   (i) 0.5 or 0.05 (ii) 0.7 or 0.5 (iii) 7 or 0.7
   (iv) 1.37 or 1.49 (v) 2.03 or 2.30 (vi) 0.8 or 0.88.
2. Express as rupees using decimals :
   (i) 7 paise (ii) 7 rupees 7 paise (iii) 77 rupees 77 paise
   (iv) 50 paise (v) 235 paise.
3. (i) Express 5 cm in metre and kilometre (ii) Express 35 mm in cm, m and km

4. Express in kg:
   (i) 200 g (ii) 3470 g (iii) 4 kg 8 g
5. Write the following decimal numbers in the expanded form:
   (i) 20.03 (ii) 2.03 (iii) 200.03 (iv) 2.034
6. Write the place value of 2 in the following decimal numbers:
   (i) 2.56 (ii) 21.37 (iii) 10.25 (iv) 9.42 (v) 63.352.
7. Dinesh went from place A to place B and from there to place C. A is 7.5 km from B and B is 12.7 km from C. Ayub went from place A to place D and from there to place C. D is 9.3 km from A and C is 11.8 km from D. Who travelled more and by how much?



8. Shyama bought 5 kg 300 g apples and 3 kg 250 g mangoes. Sarala bought 4 kg 800 g oranges and 4 kg 150 g bananas. Who bought more fruits?
9. How much less is 28 km than 42.6 km?

2.6 Multiplication of Decimal Numbers

Reshma purchased 1.5kg vegetable at the rate of ₹ 8.50 per kg. How much money should she pay? Certainly it would be ₹ (8.50 × 1.50). Both 8.5 and 1.5 are decimal numbers. So, we have come across a situation where we need to know how to multiply two decimals. Let us now learn the multiplication of two decimal numbers.

First we find 0.1 × 0.1.

Now, $0.1 = \frac{1}{10}$. So, $0.1 \times 0.1 = \frac{1}{10} \times \frac{1}{10} = \frac{1 \times 1}{10 \times 10} = \frac{1}{100} = 0.01$.

Let us see it's pictorial representation (Fig 2.13)

The fraction $\frac{1}{10}$ represents 1 part out of 10 equal parts.

The shaded part in the picture represents $\frac{1}{10}$.

We know that,

$\frac{1}{10} \times \frac{1}{10}$ means $\frac{1}{10}$ of $\frac{1}{10}$. So, divide this $\frac{1}{10}^{\text{th}}$ part into 10 equal parts and take one part out of it.

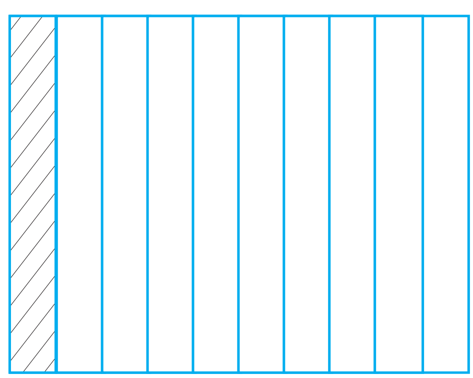


Fig 2.13

Thus, we have, (Fig 2.14).

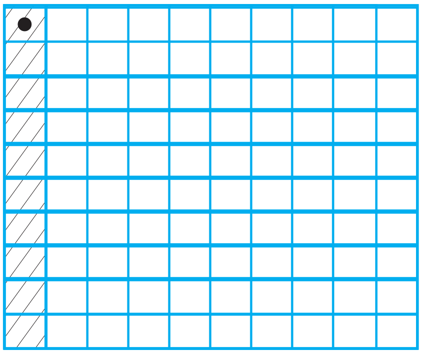


Fig 2.14

The dotted square is one part out of 10 of the $\frac{1}{10}^{\text{th}}$ part. That is, it represents

$\frac{1}{10} \times \frac{1}{10}$ or 0.1×0.1.

Can the dotted square be represented in some other way?

How many small squares do you find in Fig 2.14?

There are 100 small squares. So the dotted square represents one out of 100 or 0.01.

Hence, $0.1 \times 0.1 = 0.01$.

Note that 0.1 occurs two times in the product. In 0.1 there is one digit to the right of the decimal point. In 0.01 there are two digits (i.e., 1 + 1) to the right of the decimal point.

Let us now find 0.2×0.3.

We have, $0.2 \times 0.3 = \frac{2}{10} \times \frac{3}{10}$

As we did for $\frac{1}{10} \times \frac{1}{10}$, let us divide the square into 10 equal parts and take three parts out of it, to get $\frac{3}{10}$. Again divide each of these three equal parts into 10 equal parts and take two from each. We get $\frac{2}{10} \times \frac{3}{10}$.

The dotted squares represent $\frac{2}{10} \times \frac{3}{10}$ or 0.2×0.3. (Fig 2.15)

Since there are 6 dotted squares out of 100, so they also reprsent 0.06.

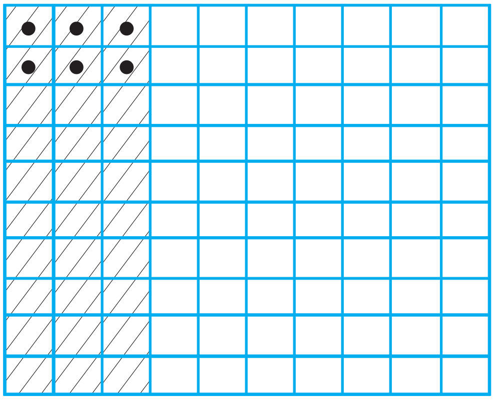


Fig 2.15

Thus, $0.2 \times 0.3 = 0.06$.

Observe that $2 \times 3 = 6$ and the number of digits to the right of the decimal point in 0.06 is 2 (= 1 + 1).

Check whether this applies to 0.1×0.1 also.

Find 0.2×0.4 by applying these observations.

While finding 0.1×0.1 and 0.2×0.3, you might have noticed that first we multiplied them as whole numbers ignoring the decimal point. In 0.1×0.1, we found 01×01 or 1×1. Similarly in 0.2×0.3 we found 02×03 or 2×3.

Then, we counted the number of digits starting from the rightmost digit and moved towards left. We then put the decimal point there. The number of digits to be counted is obtained by adding the number of digits to the right of the decimal point in the decimal numbers that are being multiplied.

Let us now find 1.2×2.5.

Multiply 12 and 25. We get 300. Both, in 1.2 and 2.5, there is 1 digit to the right of the decimal point. So, count $1 + 1 = 2$ digits from the rightmost digit (i.e., 0) in 300 and move towards left. We get 3.00 or 3.

Find in a similar way 1.5×1.6, 2.4×4.2.

While multiplying 2.5 and 1.25, you will first multiply 25 and 125. For placing the decimal in the product obtained, you will count $1 + 2 = 3$ (Why?) digits starting from the rightmost digit. Thus, $2.5 \times 1.25 = 3.225$

Find 2.7×1.35.

TRY THESE

1. Find: (i) 2.7×4 (ii) 1.8×1.2 (iii) 2.3×4.35
2. Arrange the products obtained in (1) in descending order.

EXAMPLE 7 The side of an equilateral triangle is 3.5 cm. Find its perimeter.

SOLUTION All the sides of an equilateral triangle are equal.

So, length of each side = 3.5 cm

Thus, perimeter = 3×3.5 cm = 10.5 cm

EXAMPLE 8 The length of a rectangle is 7.1 cm and its breadth is 2.5 cm. What is the area of the rectangle?

SOLUTION Length of the rectangle = 7.1 cm

Breadth of the rectangle = 2.5 cm

Therefore, area of the rectangle = 7.1×2.5 cm^2 = 17.75 cm^2

2.6.1 Multiplication of Decimal Numbers by 10, 100 and 1000

Reshma observed that $2.3 = \frac{23}{10}$ whereas $2.35 = \frac{235}{100}$. Thus, she found that depending on the position of the decimal point the decimal number can be converted to a fraction with denominator 10 or 100. She wondered what would happen if a decimal number is multiplied by 10 or 100 or 1000.

Let us see if we can find a pattern of multiplying numbers by 10 or 100 or 1000.

Have a look at the table given below and fill in the blanks:

$1.76 \times 10 = \frac{176}{100} \times 10 = 17.6$	$2.35 \times 10 =$ ___	$12.356 \times 10 =$ ___
$1.76 \times 100 = \frac{176}{100} \times 100 = 176$ or 176.0	$2.35 \times 100 =$ ___	$12.356 \times 100 =$ ___
$1.76 \times 1000 = \frac{176}{100} \times 1000 = 1760$ or 1760.0	$2.35 \times 1000 =$ ___	$12.356 \times 1000 =$ ___
$0.5 \times 10 = \frac{5}{10} \times 10 = 5$; $0.5 \times 100 =$ ___ ; $0.5 \times 1000 =$ ___		

Observe the shift of the decimal point of the products in the table. Here the numbers are multiplied by 10,100 and 1000. In $1.76 \times 10 = 17.6$, the digits are same i.e., 1, 7 and 6. Do you observe this in other products also? Observe 1.76 and 17.6. To which side has the decimal point shifted, right or left? The decimal point has shifted to the right by one place. Note that 10 has one zero over 1.

In $1.76 \times 100 = 176.0$, observe 1.76 and 176.0. To which side and by how many digits has the decimal point shifted? The decimal point has shifted to the right by two places.

Note that 100 has two zeros over one.

Do you observe similar shifting of decimal point in other products also?

So we say, *when a decimal number is multiplied by 10, 100 or 1000, the digits in the product are same as in the decimal number but the decimal point in the product is shifted to the right by as, many of places as there are zeros over one.*

Based on these observations we can now say

$0.07 \times 10 = 0.7$, $0.07 \times 100 = 7$ and $0.07 \times 1000 = 70$.

Can you now tell $2.97 \times 10 = ?$ $2.97 \times 100 = ?$ $2.97 \times 1000 = ?$

TRY THESE

Find: (i) 0.3×10
(ii) 1.2×100
(iii) 56.3×1000

Can you now help Reshma to find the total amount i.e., ₹ 8.50×150, that she has to pay?

Exercise 2.6

1. Find:

   (i) 0.2×6 (ii) 8×4.6 (iii) 2.71×5 (iv) 20.1×4
   (v) 0.05×7 (vi) 211.02×4 (vii) 2×0.86

2. Find the area of rectangle whose length is 5.7cm and breadth is 3 cm.

3. Find:

   (i) 1.3×10 (ii) 36.8×10 (iii) 153.7×10 (iv) 168.07×10
   (v) 31.1×100 (vi) 156.1×100 (vii) 3.62×100 (viii) 43.07×100
   (ix) 0.5×10 (x) 0.08×10 (xi) 0.9×100 (xii) 0.03×1000

4. A two-wheeler covers a distance of 55.3 km in one litre of petrol. How much distance will it cover in 10 litres of petrol?

5. Find:

   (i) 2.5×0.3 (ii) 0.1×51.7 (iii) 0.2×316.8 (iv) 1.3×3.1
   (v) 0.5×0.05 (vi) 11.2×0.15 (vii) 1.07×0.02
   (viii) 10.05×1.05 (ix) 101.01×0.01 (x) 100.01×1.1

2.7 Division of Decimal Numbers

Savita was preparing a design to decorate her classroom. She needed a few coloured strips of paper of length 1.9 cm each. She had a strip of coloured paper of length 9.5 cm. How many pieces of the required length will she get out of this strip? She thought it would be $\frac{9.5}{1.9}$ cm. Is she correct?

Both 9.5 and 1.9 are decimal numbers. So we need to know the division of decimal numbers too!

2.7.1 Division by 10, 100 and 1000

Let us find the division of a decimal number by 10, 100 and 1000.
Consider $31.5 \div 10$.

$$31.5 \div 10 = \frac{315}{10} \times \frac{1}{10} = \frac{315}{100} = 3.15$$

$$\text{Similarly, } 31.5 \div 100 = \frac{315}{10} \times \frac{1}{100} = \frac{315}{1000} = 0.315$$

Let us see if we can find a pattern for dividing numbers by 10, 100 or 1000. This may help us in dividing numbers by 10, 100 or 1000 in a shorter way.

$31.5 \div 10 = 3.15$	$231.5 \div 10 =$___	$1.5 \div 10 =$___	$29.36 \div 10 =$___
$31.5 \div 100 = 0.315$	$231.5 \div 10 =$___	$1.5 \div 100 =$___	$29.36 \div 100 =$___
$31.5 \div 1000 = 0.0315$	$231.5 \div 1000 =$___	$1.5 \div 1000 =$___	$29.36 \div 1000 =$___

Take 31.5 ÷ 10 = 3.15. In 31.5 and 3.15, the digits are same i.e., 3, 1, and 5 but the decimal point has shifted in the quotient. To which side and by how many digits? The decimal point has shifted to the left by one place. Note that 10 has one zero over 1.

Consider now 31.5 ÷ 100 = 0.315. In 31.5 and 0.315 the digits are same, but what about the decimal point in the quotient? It has shifted to the left by two places. Note that 100 has two zeros over1.

TRY THESE

Find: (i) 235.4 ÷ 10
(ii) 235.4 ÷100
(iii) 235.4 ÷ 1000

So we can say that, *while dividing a number by 10, 100 or 1000, the digits of the number and the quotient are same but the decimal point in the quotient shifts to the left by as many places as there are zeros over 1.* Using this observation let us now quickly find: 2.38 ÷ 10 = 0.238, 2.38 ÷ 100 = 0.0238, 2.38 ÷ 1000 = 0.00238

2.7.2 Division of a Decimal Number by a Whole Number

Let us find $\frac{6.4}{2}$. Remember we also write it as 6.4 ÷ 2.

So, $6.4 \div 2 = \frac{64}{10} \div 2 = \frac{64}{10} \times \frac{1}{2}$ as learnt in fractions.

$$= \frac{64 \times 1}{10 \times 2} = \frac{1 \times 64}{10 \times 2} = \frac{1}{10} \times \frac{64}{2} = \frac{1}{10} \times 32 = \frac{32}{10} = 3.2$$

TRY THESE

(i) 35.7 ÷ 3 = ?;
(ii) 25.5 ÷ 3 = ?

Or, let us first divide 64 by 2. We get 32. There is one digit to the right of the decimal point in 6.4. Place the decimal in 32 such that there would be one digit to its right. We get 3.2 again.

To find 19.5 ÷ 5, first find 195 ÷5. We get 39. There is one digit to the right of the decimal point in 19.5. Place the decimal point in 39 such that there would be one digit to its right. You will get 3.9.

TRY THESE

(i) 43.15 ÷ 5 = ?;
(ii) 82.44 ÷ 6 = ?

Now, $12.96 \div 4 = \frac{1296}{100} \div 4 = \frac{1296}{100} \times \frac{1}{4} = \frac{1}{100} \times \frac{1296}{4} = \frac{1}{100} \times 324 = 3.24$

Or, divide 1296 by 4. You get 324. There are two digits to the right of the decimal in 12.96. Making similar placement of the decimal in 324, you will get 3.24.

Note that here and in the next section, we have considered only those divisions in which, ignoring the decimal, the number would be completely divisible by another number to give remainder zero. Like, in 19.5 ÷ 5, the number 195 when divided by 5, leaves remainder zero.

TRY THESE

Find: (i) 15.5 ÷ 5
(ii) 126.35 ÷ 7

However, there are situations in which the number may not be completely divisible by another number, i.e., we may not get remainder zero. For example, 195 ÷ 7. We deal with such situations in later classes.

EXAMPLE 9 Find the average of 4.2, 3.8 and 7.6.

SOLUTION The average of 4.2, 3.8 and 7.6 is $\frac{4.2+3.8+7.6}{3} = \qquad = 5.2.$

2.7.3 Division of a Decimal Number by another Decimal Number

Let us find $\frac{25.5}{0.5}$ i.e., $25.5 \div 0.5$.

We have $25.5 \div 0.5 = \frac{255}{10} \div \frac{5}{10} = \frac{255}{10} \times \frac{10}{5} = 51$. Thus, $25.5 \div 0.5 = 51$

What do you observe? For $\frac{25.5}{0.5}$, we find that there is one digit to the right of the decimal in 0.5. This could be converted to whole number by dividing by 10. Accordingly 25.5 was also converted to a fraction by dividing by 10.

Or, we say the decimal point was shifted by one place to the right in 0.5 to make it 5. So, there was a shift of one decimal point to the right in 25.5 also to make it 255.

Thus, $22.5 \div 1.5 = \frac{22.5}{1.5} = \frac{225}{15} = 15$

Find $\frac{20.3}{0.7}$ and $\frac{15.2}{0.8}$ in a similar way.

TRY THESE

Find: (i) $\frac{7.75}{0.25}$ (ii) $\frac{42.8}{0.02}$ (iii) $\frac{5.6}{1.4}$

Let us now find $20.55 \div 1.5$.

We can write it is as $205.5 \div 15$, as discussed above. We get 13.7. Find $\frac{3.96}{0.4}$, $\frac{2.31}{0.3}$.

Consider now, $\frac{33.725}{0.25}$. We can write it as $\frac{3372.5}{25}$ (How?) and we get the quotient as 134.9. How will you find $\frac{27}{0.03}$? We know that 27 can be written as 27.00.

So, $\frac{27}{0.03} = \frac{27.00}{0.03} = \frac{2700}{3} = 900$

EXAMPLE 10 Each side of a regular polygon is 2.5 cm in length. The perimeter of the polygon is 12.5cm. How many sides does the polygon have?

SOLUTION The perimeter of a regular polygon is the sum of the lengths of all its equal sides = 12.5 cm.

Length of each side = 2.5 cm. Thus, the number of sides = $\frac{12.5}{2.5} = \frac{125}{25} = 5$

The polygon has 5 sides.

EXAMPLE 11 A car covers a distance of 89.1 km in 2.2 hours. What is the average distance covered by it in 1 hour?

SOLUTION Distance covered by the car = 89.1 km.

Time required to cover this distance = 2.2 hours.

So distance covered by it in 1 hour $= \frac{89.1}{2.2} = \frac{891}{22} = 40.5$ km.

EXERCISE 2.7

1. Find:
(i) $0.4 \div 2$ (ii) $0.35 \div 5$ (iii) $2.48 \div 4$ (iv) $65.4 \div 6$
(v) $651.2 \div 4$ (vi) $14.49 \div 7$ (vii) $3.96 \div 4$ (viii) $0.80 \div 5$

2. Find:
(i) $4.8 \div 10$ (ii) $52.5 \div 10$ (iii) $0.7 \div 10$ (iv) $33.1 \div 10$
(v) $272.23 \div 10$ (vi) $0.56 \div 10$ (vii) $3.97 \div 10$

3. Find:
(i) $2.7 \div 100$ (ii) $0.3 \div 100$ (iii) $0.78 \div 100$
(iv) $432.6 \div 100$ (v) $23.6 \div 100$ (vi) $98.53 \div 100$

4. Find:
(i) $7.9 \div 1000$ (ii) $26.3 \div 1000$ (iii) $38.53 \div 1000$
(iv) $128.9 \div 1000$ (v) $0.5 \div 1000$

5. Find:
(i) $7 \div 3.5$ (ii) $36 \div 0.2$ (iii) $3.25 \div 0.5$ (iv) $30.94 \div 0.7$
(v) $0.5 \div 0.25$ (vi) $7.75 \div 0.25$ (vii) $76.5 \div 0.15$ (viii) $37.8 \div 1.4$
(ix) $2.73 \div 1.3$

6. A vehicle covers a distance of 43.2 km in 2.4 litres of petrol. How much distance will it cover in one litre of petrol?

WHAT HAVE WE DISCUSSED?

1. We have learnt about fractions and decimals alongwith the operations of addition and subtraction on them, in the earlier class.

2. We now study the operations of multiplication and division on fractions as well as on decimals.

3. We have learnt how to multiply fractions. Two fractions are multiplied by multiplying their numerators and denominators seperately and writing the product as $\frac{\text{product of numerators}}{\text{product of denominators}}$. For example, $\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$.

4. A fraction acts as an operator 'of '. For example, $\frac{1}{2}$ of 2 is $\frac{1}{2} \times 2 = 1$.

5. (a) The product of two proper fractions is less than each of the fractions that are multiplied.

(b) The product of a proper and an improper fraction is less than the improper fraction and greater than the proper fraction.

(c) The product of two imporper fractions is greater than the two fractions.

6. A reciprocal of a fraction is obtained by inverting it upside down.

7. We have seen how to divide two fractions.

(a) While dividing a whole number by a fraction, we multiply the whole number with the reciprocal of that fraction.

For example, $2 \div \frac{3}{5} = 2 \times \frac{5}{3} = \frac{10}{3}$

(b) While dividing a fraction by a whole number we multiply the fraction by the reciprocal of the whole number.

For example, $\frac{2}{3} \div 7 = \frac{2}{3} \times \frac{1}{7} = \frac{2}{21}$

(c) While dividing one fraction by another fraction, we multuiply the first fraction by the reciprocal of the other. So, $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$.

8. We also learnt how to multiply two decimal numbers. While multiplying two decimal numbers, first multiply them as whole numbers. Count the number of digits to the right of the decimal point in both the decimal numbers. Add the number of digits counted. Put the decimal point in the product by counting the digits from its rightmost place. The count should be the sum obtained earlier.

For example, $0.5 \times 0.7 = 0.35$

9. To multiply a decimal number by 10, 100 or 1000, we move the decimal point in the number to the right by as many places as there are zeros over 1.

Thus $0.53 \times 10 = 5.3$, $0.53 \times 100 = 53$, $0.53 \times 1000 = 530$

10. We have seen how to divide decimal numbers.

(a) To divide a decimal number by a whole number, we first divide them as whole numbers. Then place the decimal point in the quotient as in the decimal number.

For example, $8.4 \div 4 = 2.1$

Note that here we consider only those divisions in which the remainder is zero.

(b) To divide a decimal number by 10, 100 or 1000, shift the digits in the decimal number to the left by as many places as there are zeros over 1, to get the quotient.

So, $23.9 \div 10 = 2.39, 23.9 \div 100 = 0.239$, $23.9 \div 1000 = 0.0239$

(c) While dividing two decimal numbers, first shift the decimal point to the right by equal number of places in both, to convert the divisor to a whole number. Then divide. Thus, $2.4 \div 0.2 = 24 \div 2 = 12$.

Chapter 3

Data Handling

0756CH03

3.1 Introduction

In your previous classes, you have dealt with various types of data. You have learnt to collect data, tabulate and put it in the form of bar graphs. The collection, recording and presentation of data help us organise our experiences and draw inferences from them. In this Chapter, we will take one more step towards learning how to do this. You will come across some more kinds of data and graphs. You have seen several kinds of data through newspapers, magazines, television and other sources. You also know that all data give us some sort of information. Let us look at some common forms of data that you come across:

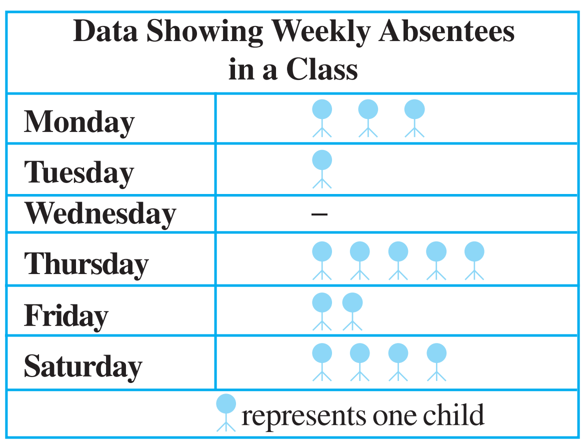
Table 3.1

Temperatures of Cities as on 20.6.2006		
City	**Max.**	**Min.**
Ahmedabad	38° C	29° C
Amritsar	37° C	26° C
Bangalore	28° C	21° C
Chennai	36° C	27° C
Delhi	38° C	28° C
Jaipur	39° C	29° C
Jammu	41° C	26° C
Mumbai	32° C	27° C

Table 3.2

Football World Cup 2006	
Ukraine beat Saudi Arabia by	4 - 0
Spain beat Tunisia by	3 - 1
Switzerland beat Togo by	2 - 0

Table 3.3



What do these collections of data tell you?

For example you can say that the highest maximum temperature was in Jammu on 20.06.2006 (Table 3.1) or we can say that, on Wednesday, no child was absent. (Table 3.3)

Can we organise and present these data in a different way, so that their analysis and interpretation becomes better? We shall address such questions in this Chapter.

3.2 Collecting Data

The data about the temperatures of cities (Table 3.1) can tell us many things, but it cannot tell us the city which had the highest maximum temperature during the year. To find that, we need to collect data regarding the highest maximum temperature reached in each of these cities during the year. In that case, the temperature chart of one particular date of the year, as given in Table 3.1 will not be sufficient.

This shows that a given collection of data may not give us a specific information related to that data. For this we need to collect data keeping in mind that specific information. In the above case the specific information needed by us, was about the highest maximum temperature of the cities during the year, which we could not get from Table 3.1

Thus, **before collecting data, we need to know what we would use it for.**

Given below are a few situations.

You want to study the

– Performance of your class in Mathematics.
– Performance of India in football or in cricket.
– Female literacy rate in a given area, or
– Number of children below the age of five in the families around you.

What kind of data would you need in the above situations? Unless and until you collect appropriate data, you cannot know the desired information. What is the appropriate data for each?

Discuss with your friends and identify the data you would need for each. Some of this data is easy to collect and some difficult.

3.3 Organisation of Data

When we collect data, we have to record and organise it. Why do we need to do that? Consider the following example.

Ms Neelam, class teacher wanted to find how children had performed in English. She writes down the marks obtained by the students in the following way:

23, 35, 48, 30, 25, 46, 13, 27, 32, 38

In this form, the data was not easy to understand. She also did not know whether her impression of the students matched their performance.

Neelam's colleague helped her organise the data in the following way (Table 3.4).

Table 3.4

Roll No.	Names	Marks Out of 50	Roll No.	Names	Marks Out of 50
1	Ajay	23	6	Govind	46
2	Armaan	35	7	Jay	13
3	Ashish	48	8	Kavita	27
4	Dipti	30	9	Manisha	32
5	Faizaan	25	10	Neeraj	38

In this form, Neelam was able to know which student has got how many marks. But she wanted more. Deepika suggested another way to organise this data (Table 3.5).

Table 3.5

Roll No.	Names	Marks Out of 50	Roll No.	Names	Marks Out of 50
3	Ashish	48	4	Dipti	30
6	Govind	46	8	Kavita	27
10	Neeraj	38	5	Faizaan	25
2	Armaan	35	1	Ajay	23
9	Manisha	32	7	Jay	13

Now Neelam was able to see who had done the best and who needed help.

Many kinds of data we come across are put in tabular form. Our school rolls, progress report, index in the notebooks, temperature record and many others are all in tabular form. Can you think of a few more data that you come across in tabular form?

When we put data in a proper table it becomes easy to understand and interpret.

Try These

Weigh (in kg) atleast 20 children (girls and boys) of your class. Organise the data, and answer the following questions using this data.

(i) Who is the heaviest of all? (ii) What is the most common weight?

(iii) What is the difference between your weight and that of your best friend?

3.4 Representative Values

You might be aware of the term *average* and would have come across statements involving the term 'average' in your day-to-day life:

- Isha spends on an average of about 5 hours daily for her studies.

- The average temperature at this time of the year is about 40 degree celsius.
- The average age of pupils in my class is 12 years.
- The average attendance of students in a school during its final examination was 98 per cent.

Many more of such statements could be there. Think about the statements given above. Do you think that the child in the first statement studies exactly for 5 hours daily? Or, is the temperature of the given place during that particular time always 40 degrees? Or, is the age of each pupil in that class 12 years? Obviously not.

Then what do these statements tell you?

By average we understand that Isha, usually, studies for 5 hours. On some days, she may study for less number of hours and on the other days she may study longer.

Similarly, the average temperature of 40 degree celsius, means that, very often, the temperature at this time of the year is around 40 degree celsius. Sometimes, it may be less than 40 degree celsius and at other times, it may be more than 40°C.

Thus, we realise that average is a number that represents or shows the central tendency of a group of observations or data. Since average lies between the highest and the lowest value of the given data so, we say average is a measure of the central tendency of the group of data. Different forms of data need different forms of representative or central value to describe it. One of these representative values is the "**Arithmetic mean**". You will learn about the other representative values in the later part of the chapter.

3.5 Arithmetic Mean

The most common representative value of a group of data is the **arithmetic mean** or the **mean**. To understand this in a better way, let us look at the following example:

Two vessels contain 20 litres and 60 litres of milk respectively. What is the amount that each vessel would have, if both share the milk equally? When we ask this question we are seeking the arithmetic mean.

In the above case, the average or the arithmetic mean would be

$$\frac{\text{Total quantity of milk}}{\text{Number of vessels}} = \frac{20+60}{2} \text{ litres} = 40 \text{ litres.}$$

Thus, each vessels would have 40 litres of milk.

The average or Arithmetic Mean (A.M.) or simply mean is defined as follows:

$$\text{mean} = \frac{\text{Sum of all observations}}{\text{number of observations}}$$

Consider these examples.

Example 1 Ashish studies for 4 hours, 5 hours and 3 hours respectively on three consecutive days. How many hours does he study daily on an average?

SOLUTION The average study time of Ashish would be

$$\frac{\text{Total number of study hours}}{\text{Number of days for which he studied}} = \frac{4+5+3}{3} \text{ hours} = 4 \text{ hours per day}$$

Thus, we can say that Ashish studies for 4 hours daily on an average.

EXAMPLE 2 A batsman scored the following number of runs in six innings:

36, 35, 50, 46, 60, 55

Calculate the mean runs scored by him in an inning.

SOLUTION Total runs = 36 + 35 + 50 + 46 + 60 + 55 = 282.

To find the mean, we find the sum of all the observations and divide it by the number of observations.

Therefore, in this case, mean = $\frac{282}{6} = 47$. Thus, the mean runs scored in an inning are 47.

Where does the arithmetic mean lie

TRY THESE

How would you find the average of your study hours for the whole week?

THINK, DISCUSS AND WRITE

Consider the data in the above examples and think on the following:

- Is the mean bigger than each of the observations?
- Is it smaller than each observation?

Discuss with your friends. Frame one more example of this type and answer the same questions.

You will find that the mean lies inbetween the greatest and the smallest observations.

In particular, the mean of two numbers will always lie between the two numbers.

For example the mean of 5 and 11 is $\frac{5+11}{2} = 8$, which lies between 5 and 11.

Can you use this idea to show that between any two fractional numbers, you can find as many fractional numbers as you like. For example between $\frac{1}{2}$ and $\frac{1}{4}$ you have their average $\frac{\frac{1}{2}+\frac{1}{4}}{2} = \frac{3}{8}$ and then between $\frac{1}{2}$ and $\frac{3}{8}$, you have their average $\frac{7}{16}$ and so on.

TRY THESE

1. Find the mean of your sleeping hours during one week.
2. Find atleast 5 numbers between $\frac{1}{2}$ and $\frac{1}{3}$.

3.5.1 Range

The difference between the highest and the lowest observation gives us an idea of the spread of the observations. This can be found by subtracting the lowest observation from the highest observation. We call the result the **range** of the observation. Look at the following example:

EXAMPLE 3 The ages in years of 10 teachers of a school are:

32, 41, 28, 54, 35, 26, 23, 33, 38, 40

(i) What is the age of the oldest teacher and that of the youngest teacher?

(ii) What is the range of the ages of the teachers?

(iii) What is the mean age of these teachers?

SOLUTION

(i) Arranging the ages in ascending order, we get:

23, 26, 28, 32, 33, 35, 38, 40, 41, 54

We find that the age of the oldest teacher is 54 years and the age of the youngest teacher is 23 years.

(ii) Range of the ages of the teachers = (54 – 23) years = 31 years

(iii) Mean age of the teachers

$$= \frac{23+26+28+32+33+35+38+40+41+54}{10} \text{ years}$$

$$= \frac{350}{10} \text{ years} = 35 \text{ years}$$

EXERCISE 3.1

1. Find the range of heights of any ten students of your class.
2. Organise the following marks in a class assessment, in a tabular form.

   4, 6, 7, 5, 3, 5, 4, 5, 2, 6, 2, 5, 1, 9, 6, 5, 8, 4, 6, 7

   (i) Which number is the highest? (ii) Which number is the lowest?

   (iii) What is the range of the data? (iv) Find the arithmetic mean.
3. Find the mean of the first five whole numbers.
4. A cricketer scores the following runs in eight innings:

   58, 76, 40, 35, 46, 45, 0, 100.

   Find the mean score.

5. Following table shows the points of each player scored in four games:

Player	Game 1	Game 2	Game 3	Game 4
A	14	16	10	10
B	0	8	6	4
C	8	11	Did not Play	13

Now answer the following questions:

(i) Find the mean to determine A's average number of points scored per game.

(ii) To find the mean number of points per game for C, would you divide the total points by 3 or by 4? Why?

(iii) B played in all the four games. How would you find the mean?

(iv) Who is the best performer?

6. The marks (out of 100) obtained by a group of students in a science test are 85, 76, 90, 85, 39, 48, 56, 95, 81 and 75. Find the:

(i) Highest and the lowest marks obtained by the students.

(ii) Range of the marks obtained.

(iii) Mean marks obtained by the group.

7. The enrolment in a school during six consecutive years was as follows:

1555, 1670, 1750, 2013, 2540, 2820

Find the mean enrolment of the school for this period.

8. The rainfall (in mm) in a city on 7 days of a certain week was recorded as follows:

Day	Mon	Tue	Wed	Thurs	Fri	Sat	Sun
Rainfall (in mm)	0.0	12.2	2.1	0.0	20.5	5.5	1.0

(i) Find the range of the rainfall in the above data.

(ii) Find the mean rainfall for the week.

(iii) On how many days was the rainfall less than the mean rainfall.

9. The heights of 10 girls were measured in cm and the results are as follows:

135, 150, 139, 128, 151, 132, 146, 149, 143, 141.

(i) What is the height of the tallest girl? (ii) What is the height of the shortest girl?

(iii) What is the range of the data? (iv) What is the mean height of the girls?

(v) How many girls have heights more than the mean height.

3.6 Mode

As we have said Mean is not the only measure of central tendency or the only form of representative value. For different requirements from a data, other measures of central tendencies are used.

Look at the following example

To find out the weekly demand for different sizes of shirt, a shopkeeper kept records of sales of sizes 90 cm, 95 cm, 100 cm, 105 cm, 110 cm. Following is the record for a week:

Size (in inches)	90 cm	95 cm	100 cm	105 cm	110 cm	**Total**
Number of Shirts Sold	8	22	32	37	6	**105**

If he found the mean number of shirts sold, do you think that he would be able to decide which shirt sizes to keep in stock?

$$\text{Mean of total shirts sold} = \frac{\text{Total number of shirts sold}}{\text{Number of different sizes of shirts}} = \frac{105}{5} = 21$$

Should he obtain 21 shirts of each size? If he does so, will he be able to cater to the needs of the customers?

The shopkeeper, on looking at the record, decides to procure shirts of sizes 95 cm, 100 cm, 105 cm. He decided to postpone the procurement of the shirts of other sizes because of their small number of buyers.

Look at another example

The owner of a readymade dress shop says, "The most popular size of dress I sell is the size 90 cm.

Observe that here also, the owner is concerned about the number of shirts of different sizes sold. She is however looking at the shirt size that is sold the most. This is another representative value for the data. The highest occuring event is the sale of size 90 cm.This representative value is called the **mode** of the data.

The mode of a set of observations is the observation that occurs most often.

Example 4 Find the mode of the given set of numbers: 1, 1, 2, 4, 3, 2, 1, 2, 2, 4

Solution Arranging the numbers with same values together, we get

1, 1, 1, 2, 2, 2, 2, 3, 4, 4

Mode of this data is 2 because it occurs more frequently than other observations.

3.6.1 Mode of Large Data

Putting the same observations together and counting them is not easy if the number of observations is large. In such cases we tabulate the data. Tabulation can begin by putting tally marks and finding the frequency, as you did in your previous class.

Look at the following example:

EXAMPLE 5 Following are the margins of victory in the football matches of a league.

1, 3, 2, 5, 1, 4, 6, 2, 5, 2, 2, 2, 4, 1, 2, 3, 1, 1, 2, 3, 2, 6, 4, 3, 2, 1, 1, 4, 2, 1, 5, 3, 3, 2, 3, 2, 4, 2, 1, 2

Find the mode of this data.

SOLUTION Let us put the data in a tabular form:

Margins of Victory	Tally Bars	Number of Matches
1	𝍸 ||||	9
2	𝍸 𝍸 ||||	14
3	𝍸 ||	7
4	𝍸	5
5	|||	3
6	||	2
	Total	**40**

Looking at the table, we can quickly say that 2 is the 'mode' since 2 has occured the highest number of times. Thus, most of the matches have been won with a victory margin of 2 goals.

TRY THESE

Find the mode of

(i) 2, 6, 5, 3, 0, 3, 4, 3, 2, 4, 5, 2, 4

(ii) 2, 14, 16, 12, 14, 14, 16, 14, 10, 14, 18, 14

THINK, DISCUSS AND WRITE

Can a set of numbers have more than one mode?

EXAMPLE 6 Find the mode of the numbers: 2, 2, 2, 3, 3, 4, 5, 5, 5, 6, 6, 8

SOLUTION Here, 2 and 5 both occur three times. Therefore, they both are modes of the data.

DO THIS

1. Record the age in years of all your classmates. Tabulate the data and find the mode.
2. Record the heights in centimetres of your classmates and find the mode.

TRY THESE

1. Find the mode of the following data:

12, 14, 12, 16, 15, 13, 14, 18, 19, 12, 14, 15, 16, 15, 16, 16, 15, 17, 13, 16, 16, 15, 15, 13, 15, 17, 15, 14, 15, 13, 15, 14

2. Heights (in cm) of 25 children are given below:

168, 165, 163, 160, 163, 161, 162, 164, 163, 162, 164, 163, 160, 163, 160, 165, 163, 162, 163, 164, 163, 160, 165, 163, 162

What is the mode of their heights? What do we understand by mode here?

Whereas mean gives us the average of all observations of the data, the mode gives that observation which occurs most frequently in the data.

Let us consider the following examples:

(a) You have to decide upon the number of chapattis needed for 25 people called for a feast.

(b) A shopkeeper selling shirts has decided to replenish her stock.

(c) We need to find the height of the door needed in our house.

(d) When going on a picnic, if only one fruit can be bought for everyone, which is the fruit that we would get.

In which of these situations can we use the mode as a good estimate?

Consider the first statement. Suppose the number of chapattis needed by each person is 2, 3, 2, 3, 2, 1, 2, 3, 2, 2, 4, 2, 2, 3, 2, 4, 4, 2, 3, 2, 4, 2, 4, 3, 5

The mode of the data is 2 chapattis. If we use mode as the representative value for this data, then we need 50 chapattis only, 2 for each of the 25 persons. However the total number would clearly be inadequate. Would **mean** be an appropriate representative value?

For the third statement the height of the door is related to the height of the persons using that door. Suppose there are 5 children and 4 adults using the door and the height of each of 5 children is around 135 cm. The mode for the heights is 135 cm. Should we get a door that is 144 cm high? Would all the adults be able to go through that door? It is clear that mode is not the appropriate representative value for this data. Would **mean** be an appropriate representative value here?

Why not? Which representative value of height should be used to decide the doorheight?

Similarly analyse the rest of the statements and find the representative value useful for that issue.

Try These

Discuss with your friends and give

(a) Two situations where mean would be an appropriate representative value to use, and

(b) Two situations where mode would be an appropriate representative value to use.

3.7 MEDIAN

We have seen that in some situations, arithmetic mean is an appropriate measure of central tendency whereas in some other situations, mode is the appropriate measure of central tendency.

Let us now look at another example. Consider a group of 17 students with the following heights (in cm): 106, 110, 123, 125, 117, 120, 112, 115, 110, 120, 115, 102, 115, 115, 109, 115, 101.

The games teacher wants to divide the class into two groups so that each group has equal number of students, one group has students with height lesser than a particular height and the other group has students with heights greater than the particular height. How would she do that?

Let us see the various options she has:

(i) She can find the mean. The mean is

$$\frac{106+110+123+125+117+120+112+115+110+120+115+102+115+115+109+115+101}{17}$$

$$= \frac{1930}{17} = 113.5$$

So, if the teacher divides the students into two groups on the basis of this mean height, such that one group has students of height less than the mean height and the other group has students with height more than the mean height, then the groups would be of unequal size. They would have 7 and 10 members respectively.

(ii) The second option for her is to find mode. The observation with highest frequency is 115 cm, which would be taken as mode.

There are 7 children below the mode and 10 children at the mode and above the mode. Therefore, we cannot divide the group into equal parts.

Let us therefore think of an alternative representative value or measure of central tendency. For doing this we again look at the given heights (in cm) of students and arrange them in ascending order. We have the following observations:

101, 102, 106, 109, 110, 110, 112, 115, 115, 115, 115, 115, 117, 120, 120, 123, 125

The middle value in this data is 115 because this value divides the students into two equal groups of 8 students each. This value is called as **Median**. Median refers to the value which lies in the middle of the data (when arranged in an increasing or decreasing order) with half of the observations above it and the other half below it. The games teacher decides to keep the middle student as a refree in the game.

Here, we consider only those cases where number of observations is odd.

Thus, in a given data, arranged in ascending or descending order, the **median** gives us the middle observation.

TRY THESE

Your friend found the median and the mode of a given data. Describe and correct your friends error if any:

35, 32, 35, 42, 38, 32, 34

Median = 42, Mode = 32

Note that in general, we may not get the same value for median and mode.

Thus we realise that mean, mode and median are the numbers that are the representative values of a group of observations or data. They lie between the minimum and maximum values of the data. They are also called the measures of the central tendency.

Example 7 Find the median of the data: 24, 36, 46, 17, 18, 25, 35

Solution We arrange the data in ascending order, we get 17, 18, 24, 25, 35, 36, 46
Median is the middle observation. Therefore 25 is the median.

Exercise 3.2

1. The scores in mathematics test (out of 25) of 15 students is as follows:

   19, 25, 23, 20, 9, 20, 15, 10, 5, 16, 25, 20, 24, 12, 20

   Find the mode and median of this data. Are they same?
2. The runs scored in a cricket match by 11 players is as follows:

   6, 15, 120, 50, 100, 80, 10, 15, 8, 10, 15

   Find the mean, mode and median of this data. Are the three same?
3. The weights (in kg.) of 15 students of a class are:

   38, 42, 35, 37, 45, 50, 32, 43, 43, 40, 36, 38, 43, 38, 47

   (i) Find the mode and median of this data.
   (ii) Is there more than one mode?
4. Find the mode and median of the data: 13, 16, 12, 14, 19, 12, 14, 13, 14
5. Tell whether the statement is true or false:
   (i) The mode is always one of the numbers in a data.
   (ii) The mean is one of the numbers in a data.
   (iii) The median is always one of the numbers in a data.
   (iv) The data 6, 4, 3, 8, 9, 12, 13, 9 has mean 9.

3.8 Use of Bar Graphs with a Different Purpose

We have seen last year how information collected could be first arranged in a frequency distribution table and then this information could be put as a visual representation in the form of pictographs or bar graphs. You can look at the bar graphs and make deductions about the data. You can also get information based on these bar graphs. For example, you can say that the mode is the longest bar if the bar represents the frequency.

3.8.1 Choosing a Scale

We know that a bar graph is a representation of numbers using bars of uniform width and the lengths of the bars depend upon the frequency and the scale you have chosen. For example, in a bar graph where numbers in units are to be shown, the graph represents one unit length for one observation and if it has to show numbers in tens or hundreds, one unit length can represent 10 or 100 observations. Consider the following examples:

EXAMPLE 8 Two hundred students of 6th and 7th classes were asked to name their favourite colour so as to decide upon what should be the colour of their school building. The results are shown in the following table. Represent the given data on a bar graph.

Favourite Colour	Red	Green	Blue	Yellow	Orange
Number of Students	43	19	55	49	34

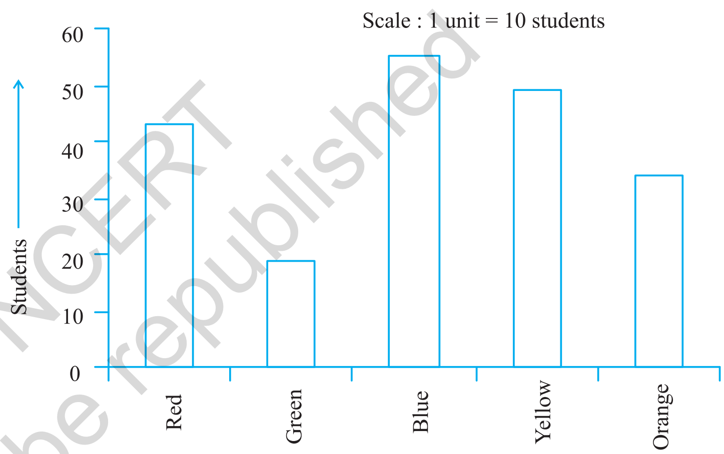
Answer the following questions with the help of the bar graph:

(i) Which is the most preferred colour and which is the least preferred?

(ii) How many colours are there in all? What are they?

SOLUTION Choose a suitable scale as follows:

Start the scale at 0. The greatest value in the data is 55, so end the scale at a value greater than 55, such as 60. Use equal divisions along the axes, such as increments of 10. You know that all the bars would lie between 0 and 60. We choose the scale such that the length between 0 and 60 is neither too long nor too small. Here we take 1 unit for 10 students.



We then draw and label the graph as shown.

From the bar graph we conclude that

(i) Blue is the most preferred colour (Because the bar representing Blue is the tallest).

(ii) Green is the least preferred colour. (Because the bar representing Green is the shortest).

(iii) There are five colours. They are Red, Green, Blue, Yellow and Orange. (These are observed on the horizontal line)

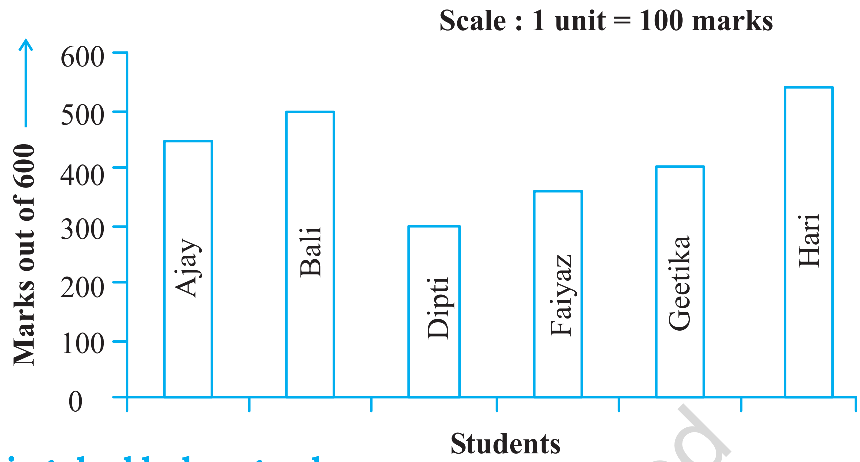
EXAMPLE 9 Following data gives total marks (out of 600) obtained by six children of a particular class. Represent the data on a bar graph.

Students	Ajay	Bali	Dipti	Faiyaz	Geetika	Hari
Marks Obtained	450	500	300	360	400	540

SOLUTION

(i) To choose an appropriate scale we make equal divisions taking increments of 100. Thus 1 unit will represent 100 marks. (What would be the difficulty if we choose one unit to represent 10 marks?)

(ii) Now represent the data on the bar graph.



Drawing double bar graph

Consider the following two collections of data giving the average daily hours of sunshine in two cities Aberdeen and Margate for all the twelve months of the year. These cities are near the south pole and hence have only a few hours of sunshine each day.

In Margate												
	Jan.	**Feb.**	**Mar.**	**April**	**May**	**June**	**July**	**Aug.**	**Sept.**	**Oct.**	**Nov.**	**Dec.**
Average hours of Sunshine	2	$3\frac{1}{4}$	4	4	$7\frac{3}{4}$	8	$7\frac{1}{2}$	7	$6\frac{1}{4}$	6	4	2
In Aberdeen												
Average hours of Sunshine	$1\frac{1}{2}$	3	$3\frac{1}{2}$	6	$5\frac{1}{2}$	$6\frac{1}{2}$	$5\frac{1}{2}$	5	$4\frac{1}{2}$	4	3	$1\frac{3}{4}$

By drawing individual bar graphs you could answer questions like

(i) In which month does each city has maximum sunlight? or

(ii) In which months does each city has minimum sunlight?

However, to answer questions like "In a particular month, which city has more sunshine hours", we need to compare the average hours of sunshine of both the cities. To do this we will learn to draw what is called a double bar graph giving the information of both cities side-by-side.

This bar graph (Fig 3.1) shows the average sunshine of both the cities.

For each month we have two bars, the heights of which give the average hours of sunshine in each city. From this we can infer that except for the month of April, there is always more sunshine in Margate than in Aberdeen. You could put together a similiar bar graph for your area or for your city.

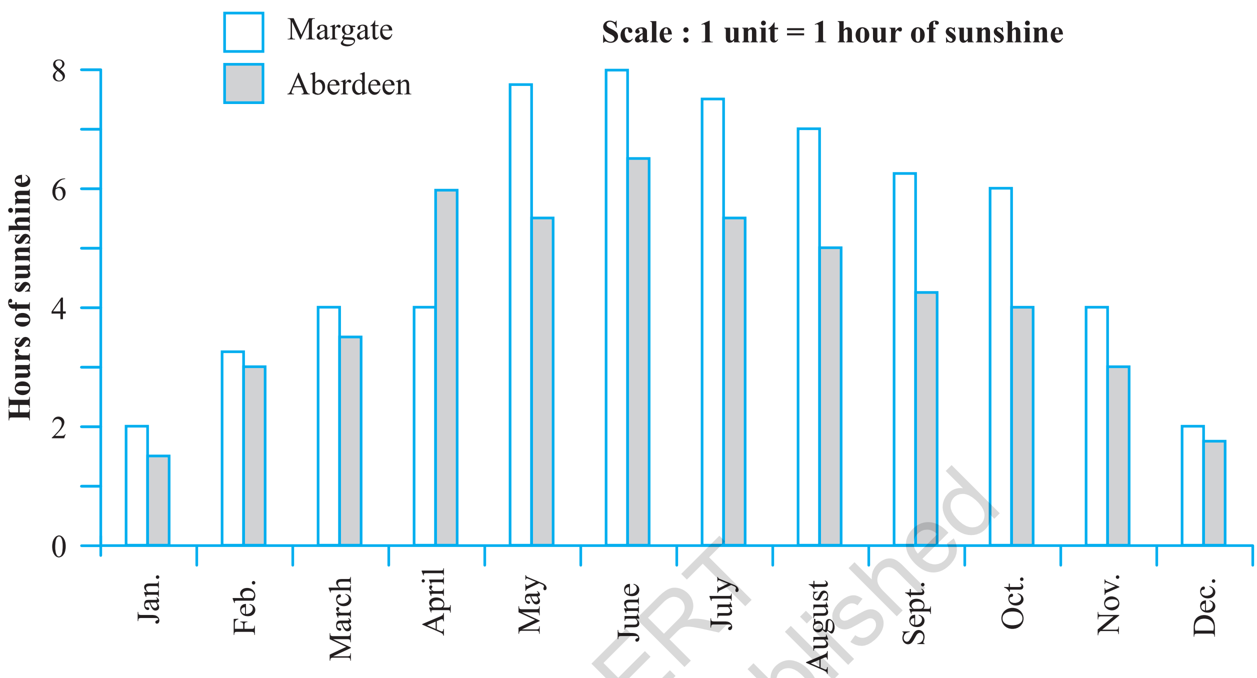
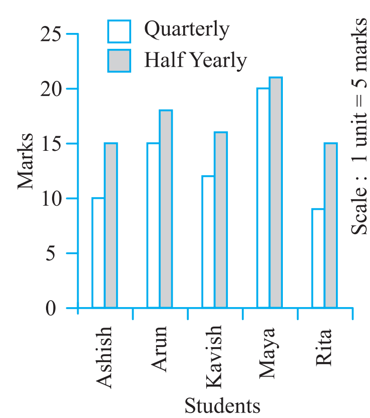


Fig 3.1

Let us look at another example more related to us.

EXAMPLE 10 A mathematics teacher wants to see, whether the new technique of teaching she applied after quarterly test was effective or not. She takes the scores of the 5 weakest children in the quarterly test (out of 25) and in the half yearly test (out of 25):

Students	Ashish	Arun	Kavish	Maya	Rita
Quarterly	10	15	12	20	9
Half yearly	15	18	16	21	15



SOLUTION She draws the adjoining double bar graph and finds a marked improvement in most of the students, the teacher decides that she should continue to use the new technique of teaching.

Can you think of a few more situations where you could use double bar graphs?

TRY THESE

1. The bar graph (Fig 3.2) shows the result of a survey to test water resistant watches made by different companies.
   Each of these companies claimed that their watches were water resistant. After a test the above results were revealed.

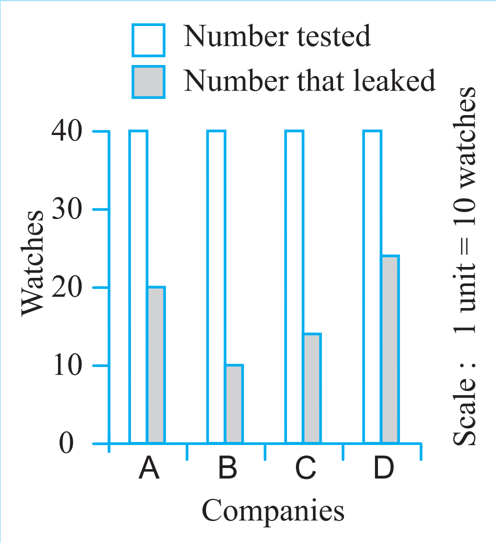


Fig 3.2

(a) Can you work out a fraction of the number of watches that leaked to the number tested for each company?

(b) Could you tell on this basis which company has better watches?

2. Sale of English and Hindi books in the years 1995, 1996, 1997 and 1998 are given below:

Years	**1995**	**1996**	**1997**	**1998**
English	350	400	450	620
Hindi	500	525	600	650

Draw a double bar graph and answer the following questions:

(a) In which year was the difference in the sale of the two language books least?.

(b) Can you say that the demand for English books rose faster? Justify.

EXERCISE 3.3

1. Use the bar graph (Fig 3.3) to answer the following questions.

(a) Which is the most popular pet? (b) How many students have dog as a pet?

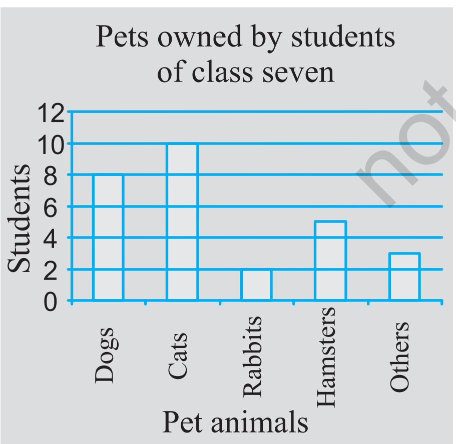


Fig 3.3

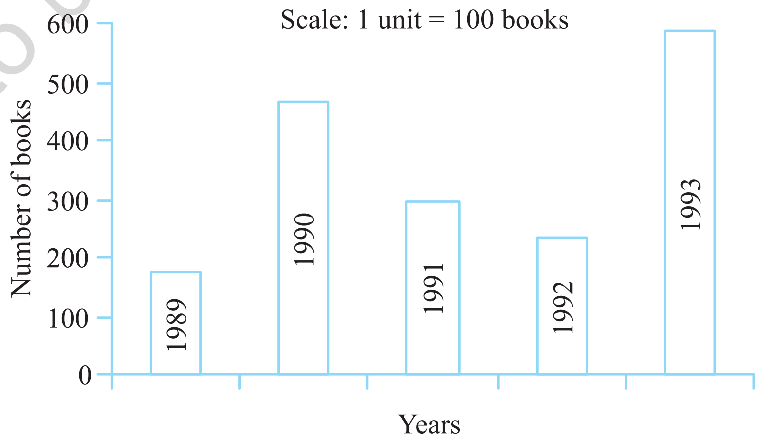


Fig 3.4

2. Read the bar graph (Fig 3.4) which shows the number of books sold by a bookstore during five consecutive years and answer the following questions:

(i) About how many books were sold in 1989? 1990? 1992?

(ii) In which year were about 475 books sold? About 225 books sold?

(iii) In which years were fewer than 250 books sold?

(iv) Can you explain how you would estimate the number of books sold in 1989?

3. Number of children in six different classes are given below. Represent the data on a bar graph.

Class	Fifth	Sixth	Seventh	Eighth	Ninth	Tenth
Number of Children	135	120	95	100	90	80

(a) How would you choose a scale?

(b) Answer the following questions:

(i) Which class has the maximum number of children? And the minimum?

(ii) Find the ratio of students of class sixth to the students of class eight.

4. The performance of a student in 1st Term and 2nd Term is given. Draw a double bar graph choosing appropriate scale and answer the following:

Subject	English	Hindi	Maths	Science	S. Science
1st Term (M.M. 100)	67	72	88	81	73
2nd Term (M.M. 100)	70	65	95	85	75

(i) In which subject, has the child improved his performance the most?

(ii) In which subject is the improvement the least?

(iii) Has the performance gone down in any subject?

5. Consider this data collected from a survey of a colony.

Favourite Sport	Cricket	Basket Ball	Swimming	Hockey	Athletics
Watching	1240	470	510	430	250
Participating	620	320	320	250	105

(i) Draw a double bar graph choosing an appropriate scale. What do you infer from the bar graph?

(ii) Which sport is most popular?

(iii) Which is more preferred, watching or participating in sports?

6. Take the data giving the minimum and the maximum temperature of various cities given in the beginning of this Chapter (Table 3.1). Plot a double bar graph using the data and answer the following:

(i) Which city has the largest difference in the minimum and maximum temperature on the given date?

(ii) Which is the hottest city and which is the coldest city?

(iii) Name two cities where maximum temperature of one was less than the minimum temperature of the other.

(iv) Name the city which has the least difference between its minimum and the maximum temperature.

TRY THESE

Think of some situations, atleast 3 examples of each, that are certain to happen, some that are impossible and some that may or may not happen i.e., situations that have some chance of happening.

3.9 Chance and Probability

These words often come up in our daily life. We often say, "there is no chance of it raining today" and also say things like "it is quite probable that India will win the World Cup." Let us try and understand these terms a bit more. Consider the statements;

(i) The Sun coming up from the West (ii) An ant growing to 3 m height.

(iii) If you take a cube of larger volume its side will also be larger.

(iv) If you take a circle with larger area then it's radius will also be larger.

(v) India winning the next test series.

If we look at the statements given above you would say that the Sun coming up from the West is impossible, an ant growing to 3 m is also not possible. On the other hand if the circle is of a larger area it is certain that it will have a larger radius. You can say the same about the larger volume of the cube and the larger side. On the other hand India can win the next test series or lose it. Both are possible.

3.9.1 Chance

If you toss a coin, can you always correctly predict what you will get? Try tossing a coin and predicting the outcome each time. Write your observations in the following table:

Toss Number	Prediction	Outcome

Do this 10 times. Look at the observed outcomes. Can you see a pattern in them? What do you get after each head? Is it that you get head all the time? Repeat the observation for another 10 tosses and write the observations in the table.

You will find that the observations show no clear pattern. In the table below we give you observations generated in 25 tosses by Sushila and Salma. Here H represents Head and T represents Tail.

Numbers	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Outcome	H	T	T	H	T	T	T	H	T	T	H	H	H	H	H
Numbers	16	17	18	19	20	21	22	23	24	25					
Outcome	T	T	H	T	T	T	T	T	T	T					

What does this data tell you? Can you find a predictable pattern for head and tail? Clearly there is no fixed pattern of occurrence of head and tail. When you throw the coin each time the outcome of every throw can be either head or tail. It is a matter of chance that in one particular throw you get either of these.

In the above data, count the number of heads and the number of tails. Throw the coin some more times and keep recording what you obtain. Find out the total number of times you get a head and the total number of times you get a tail.

You also might have played with a die. The die has six faces. When you throw a die, can you predict the number that will be obtained? While playing ludo or snake and ladders you may have often wished that in a throw you get a particular outcome.

Does the die always fall according to your wishes? Take a die and throw it 150 times and fill the data in the following table:

Number on Die	Tally Marks	Number of Times it Occured
1		
2		

Make a tally mark each time you get the outcome, against the appropriate number. For example in the first throw you get 5. Put a tally in front of 5. The next throw gives you 1. Make a tally for 1. Keep on putting tally marks for the appropriate number. Repeat this exercise for 150 throws and find out the number of each outcome for 150 throws.

Make bar graph using the above data showing the number of times 1, 2, 3, 4, 5, 6 have occured in the data.

Try These

(Do in a group)

1. Toss a coin 100 times and record the data. Find the number of times heads and tails occur in it.
2. Aftaab threw a die 250 times and got the following table. Draw a bar graph for this data.

Number on the Die	Tally Marks
1	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸𝍷𝍷𝍷
2	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸
3	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸𝍷𝍷
4	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸𝍷𝍷
5	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸𝍷𝍷𝍷
6	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸

3. Throw a die 100 times and record the data. Find the number of times 1, 2, 3, 4, 5, 6 occur.

What is probability?

We know that when a coin is thrown, it has two possible outcomes, Head or Tail and for a die we have 6 possible outcomes. We also know from experience that for a coin, Head or Tail is equally likely to be obtained. We say that the probability of getting Head or Tail is equal and is $\frac{1}{2}$ for each.

For a die, possibility of getting either of 1, 2, 3, 4, 5 or 6 is equal. That is for a die there are 6 equally likely possible outcomes. We say each of 1, 2, 3, 4, 5, 6 has one-sixth ($\frac{1}{6}$) probability. We will learn about this in the later classes. But from what we

TRY THESE

Construct or think of five situations where outcomes do not have equal chances.

have done, it may perhaps be obvious that events that have many possibilities can have probability between 0 and 1. Those which have no chance of happening have probability 0 and those that are bound to happen have probability 1.

Given any situation we need to understand the different possible outcomes and study the possible chances for each outcome. It may be possible that the outcomes may not have equal chance of occuring unlike the cases of the coin and die. For example, if a container has 15 red balls and 9 white balls and if a ball is pulled out without seeing, the chances of getting a red ball are much more. Can you see why? How many times are the chances of getting a red ball than getting a white ball, probabilities for both being between 0 and 1.

EXERCISE 3.4

1. Tell whether the following is certain to happen, impossible, can happen but not certain.
   (i) You are older today than yesterday. (ii) A tossed coin will land heads up.
   (iii) A die when tossed shall land up with 8 on top.
   (iv) The next traffic light seen will be green. (v) Tomorrow will be a cloudy day.
2. There are 6 marbles in a box with numbers from 1 to 6 marked on each of them.
   (i) What is the probability of drawing a marble with number 2?
   (ii) What is the probability of drawing a marble with number 5?
3. A coin is flipped to decide which team starts the game. What is the probability that your team will start?

WHAT HAVE WE DISCUSSED?

1. The collection, recording and presentation of data help us organise our experiences and draw inferences from them.
2. Before collecting data we need to know what we would use it for.
3. The data that is collected needs to be organised in a proper table, so that it becomes easy to understand and interpret.
4. Average is a number that represents or shows the central tendency of a group of observations or data.
5. Arithmetic mean is one of the representative values of data.
6. Mode is another form of central tendency or representative value. The mode of a set of observations is the observation that occurs most often.
7. Median is also a form of representative value. It refers to the value which lies in the middle of the data with half of the observations above it and the other half below it.
8. A bar graph is a representation of numbers using bars of uniform widths.
9. Double bar graphs help to compare two collections of data at a glance.
10. There are situations in our life, that are certain to happen, some that are impossible and some that may or may not happen. The situation that may or may not happen has a chance of happening.

Chapter 4

Simple Equations

0756CH04

4.1 A Mind-reading Game!

The teacher has said that she would be starting a new chapter in mathematics and it is going to be simple equations. Appu, Sarita and Ameena have revised what they learnt in algebra chapter in Class VI. Have you? Appu, Sarita and Ameena are excited because they have constructed a game which they call mind reader and they want to present it to the whole class.

The teacher appreciates their enthusiasm and invites them to present their game. Ameena begins; she asks Sara to think of a number, multiply it by 4 and add 5 to the product. Then, she asks Sara to tell the result. She says it is 65. Ameena instantly declares that the number Sara had thought of is 15. Sara nods. The whole class including Sara is surprised.

It is Appu's turn now. He asks Balu to think of a number, multiply it by 10 and subtract 20 from the product. He then asks Balu what his result is? Balu says it is 50. Appu immediately tells the number thought by Balu. It is 7, Balu confirms it.

Everybody wants to know how the 'mind reader' presented by Appu, Sarita and Ameena works. Can you see how it works? After studying this chapter and chapter 12, you will very well know how the game works.

4.2 Setting up of an Equation

Let us take Ameena's example. Ameena asks Sara to think of a number. Ameena does not know the number. For her, it could be anything 1, 2, 3, . . ., 11, . . . , 100, ... Let us denote this unknown number by a letter, say x. You may use y or t or some other letter in place of x. It does not matter which letter we use to denote the unknown number Sara has thought of. When Sara multiplies the number by 4, she gets $4x$. She then adds 5 to the product, which gives $4x + 5$. The value of $(4x + 5)$ depends on the value of x. Thus if $x = 1$, $4x + 5 = 4 \times 1 + 5 = 9$. This means that if Sara had 1 in her mind, her result would have been 9. Similarly, if she thought of 5, then for $x = 5$, $4x + 5 = 4 \times 5 + 5 = 25$; Thus if Sara had chosen 5, the result would have been 25.

To find the number thought by Sara let us work backward from her answer 65. We have to find x such that

$$4x + 5 = 65 \qquad (4.1)$$

Solution to the equation will give us the number which Sara held in her mind.

Let us similarly look at Appu's example. Let us call the number Balu chose as y. Appu asks Balu to multiply the number by 10 and subtract 20 from the product. That is, from y, Balu first gets $10y$ and from there $(10y – 20)$. The result is known to be 50.

Therefore, $$10y – 20 = 50 \qquad (4.2)$$

The solution of this equation will give us the number Balu had thought of.

4.3 Review of what We Know

Note, (4.1) and (4.2) are equations. Let us recall what we learnt about equations in Class VI. *An equation is a condition on a variable.* In equation (4.1), the variable is x; in equation (4.2), the variable is y.

The word *variable* means something that can vary, i.e. change. A **variable** *takes on different numerical values; its value is not fixed. Variables are denoted usually by letters of the alphabets, such as x, y, z, l, m, n, p, etc. From variables, we form expressions*. The expressions are formed by performing operations like addition, subtraction, multiplication and division on the variables. From x, we formed the expression $(4x + 5)$. For this, first we multiplied x by 4 and then added 5 to the product. Similarly, from y, we formed the expression $(10y – 20)$. For this, we multiplied y by 10 and then subtracted 20 from the product. All these are examples of expressions.

The value of an expression thus formed depends upon the chosen value of the variable. As we have already seen, when $x = 1$, $4x + 5 = 9$; when $x = 5$, $4x + 5 = 25$. Similarly,

when $\quad x = 15,\; 4x + 5 = 4 \times 15 + 5 = 65;$

when $\quad x = 0,\; 4x + 5 = 4 \times 0 + 5 = 5;$ and so on.

Equation (4.1) is a condition on the variable x. It states that the value of the expression $(4x + 5)$ is 65. The condition is satisfied when $x = 15$. It is the solution to the equation $4x + 5 = 65$. When $x = 5$, $4x + 5 = 25$ and not 65. Thus $x = 5$ is not a solution to the equation. Similarly, $x = 0$ is not a solution to the equation. No value of x other than 15 satisfies the condition $4x + 5 = 65$.

Try These

The value of the expression $(10y – 20)$ depends on the value of y. Verify this by giving five different values to y and finding for each y the value of $(10y – 20)$. From the different values of $(10y – 20)$ you obtain, do you see a solution to $10y – 20 = 50$? If there is no solution, try giving more values to y and find whether the condition $10y – 20 = 50$ is met.

4.4 What Equation is?

In an equation there is always an **equality** sign. The equality sign shows that the value of the expression to the left of the sign (the left hand side or LHS) is equal to the value of the expression to the right of the sign (the right hand side or RHS). In equation (4.1), the LHS is $(4x + 5)$ and the RHS is 65. In equation (4.2), the LHS is $(10y - 20)$ and the RHS is 50.

> If there is some sign other than the equality sign between the LHS and the RHS, it is not an equation. Thus, $4x + 5 > 65$ is not an equation.
>
> It says that, the value of $(4x + 5)$ is greater than 65.
>
> Similarly, $4x + 5 < 65$ is not an equation. It says that the value of $(4x + 5)$ is smaller than 65.

In equations, we often find that the RHS is just a number. In Equation (4.1), it is 65 and in equation (4.2), it is 50. But this need not be always so. The RHS of an equation may be an expression containing the variable. For example, the equation

$$4x + 5 = 6x - 25$$

has the expression $(4x + 5)$ on the left and $(6x - 25)$ on the right of the equality sign.

In short, an equation is a condition on a variable. The condition is that two expressions should have equal value. Note that at least one of the two expressions must contain the variable.

We also note a simple and useful property of equations. The equation $4x + 5 = 65$ is the same as $65 = 4x + 5$. Similarly, the equation $6x - 25 = 4x + 5$ is the same as $4x + 5 = 6x - 25$. *An equation remains the same, when the expressions on the left and on the right are interchanged.* This property is often useful in solving equations.

Example 1 Write the following statements in the form of equations:

(i) The sum of three times x and 11 is 32.

(ii) If you subtract 5 from 6 times a number, you get 7.

(iii) One fourth of m is 3 more than 7.

(iv) One third of a number plus 5 is 8.

Solution

(i) Three times x is $3x$.

Sum of $3x$ and 11 is $3x + 11$. The sum is 32.

The equation is $3x + 11 = 32$.

(ii) Let us say the number is z; z multiplied by 6 is $6z$.

Subtracting 5 from $6z$, one gets $6z - 5$. The result is 7.

The equation is $6z - 5 = 7$

(iii) One fourth of m is $\frac{m}{4}$.

It is greater than 7 by 3. This means the difference ($\frac{m}{4} - 7$) is 3.

The equation is $\frac{m}{4} - 7 = 3$.

(iv) Take the number to be n. One third of n is $\frac{n}{3}$.

This one-third plus 5 is $\frac{n}{3} + 5$. It is 8.

The equation is $\frac{n}{3} + 5 = 8$.

EXAMPLE 2 Convert the following equations in statement form:

(i) $x - 5 = 9$ (ii) $5p = 20$ (iii) $3n + 7 = 1$ (iv) $\frac{m}{5} - 2 = 6$

SOLUTION (i) Taking away 5 from x gives 9.

(ii) Five times a number p is 20.

(iii) Add 7 to three times n to get 1.

(iv) You get 6, when you subtract 2 from one-fifth of a number m.

What is important to note is that for a given equation, **not just one, but many** statement forms can be given. For example, for Equation (i) above, you can say:

Subtract 5 from x, you get 9.

or The number x is 5 more than 9.

or The number x is greater by 5 than 9.

or The difference between x and 5 is 9, and so on.

TRY THESE

Write atleast one other form for each equation (ii), (iii) and (iv).

EXAMPLE 3 Consider the following situation:

Raju's father's age is 5 years more than three times Raju's age. Raju's father is 44 years old. Set up an equation to find Raju's age.

SOLUTION We do not know Raju's age. Let us take it to be y years. Three times Raju's age is $3y$ years. Raju's father's age is 5 years more than $3y$; that is, Raju's father is $(3y + 5)$ years old. It is also given that Raju's father is 44 years old.

Therefore, $$3y + 5 = 44 \qquad (4.3)$$

This is an equation in y. It will give Raju's age when solved.

EXAMPLE 4 A shopkeeper sells mangoes in two types of boxes, one small and one large. A large box contains as many as 8 small boxes plus 4 loose mangoes. Set up an equation which gives the number of mangoes in each small box. The number of mangoes in a large box is given to be 100.

SOLUTION Let a small box contain m mangoes. A large box contains 4 more than 8 times m, that is, 8m + 4 mangoes. But this is given to be 100. Thus

$$8m + 4 = 100 \qquad (4.4)$$

You can get the number of mangoes in a small box by solving this equation.

EXERCISE 4.1

1. Complete the last column of the table.

S. No.	Equation	Value	Say, whether the Equation is Satisfied. (Yes/ No)
(i)	$x + 3 = 0$	$x = 3$	
(ii)	$x + 3 = 0$	$x = 0$	
(iii)	$x + 3 = 0$	$x = -3$	
(iv)	$x - 7 = 1$	$x = 7$	
(v)	$x - 7 = 1$	$x = 8$	
(vi)	$5x = 25$	$x = 0$	
(vii)	$5x = 25$	$x = 5$	
(viii)	$5x = 25$	$x = -5$	
(ix)	$\frac{m}{3} = 2$	$m = -6$	
(x)	$\frac{m}{3} = 2$	$m = 0$	
(xi)	$\frac{m}{3} = 2$	$m = 6$	

2. Check whether the value given in the brackets is a solution to the given equation or not:

(a) $n + 5 = 19$ $(n = 1)$ (b) $7n + 5 = 19$ $(n = -2)$ (c) $7n + 5 = 19$ $(n = 2)$

(d) $4p - 3 = 13$ $(p = 1)$ (e) $4p - 3 = 13$ $(p = -4)$ (f) $4p - 3 = 13$ $(p = 0)$

3. Solve the following equations by trial and error method:

(i) $5p + 2 = 17$ (ii) $3m - 14 = 4$

4. Write equations for the following statements:

(i) The sum of numbers x and 4 is 9. (ii) 2 subtracted from y is 8.

(iii) Ten times a is 70. (iv) The number b divided by 5 gives 6.

(v) Three-fourth of t is 15. (vi) Seven times m plus 7 gets you 77.

(vii) One-fourth of a number x minus 4 gives 4.

(viii) If you take away 6 from 6 times y, you get 60.

(ix) If you add 3 to one-third of z, you get 30.

5. Write the following equations in statement forms:

(i) $p + 4 = 15$ (ii) $m - 7 = 3$ (iii) $2m = 7$ (iv) $\frac{m}{5} = 3$

(v) $\frac{3m}{5} = 6$ (vi) $3p + 4 = 25$ (vii) $4p - 2 = 18$ (viii) $\frac{p}{2} + 2 = 8$

6. Set up an equation in the following cases:

(i) Irfan says that he has 7 marbles more than five times the marbles Parmit has. Irfan has 37 marbles. (Take m to be the number of Parmit's marbles.)

(ii) Laxmi's father is 49 years old. He is 4 years older than three times Laxmi's age. (Take Laxmi's age to be y years.)

(iii) The teacher tells the class that the highest marks obtained by a student in her class is twice the lowest marks plus 7. The highest score is 87. (Take the lowest score to be l.)

(iv) In an isosceles triangle, the vertex angle is twice either base angle. (Let the base angle be b in degrees. Remember that the sum of angles of a triangle is 180 degrees).

4.4.1 Solving an Equation

Consider an equality $\quad 8 - 3 = 4 + 1 \quad$ (4.5)

The equality (4.5) holds, since both its sides are equal (each is equal to 5).

- Let us now add 2 to both sides; as a result

  LHS = $8 - 3 + 2 = 5 + 2 = 7$ RHS = $4 + 1 + 2 = 5 + 2 = 7$.

  Again the equality holds (i.e., its LHS and RHS are equal).

  Thus *if we add the same number to both sides of an equality, it still holds.*

- Let us now subtract 2 from both the sides; as a result,

  LHS = $8 - 3 - 2 = 5 - 2 = 3$ RHS = $4 + 1 - 2 = 5 - 2 = 3$.

  Again, the equality holds.

  Thus *if we subtract the same number from both sides of an equality, it still holds.*

- Similarly, *if we multiply or divide both sides of the equality by the same non-zero number, it still holds.*

  For example, let us multiply both the sides of the equality by 3, we get

  LHS = $3 \times (8 - 3) = 3 \times 5 = 15$, RHS = $3 \times (4 + 1) = 3 \times 5 = 15$.

  The equality holds.

  Let us now divide both sides of the equality by 2.

  $$\text{LHS} = (8 - 3) \div 2 = 5 \div 2 = \frac{5}{2}$$

  $$\text{RHS} = (4+1) \div 2 = 5 \div 2 = \frac{5}{2} = \text{LHS}$$

  Again, the equality holds.

If we take any other equality, we shall find the same conclusions.

Suppose, we do not observe these rules. Specificially, suppose we add different numbers, to the two sides of an equality. We shall find in this case that the equality does not hold (i.e., its both sides are not equal). For example, let us take again equality (4.5),

$$8 - 3 = 4 + 1$$

add 2 to the LHS and 3 to the RHS. The new LHS is $8 - 3 + 2 = 5 + 2 = 7$ and the new RHS is $4 + 1 + 3 = 5 + 3 = 8$. The equality does not hold, because the new LHS and RHS are not equal.

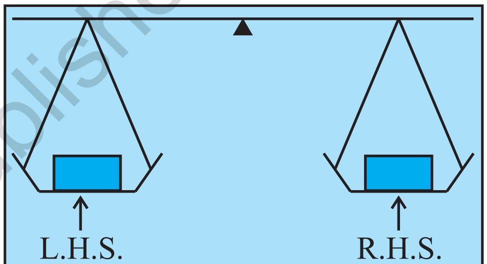
Thus if we fail to do the same mathematical operation with same number on both sides of an equality, the equality may not hold.

The equality that involves variables is an equation.

These conclusions are also valid for equations, as in each equation variable represents a number only.

Often an equation is said to be like a weighing balance. Doing a mathematical operation on an equation is like adding weights to or removing weights from the pans of a weighing balance.

An equation is like a weighing balance with equal weights on both its pans, in which case the arm of the balance is exactly horizontal. If we add the same weights to both the pans, the arm remains horizontal. Similarly, if we remove the same weights from both the pans, the arm remains horizontal. On the other hand if we add different weights to the pans or remove different weights from them, the balance is tilted; that is, the arm of the balance does not remain horizontal.



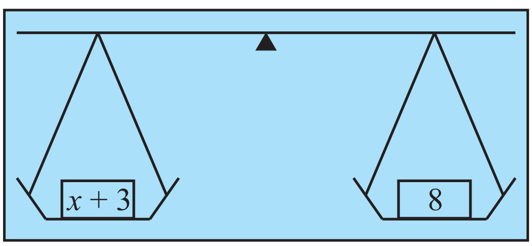
A balanced equation is like a weighing balance with equal weights in the two pans.

We use this principle for solving an equation. Here, ofcourse, the balance is imaginary and numbers can be used as weights that can be physically balanced against each other. This is the real purpose in presenting the principle. Let us take some examples.

- Consider the equation: $x + 3 = 8$ (4.6)

  We shall subtract 3 from both sides of this equation.

  The new LHS is $x + 3 - 3 = x$ and the new RHS is $8 - 3 = 5$



Why should we subtract 3, and not some other number? Try adding 3. Will it help? Why not?

It is because subtracting 3 reduces the LHS to x.

Since this does not disturb the balance, we have

$$\text{New LHS} = \text{New RHS} \quad \text{or} \quad x = 5$$

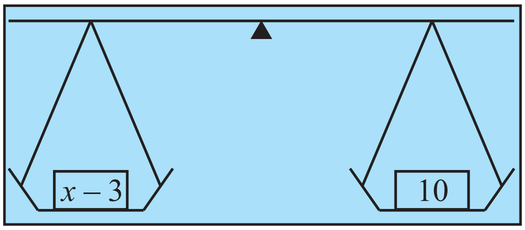
which is exactly what we want, the solution of the equation (4.6).

To confirm whether we are right, we shall put $x = 5$ in the original equation. We get LHS $= x + 3 = 5 + 3 = 8$, which is equal to the RHS as required.

By doing the right mathematical operation (i.e., subtracting 3) on both the sides of the equation, we arrived at the solution of the equation.

- Let us look at another equation $\quad x - 3 = 10 \quad$ (4.7)

  What should we do here? We should add 3 to both the sides, By doing so, we shall retain the balance and also the LHS will reduce to just x.



New LHS $= x - 3 + 3 = x$, New RHS $= 10 + 3 = 13$

Therefore, $x = 13$, which is the required solution.

By putting $x = 13$ in the original equation (4.7) we confirm that the solution is correct:

LHS of original equation $= x - 3 = 13 - 3 = 10$

This is equal to the RHS as required.

- Similarly, let us look at the equations

$$5y = 35 \tag{4.8}$$

$$\frac{m}{2} = 5 \tag{4.9}$$

In the first case, we shall divide both the sides by 5. This will give us just y on LHS

$$\text{New LHS} = \frac{5y}{5} = \frac{5 \times y}{5} = y\,, \qquad \text{New RHS} = \frac{35}{5} = \frac{5 \times 7}{5} = 7$$

Therefore, $\quad y = 7$

This is the required solution. We can substitute $y = 7$ in Eq. (4.8) and check that it is satisfied.

In the second case, we shall multiply both sides by 2. This will give us just m on the LHS

The new LHS $= \frac{m}{2} \times 2 = m$. The new RHS $= 5 \times 2 = 10$.

Hence, $m = 10$ (It is the required solution. You can check whether the solution is correct).

One can see that in the above examples, the operation we need to perform depends on the equation. Our attempt should be to get the variable in the equation separated. Sometimes, for doing so we may have to carry out more than one mathematical operation. Let us solve some more equations with this in mind.

EXAMPLE 5 Solve: (a) $3n + 7 = 25$ (4.10)

(b) $2p - 1 = 23$ (4.11)

SOLUTION

(a) We go stepwise to separate the variable n on the LHS of the equation. The LHS is $3n + 7$. We shall first subtract 7 from it so that we get $3n$. From this, in the next step we shall divide by 3 to get n. Remember we must do the same operation on both sides of the equation. Therefore, subtracting 7 from both sides,

$$3n + 7 - 7 = 25 - 7 \qquad \text{(Step 1)}$$

or $\quad 3n = 18$

Now divide both sides by 3,

$$\frac{3n}{3} = \frac{18}{3} \qquad \text{(Step 2)}$$

or $\quad n = 6$, which is the solution.

(b) What should we do here? First we shall add 1 to both the sides:

$$2p - 1 + 1 = 23 + 1 \qquad \text{(Step 1)}$$

or $\quad 2p = 24$

Now divide both sides by 2, we get $\frac{2p}{2} = \frac{24}{2}$ (Step 2)

or $\quad p = 12$, which is the solution.

One good practice you should develop is to check the solution you have obtained. Although we have not done this for (a) above, let us do it for this example.

Let us put the solution $p = 12$ back into the equation.

$$\text{LHS} = 2p - 1 = 2 \times 12 - 1 = 24 - 1$$

$$= 23 = \text{RHS}$$

The solution is thus checked for its correctness.

Why do you not check the solution of (a) also?

We are now in a position to go back to the mind-reading game presented by Appu, Sarita, and Ameena and understand how they got their answers. For this purpose, let us look at the equations (4.1) and (4.2) which correspond respectively to Ameena's and Appu's examples.

- First consider the equation $4x + 5 = 65$. (4.1)

  Subtracting 5 from both sides, $4x + 5 - 5 = 65 - 5$.

  i.e. $\quad 4x = 60$

  Divide both sides by 4; this will separate x. We get $\frac{4x}{4} = \frac{60}{4}$

  or $\quad x = 15$, which is the solution. (Check, if it is correct.)

- Now consider, $10y - 20 = 50$ (4.2)

  Adding 20 to both sides, we get $10y - 20 + 20 = 50 + 20$ or $10y = 70$

  Dividing both sides by 10, we get $\frac{10y}{10} = \frac{70}{10}$

  or $\quad y = 7$, which is the solution. (Check if it is correct.)

You will realise that exactly these were the answers given by Appu, Sarita and Ameena. They had learnt to set up equations and solve them. That is why they could construct their mind reader game and impress the whole class. We shall come back to this in Section 4.7.

Exercise 4.2

1. Give first the step you will use to separate the variable and then solve the equation:

(a) $x - 1 = 0$ (b) $x + 1 = 0$ (c) $x - 1 = 5$ (d) $x + 6 = 2$

(e) $y - 4 = -7$ (f) $y - 4 = 4$ (g) $y + 4 = 4$ (h) $y + 4 = -4$

2. Give first the step you will use to separate the variable and then solve the equation:

(a) $3l = 42$ (b) $\frac{b}{2} = 6$ (c) $\frac{p}{7} = 4$ (d) $4x = 25$

(e) $8y = 36$ (f) $\frac{z}{3} = \frac{5}{4}$ (g) $\frac{a}{5} = \frac{7}{15}$ (h) $20t = -10$

3. Give the steps you will use to separate the variable and then solve the equation:

(a) $3n - 2 = 46$ (b) $5m + 7 = 17$ (c) $\frac{20p}{3} = 40$ (d) $\frac{3p}{10} = 6$

4. Solve the following equations:

(a) $10p = 100$ (b) $10p + 10 = 100$ (c) $\frac{p}{4} = 5$ (d) $\frac{-p}{3} = 5$

(e) $\frac{3p}{4} = 6$ (f) $3s = -9$ (g) $3s + 12 = 0$ (h) $3s = 0$

(i) $2q = 6$ (j) $2q - 6 = 0$ (k) $2q + 6 = 0$ (l) $2q + 6 = 12$

4.5 More Equations

Let us practise solving some more equations. While solving these equations, we shall learn about transposing a number, i.e., moving it from one side to the other. We can transpose a number instead of adding or subtracting it from both sides of the equation.

Example 6 Solve: $12p - 5 = 25$ (4.12)

Solution

- Adding 5 on both sides of the equation,

  $12p - 5 + 5 = 25 + 5$ or $12p = 30$

- Dividing both sides by 12,

  $\frac{12p}{12} = \frac{30}{12}$ or $p = \frac{5}{2}$

Note, adding 5 to both sides is the same as changing side of (– 5).

$12p - 5 = 25$

$12p = 25 + 5$

Changing side is called ***transposing****. While transposing a number, we change its sign.*

Check Putting $p = \frac{5}{2}$ in the LHS of equation 4.12,

$$\text{LHS} = 12 \times \frac{5}{2} - 5 = 6 \times 5 - 5$$

$$= 30 - 5 = 25 = \text{RHS}$$

As we have seen, while solving equations one commonly used operation is adding or subtracting the same number on both sides of the equation. *Transposing a number (i.e., changing the side of the number) is the same as adding or subtracting the number from both sides.* In doing so, the sign of the number has to be changed. What applies to numbers also applies to expressions. Let us take two more examples of transposing.

Adding or Subtracting on both sides	**Transposing**
(i) $3p - 10 = 5$	(i) $3p - 10 = 5$
Add 10 to both sides	Transpose (–10) from LHS to RHS
$3p - 10 + 10 = 5 + 10$	(On transposing – 10 becomes + 10).
or $3p = 15$	$3p = 5 + 10$ or $3p = 15$
(ii) $5x + 12 = 27$	(ii) $5x + 12 = 27$
Subtract 12 from both sides	Transposing + 12
	(On transposing + 12 becomes – 12)
$5x + 12 - 12 = 27 - 12$	$5x = 27 - 12$
or $5x = 15$	or $5x = 15$

We shall now solve two more equations. As you can see they involve brackets, which have to be solved before proceeding.

EXAMPLE 7 Solve

(a) $4(m + 3) = 18$ (b) $-2(x + 3) = 8$

SOLUTION

(a) $4(m + 3) = 18$

Let us divide both the sides by 4. This will remove the brackets in the LHS We get,

$$m+3=\frac{18}{4} \quad \text{or} \quad m+3=\frac{9}{2}$$

or $m=\frac{9}{2}-3$ (transposing 3 to RHS)

or $m=\frac{3}{2}$ (required solution) $\left(\text{as } \frac{9}{2}-3=\frac{9}{2}-\frac{6}{2}=\frac{3}{2}\right)$

Check LHS = $4\left[\frac{3}{2}+3\right]=4\times\frac{3}{2}+4\times 3=2\times 3+4\times 3$ [put $m = \frac{3}{2}$]

$= 6 + 12 = 18 =$ RHS

(b) $-2(x + 3) = 8$

We divide both sides by (– 2), so as to remove the brackets in the LHS, we get,

$x + 3 = -\frac{8}{2}$ or $x + 3 = -4$

i.e., $x = -4 - 3$ (transposing 3 to RHS) or $x = -7$ (required solution)

Check LHS = $-2(-7+3) = -2(-4)$

$= 8 = \text{RHS}$ as required.

4.6 From Solution to Equation

Atul always thinks differently. He looks at successive steps that one takes to solve an equation. He wonders why not follow the reverse path:

Equation	$\longrightarrow$	Solution	(normal path)
Solution	$\longrightarrow$	Equation	(reverse path)

He follows the path given below:

Start with	$\downarrow$	$x = 5$	$\uparrow$	
Multiply both sides by 4,		$4x = 20$		Divide both sides by 4.
Subtract 3 from both sides,	$\downarrow$	$4x - 3 = 17$	$\uparrow$	Add 3 to both sides.

This has resulted in an equation. If we follow the reverse path with each step, as shown on the right, we get the solution of the equation.

Try These

Start with the same step $x = 5$ and make two different equations. Ask two of your classmates to solve the equations. Check whether they get the solution $x = 5$.

Hetal feels interested. She starts with the same first step and builds up another equation.

	$x = 5$
Multiply both sides by 3	$3x = 15$
Add 4 to both sides	$3x + 4 = 19$

Start with $y = 4$ and make two different equations. Ask three of your friends to do the same. Are their equations different from yours?

Is it not nice that not only can you solve an equation, but you can make equations? Further, did you notice that given an equation, you get one solution; but given a solution, you can make many equations?

Now, Sara wants the class to know what she is thinking. She says, "I shall take Hetal's equation and put it into a statement form and that makes a puzzle. For example, think of a number; multiply it by 3 and add 4 to the product. Tell me the sum you get.

If the sum is 19, the equation Hetal got will give us the solution to the puzzle. In fact, we know it is 5, because Hetal started with it."

She turns to Appu, Ameena and Sarita to check whether they made their puzzle this way. All three say, "Yes!"

We now know how to create number puzzles and many other similar problems.

Try These

Try to make two number puzzles, one with the solution 11 and another with 100

EXERCISE 4.3

1. Solve the following equations:

(a) $2y+\frac{5}{2}=\frac{37}{2}$ (b) $5t + 28 = 10$ (c) $\frac{a}{5}+3=2$ (d) $\frac{q}{4}+7=5$

(e) $\frac{5}{2}x=-5$ (f) $\frac{5}{2}x=\frac{25}{4}$ (g) $7m+\frac{19}{2}=13$ (h) $6z + 10 = -2$

(i) $\frac{3l}{2}=\frac{2}{3}$ (j) $\frac{2b}{3}-5=3$

2. Solve the following equations:

(a) $2(x + 4) = 12$ (b) $3(n - 5) = 21$ (c) $3(n - 5) = - 21$

(d) $- 4(2 + x) = 8$ (e) $4(2 - x) = 8$

3. Solve the following equations:

(a) $4 = 5(p - 2)$ (b) $- 4 = 5(p - 2)$ (c) $16 = 4 + 3(t + 2)$

(d) $4 + 5(p - 1) =34$ (e) $0 = 16 + 4(m - 6)$

4. (a) Construct 3 equations starting with $x = 2$

(b) Construct 3 equations starting with $x = - 2$

4.7 APPLICATIONS OF SIMPLE EQUATIONS TO PRACTICAL SITUATIONS

We have already seen examples in which we have taken statements in everyday language and converted them into simple equations. We also have learnt how to solve simple equations. Thus we are ready to solve puzzles/problems from practical situations. The method is first to form equations corresponding to such situations and then to solve those equations to give the solution to the puzzles/problems. We begin with what we have already seen [Example 1 (i) and (iii), Section 4.2].

EXAMPLE 8 The sum of three times a number and 11 is 32. Find the number.

SOLUTION

- If the unknown number is taken to be x, then three times the number is $3x$ and the sum of $3x$ and 11 is 32. That is, $3x + 11 = 32$
- To solve this equation, we transpose 11 to RHS, so that

  $3x = 32 - 11$ or $3x = 21$

  Now, divide both sides by 3

  So $x = \frac{21}{3} = 7$

This equation was obtained earlier in Section 4.2, Example 1.

The required number is 7. (We may check it by taking 3 times 7 and adding 11 to it. It gives 32 as required.)

EXAMPLE 9 Find a number, such that one-fourth of the number is 3 more than 7.

SOLUTION

- Let us take the unknown number to be y; one-fourth of y is $\frac{y}{4}$.

  This number $\left(\frac{y}{4}\right)$ is more than 7 by 3.

  Hence we get the equation for y as $\frac{y}{4} - 7 = 3$

TRY THESE

(i) When you multiply a number by 6 and subtract 5 from the product, you get 7. Can you tell what the number is?

(ii) What is that number one third of which added to 5 gives 8?

- To solve this equation, first transpose 7 to RHS We get,

  $\frac{y}{4} = 3 + 7 = 10.$

  We then multiply both sides of the equation by 4, to get

  $\frac{y}{4} \times 4 = 10 \times 4$ or $y = 40$ (the required number)

  Let us check the equation formed. Putting the value of y in the equation,

  $\text{LHS} = \frac{40}{4} - 7 = 10 - 7 = 3 = \text{RHS}$, as required.

EXAMPLE 10 Raju's father's age is 5 years more than three times Raju's age. Find Raju's age, if his father is 44 years old.

SOLUTION

- As given in Example 3 earlier, the equation that gives Raju's age is

  $$3y + 5 = 44$$

- To solve it, we first transpose 5, to get $3y = 44 - 5 = 39$

  Dividing both sides by 3, we get $y = 13$

  That is, Raju's age is 13 years. (You may check the answer.)

TRY THESE

There are two types of boxes containing mangoes. Each box of the larger type contains 4 more mangoes than the number of mangoes contained in 8 boxes of the smaller type. Each larger box contains 100 mangoes. Find the number of mangoes contained in the smaller box?

EXERCISE 4.4

1. Set up equations and solve them to find the unknown numbers in the following cases:
   (a) Add 4 to eight times a number; you get 60.
   (b) One-fifth of a number minus 4 gives 3.
   (c) If I take three-fourths of a number and add 3 to it, I get 21.
   (d) When I subtracted 11 from twice a number, the result was 15.
   (e) Munna subtracts thrice the number of notebooks he has from 50, he finds the result to be 8.
   (f) Ibenhal thinks of a number. If she adds 19 to it and divides the sum by 5, she will get 8.
   (g) Anwar thinks of a number. If he takes away 7 from $\frac{5}{2}$ of the number, the result is 23.

2. Solve the following:
   (a) The teacher tells the class that the highest marks obtained by a student in her class is twice the lowest marks plus 7. The highest score is 87. What is the lowest score?
   (b) In an isosceles triangle, the base angles are equal. The vertex angle is 40°. What are the base angles of the triangle? (Remember, the sum of three angles of a triangle is 180°).
   (c) Sachin scored twice as many runs as Rahul. Together, their runs fell two short of a double century. How many runs did each one score?

3. Solve the following:
   (i) Irfan says that he has 7 marbles more than five times the marbles Parmit has. Irfan has 37 marbles. How many marbles does Parmit have?
   (ii) Laxmi's father is 49 years old. He is 4 years older than three times Laxmi's age. What is Laxmi's age?
   (iii) People of Sundargram planted trees in the village garden. Some of the trees were fruit trees. The number of non-fruit trees were two more than three times the number of fruit trees. What was the number of fruit trees planted if the number of non-fruit trees planted was 77?

4. Solve the following riddle:

I am a number,
Tell my identity!
Take me seven times over
And add a fifty!
To reach a triple century
You still need forty!

What have We Discussed?

1. An equation is a condition on a variable such that two expressions in the variable should have equal value.
2. The value of the variable for which the equation is satisfied is called the solution of the equation.
3. An equation remains the same if the LHS and the RHS are interchanged.
4. In case of the balanced equation, if we

   (i) add the same number to both the sides, or (ii) subtract the same number from both the sides, or (iii) multiply both sides by the same number, or (iv) divide both sides by the same number, the balance remains undisturbed, i.e., the value of the LHS remains equal to the value of the RHS
5. The above property gives a systematic method of solving an equation. We carry out a series of identical mathematical operations on the two sides of the equation in such a way that on one of the sides we get just the variable. The last step is the solution of the equation.
6. Transposing means moving to the other side. Transposition of a number has the same effect as adding same number to (or subtracting the same number from) both sides of the equation. When you transpose a number from one side of the equation to the other side, you change its sign. For example, transposing +3 from the LHS to the RHS in equation $x + 3 = 8$ gives $x = 8 – 3$ (= 5). We can carry out the transposition of an expression in the same way as the transposition of a number.
7. We have learnt how to construct simple algebraic expressions corresponding to practical situations.
8. We also learnt how, using the technique of doing the same mathematical operation (for example adding the same number) on both sides, we could build an equation starting from its solution. Further, we also learnt that we could relate a given equation to some appropriate practical situation and build a practical word problem/puzzle from the equation.

Chapter 5

Lines and Angles

5.1 Introduction

You already know how to identify different lines, line segments and angles in a given shape. Can you identify the different line segments and angles formed in the following figures? (Fig 5.1)

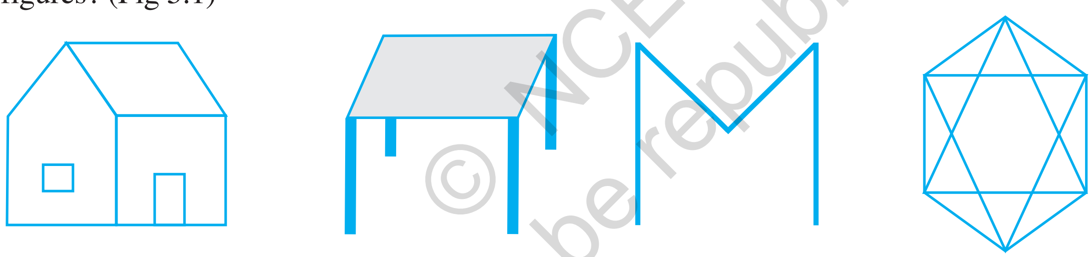


Fig 5.1

Can you also identify whether the angles made are acute or obtuse or right?

Recall that a **line segment** has two end **points**. If we extend the two end points in either direction endlessly, we get a **line**. Thus, we can say that a line has no end points. On the other hand, recall that a ray has one end point (namely its starting point). For example, look at the figures given below:

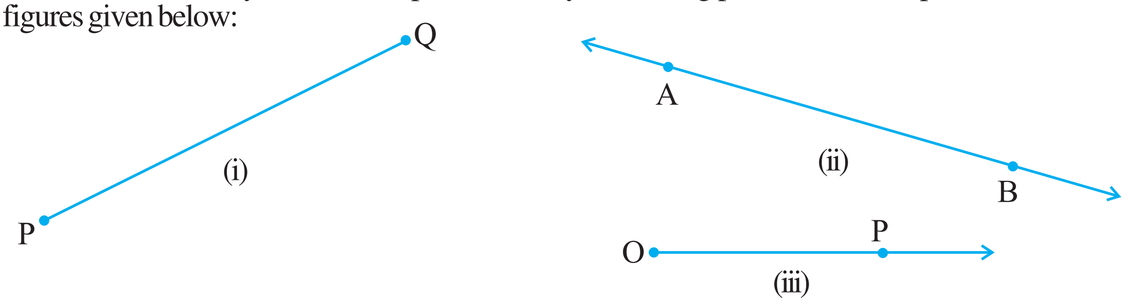


Fig 5.2

Here, Fig 5.2 (i) shows a **line segment,** Fig 5.2 (ii) shows a **line** and Fig 5.2 (iii) is that of a **ray**. A line segment PQ is generally denoted by the symbol $\overline{PQ}$, a line AB is denoted by the symbol $\overleftrightarrow{AB}$ and the ray OP is denoted by $\overrightarrow{OP}$. Give some examples of line segments and rays from your daily life and discuss them with your friends.

Again recall that an **angle** is formed when lines or line segments meet. In Fig 5.1, observe the corners. These corners are formed when two lines or line segments intersect at a point. For example, look at the figures given below:

Fig 5.3

In Fig 5.3 (i) line segments AB and BC intersect at B to form angle ABC, and again line segments BC and AC intersect at C to form angle ACB and so on. Whereas, in Fig 5.3 (ii) lines PQ and RS intersect at O to form four angles POS, SOQ, QOR and ROP. An angle ABC is represented by the symbol ∠ABC. Thus, in Fig 5.3 (i), the three angles formed are ∠ABC, ∠BCA and ∠BAC, and in Fig 5.3 (ii), the four angles formed are ∠POS, ∠SOQ, ∠QOR and ∠POR. You have already studied how to classify the angles as acute, obtuse or right angle.

TRY THESE

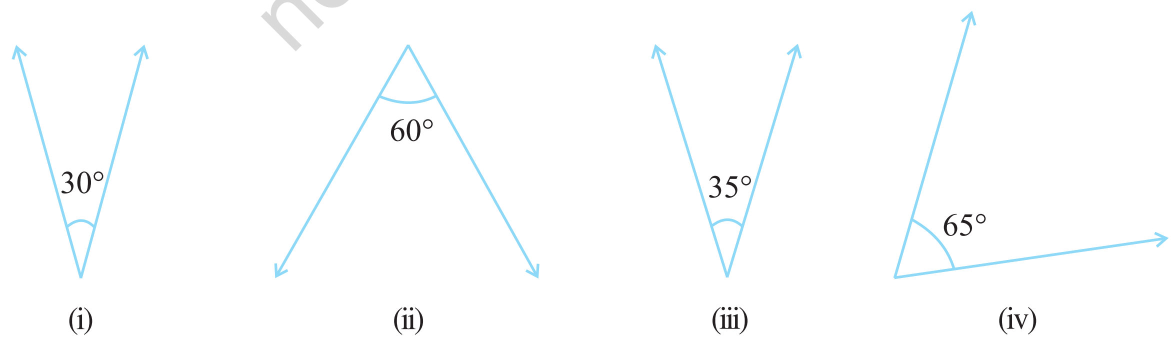
List ten figures around you and identify the acute, obtuse and right angles found in them.

Note: While referring to the measure of an angle ABC, we shall write m∠ABC as simply ∠ABC. The context will make it clear, whether we are referring to the angle or its measure.

5.2 RELATED ANGLES

5.2.1 Complementary Angles

When the sum of the measures of two angles is 90°, the angles are called **complementary angles.**



Are these two angles complementary?
Yes

Are these two angles complementary?
No

Fig 5.4

Whenever two angles are complementary, each angle is said to be the **complement** of the other angle. In the above diagram (Fig 5.4), the '30° angle' is the complement of the '60° angle' and vice versa.

Think, Discuss and Write

1. Can two acute angles be complement to each other?
2. Can two obtuse angles be complement to each other?
3. Can two right angles be complement to each other?

Try These

1. Which pairs of following angles are complementary? (Fig 5.5)

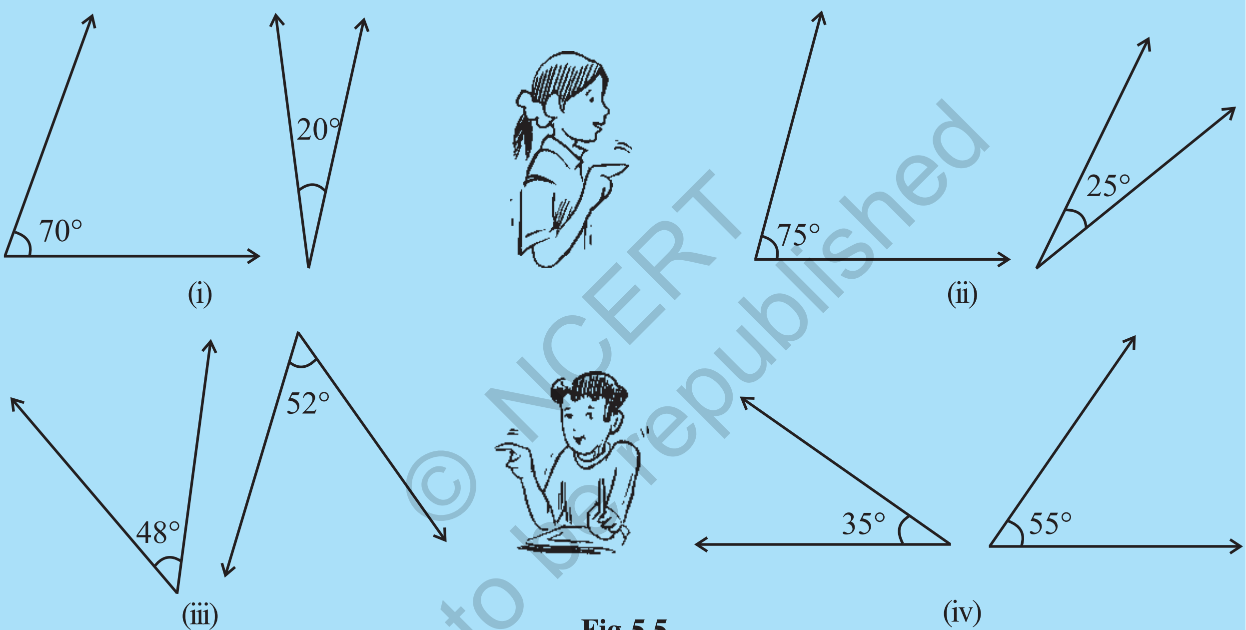
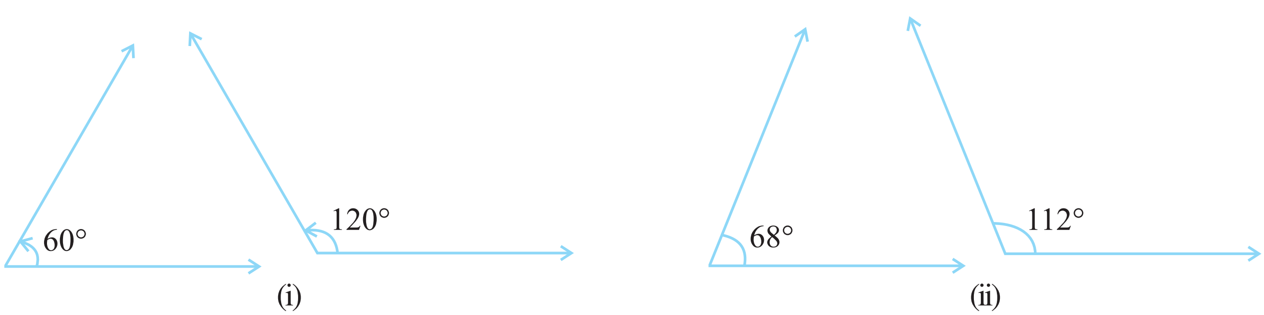


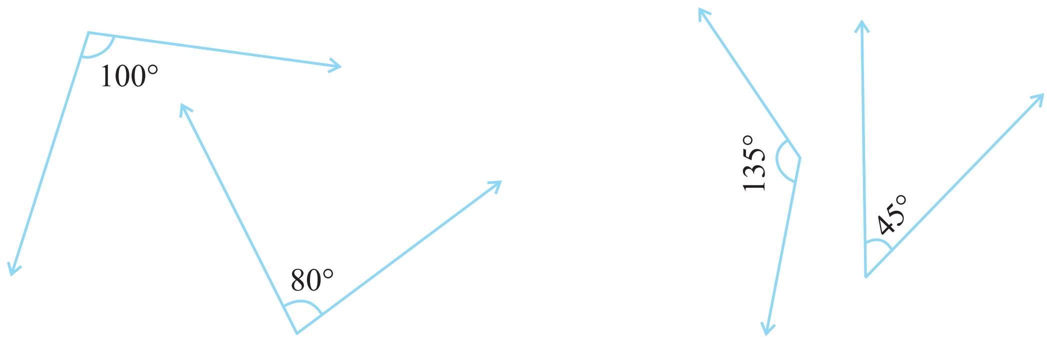
Fig 5.5

2. What is the measure of the complement of each of the following angles?
   (i) 45º (ii) 65º (iii) 41º (iv) 54º
3. The difference in the measures of two complementary angles is 12º. Find the measures of the angles.

5.2.2 Supplementary Angles

Let us now look at the following pairs of angles (Fig 5.6):





(iii) **Fig 5.6** (iv)

Do you notice that the sum of the measures of the angles in each of the above pairs (Fig 5.6) comes out to be 180º? Such pairs of angles are called **supplementary angles**. When two angles are supplementary, each angle is said to be the **supplement** of the other.

Think, Discuss and Write

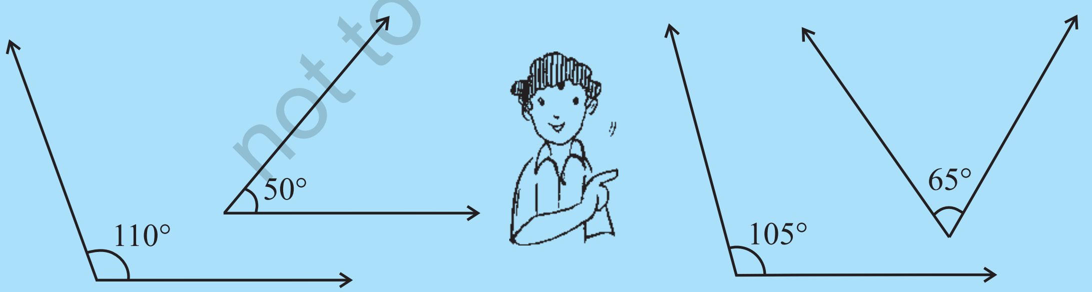
1. Can two obtuse angles be supplementary?

2. Can two acute angles be supplementary?

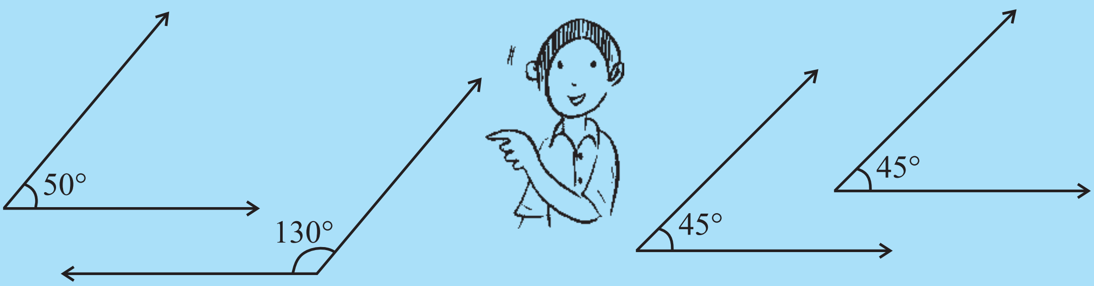
3. Can two right angles be supplementary?

Try These

1. Find the pairs of supplementary angles in Fig 5.7:



(i) (ii)

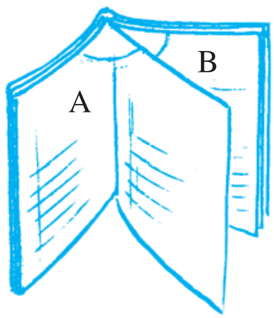


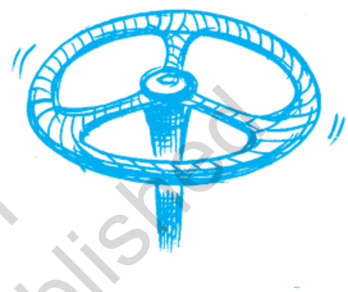
(iii) **Fig 5.7** (iv)

2. What will be the measure of the supplement of each one of the following angles?
   (i) 100º (ii) 90º (iii) 55º (iv) 125º
3. Among two supplementary angles the measure of the larger angle is 44º more than the measure of the smaller. Find their measures.

5.2.3. Adjacent Angles

Look at the following figures:





When you open a book it looks like the above figure. In A and B, we find a pair of angles, placed next to each other.

Look at this steering wheel of a car. At the centre of the wheel you find three angles being formed, lying next to one another.

Fig 5.8

At both the vertices A and B, we find, a pair of angles are placed next to each other.

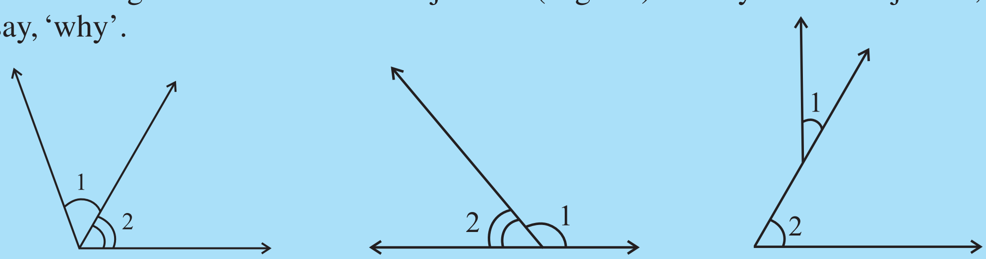
These angles are such that:

(i) they have a common vertex;
(ii) they have a common arm; and
(iii) the non-common arms are on either side of the common arm.

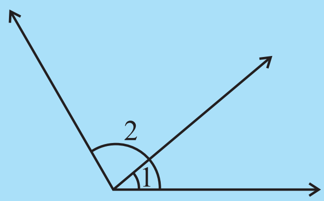
Such pairs of angles are called **adjacent angles.** Adjacent angles have a common vertex and a common arm but no common interior points.

TRY THESE

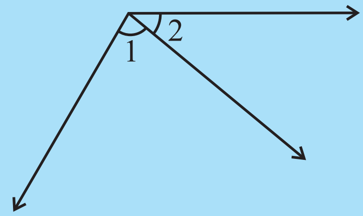
1. Are the angles marked 1 and 2 adjacent? (Fig 5.9). If they are not adjacent, say, 'why'.



(i) (ii) (iii)



(iv)



(v)

Fig 5.9

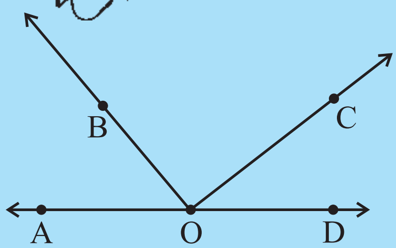


Fig 5.10

2. In the given Fig 5.10, are the following adjacent angles?

(a) $\angle AOB$ and $\angle BOC$

(b) $\angle BOD$ and $\angle BOC$

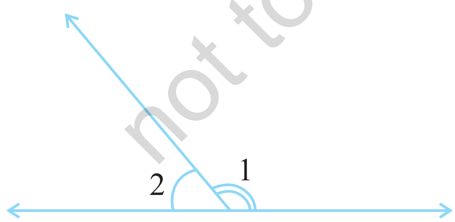
Justify your answer.

Think, Discuss and Write

1. Can two adjacent angles be supplementary?
2. Can two adjacent angles be complementary?
3. Can two obtuse angles be adjacent angles?
4. Can an acute angle be adjacent to an obtuse angle?

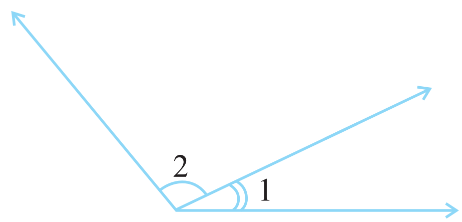
5.2.4 Linear Pair

A linear pair is a pair of adjacent angles whose non-common sides are opposite rays.



Are $\angle 1$, $\angle 2$ a linear pair? Yes

(i)



Are $\angle 1$, $\angle 2$ a linear pair? No! (Why?)

(ii)

Fig 5.11

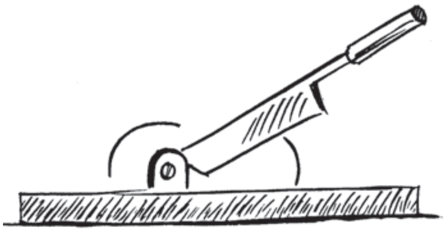
In Fig 5.11 (i) above, observe that the opposite rays (which are the non-common sides of $\angle 1$ and $\angle 2$) form a line. Thus, $\angle 1 + \angle 2$ amounts to 180°.

The angles in a linear pair are supplementary.

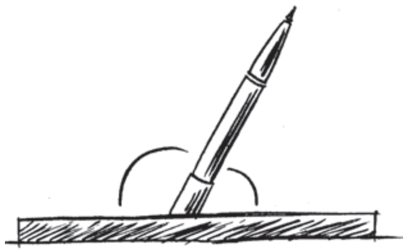
Have you noticed models of a linear pair in your environment?

Note carefully that a pair of supplementary angles form a linear pair when placed adjacent to each other. Do you find examples of linear pair in your daily life?

Observe a vegetable chopping board (Fig 5.12).



A vegetable chopping board
The chopping blade makes a linear pair of angles with the board.



A pen stand
The pen makes a linear pair of angles with the stand.

Fig 5.12

THINK, DISCUSS AND WRITE

1. Can two acute angles form a linear pair?
2. Can two obtuse angles form a linear pair?
3. Can two right angles form a linear pair?

TRY THESE

Check which of the following pairs of angles form a linear pair (Fig 5.13):

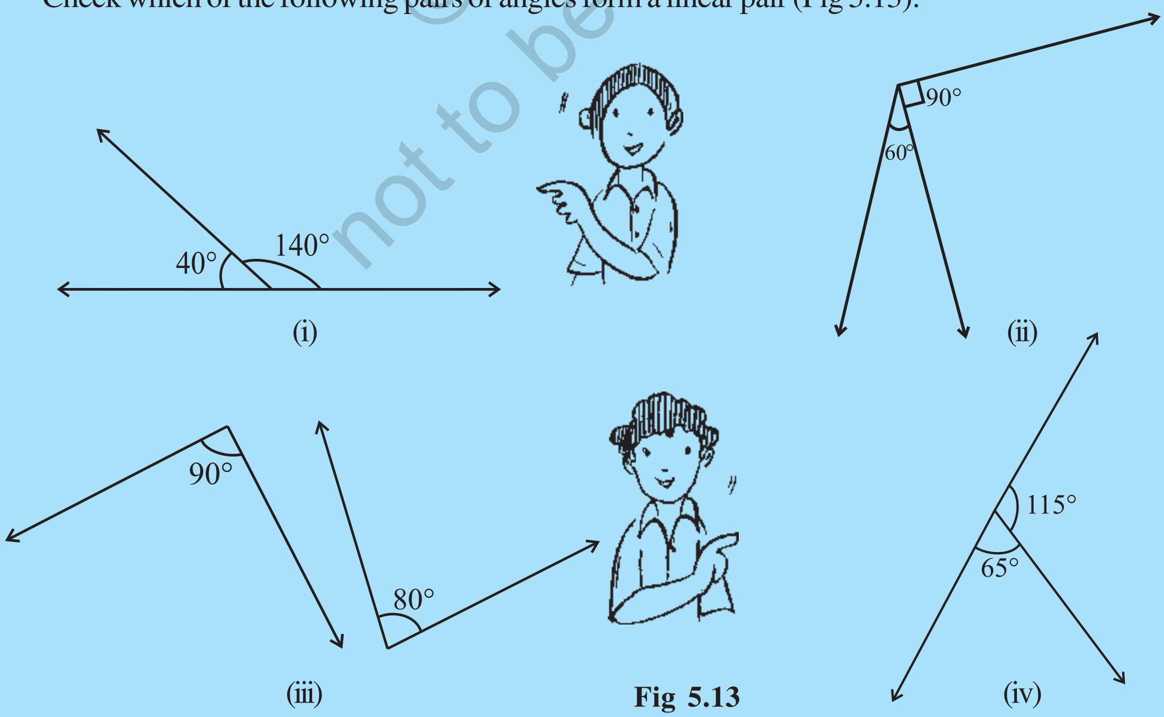


Fig 5.13

5.2.5 Vertically Opposite Angles

Next take two pencils and tie them with the help of a rubber band at the middle as shown (Fig 5.14).

Look at the four angles formed ∠1, ∠2, ∠3 and ∠4.

∠1 is vertically opposite to ∠3.

and ∠2 is vertically opposite to ∠4.

We call ∠1 and ∠3, a pair of vertically opposite angles.

Can you name the other pair of vertically opposite angles?

Does ∠1 appear to be equal to ∠3? Does ∠2 appear to be equal to ∠4?

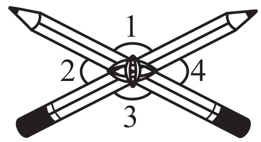


Fig 5.14

Before checking this, let us see some real life examples for vertically opposite angles (Fig 5.15).

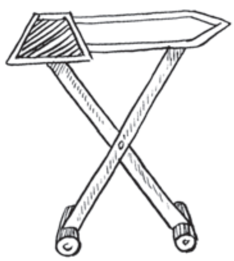
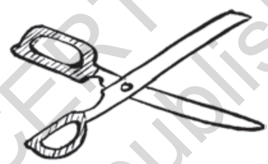




Fig 5.15

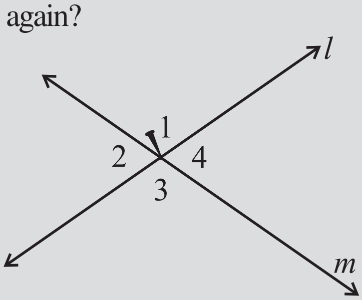
Do This

Draw two lines l and m, intersecting at a point. You can now mark ∠1, ∠2, ∠3 and ∠4 as in the Fig (5.16).

Take a tracecopy of the figure on a transparent sheet.

Place the copy on the original such that ∠1 matches with its copy, ∠2 matches with its copy, ... etc.

Fix a pin at the point of intersection. Rotate the copy by 180°. Do the lines coincide again?



can be rotated to get

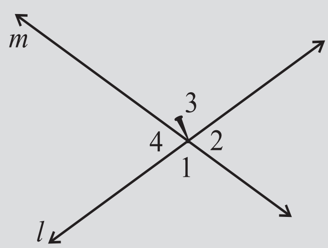


Fig 5.16

You find that ∠1 and ∠3 have interchanged their positions and so have ∠2 and ∠4. This has been done without disturbing the position of the lines.

Thus, ∠1 = ∠3 and ∠2 = ∠4.

We conclude that **when two lines intersect, the vertically opposite angles so formed are equal.**

Let us try to prove this using Geometrical Idea.

Let us consider two lines l and m. (Fig 5.17)

We can arrive at this result through logical reasoning as follows:

Let l and m be two lines, which intersect at O, making angles $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$.

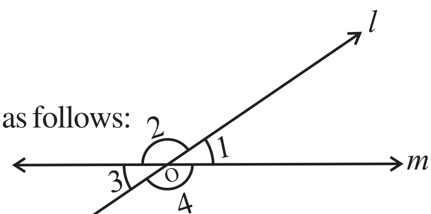


Fig 5.17

We want to prove that $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$

Now, $\angle 1 = 180^\circ - \angle 2$ (Because $\angle 1$, $\angle 2$ form a linear pair, so, $\angle 1 + \angle 2 = 180^\circ$) (i)

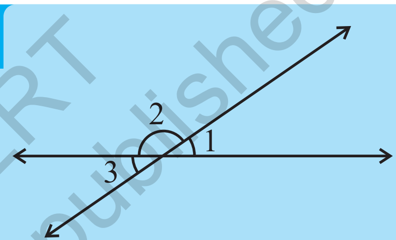
Similarly, $\angle 3 = 180^\circ - \angle 2$ (Since $\angle 2$, $\angle 3$ form a linear pair, so, $\angle 2 + \angle 3 = 180^\circ$) (ii)

Therfore, $\angle 1 = \angle 3$ [By (i) and (ii)]

Similarly, we can prove that $\angle 2 = \angle 4$, (Try it!)

Try These

1. In the given figure, if $\angle 1 = 30^\circ$, find $\angle 2$ and $\angle 3$.
2. Give an example for vertically opposite angles in your surroundings.



Example 1 In Fig (5.18) identify:

(i) Five pairs of adjacent angles. (ii) Three linear pairs.

(iii) Two pairs of vertically opposite angles.

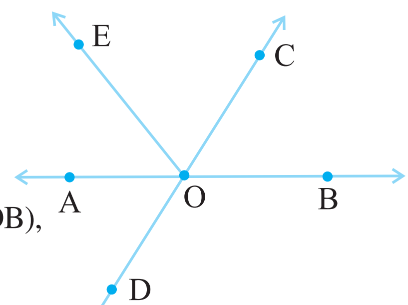


Fig 5.18

Solution

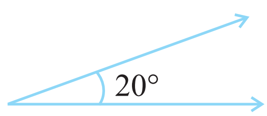
(i) Five pairs of adjacent angles are ($\angle$AOE, $\angle$EOC), ($\angle$EOC, $\angle$COB), ($\angle$AOC, $\angle$COB), ($\angle$COB, $\angle$BOD), ($\angle$EOB, $\angle$BOD)

(ii) Linear pairs are ($\angle$AOE, $\angle$EOB), ($\angle$AOC, $\angle$COB), ($\angle$COB, $\angle$BOD)

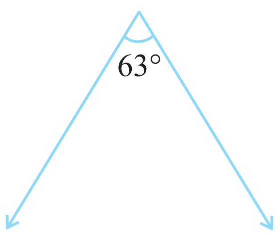
(iii) Vertically opposite angles are: ($\angle$COB, $\angle$AOD), and ($\angle$AOC, $\angle$BOD)

Exercise 5.1

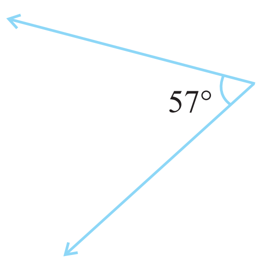
1. Find the complement of each of the following angles:



(i)

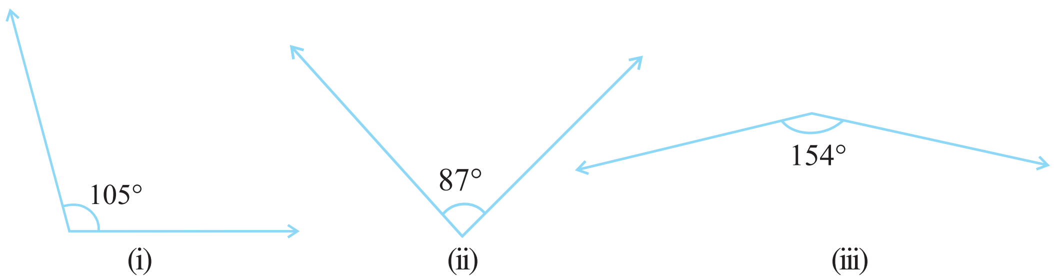


(ii)



(iii)

2. Find the supplement of each of the following angles:



3. Identify which of the following pairs of angles are complementary and which are supplementary.

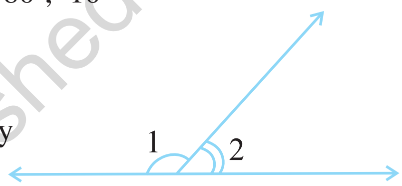
(i) 65º, 115º (ii) 63º, 27º (iii) 112º, 68º

(iv) 130º, 50º (v) 45º, 45º (vi) 80º, 10º

4. Find the angle which is equal to its complement.

5. Find the angle which is equal to its supplement.

6. In the given figure, ∠1 and ∠2 are supplementary angles.



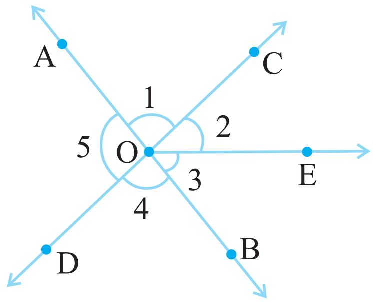
If ∠1 is decreased, what changes should take place in ∠2 so that both the angles still remain supplementary.

7. Can two angles be supplementary if both of them are:

(i) acute? (ii) obtuse? (iii) right?

8. An angle is greater than 45º. Is its complementary angle greater than 45º or equal to 45º or less than 45º?

9. In the adjoining figure:



(i) Is ∠1 adjacent to ∠2?

(ii) Is ∠AOC adjacent to ∠AOE?

(iii) Do ∠COE and ∠EOD form a linear pair?

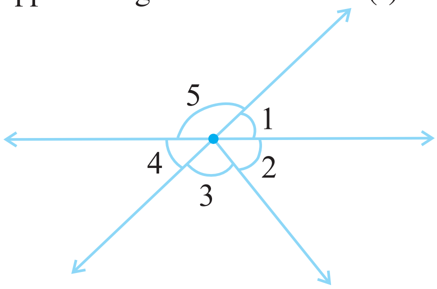
(iv) Are ∠BOD and ∠DOA supplementary?

(v) Is ∠1 vertically opposite to ∠4?

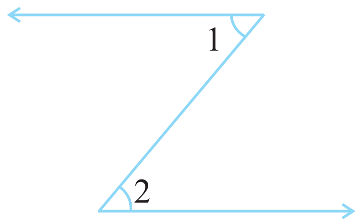
(vi) What is the vertically opposite angle of ∠5?

10. Indicate which pairs of angles are:

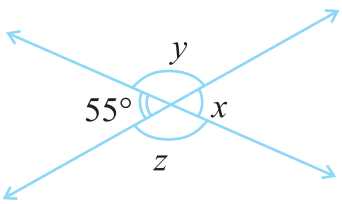
(i) Vertically opposite angles. (ii) Linear pairs.



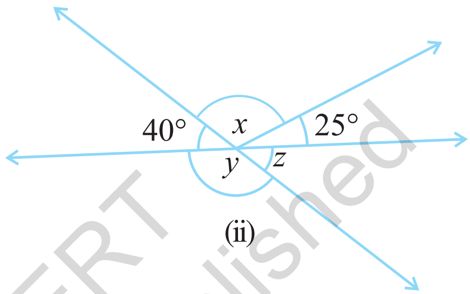
11. In the following figure, is $\angle 1$ adjacent to $\angle 2$? Give reasons.



12. Find the values of the angles x, y, and z in each of the following:



(i)



(ii)

13. Fill in the blanks:

(i) If two angles are complementary, then the sum of their measures is ________.

(ii) If two angles are supplementary, then the sum of their measures is _______.

(iii) Two angles forming a linear pair are ________________.

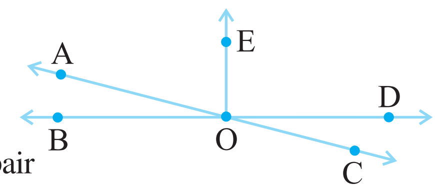
(iv) If two adjacent angles are supplementary, they form a ____________.

(v) If two lines intersect at a point, then the vertically opposite angles are always ______________.

(vi) If two lines intersect at a point, and if one pair of vertically opposite angles are acute angles, then the other pair of vertically opposite angles are ___________.

14. In the adjoining figure, name the following pairs of angles.

(i) Obtuse vertically opposite angles
(ii) Adjacent complementary angles
(iii) Equal supplementary angles
(iv) Unequal supplementary angles
(v) Adjacent angles that do not form a linear pair



5.3 Pairs of Lines

5.3.1 Intersecting Lines

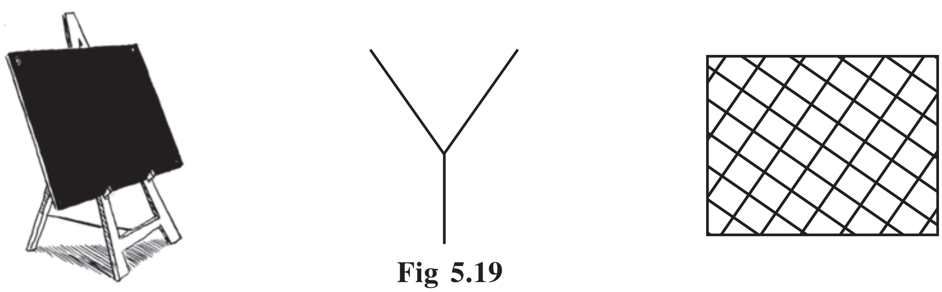
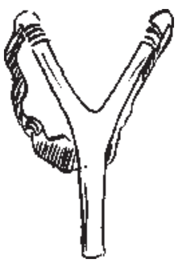


Fig 5.19

The blackboard on its stand, the letter Y made up of line segments and the grill-door of a window (Fig 5.19), what do all these have in common? They are examples of **intersecting lines**.

Two lines *l* and m intersect if they have a point in common. This common point O is their **point of intersection**.

THINK, DISCUSS AND WRITE



In Fig 5.20, AC and BE intersect at P.

AC and BC intersect at C, AC and EC intersect at C.

Try to find another ten pairs of intersecting line segments.

Should any two lines or line segments necessarily intersect? Can you find two pairs of non-intersecting line segments in the figure?

Can two lines intersect in more than one point? Think about it.

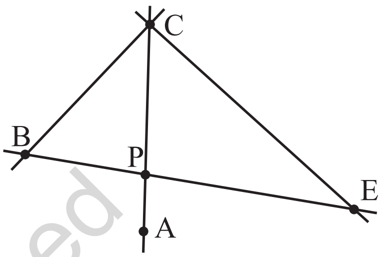


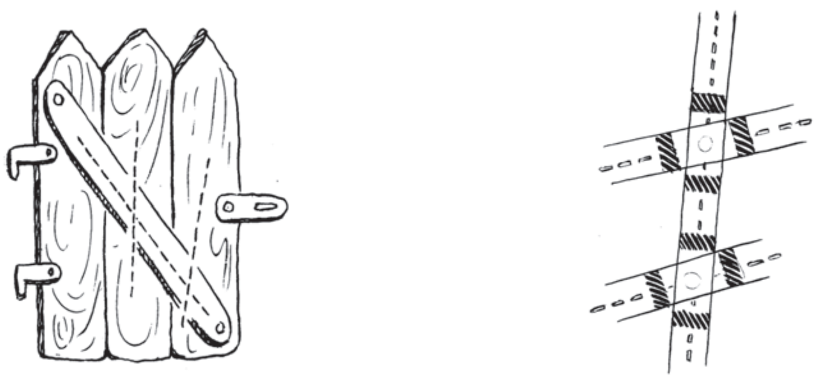
Fig 5.20

TRY THESE

1. Find examples from your surroundings where lines intersect at right angles.
2. Find the measures of the angles made by the intersecting lines at the vertices of an equilateral triangle.
3. Draw any rectangle and find the measures of angles at the four vertices made by the intersecting lines.
4. If two lines intersect, do they always intersect at right angles?

5.3.2 Transversal

You might have seen a road crossing two or more roads or a railway line crossing several other lines (Fig 5.21). These give an idea of a transversal.



(i) **Fig 5.21** (ii)

A line that intersects two or more lines at **distinct** points is called a **transversal**.

In the Fig 5.22, *p* is a transversal to the lines *l* and *m*.

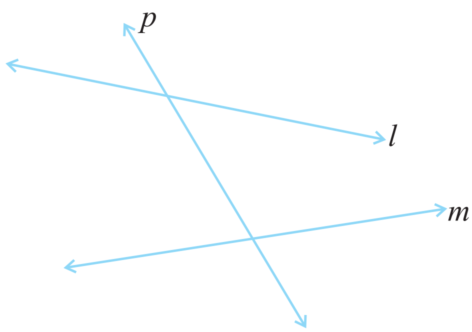


Fig 5.22

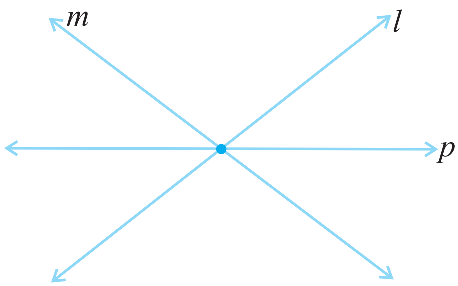


Fig 5.23

In Fig 5.23 the line *p* is not a transversal, although it cuts two lines *l* and *m*. Can you say, 'why'?

5.3.3. Angles made by a Transversal

In Fig 5.24, you see lines *l* and *m* cut by transversal *p*. The eight angles marked 1 to 8 have their special names:

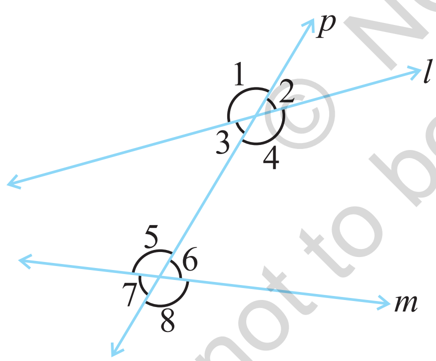


Fig 5.24

TRY THESE

1. Suppose two lines are given. How many transversals can you draw for these lines?
2. If a line is a transversal to three lines, how many points of intersections are there?
3. Try to identify a few transversals in your surroundings.

Interior angles	$\angle 3, \angle 4, \angle 5, \angle 6$
Exterior angles	$\angle 1, \angle 2, \angle 7, \angle 8$
Pairs of Corresponding angles	$\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$
Pairs of Alternate interior angles	$\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$
Pairs of Alternate exterior angles	$\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$
Pairs of interior angles on the same side of the transversal	$\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$

Note: Corresponding angles (like $\angle 1$ and $\angle 5$ in Fig 5.25) include

(i) different vertices (ii) are on the same side of the transversal and

(iii) are in ‘corresponding’ positions (above or below, left or right) relative to the two lines.

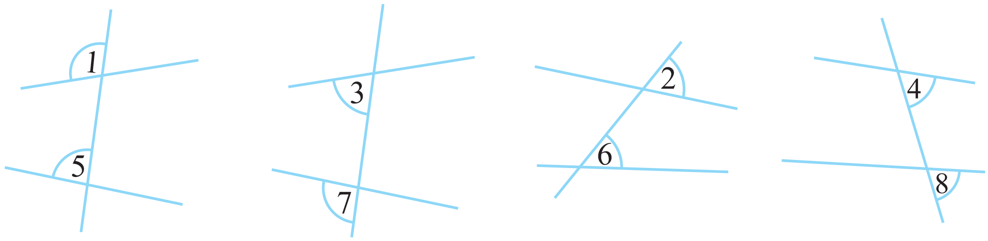


Fig 5.25

Alternate interior angles (like ∠3 and ∠6 in Fig 5.26)

(i) have different vertices

(ii) are on opposite sides of the transversal and

(iii) lie ‘between’ the two lines.

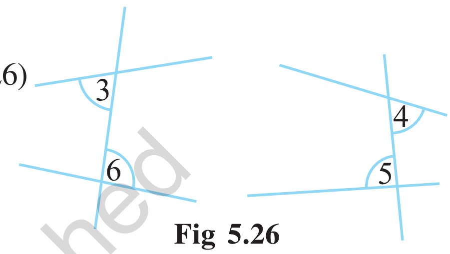
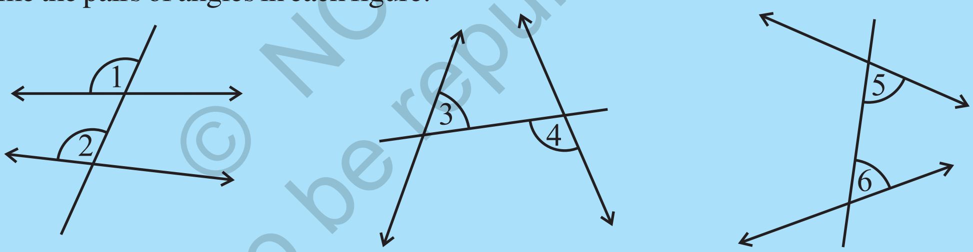
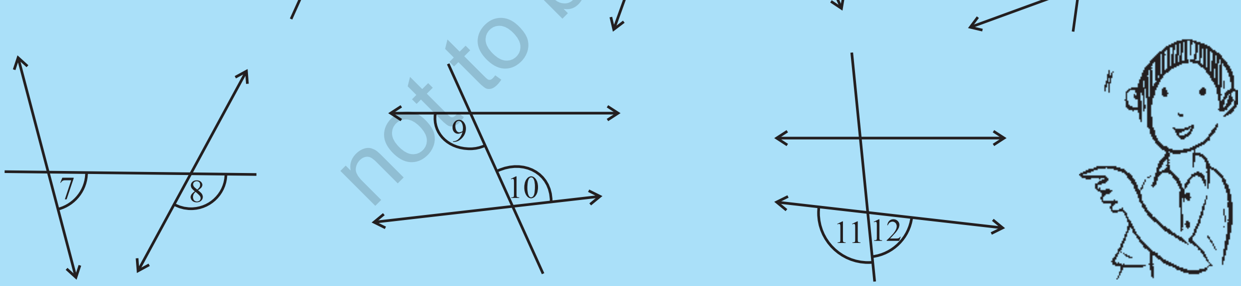


Fig 5.26

TRY THESE

Name the pairs of angles in each figure:

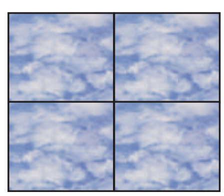


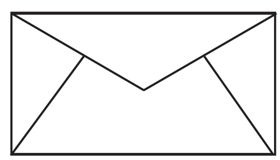


5.3.4 Transversal of Parallel Lines

Do you remember what parallel lines are? They are lines on a plane that do not meet anywhere. Can you identify parallel lines in the following figures? (Fig 5.27)

Fig 5.27





Transversals of parallel lines give rise to quite interesting results.

Do This

Take a ruled sheet of paper. Draw (in thick colour) two parallel lines *l* and *m*.

Draw a transversal *t* to the lines *l* and *m*. Label ∠1 and ∠2 as shown [Fig 5.28(i)].

Place a tracing paper over the figure drawn. Trace the lines *l*, *m* and *t*.

Slide the tracing paper along *t*, until *l* coincides with *m*.

You find that ∠1 on the traced figure coincides with ∠2 of the original figure.

In fact, you can see all the following results by similar tracing and sliding activity.

(i) ∠1 = ∠2 (ii) ∠3 = ∠4 (iii) ∠5 = ∠6 (iv) ∠7 = ∠8

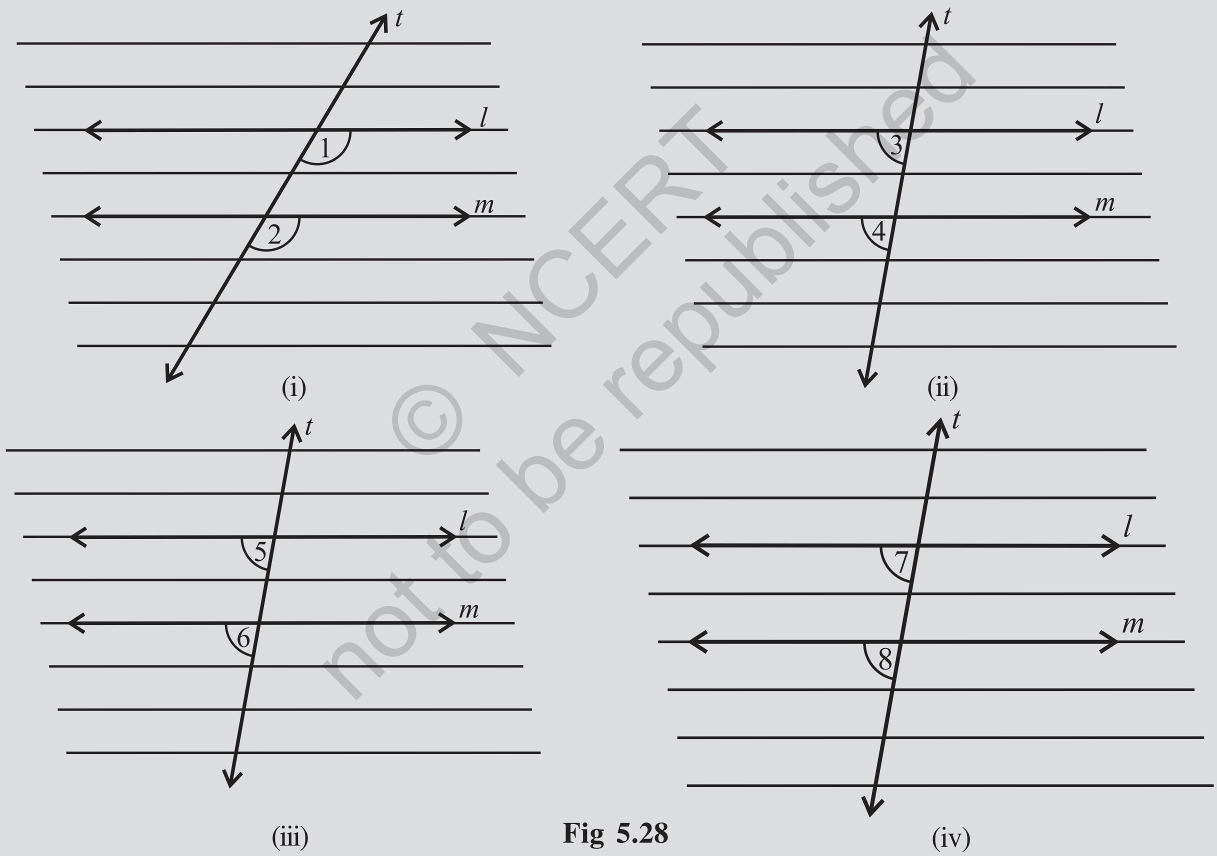


Fig 5.28

This activity illustrates the following fact:

> If two parallel lines are cut by a transversal, each pair of corresponding angles are equal in measure.

We use this result to get another interesting result. Look at Fig 5.29.

When *t* cuts the parallel lines, *l*, *m*, we get, ∠3 = ∠7 (vertically opposite angles).

But ∠7 = ∠8 (corresponding angles). Therefore, ∠3 = ∠8

You can similarly show that $\angle 1 = \angle 6$. Thus, we have the following result :

> If two parallel lines are cut by a transversal, each pair of alternate interior angles are equal.

This second result leads to another interesting property. Again, from Fig 5.29.

$\angle 3 + \angle 1 = 180°$ ($\angle 3$ and $\angle 1$ form a linear pair)

But $\angle 1 = \angle 6$ (A pair of alternate interior angles)

Therefore, we can say that $\angle 3 + \angle 6 = 180°$.

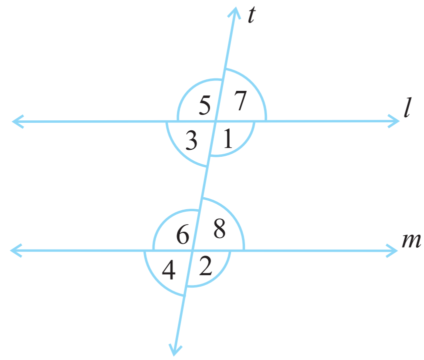


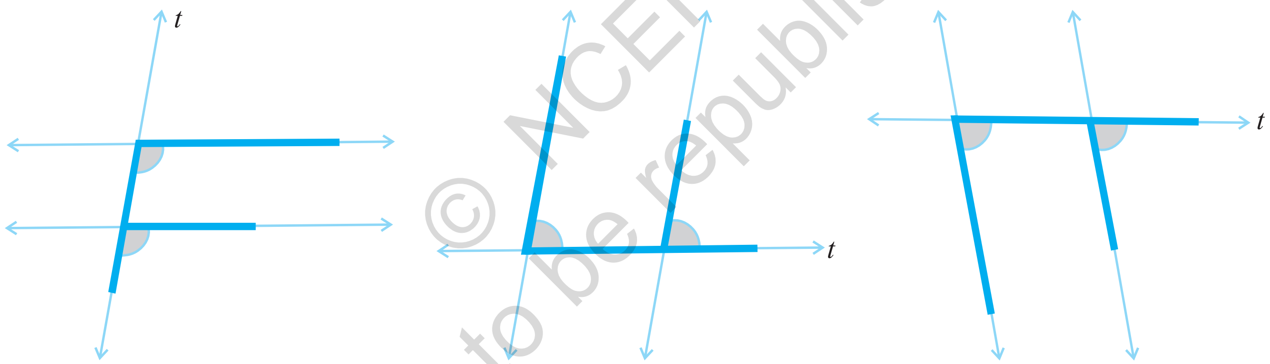
Fig 5.29

Similarly, $\angle 1 + \angle 8 = 180°$. Thus, we obtain the following result:

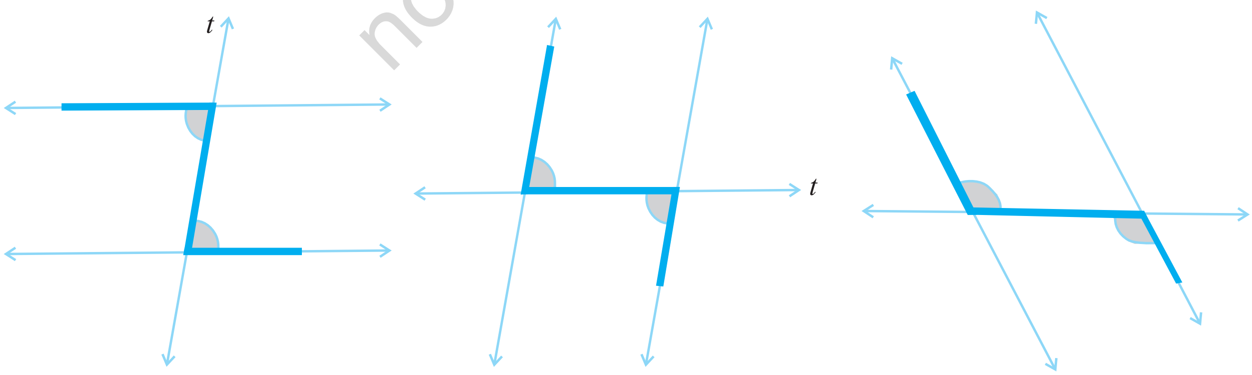
> If two parallel lines are cut by a transversal, then each pair of interior angles on the same side of the transversal are supplementary.

You can very easily remember these results if you can look for relevant 'shapes'.

The F-shape stands for corresponding angles:



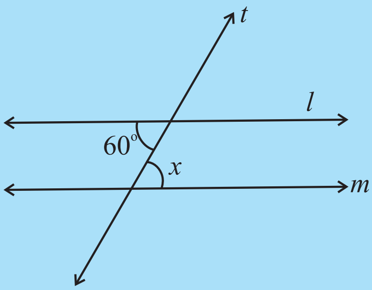
The Z - shape stands for alternate angles.



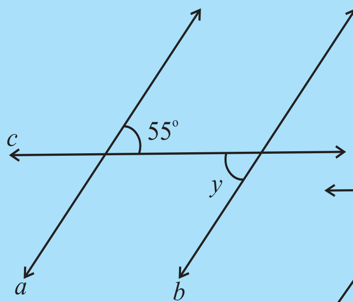
Do This

Draw a pair of parallel lines and a transversal. Verify the above three statements by actually measuring the angles.

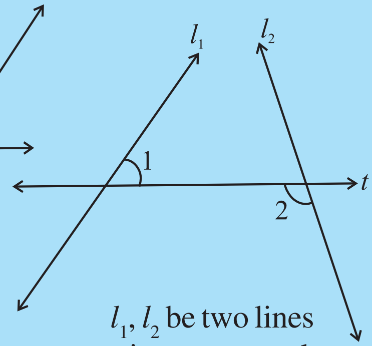
TRY THESE



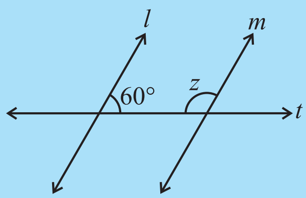
Lines $l \parallel m$;
t is a transversal
$\angle x = ?$



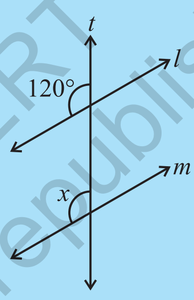
Lines $a \parallel b$;
c is a transversal
$\angle y = ?$



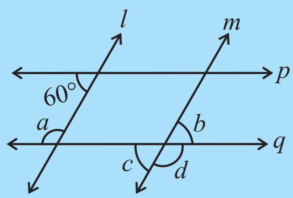
l_1, l_2 be two lines
t is a transversal
Is $\angle 1 = \angle 2$?



Lines $l \parallel m$;
t is a transversal
$\angle z = ?$



Lines $l \parallel m$;
t is a transversal
$\angle x = ?$



Lines $l \parallel m$, $p \parallel q$;
Find a, b, c, d

5.4 Checking for Parallel Lines

If two lines are parallel, then you know that a transversal gives rise to pairs of equal corresponding angles, equal alternate interior angles and interior angles on the same side of the transversal being supplementary.

When two lines are given, is there any method to check if they are parallel or not? You need this skill in many life-oriented situations.

A draftsman uses a carpenter's square and a straight edge (ruler) to draw these segments (Fig 5.30). He claims they are parallel. How?

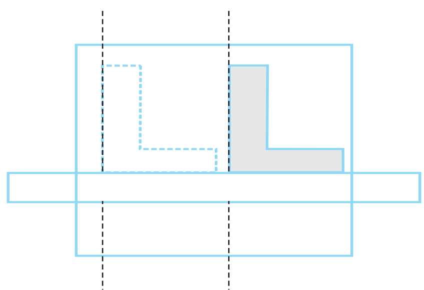


Fig 5.30

Are you able to see that he has kept the corresponding angles to be equal? (What is the transversal here?)

Thus, *when a transversal cuts two lines, such that pairs of corresponding angles are equal, then the lines have to be parallel.*

Look at the letter Z(Fig 5.31). The horizontal segments here are parallel, because the alternate angles are equal.

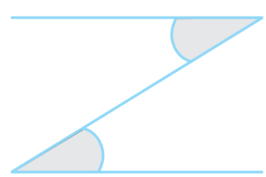


Fig 5.31

When a transversal cuts two lines, such that pairs of alternate interior angles are equal, the lines have to be parallel.

Draw a line l (Fig 5.32).

Draw a line m, perpendicular to l. Again draw a line p, such that p is perpendicular to m.

Thus, p is perpendicular to a perpendicular to l.

You find $p \parallel l$. How? This is because you draw p such that $\angle 1 + \angle 2 = 180°$.

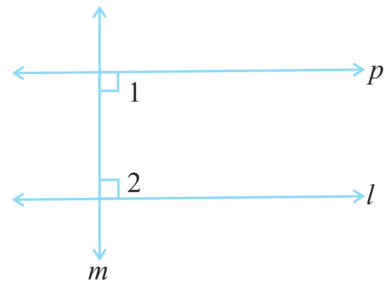
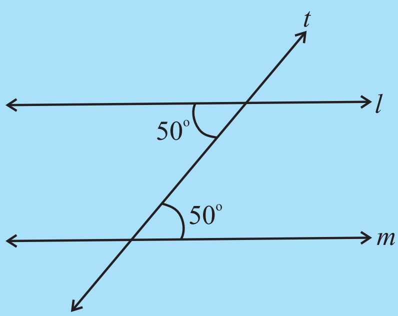


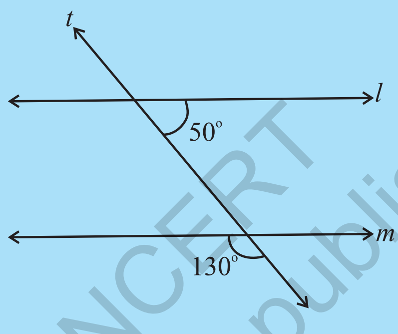
Fig 5.32

Thus, *when a transversal cuts two lines, such that pairs of interior angles on the same side of the transversal are supplementary, the lines have to be parallel.*

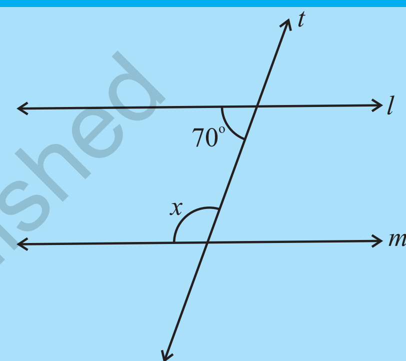
Try These



Is $l \parallel m$? Why?



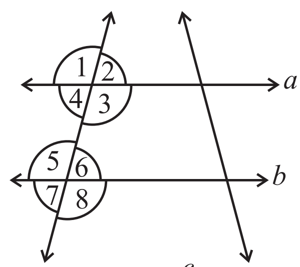
Is $l \parallel m$? Why?



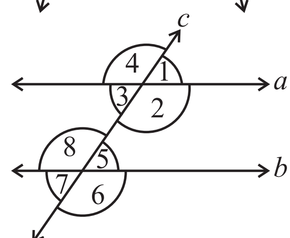
If $l \parallel m$, what is $\angle x$?

Exercise 5.2

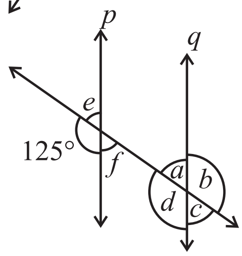
1. State the property that is used in each of the following statements?
   (i) If $a \parallel b$, then $\angle 1 = \angle 5$.
   (ii) If $\angle 4 = \angle 6$, then $a \parallel b$.
   (iii) If $\angle 4 + \angle 5 = 180°$, then $a \parallel b$.



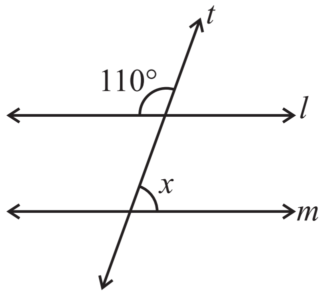
2. In the adjoining figure, identify
   (i) the pairs of corresponding angles.
   (ii) the pairs of alternate interior angles.
   (iii) the pairs of interior angles on the same side of the transversal.
   (iv) the vertically opposite angles.



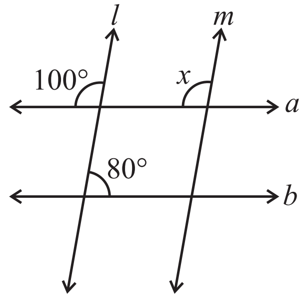
3. In the adjoining figure, $p \parallel q$. Find the unknown angles.



4. Find the value of x in each of the following figures if $l \parallel m$.

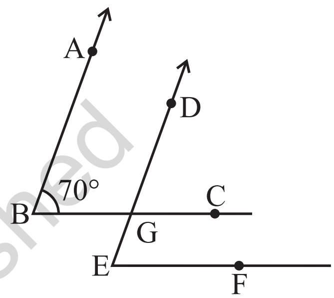


(i)

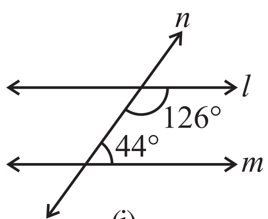


(ii)

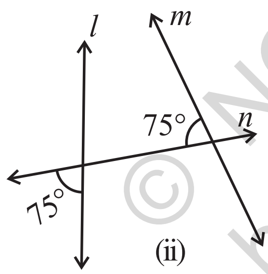
5. In the given figure, the arms of two angles are parallel.
If $\angle ABC = 70^\circ$, then find
(i) $\angle DGC$
(ii) $\angle DEF$



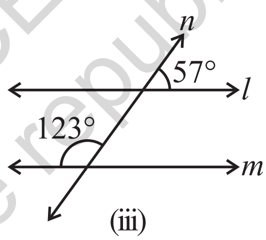
6. In the given figures below, decide whether l is parallel to m.



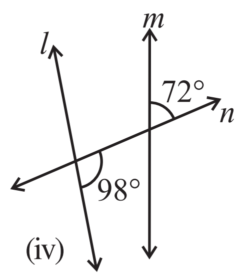
(i)



(ii)



(iii)



(iv)

WHAT HAVE WE DISCUSSED?

1. We recall that (i) A line-segment has two end points.
(ii) A ray has only one end point (its initial point); and
(iii) A line has no end points on either side.

2. An angle is formed when two lines (or rays or line-segments) meet.

Pairs of Angles	Condition
Two complementary angles	Measures add up to 90°
Two supplementary angles	Measures add up to 180°
Two adjacent angles	Have a common vertex and a common arm but no common interior.
Linear pair	Adjacent and supplementary

3. When two lines l and m meet, we say they *intersect*; the meeting point is called the point of intersection.
When lines drawn on a sheet of paper do not meet, however far produced, we call them to be *parallel* lines.

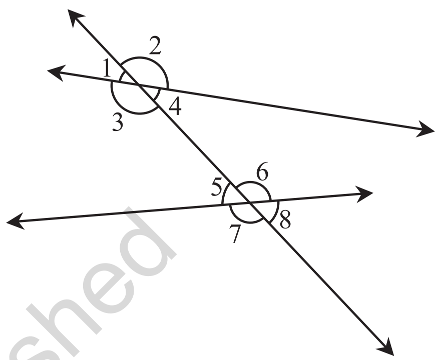
4. (i) When two lines intersect (looking like the letter X) we have two pairs of opposite angles. They are called *vertically opposite angles*. They are equal in measure.

(ii) A transversal is a line that intersects two or more lines at distinct points.

(iii) A transversal gives rise to several types of angles.

(iv) In the figure, we have

Types of Angles	Angles Shown
Interior	$\angle 3, \angle 4, \angle 5, \angle 6$
Exterior	$\angle 1, \angle 2, \angle 7, \angle 8$
Corresponding	$\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$
Alternate interior	$\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$
Alternate exterior	$\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$
Interior, on the same side of transversal	$\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$



(v) When a transversal cuts two *parallel* lines, we have the following interesting relationships:

Each pair of corresponding angles are equal.

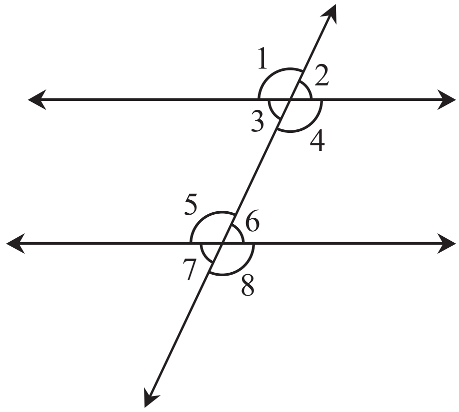
$$\angle 1 = \angle 5,\ \angle 3 = \angle 7,\ \angle 2 = \angle 6,\ \angle 4 = \angle 8$$

Each pair of alternate interior angles are equal.

$$\angle 3 = \angle 6,\ \angle 4 = \angle 5$$

Each pair of interior angles on the same side of transversal are supplementary.

$$\angle 3 + \angle 5 = 180°,\ \angle 4 + \angle 6 = 180°$$



Chapter 6

The Triangle and its Properties

0756CH06

6.1 Introduction

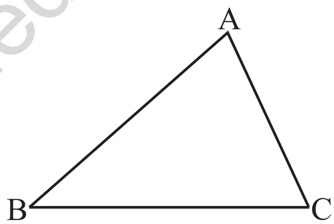


Fig 6.1

A triangle, you have seen, is a simple closed curve made of three line segments. It has three vertices, three sides and three angles.

Here is ΔABC (Fig 6.1). It has

Sides: $\overline{AB}$, $\overline{BC}$, $\overline{CA}$

Angles: $\angle BAC$, $\angle ABC$, $\angle BCA$

Vertices: A, B, C

The side opposite to the vertex A is BC. Can you name the angle opposite to the side AB?

You know how to classify triangles based on the (i) sides (ii) angles.

(i) Based on Sides: Scalene, Isosceles and Equilateral triangles.

(ii) Based on Angles: Acute-angled, Obtuse-angled and Right-angled triangles.

Make paper-cut models of the above triangular shapes. Compare your models with those of your friends and discuss about them.

Try These

1. Write the six elements (i.e., the 3 sides and the 3 angles) of ΔABC.

2. Write the:

(i) Side opposite to the vertex Q of ΔPQR

(ii) Angle opposite to the side LM of ΔLMN

(iii) Vertex opposite to the side RT of ΔRST

3. Look at Fig 6.2 and classify each of the triangles according to its

(a) Sides

(b) Angles

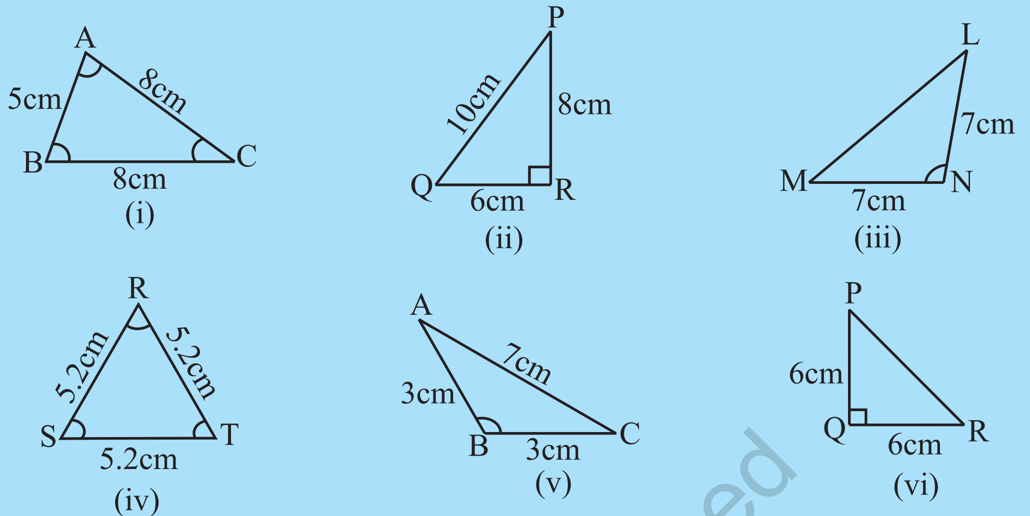


Fig 6.2

Now, let us try to explore something more about triangles.

6.2 Medians of a Triangle

Given a line segment, you know how to find its perpendicular bisector by paper folding. Cut out a triangle ABC from a piece of paper (Fig 6.3). Consider any one of its sides, say, $\overline{BC}$. By paper-folding, locate the perpendicular bisector of $\overline{BC}$. The folded crease meets $\overline{BC}$ at D, its mid-point. Join $\overline{AD}$.

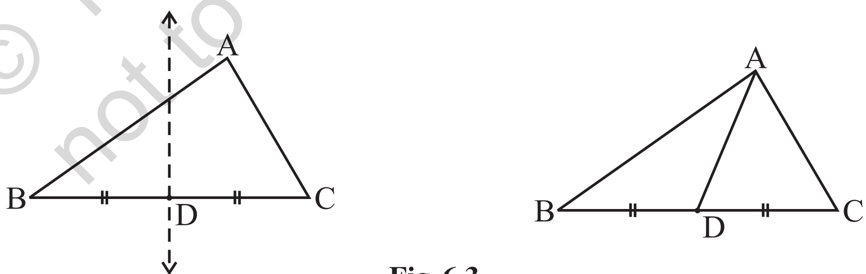


Fig 6.3

The line segment AD, joining the mid-point of $\overline{BC}$ to its opposite vertex A is called a **median** of the triangle.

Consider the sides $\overline{AB}$ and $\overline{CA}$ and find two more medians of the triangle.
A median connects a vertex of a triangle to the mid-point of the opposite side.

Think, Discuss and Write

1. How many medians can a triangle have?
2. Does a median lie wholly in the interior of the triangle? (If you think that this is not true, draw a figure to show such a case).

6.3 Altitudes of a Triangle

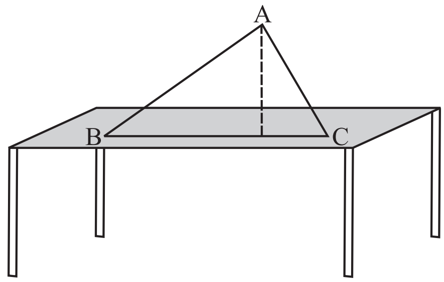


Fig 6.4

Make a triangular shaped cardboard ABC. Place it upright on a table. How 'tall' is the triangle? The **height** is the distance from vertex A (in the Fig 6.4) to the base $\overline{BC}$.

From A to $\overline{BC}$, you can think of many line segments (see the next Fig 6.5). Which among them will represent its height?

The **height** is given by the line segment that starts from A, comes straight down to $\overline{BC}$, and is perpendicular to $\overline{BC}$. This line segment $\overline{AL}$ is an **altitude** of the triangle.

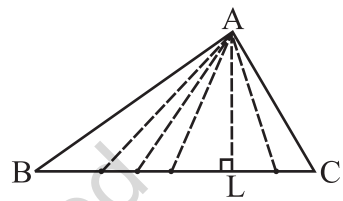
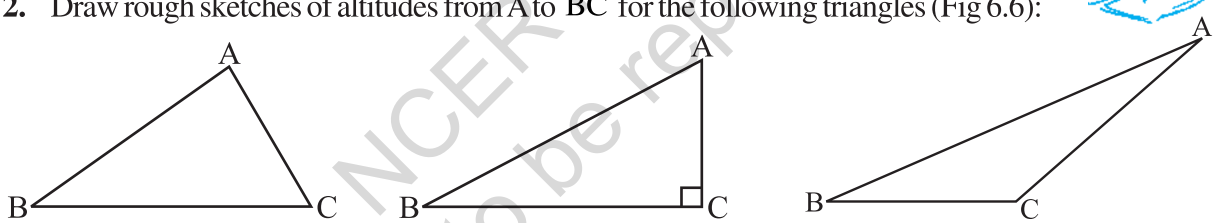


Fig 6.5

An altitude has one end point at a vertex of the triangle and the other on the line containing the opposite side. Through each vertex, an altitude can be drawn.

Think, Discuss and Write

1. How many altitudes can a triangle have?
2. Draw rough sketches of altitudes from A to $\overline{BC}$ for the following triangles (Fig 6.6):



Acute-angled (i) | Right-angled (ii) | Obtuse-angled (iii)

Fig 6.6

3. Will an altitude always lie in the interior of a triangle? If you think that this need not be true, draw a rough sketch to show such a case.
4. Can you think of a triangle in which two altitudes of the triangle are two of its sides?
5. Can the altitude and median be same for a triangle?

(Hint: For Q.No. 4 and 5, investigate by drawing the altitudes for every type of triangle).

Do This

Take several cut-outs of

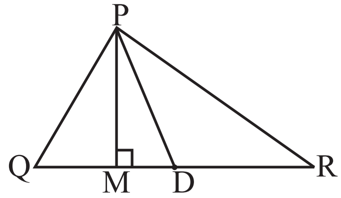
(i) an equilateral triangle (ii) an isosceles triangle and

(iii) a scalene triangle.

Find their altitudes and medians. Do you find anything special about them? Discuss it with your friends.

Exercise 6.1

1. In Δ PQR, D is the mid-point of $\overline{QR}$.



$\overline{PM}$ is _________________.

PD is _________________.

Is QM = MR?

2. Draw rough sketches for the following:
   (a) In ΔABC, BE is a median.
   (b) In ΔPQR, PQ and PR are altitudes of the triangle.
   (c) In ΔXYZ, YL is an altitude in the exterior of the triangle.
3. Verify by drawing a diagram if the median and altitude of an isosceles triangle can be same.

6.4 Exterior Angle of a Triangle and its Property

Do This

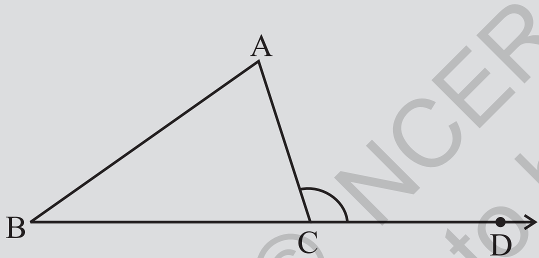


Fig 6.7

1. Draw a triangle ABC and produce one of its sides, say BC as shown in Fig 6.7. Observe the angle ACD formed at the point C. This angle lies in the exterior of ΔABC. We call it an **exterior angle** of the ΔABC formed at vertex C.

   Clearly $\angle$BCA is an adjacent angle to $\angle$ACD. The remaining two angles of the triangle namely $\angle$A and $\angle$B are called the two **interior opposite angles** or the two remote interior angles of $\angle$ACD. Now cut out (or make trace copies of) $\angle$A and $\angle$B and place them adjacent to each other as shown in Fig 6.8.

   Do these two pieces together entirely cover $\angle$ACD?

   Can you say that

   $m\,\angle ACD = m\,\angle A + m\,\angle B$?

2. As done earlier, draw a triangle ABC and form an exterior angle ACD. Now take a protractor and measure $\angle$ACD, $\angle$A and $\angle$B.

   Find the sum $\angle$A + $\angle$B and compare it with the measure of $\angle$ACD. Do you observe that $\angle$ACD is equal (or nearly equal, if there is an error in measurement) to $\angle$A + $\angle$B?

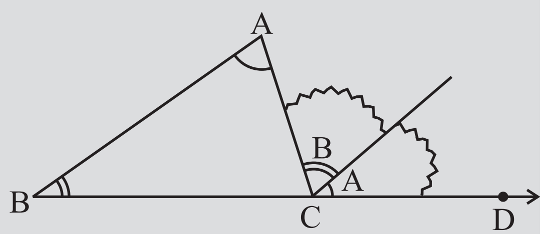


Fig 6.8

You may repeat the two activities as mentioned by drawing some more triangles along with their exterior angles. Every time, you will find that the exterior angle of a triangle is equal to the sum of its two interior opposite angles.

A logical step-by-step argument can further confirm this fact.

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

Given: Consider ΔABC.

$\angle ACD$ is an exterior angle.

To Show: $m\angle ACD = m\angle A + m\angle B$

Through C draw $\overline{CE}$, parallel to $\overline{BA}$.

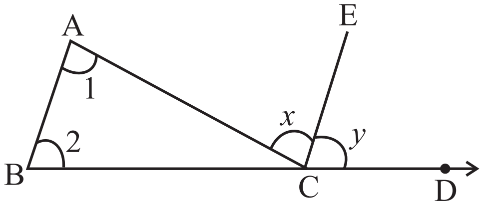


Fig 6.9

Justification

	Steps	**Reasons**
(a)	$\angle 1 = \angle x$	$\overline{BA} \parallel \overline{CE}$ and $\overline{AC}$ is a transversal. Therefore, alternate angles should be equal.
(b)	$\angle 2 = \angle y$	$\overline{BA} \parallel \overline{CE}$ and $\overline{BD}$ is a transversal. Therefore, corresponding angles should be equal.
(c)	$\angle 1 + \angle 2 = \angle x + \angle y$	
(d)	Now, $\angle x + \angle y = m\angle ACD$	From Fig 6.9
	Hence, $\angle 1 + \angle 2 = \angle ACD$	

The above relation between an exterior angle and its two interior opposite angles is referred to as the **Exterior Angle Property of a triangle.**

Think, Discuss and Write

1. Exterior angles can be formed for a triangle in many ways. Three of them are shown here (Fig 6.10)

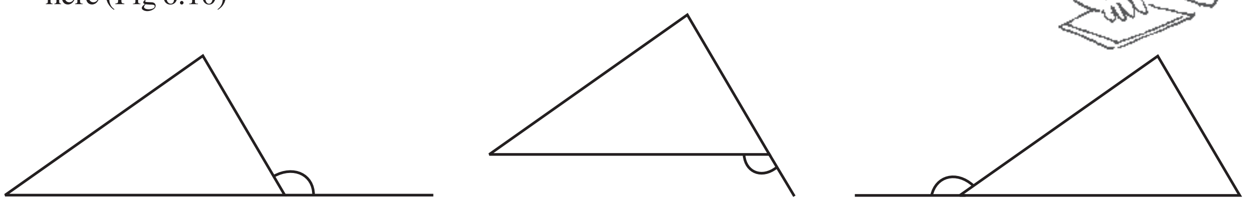


Fig 6.10

There are three more ways of getting exterior angles. Try to produce those rough sketches.

2. Are the exterior angles formed at each vertex of a triangle equal?
3. What can you say about the sum of an exterior angle of a triangle and its adjacent interior angle?

EXAMPLE 1 Find angle x in Fig 6.11.

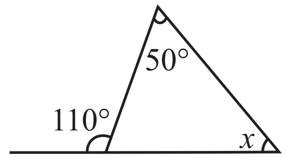


Fig 6.11

SOLUTION Sum of interior opposite angles = Exterior angle

or $50° + x = 110°$

or $x = 60°$

THINK, DISCUSS AND WRITE

1. What can you say about each of the interior opposite angles, when the exterior angle is
   (i) a right angle? (ii) an obtuse angle? (iii) an acute angle?
2. Can the exterior angle of a triangle be a straight angle?

TRY THESE

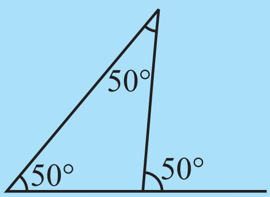
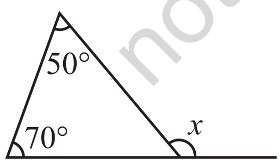


Fig 6.12

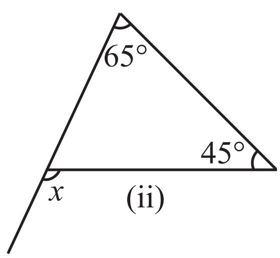
1. An exterior angle of a triangle is of measure 70º and one of its interior opposite angles is of measure 25º. Find the measure of the other interior opposite angle.
2. The two interior opposite angles of an exterior angle of a triangle are 60º and 80º. Find the measure of the exterior angle.
3. Is something wrong in this diagram (Fig 6.12)? Comment.

EXERCISE 6.2

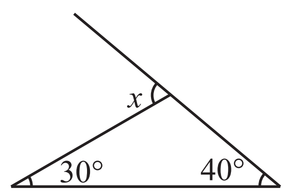
1. Find the value of the unknown exterior angle x in the following diagrams:



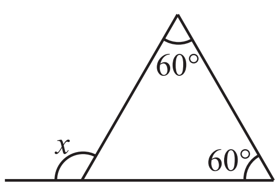
(i)



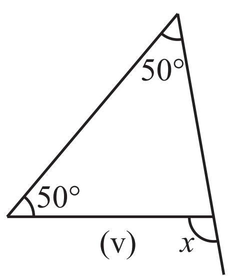
(ii)



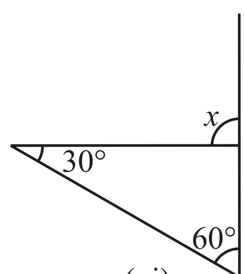
(iii)



(iv)

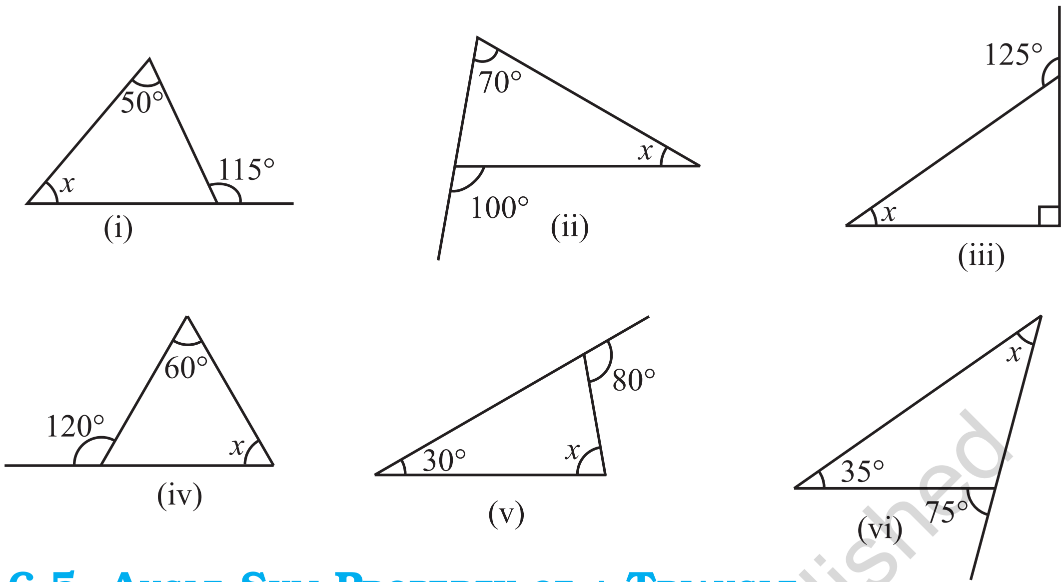


(v)



(vi)

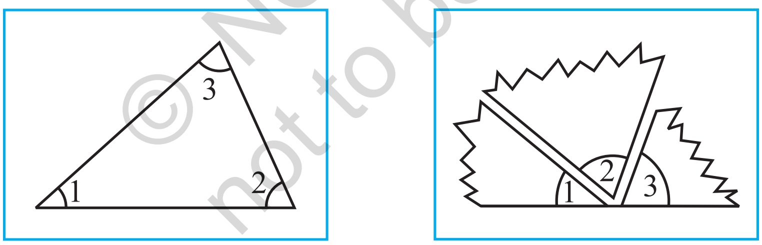
2. Find the value of the unknown interior angle x in the following figures:



6.5 Angle Sum Property of a Triangle

There is a remarkable property connecting the three angles of a triangle. You are going to see this through the following four activities.

1. Draw a triangle. Cut on the three angles. Rearrange them as shown in Fig 6.13 (i), (ii). The three angles now constitute one angle. This angle is a straight angle and so has measure 180°.



(i) (ii)

Fig 6.13

Thus, the sum of the measures of the three angles of a triangle is 180°.

2. The same fact you can observe in a different way also. Take three copies of any triangle, say ΔABC (Fig 6.14).

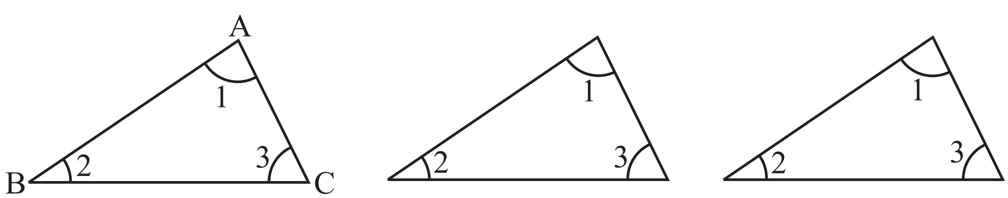


Fig 6.14

Arrange them as in Fig 6.15.

What do you observe about $\angle 1 + \angle 2 + \angle 3$?

(Do you also see the 'exterior angle property'?)

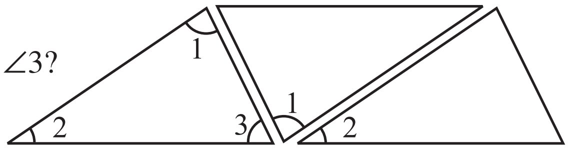
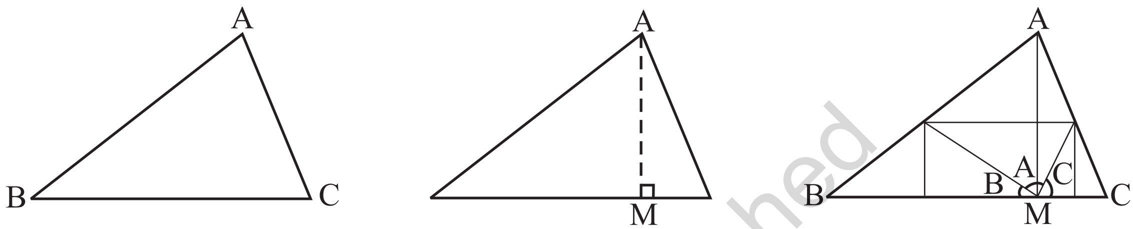


Fig 6.15

3. Take a piece of paper and cut out a triangle, say, ΔABC (Fig 6.16).

   Make the altitude AM by folding ΔABC such that it passes through A.

   Fold now the three corners such that all the three vertices A, B and C touch at M.



(i) (ii) (iii)

Fig 6.16

You find that all the three angles form together a straight angle. This again shows that the sum of the measures of the three angles of a triangle is 180°.

4. Draw any three triangles, say ΔABC, ΔPQR and ΔXYZ in your notebook.

   Use your protractor and measure each of the angles of these triangles.

   Tabulate your results

Name of Δ	Measures of Angles			Sum of the Measures of the three Angles
ΔABC	$m\angle A =$	$m\angle B =$	$m\angle C =$	$m\angle A + m\angle B + m\angle C =$
ΔPQR	$m\angle P =$	$m\angle Q =$	$m\angle R =$	$m\angle P + m\angle Q + m\angle R =$
ΔXYZ	$m\angle X =$	$m\angle Y =$	$m\angle Z =$	$m\angle X + m\angle Y + m\angle Z =$

Allowing marginal errors in measurement, you will find that the last column always gives 180° (or nearly 180°).

When perfect precision is possible, this will also show that the sum of the measures of the three angles of a triangle is 180°.

You are now ready to give a formal justification of your assertion through logical argument.

Statement **The total measure of the three angles of a triangle is 180°.**

To justify this let us use the exterior angle property of a triangle.

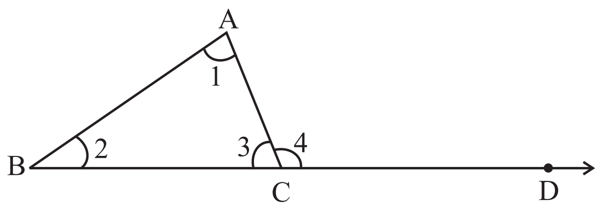


Fig 6.17

Given $\angle 1$, $\angle 2$, $\angle 3$ are angles of ΔABC (Fig 6.17).

$\angle 4$ is the exterior angle when BC is extended to D.

Justification $\angle 1 + \angle 2 = \angle 4$ (by exterior angle property)

$\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 3$ (adding $\angle 3$ to both the sides)

But $\angle 4$ and $\angle 3$ form a linear pair so it is 180°. Therefore, $\angle 1 + \angle 2 + \angle 3 = 180°$.

Let us see how we can use this property in a number of ways.

EXAMPLE 2 In the given figure (Fig 6.18) find $m\angle P$.

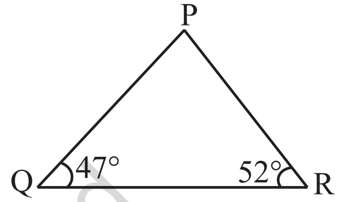


Fig 6.18

SOLUTION By angle sum property of a triangle,

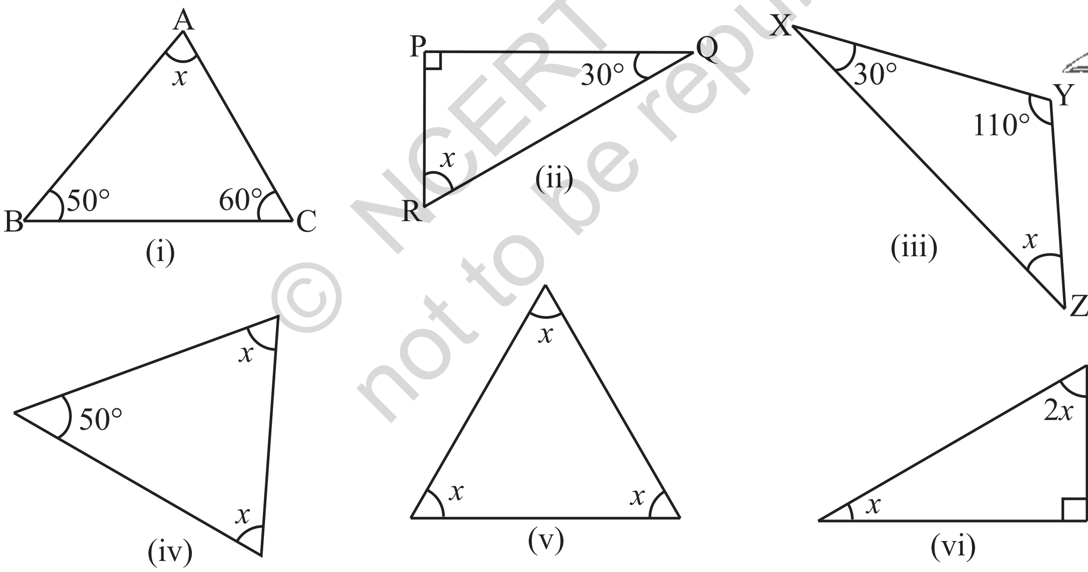
$$m\angle P + 47° + 52° = 180°$$

Therefore $$m\angle P = 180° - 47° - 52°$$

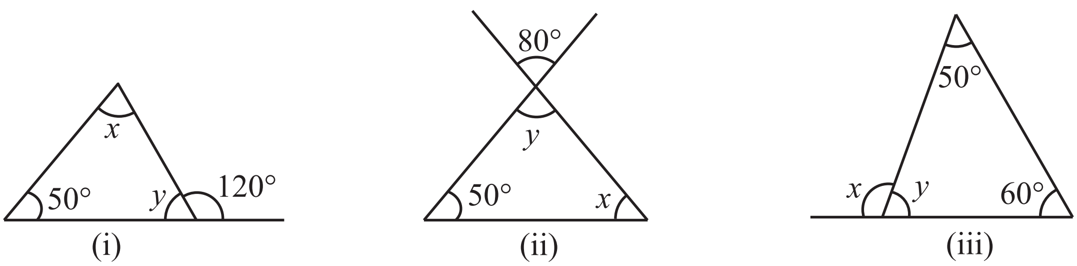
$$= 180° - 99° = 81°$$

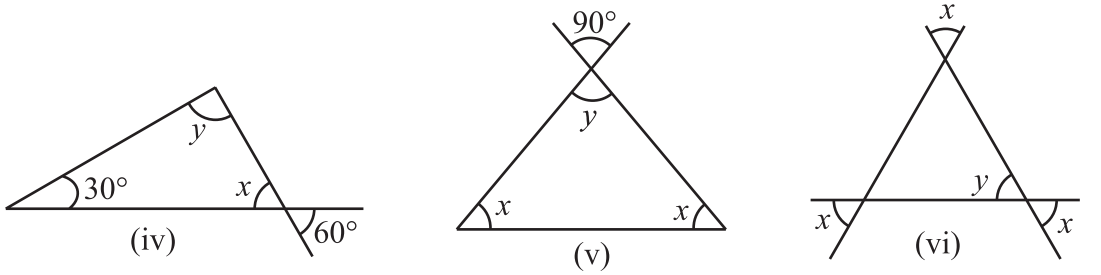
EXERCISE 6.3

1. Find the value of the unknown x in the following diagrams:



2. Find the values of the unknowns x and y in the following diagrams:





Try These

1. Two angles of a triangle are 30º and 80º. Find the third angle.
2. One of the angles of a triangle is 80º and the other two angles are equal. Find the measure of each of the equal angles.
3. The three angles of a triangle are in the ratio 1:2:1. Find all the angles of the triangle. Classify the triangle in two different ways.

Think, Discuss and Write

1. Can you have a triangle with two right angles?
2. Can you have a triangle with two obtuse angles?
3. Can you have a triangle with two acute angles?
4. Can you have a triangle with all the three angles greater than 60º?
5. Can you have a triangle with all the three angles equal to 60º?
6. Can you have a triangle with all the three angles less than 60º?

6.6 Two Special Triangles : Equilateral and Isosceles

A triangle in which all the three sides are of equal lengths is called an equilateral triangle.

Take two copies of an equilateral triangle ABC (Fig 6.19). Keep one of them fixed. Place the second triangle on it. It fits exactly into the first. Turn it round in any way and still they fit with one another exactly. Are you able to see that when the three sides of a triangle have equal lengths then the three angles are also of the same size?

We conclude that in an equilateral triangle:

(i) all sides have same length.

(ii) each angle has measure 60°.

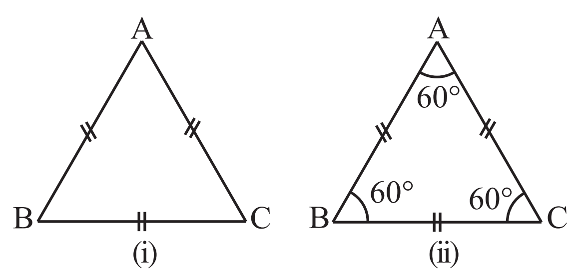


Fig 6.19

A triangle in which two sides are of equal lengths is called an isosceles triangle.

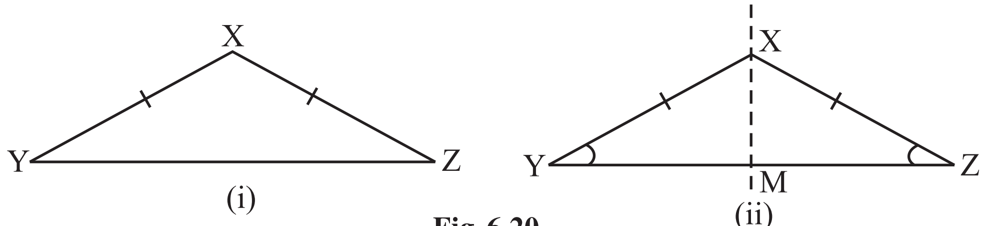


Fig 6.20

From a piece of paper cut out an isosceles triangle XYZ, with XY=XZ (Fig 6.20). Fold it such that Z lies on Y. The line XM through X is now the axis of symmetry (which you will read in Chapter 14). You find that ∠Y and ∠Z fit on each other exactly. XY and XZ are called equal sides; YZ is called the base; ∠Y and ∠Z are called base angles and these are also equal.

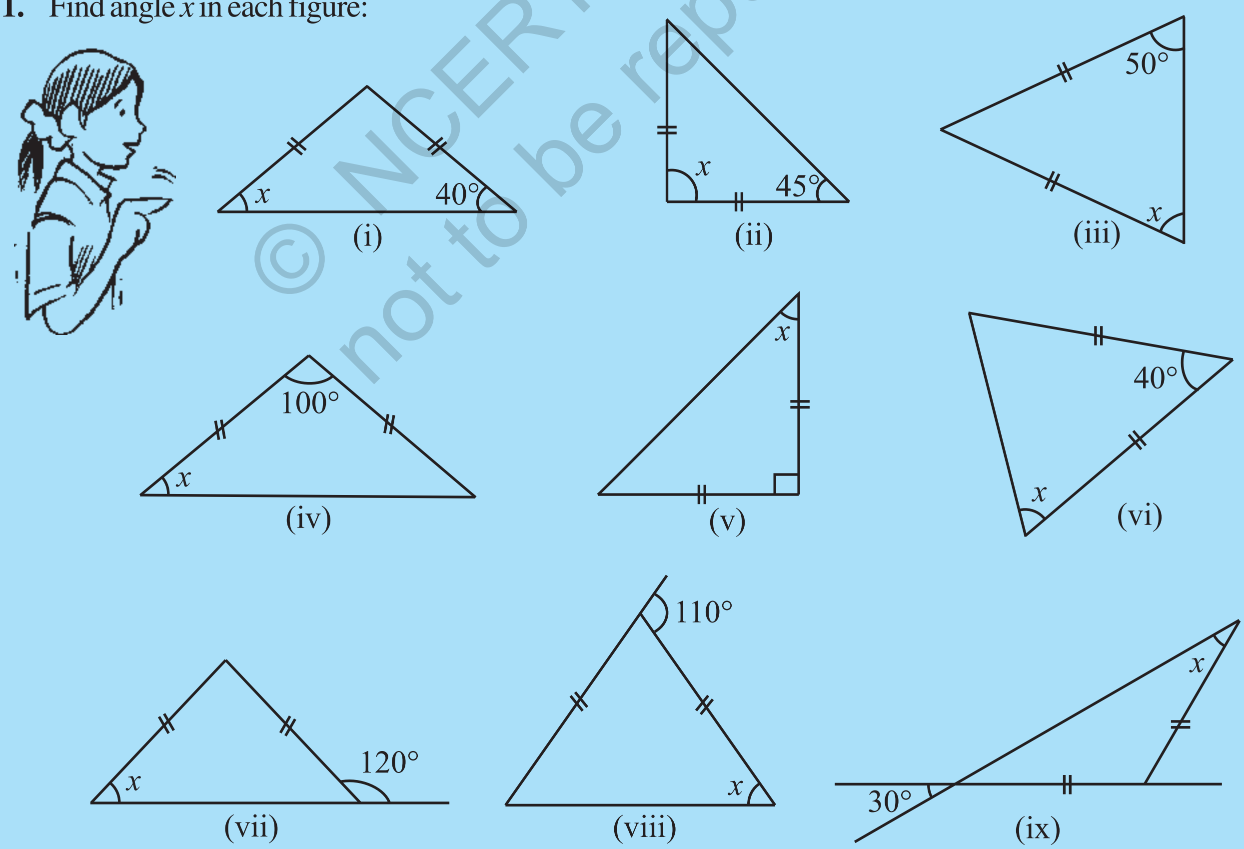
Thus, in an isosceles triangle:

(i) two sides have same length.

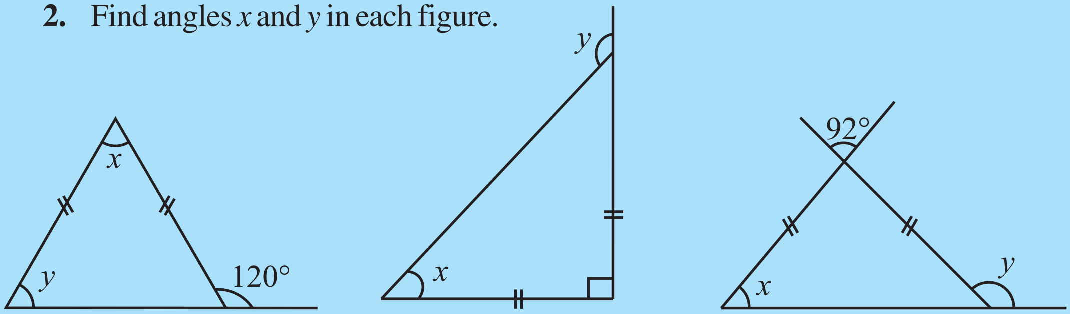
(ii) base angles opposite to the equal sides are equal.

Try These

1. Find angle x in each figure:



2. Find angles x and y in each figure.



(i) (ii) (iii)

6.7 Sum of the Lengths of Two Sides of a Triangle

1. Mark three non-collinear spots A, B and C in your playground. Using lime powder mark the paths AB, BC and AC.

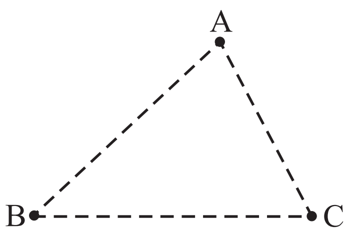


Fig 6.21

Ask your friend to start from A and reach C, walking along one or more of these paths. She can, for example, walk first along $\overline{AB}$ and then along $\overline{BC}$ to reach C; or she can walk straight along $\overline{AC}$. She will naturally prefer the direct path AC. If she takes the other path ($\overline{AB}$ and then $\overline{BC}$), she will have to walk more. In other words,

$$AB + BC > AC \qquad \text{(i)}$$

Similarly, if one were to start from B and go to A, he or she will not take the route $\overline{BC}$ and $\overline{CA}$ but will prefer $\overline{BA}$ This is because

$$BC + CA > AB \qquad \text{(ii)}$$

By a similar argument, you find that

$$CA + AB > BC \qquad \text{(iii)}$$

These observations suggest that **the sum of the lengths of any two sides of a triangle is greater than the third side**.

2. Collect fifteen small sticks (or strips) of different lengths, say, 6 cm, 7 cm, 8 cm, 9 cm, ..., 20 cm.

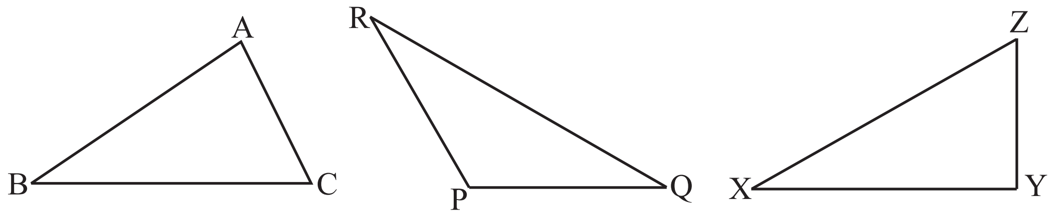
Take any three of these sticks and try to form a triangle. Repeat this by choosing different combinations of three sticks.

Suppose you first choose two sticks of length 6 cm and 12 cm. Your third stick has to be of length more than 12 – 6 = 6 cm and less than 12 + 6 = 18 cm. Try this and find out why it is so.

To form a triangle you will need any three sticks such that the sum of the lengths of any two of them will always be greater than the length of the third stick.

This also suggests that the sum of the lengths of any two sides of a triangle is greater than the third side.

3. Draw any three triangles, say ΔABC, ΔPQR and ΔXYZ in your notebook (Fig 6.22).



(i) (ii) (iii)

Fig 6.22

Use your ruler to find the lengths of their sides and then tabulate your results as follows:

Name of Δ	Lengths of Sides	Is this True?	
Δ ABC	AB ___	AB – BC < CA	(Yes/No)
		___ + ___ > ___	
	BC ___	BC – CA < AB	(Yes/No)
		___ + ___ > ___	
	CA ___	CA – AB < BC	(Yes/No)
		___ + ___ > ___	
Δ PQR	PQ ___	PQ – QR < RP	(Yes/No)
		___ + ___ > ___	
	QR ___	QR – RP < PQ	(Yes/No)
		___ + ___ > ___	
	RP ___	RP – PQ < QR	(Yes/No)
		___ + ___ > ___	
Δ XYZ	XY ___	XY – YZ < ZX	(Yes/No)
		___ + ___ > ___	
	YZ ___	YZ – ZX < XY	(Yes/No)
		___ + ___ > ___	
	ZX ___	ZX – XY < YZ	(Yes/No)
		___ + ___ > ___	

This also strengthens our earlier guess. Therefore, we conclude that **sum of the lengths of any two sides of a triangle is greater than the length of the third side.**

We also find that the difference between the length of any two sides of a triangle is smaller than the length of the third side.

EXAMPLE 3 Is there a triangle whose sides have lengths 10.2 cm, 5.8 cm and 4.5 cm?

SOLUTION Suppose such a triangle is possible. Then the sum of the lengths of any two sides would be greater than the length of the third side. Let us check this.

Is $4.5 + 5.8 > 10.2$? Yes

Is $5.8 + 10.2 > 4.5$? Yes

Is $10.2 + 4.5 > 5.8$? Yes

Therefore, the triangle is possible.

EXAMPLE 4 The lengths of two sides of a triangle are 6 cm and 8 cm. Between which two numbers can length of the third side fall?

SOLUTION We know that the sum of two sides of a triangle is always greater than the third.

Therefore, third side has to be less than the sum of the two sides. The third side is thus, less than $8 + 6 = 14$ cm.

The side cannot be less than the difference of the two sides. Thus, the third side has to be more than $8 - 6 = 2$ cm.

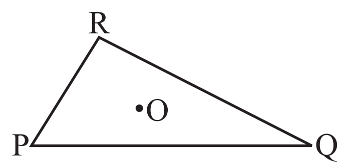
The length of the third side could be any length greater than 2 and less than 14 cm.

EXERCISE 6.4

1. Is it possible to have a triangle with the following sides?

   (i) 2 cm, 3 cm, 5 cm (ii) 3 cm, 6 cm, 7 cm

   (iii) 6 cm, 3 cm, 2 cm

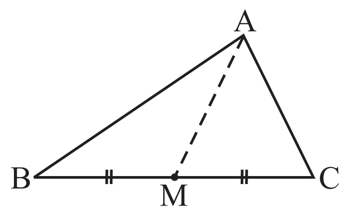


2. Take any point O in the interior of a triangle PQR. Is

   (i) $OP + OQ > PQ$?

   (ii) $OQ + OR > QR$?

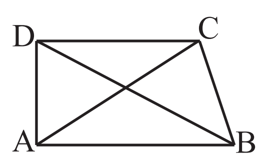
   (iii) $OR + OP > RP$?



3. AM is a median of a triangle ABC.

   Is $AB + BC + CA > 2\ AM$?

   (Consider the sides of triangles ΔABM and ΔAMC.)



4. ABCD is a quadrilateral.

   Is $AB + BC + CD + DA > AC + BD$?

5. ABCD is quadrilateral. Is

   $AB + BC + CD + DA < 2\ (AC + BD)$?

6. The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?

Think, Discuss and Write

1. Is the sum of any two angles of a triangle always greater than the third angle?

6.8 Right-angled Triangles and Pythagoras Property

Pythagoras, a Greek philosopher of sixth century B.C. is said to have found a very important and useful property of right-angled triangles given in this section. The property is, hence, named after him. In fact, this property was known to people of many other countries too. The Indian mathematician Baudhayan has also given an equivalent form of this property. We now try to explain the Pythagoras property.

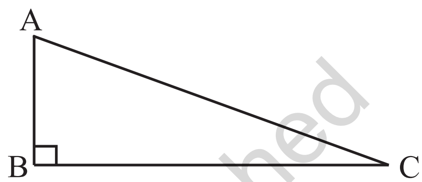


Fig 6.23

In a right-angled triangle, the sides have some special names. The side opposite to the right angle is called the **hypotenuse**; the other two sides are known as the **legs** of the right-angled triangle.

In ΔABC (Fig 6.23), the right-angle is at B. So, AC is the hypotenuse. $\overline{\text{AB}}$ and $\overline{\text{BC}}$ are the legs of ΔABC.

Make eight identical copies of right angled triangle of any size you prefer. For example, you make a right-angled triangle whose hypotenuse is a units long and the legs are of lengths b units and c units (Fig 6.24).

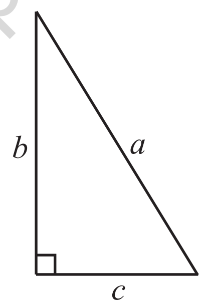
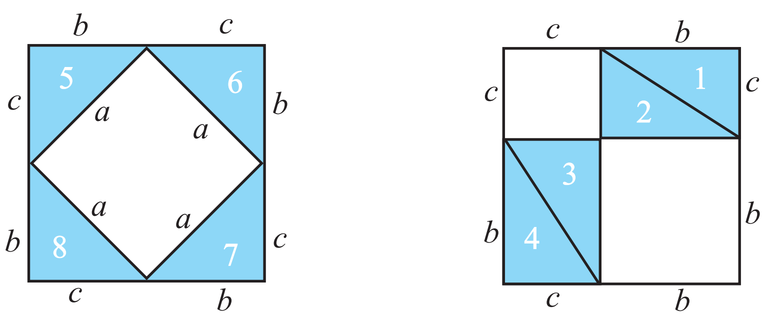


Fig 6.24

Draw two identical squares on a sheet with sides of lengths $b + c$.

You are to place four triangles in one square and the remaining four triangles in the other square, as shown in the following diagram (Fig 6.25).



Square A **Fig 6.25** Square B

The squares are identical; the eight triangles inserted are also identical.

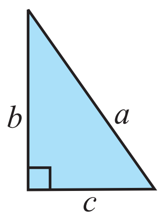
Hence the uncovered area of square A = Uncovered area of square B.

i.e., Area of inner square of square A = The total area of two uncovered squares in square B.

$$a^2 = b^2 + c^2$$

This is Pythagoras property. It may be stated as follows:

> In a right-angled triangle,
> the square on the hypotenuse = sum of the squares on the legs.



Pythagoras property is a very useful tool in mathematics. It is formally proved as a theorem in later classes. You should be clear about its meaning.

It says that for any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the legs.

Draw a right triangle, preferably on a square sheet, construct squares on its sides, compute the area of these squares and verify the theorem practically (Fig 6.26).

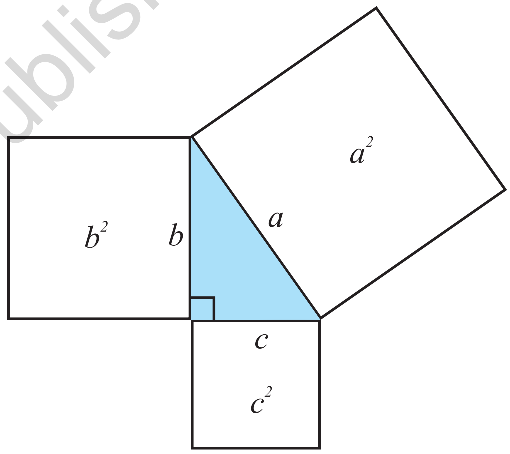


Fig 6.26

If you have a right-angled triangle, the Pythagoras property holds. If the Pythagoras property holds for some triangle, will the triangle be right-angled? (Such problems are known as converse problems). We will try to answer this. Now, we will show that, if there is a triangle such that sum of the squares on two of its sides is equal to the square of the third side, it must be a right-angled triangle.

Do This

1. Have cut-outs of squares with sides 4 cm, 5 cm, 6 cm long. Arrange to get a triangular shape by placing the corners of the squares suitably as shown in the figure (Fig 6.27). Trace out the triangle formed. Measure each angle of the triangle. You find that there is no right angle at all.

In fact, in this case each angle will be acute! Note that $4^2 + 5^2 \neq 6^2$, $5^2 + 6^2 \neq 4^2$ and $6^2 + 4^2 \neq 5^2$.

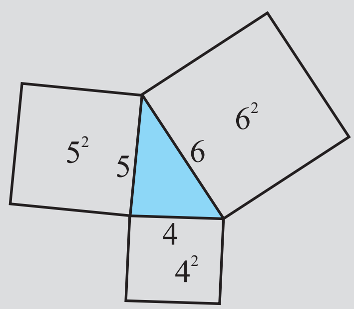


Fig 6.27

2. Repeat the above activity with squares whose sides have lengths 4 cm, 5 cm and 7 cm. You get an obtuse-angled triangle! Note that

$$4^2 + 5^2 \neq 7^2 \text{ etc.}$$

This shows that Pythagoras property holds if and only if the triangle is right-angled. Hence we get this fact:

If the Pythagoras property holds, the triangle must be right-angled.

EXAMPLE 5 Determine whether the triangle whose lengths of sides are 3 cm, 4 cm, 5 cm is a right-angled triangle.

SOLUTION $3^2 = 3 \times 3 = 9$; $4^2 = 4 \times 4 = 16$; $5^2 = 5 \times 5 = 25$

We find $3^2 + 4^2 = 5^2$.

Therefore, the triangle is right-angled.

Note: In any right-angled triangle, the hypotenuse happens to be the longest side. In this example, the side with length 5 cm is the hypotenuse.

EXAMPLE 6 Δ ABC is right-angled at C. If AC = 5 cm and BC = 12 cm find the length of AB.

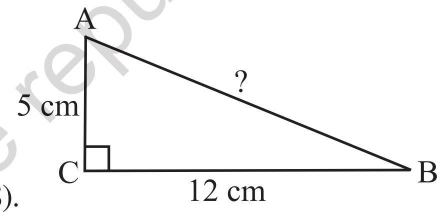


Fig 6.28

SOLUTION A rough figure will help us (Fig 6.28).

By Pythagoras property,

$$AB^2 = AC^2 + BC^2$$
$$= 5^2 + 12^2 = 25 + 144 = 169 = 13^2$$

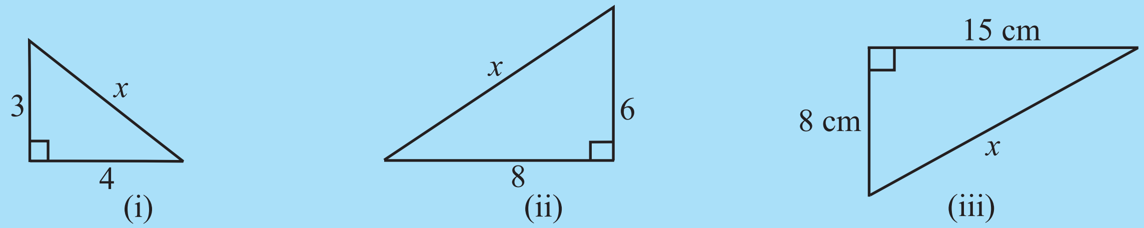
or $AB^2 = 13^2$. So, AB = 13

or the length of AB is 13 cm.

Note: To identify perfect squares, you may use prime factorisation technique.

TRY THESE

Find the unknown length x in the following figures (Fig 6.29):



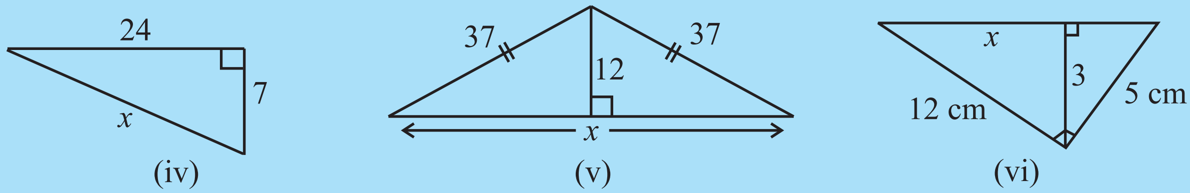
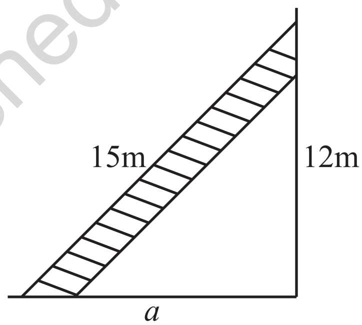


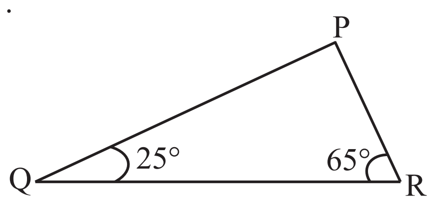
Fig 6.29

EXERCISE 6.5

1. PQR is a triangle, right-angled at P. If PQ = 10 cm and PR = 24 cm, find QR.
2. ABC is a triangle, right-angled at C. If AB = 25 cm and AC = 7 cm, find BC.
3. A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance a. Find the distance of the foot of the ladder from the wall.



4. Which of the following can be the sides of a right triangle?
   (i) 2.5 cm, 6.5 cm, 6 cm.
   (ii) 2 cm, 2 cm, 5 cm.
   (iii) 1.5 cm, 2cm, 2.5 cm.
   In the case of right-angled triangles, identify the right angles.
5. A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.
6. Angles Q and R of a ΔPQR are 25° and 65°. Write which of the following is true:
   (i) $PQ^2 + QR^2 = RP^2$
   (ii) $PQ^2 + RP^2 = QR^2$
   (iii) $RP^2 + QR^2 = PQ^2$



7. Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.
8. The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.

THINK, DISCUSS AND WRITE

1. Which is the longest side in the triangle PQR, right-angled at P?
2. Which is the longest side in the triangle ABC, right-angled at B?
3. Which is the longest side of a right triangle?
4. 'The diagonal of a rectangle produce by itself the same area as produced by its length and breadth'– This is Baudhayan Theorem. Compare it with the Pythagoras property.

DO THIS

Enrichment activity

There are many proofs for Pythagoras theorem, using 'dissection' and 'rearrangement' procedure. Try to collect a few of them and draw charts explaining them.

WHAT HAVE WE DISCUSSED?

1. The **six elements** of a triangle are its **three angles** and the **three sides**.
2. The line segment joining a vertex of a triangle to the mid point of its opposite side is called a **median** of the triangle. A triangle has 3 medians.
3. The perpendicular line segment from a vertex of a triangle to its opposite side is called an **altitude** of the triangle. A triangle has 3 altitudes.
4. An **exterior angle** of a triangle is formed, when a side of a triangle is produced. At each vertex, you have two ways of forming an exterior angle.
5. A property of exterior angles:

   The measure of any exterior angle of a triangle is equal to the sum of the measures of its interior opposite angles.
6. The angle sum property of a triangle:

   The total measure of the three angles of a triangle is 180°.
7. A triangle is said to be **equilateral,** if each one of its sides has the same length.

   In an equilateral triangle, each angle has measure 60°.
8. A triangle is said to be **isosceles,** if atleast any two of its sides are of same length.

   The non-equal side of an isosceles triangle is called its **base**; the base angles of an isosceles triangle have equal measure.
9. Property of the lengths of sides of a triangle:

   The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

   The difference between the lengths of any two sides is smaller than the length of the third side.

This property is useful to know if it is possible to draw a triangle when the lengths of the three sides are known.

10. In a right angled triangle, the side opposite to the right angle is called the **hypotenuse** and the other two sides are called its **legs**.

11. **Pythagoras property**:

In a right-angled triangle,

the square on the hypotenuse = the sum of the squares on its legs.

If a triangle is not right-angled, this property does not hold good. This property is useful to decide whether a given triangle is right-angled or not.

Chapter 7

Congruence of Triangles

0756CH07

7.1 Introduction

You are now ready to learn a very important geometrical idea, **Congruence**. In particular, you will study a lot about congruence of triangles.

To understand what congruence is, we turn to some activities.

Do This

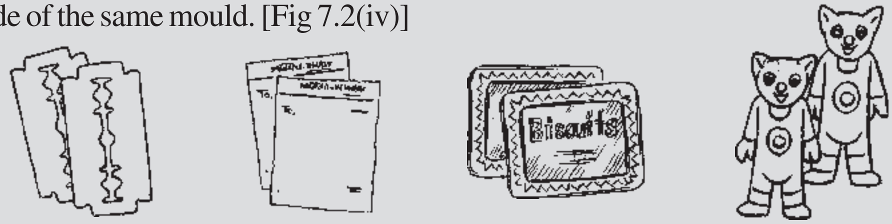
Take two stamps (Fig 7.1) of same denomination. Place one stamp over the other. What do you observe?

Fig 7.1

One stamp covers the other completely and exactly. This means that the two stamps are of the same shape and same size. Such objects are said to be congruent. The two stamps used by you are congruent to one another. Congruent objects are exact copies of one another.

Can you, now, say if the following objects are congruent or not?

1. Shaving blades of the same company [Fig 7.2 (i)].
2. Sheets of the same letter-pad [Fig 7.2 (ii)].
3. Biscuits in the same packet [Fig 7.2 (iii)].
4. Toys made of the same mould. [Fig 7.2(iv)]



(i) (ii) (iii) (iv)

Fig 7.2

The relation of two objects being congruent is called **congruence.** For the present, we will deal with plane figures only, although congruence is a general idea applicable to three-dimensional shapes also. We will try to learn a precise meaning of the congruence of plane figures already known.

7.2 Congruence of Plane Figures

Look at the two figures given here (Fig 7.3). Are they congruent?

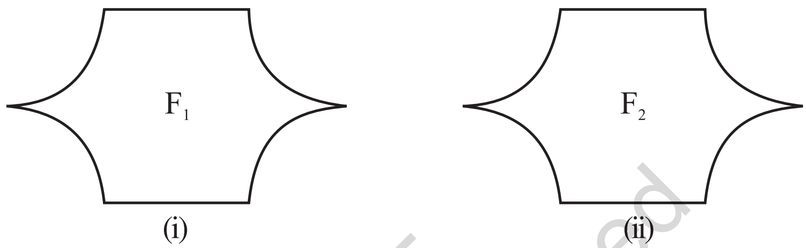


Fig 7.3

You can use the method of superposition. Take a trace-copy of one of them and place it over the other. If the figures cover each other completely, they are congruent. Alternatively, you may cut out one of them and place it over the other. Beware! You are not allowed to bend, twist or stretch the figure that is cut out (or traced out).

In Fig 7.3, if figure F_1 is congruent to figure F_2, we write $F_1 \cong F_2$.

7.3 Congruence Among Line Segments

When are two line segments congruent? Observe the two pairs of line segments given here (Fig 7.4).

Fig 7.4

Use the 'trace-copy' superposition method for the pair of line segments in [Fig 7.4(i)]. Copy $\overline{CD}$ and place it on $\overline{AB}$. You find that $\overline{CD}$ covers $\overline{AB}$, with C on A and D on B. Hence, the line segments are congruent. We write $\overline{AB} \cong \overline{CD}$.

Repeat this activity for the pair of line segments in [Fig 7.4(ii)]. What do you find? They are not congruent. How do you know it? It is because the line segments do not coincide when placed one over other.

You should have by now noticed that the pair of line segments in [Fig 7.4(i)] matched with each other because they had same length; and this was not the case in [Fig 7.4(ii)].

If two line segments have the same (i.e., equal) length, they are congruent. Also, if two line segments are congruent, they have the same length.

In view of the above fact, when two line segments are congruent, we sometimes just say that the line segments are equal; and we also write AB = CD. (What we actually mean is $\overline{AB} \cong \overline{CD}$).

7.4 Congruence of Angles

Look at the four angles given here (Fig 7.5).

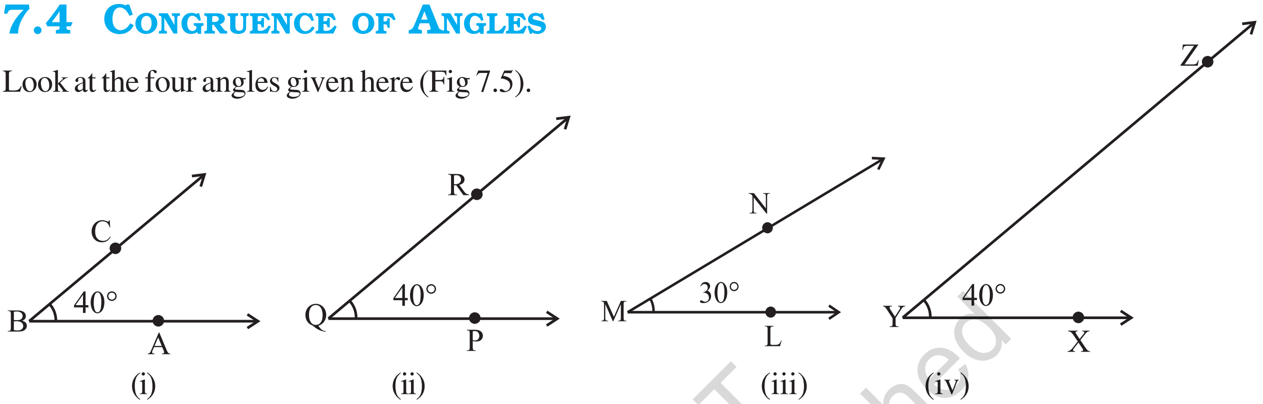


Fig 7.5

Make a trace-copy of ∠PQR. Try to superpose it on ∠ABC. For this, first place Q on B and $\overrightarrow{QP}$ along $\overrightarrow{BA}$. Where does $\overrightarrow{QR}$ fall? It falls on $\overrightarrow{BC}$.

Thus, ∠PQR matches exactly with ∠ABC.

That is, ∠ABC and ∠PQR are congruent.

(Note that the measurement of these two congruent angles are same).

We write $\angle ABC \cong \angle PQR$ (i)

or $m\angle ABC = m\angle PQR$ (In this case, measure is 40°).

Now, you take a trace-copy of ∠LMN. Try to superpose it on ∠ABC. Place M on B and $\overrightarrow{ML}$ along $\overrightarrow{BA}$. Does $\overrightarrow{MN}$ fall on $\overrightarrow{BC}$? No, in this case it does not happen. You find that ∠ABC and ∠LMN do not cover each other exactly. So, they are not congruent.

(Note that, in this case, the measures of ∠ABC and ∠LMN are not equal).

What about angles ∠XYZ and ∠ABC? The rays $\overrightarrow{YX}$ and $\overrightarrow{YZ}$, respectively appear [in Fig 7.5 (iv)] to be longer than $\overrightarrow{BA}$ and $\overrightarrow{BC}$. You may, hence, think that ∠ABC is 'smaller' than ∠XYZ. But remember that the rays in the figure only indicate the direction and not any length. On superposition, you will find that these two angles are also congruent.

We write $\angle ABC \cong \angle XYZ$ (ii)

or $m\angle ABC = m\angle XYZ$

In view of (i) and (ii), we may even write

$$\angle ABC \cong \angle PQR \cong \angle XYZ$$

If two angles have the same measure, they are congruent. Also, if two angles are congruent, their measures are same.

As in the case of line segments, congruency of angles entirely depends on the equality of their measures. So, to say that two angles are congruent, we sometimes just say that the angles are equal; and we write

$$\angle ABC = \angle PQR \text{ (to mean } \angle ABC \cong \angle PQR).$$

7.5 Congruence of Triangles

We saw that two line segments are congruent where one of them, is just a copy of the other. Similarly, two angles are congruent if one of them is a copy of the other. We extend this idea to triangles.

Two triangles are congruent if they are copies of each other and when superposed, they cover each other exactly.

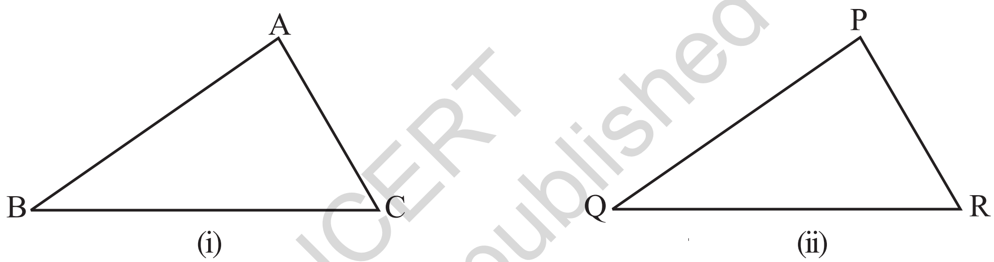


Fig 7.6

ΔABC and ΔPQR have the same size and shape. They are congruent. So, we would express this as

$$\Delta ABC \cong \Delta PQR$$

This means that, when you place ΔPQR on ΔABC, P falls on A, Q falls on B and R falls on C, also falls along $\overline{AB}$, $\overline{QR}$ falls along $\overline{BC}$ and $\overline{PR}$ falls along $\overline{AC}$. If, under a given correspondence, two triangles are congruent, then their corresponding parts (i.e., angles and sides) that match one another are equal. Thus, in these two congruent triangles, we have:

Corresponding vertices : A and P, B and Q, C and R.

Corresponding sides : $\overline{AB}$ and $\overline{PQ}$, $\overline{BC}$ and $\overline{QR}$, $\overline{AC}$ and $\overline{PR}$.

Corresponding angles : $\angle A$ and $\angle P$, $\angle B$ and $\angle Q$, $\angle C$ and $\angle R$.

If you place ΔPQR on ΔABC such that P falls on B, then, should the other vertices also correspond suitably? *It need not happen*! Take trace, copies of the triangles and try to find out.

This shows that while talking about congruence of triangles, not only the measures of angles and lengths of sides matter, but also the matching of vertices. In the above case, the correspondence is

$$A \leftrightarrow P,\ B \leftrightarrow Q,\ C \leftrightarrow R$$

We may write this as $ABC \leftrightarrow PQR$

EXAMPLE 1 ΔABC and ΔPQR are congruent under the correspondence:

$$ABC \leftrightarrow RQP$$

Write the parts of ΔABC that correspond to

(i) $\overline{PQ}$ (ii) $\angle Q$ (iii) $\overline{RP}$

SOLUTION For better understanding of the correspondence, let us use a diagram (Fig 7.7).

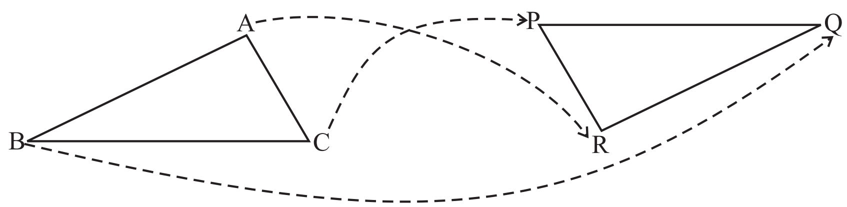


Fig 7.7

The correspondence is $ABC \leftrightarrow RQP$. This means

$$A \leftrightarrow R \;;\quad B \leftrightarrow Q;\ \text{and}\ C \leftrightarrow P.$$

So, (i) $\overline{PQ} \leftrightarrow \overline{CB}$ (ii) $\angle Q \leftrightarrow \angle B$ and (iii) $\overline{RP} \leftrightarrow \overline{AC}$

THINK, DISCUSS AND WRITE

When two triangles, say ABC and PQR are given, there are, in all, six possible matchings or correspondences. Two of them are

(i) $ABC \leftrightarrow PQR$ and (ii) $ABC \leftrightarrow QRP$.

Find the other four correspondences by using two cutouts of triangles. Will all these correspondences lead to congruence? Think about it.

EXERCISE 7.1

1. Complete the following statements:
   (a) Two line segments are congruent if ___________.
   (b) Among two congruent angles, one has a measure of 70°; the measure of the other angle is ___________.
   (c) When we write $\angle A = \angle B$, we actually mean ___________.
2. Give any two real-life examples for congruent shapes.
3. If $\Delta ABC \cong \Delta FED$ under the correspondence $ABC \leftrightarrow FED$, write all the corresponding congruent parts of the triangles.
4. If $\Delta DEF \cong \Delta BCA$, write the part(s) of ΔBCA that correspond to

   (i) $\angle E$ (ii) $\overline{EF}$ (iii) $\angle F$ (iv) $\overline{DF}$

7.6 Criteria for Congruence of Triangles

We make use of triangular structures and patterns frequently in day-to-day life. So, it is rewarding to find out when two triangular shapes will be congruent. If you have two triangles drawn in your notebook and want to verify if they are congruent, you cannot everytime cut out one of them and use method of superposition. Instead, if we can judge congruency in terms of approrpriate measures, it would be quite useful. Let us try to do this.

A Game

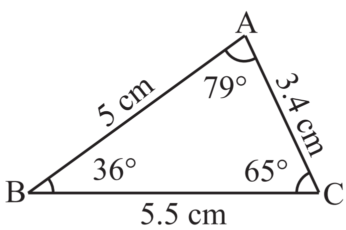


Fig 7.8
Triangle drawn by Appu

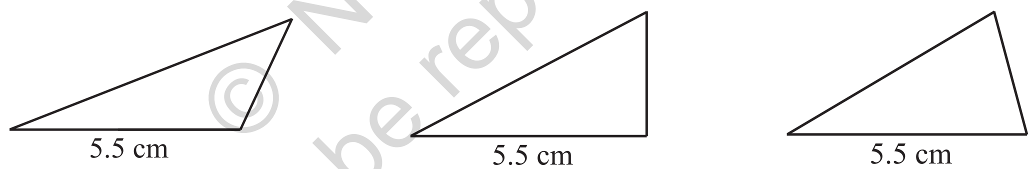
Appu and Tippu play a game. Appu has drawn a triangle ABC (Fig 7.8) and has noted the length of each of its sides and measure of each of its angles. Tippu has not seen it. Appu challenges Tippu if he can draw a copy of his ΔABC based on bits of information that Appu would give. Tippu attempts to draw a triangle congruent to ΔABC, using the information provided by Appu.

The game starts. Carefully observe their conversation and their games.

SSS Game

Appu : One side of ΔABC is 5.5 cm.

Tippu : With this information, I can draw any number of triangles (Fig 7.9) but they need not be copies of ΔABC. The triangle I draw may be obtuse-angled or right-angled or acute-angled. For example, here are a few.



(Obtuse-angled) (Right-angled) (Acute-angled)

Fig 7.9

I have used some arbitrary lengths for other sides. This gives me many triangles with length of base 5.5 cm.

So, giving only one side-length will not help me to produce a copy of ΔABC.

Appu : Okay. I will give you the length of one more side. Take two sides of ΔABC to be of lengths 5.5 cm and 3.4 cm.

Tippu : Even this will not be sufficient for the purpose. I can draw several triangles (Fig 7.10) with the given information which may not be copies of ΔABC. Here are a few to support my argument:

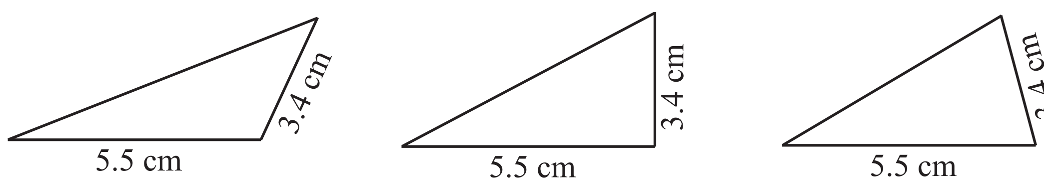


Fig 7.10

One cannot draw an exact copy of your triangle, if only the lengths of two sides are given.

Appu : Alright. Let me give the lengths of all the three sides. In ΔABC, I have AB = 5cm, BC = 5.5 cm and AC = 3.4 cm.

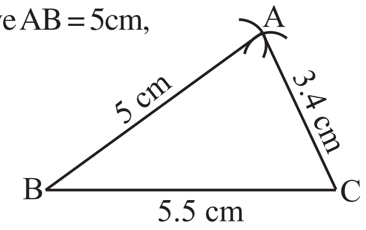


Fig 7.11

Tippu : I think it should be possible. Let me try now.

First I draw a rough figure so that I can remember the lengths easily.

I draw $\overline{BC}$ with length 5.5 cm.

With B as centre, I draw an arc of radius 5 cm. The point A has to be somewhere on this arc. With C as centre, I draw an arc of radius 3.4 cm. The point A has to be on this arc also.

So, A lies on both the arcs drawn. This means A is the point of intersection of the arcs.

I know now the positions of points A, B and C. Aha! I can join them and get ΔABC (Fig 7.11).

Appu : Excellent. So, to draw a copy of a given ΔABC (i.e., to draw a triangle congruent to ΔABC), we need the lengths of three sides. Shall we call this condition as side-side-side criterion?

Tippu : Why not we call it SSS criterion, to be short?

> **SSS Congruence criterion:**
> If under a given correspondence, the three sides of one triangle are equal to the three corresponding sides of another triangle, then the triangles are congruent.

EXAMPLE 2 In triangles ABC and PQR, AB = 3.5 cm, BC = 7.1 cm, AC = 5 cm, PQ = 7.1 cm, QR = 5 cm and PR = 3.5 cm. Examine whether the two triangles are congruent or not. If yes, write the congruence relation in symbolic form.

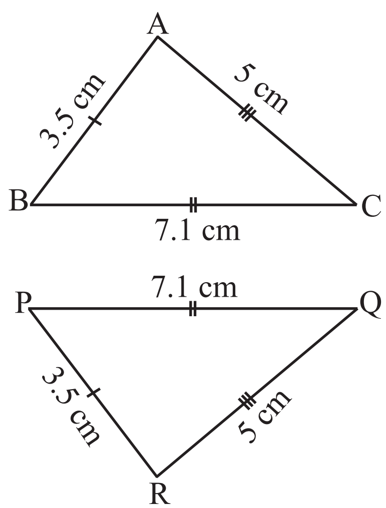


Fig 7.12

SOLUTION Here, AB = PR (= 3.5 cm),

BC = PQ (= 7.1 cm)

and AC = QR (= 5 cm)

This shows that the three sides of one triangle are equal to the three sides of the other triangle. So, by SSS congruence rule, the two triangles are congruent. From the above three equality relations, it can be easily seen that A $\leftrightarrow$ R, B $\leftrightarrow$ P and C $\leftrightarrow$ Q.

So, we have $\Delta ABC \cong \Delta RPQ$

Important note: The order of the letters in the names of congruent triangles displays the corresponding relationships. Thus, when you write $\Delta ABC \cong \Delta RPQ$, you would know that A lies on R, B on P, C on Q, $\overline{AB}$ along $\overline{RP}$, $\overline{BC}$ along $\overline{PQ}$ and $\overline{AC}$ along $\overline{RQ}$.

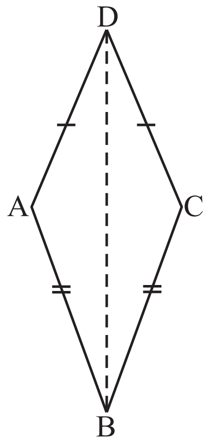


Fig 7.13

EXAMPLE 3 In Fig 7.13, AD = CD and AB = CB.

(i) State the three pairs of equal parts in ΔABD and ΔCBD.

(ii) Is ΔABD ≅ ΔCBD? Why or why not?

(iii) Does BD bisect ∠ABC? Give reasons.

SOLUTION

(i) In ΔABD and ΔCBD, the three pairs of equal parts are as given below:

AB = CB (Given)

AD = CD (Given)

and BD = BD (Common in both)

(ii) From (i) above, ΔABD ≅ ΔCBD (By SSS congruence rule)

(iii) ∠ABD = ∠CBD (Corresponding parts of congruent triangles)

So, BD bisects ∠ABC.

TRY THESE

1. In Fig 7.14, lengths of the sides of the triangles are indicated. By applying the SSS congruence rule, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form:

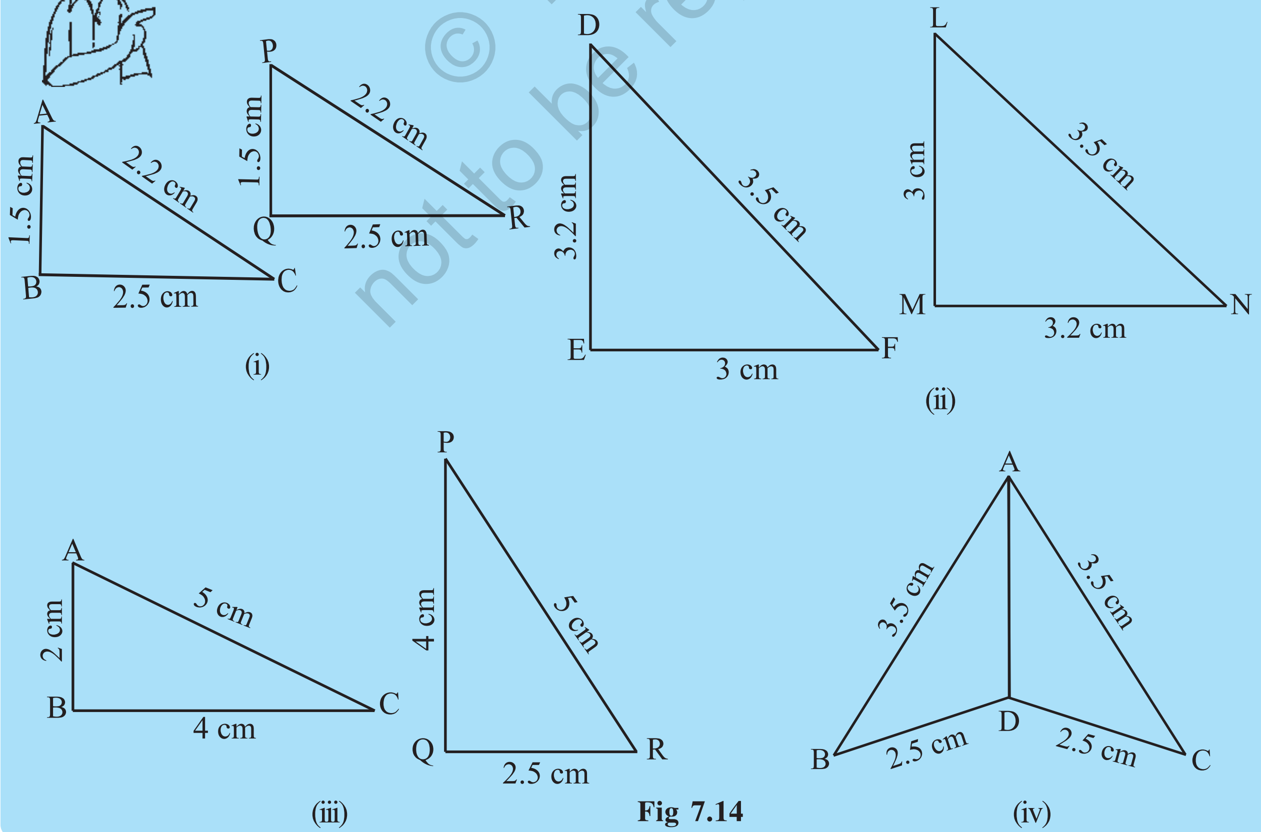


Fig 7.14

2. In Fig 7.15, AB = AC and D is the mid-point of $\overline{BC}$.

(i) State the three pairs of equal parts in ΔADB and ΔADC.

(ii) Is $\Delta ADB \cong \Delta ADC$? Give reasons.

(iii) Is $\angle B = \angle C$? Why?

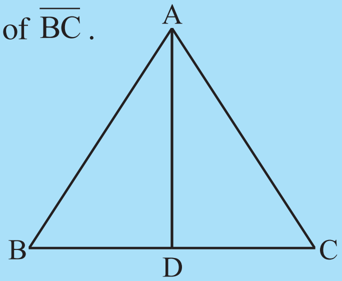


Fig 7.15

3. In Fig 7.16, AC = BD and AD = BC. Which of the following statements is meaningfully written?

(i) $\Delta ABC \cong \Delta ABD$ (ii) $\Delta ABC \cong \Delta BAD$.

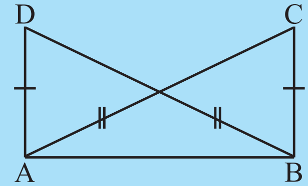


Fig 7.16

Think, Discuss and Write

ABC is an isosceles triangle with AB = AC (Fig 7.17).
Take a trace-copy of ΔABC and also name it as ΔABC.

(i) State the three pairs of equal parts in ΔABC and ΔACB.

(ii) Is $\Delta ABC \cong \Delta ACB$? Why or why not?

(iii) Is $\angle B = \angle C$? Why or why not?

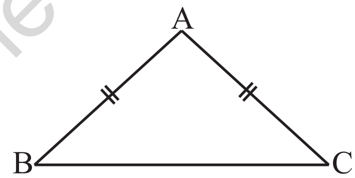


Fig 7.17

Appu and Tippu now turn to playing the game with a slight modification.

SAS Game

Appu : Let me now change the rules of the triangle-copying game.

Tippu : Right, go ahead.

Appu : You have already found that giving the length of only one side is useless.

Tippu : Of course, yes.

Appu : In that case, let me tell that in ΔABC, one side is 5.5 cm and one angle is 65°.

Tippu : This again is not sufficient for the job. I can find many triangles satisfying your information, but are not copies of ΔABC. For example, I have given here some of them (Fig 7.18):

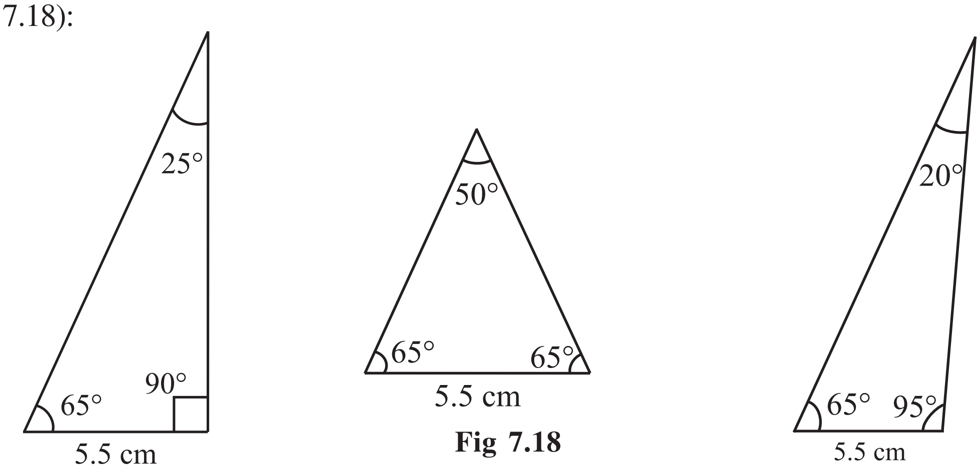


Fig 7.18

Appu : So, what shall we do?

Tippu : More information is needed.

Appu : Then, let me modify my earlier statement. In ΔABC, the length of two sides are 5.5 cm and 3.4 cm, and the angle between these two sides is 65°.

Tippu : This information should help me. Let me try. I draw first $\overline{BC}$ of length 5.5. cm [Fig 7.19 (i)]. Now I make 65° at C [Fig 7.19 (ii)].

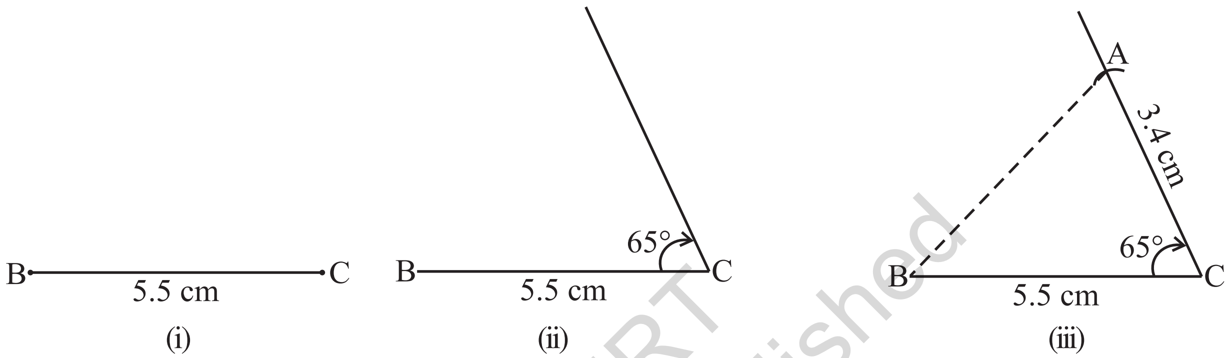


Fig 7.19

Yes, I got it, A must be 3.4 cm away from C along this angular line through C.

I draw an arc of 3.4 cm with C as centre. It cuts the 65° line at A.

Now, I join AB and get ΔABC [Fig 7.19(iii)].

Appu : You have used side-angle-side, where the angle is 'included' between the sides!

Tippu : Yes. How shall we name this criterion?

Appu : It is SAS criterion. Do you follow it?

Tippu : Yes, of course.

> **SAS Congruence criterion:**
> If under a correspondence, two sides and the angle included between them of a triangle are equal to two corresponding sides and the angle included between them of another triangle, then the triangles are congruent.

EXAMPLE 4 Given below are measurements of some parts of two triangles. Examine whether the two triangles are congruent or not, by using SAS congruence rule. If the triangles are congruent, write them in symbolic form.

	ΔABC	ΔDEF
(a)	AB = 7 cm, BC = 5 cm, $\angle B = 50°$	DE = 5 cm, EF = 7 cm, $\angle E = 50°$
(b)	AB = 4.5 cm, AC = 4 cm, $\angle A = 60°$	DE = 4 cm, FD = 4.5 cm, $\angle D = 55°$
(c)	BC = 6 cm, AC = 4 cm, $\angle B = 35°$	DF = 4 cm, EF = 6 cm, $\angle E = 35°$

(It will be always helpful to draw a rough figure, mark the measurements and then probe the question).

SOLUTION

(a) Here, AB = EF (= 7 cm), BC = DE (= 5 cm) and included $\angle$B = included $\angle$E (= 50°). Also, A $\leftrightarrow$ F B $\leftrightarrow$ E and C $\leftrightarrow$ D.

Therefore, ΔABC $\cong$ ΔFED (By SAS congruence rule) (Fig 7.20)

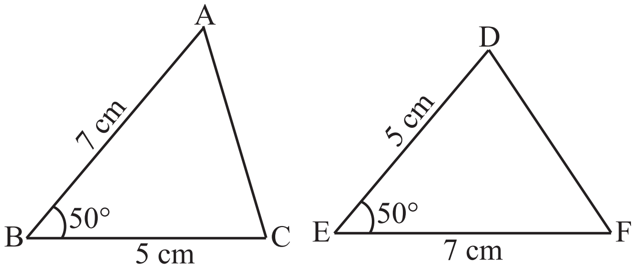


Fig 7.20

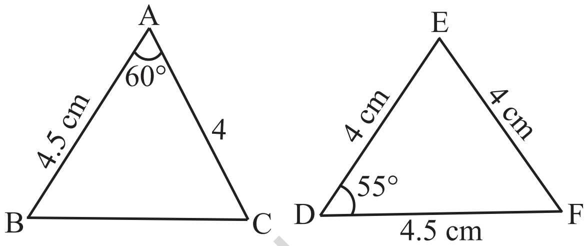


Fig 7.21

(b) Here, AB = FD and AC = DE (Fig 7.21).

But included $\angle$A $\neq$ included $\angle$D. So, we cannot say that the triangles are congruent.

(c) Here, BC = EF, AC = DF and $\angle$B = $\angle$E.

But $\angle$B is not the included angle between the sides AC and BC.

Similarly, $\angle$E is not the included angle between the sides EF and DF.

So, SAS congruence rule cannot be applied and we cannot conclude that the two triangles are congruent.

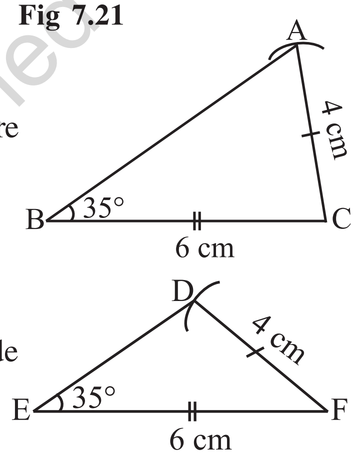


Fig 7.22

EXAMPLE 5 In Fig 7.23, AB = AC and AD is the bisector of $\angle$BAC.

(i) State three pairs of equal parts in triangles ADB and ADC.

(ii) Is ΔADB $\cong$ ΔADC? Give reasons.

(iii) Is $\angle$B = $\angle$C? Give reasons.

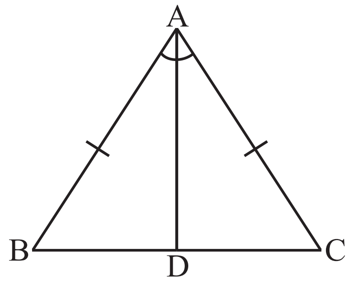


Fig 7.23

SOLUTION

(i) The three pairs of equal parts are as follows:

AB = AC (Given)

$\angle$BAD = $\angle$CAD (AD bisects $\angle$BAC) and AD = AD (common)

(ii) Yes, ΔADB $\cong$ ΔADC (By SAS congruence rule)

(iii) $\angle$B = $\angle$C (Corresponding parts of congruent triangles)

TRY THESE

1. Which angle is included between the sides $\overline{DE}$ and $\overline{EF}$ of ΔDEF?
2. By applying SAS congruence rule, you want to establish that ΔPQR $\cong$ ΔFED. It is given that PQ = FE and RP = DF. What additional information is needed to establish the congruence?

3. In Fig 7.24, measures of some parts of the triangles are indicated. By applying SAS congruence rule, state the pairs of congruent triangles, if any, in each case. In case of congruent triangles, write them in symbolic form.

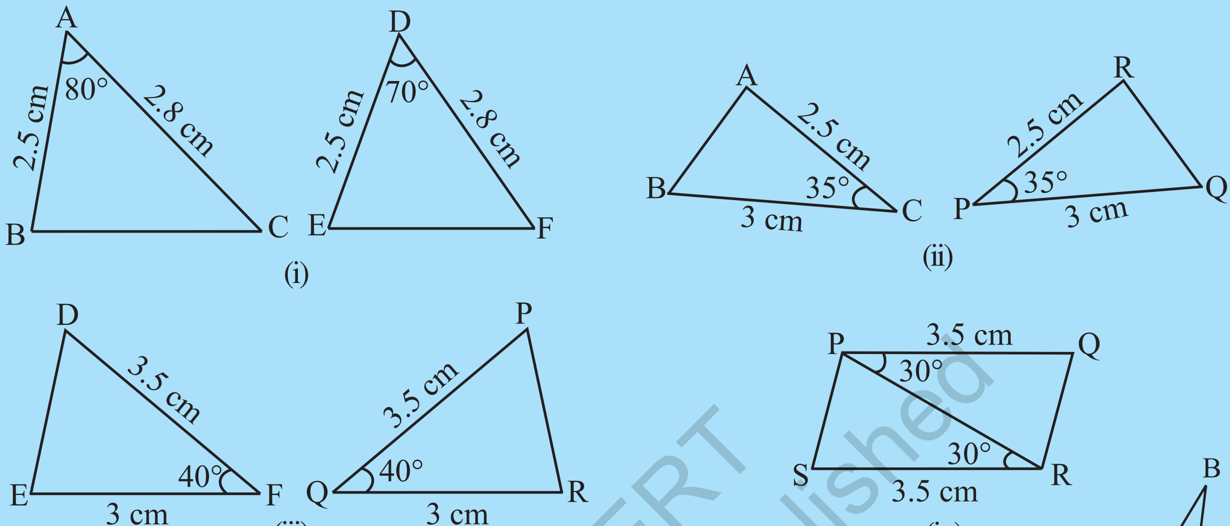


Fig 7.24

4. In Fig 7.25, $\overline{AB}$ and $\overline{CD}$ bisect each other at O.

   (i) State the three pairs of equal parts in two triangles AOC and BOD.

   (ii) Which of the following statements are true?

      (a) $\Delta AOC \cong \Delta DOB$

      (b) $\Delta AOC \cong \Delta BOD$

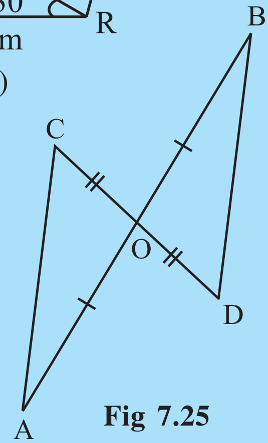


Fig 7.25

ASA Game

Can you draw Appu's triangle, if you know

(i) only one of its angles? (ii) only two of its angles?

(iii) two angles and any one side?

(iv) two angles and the side included between them?

Attempts to solve the above questions lead us to the following criterion:

ASA Congruence criterion:

If under a correspondence, two angles and the included side of a triangle are equal to two corresponding angles and the included side of another triangle, then the triangles are congruent.

EXAMPLE 6 By applying ASA congruence rule, it is to be established that $\Delta ABC \cong \Delta QRP$ and it is given that BC = RP. What additional information is needed to establish the congruence?

SOLUTION For ASA congruence rule, we need the two angles between which the two sides BC and RP are included. So, the additional information is as follows:

$$\angle B = \angle R$$

and $$\angle C = \angle P$$

EXAMPLE 7 In Fig 7.26, can you use ASA congruence rule and conclude that $\Delta AOC \cong \Delta BOD$?

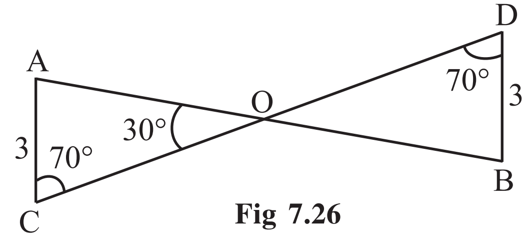


Fig 7.26

SOLUTION In the two triangles AOC and BOD, $\angle C = \angle D$ (each 70°)

Also, $\angle AOC = \angle BOD = 30°$ (vertically opposite angles)

So, $\angle A$ of $\Delta AOC = 180° - (70° + 30°) = 80°$

(using angle sum property of a triangle)

Similarly, $\angle B$ of $\Delta BOD = 180° - (70° + 30°) = 80°$

Thus, we have $\angle A = \angle B$, $AC = BD$ and $\angle C = \angle D$

Now, side AC is between $\angle A$ and $\angle C$ and side BD is between $\angle B$ and $\angle D$.

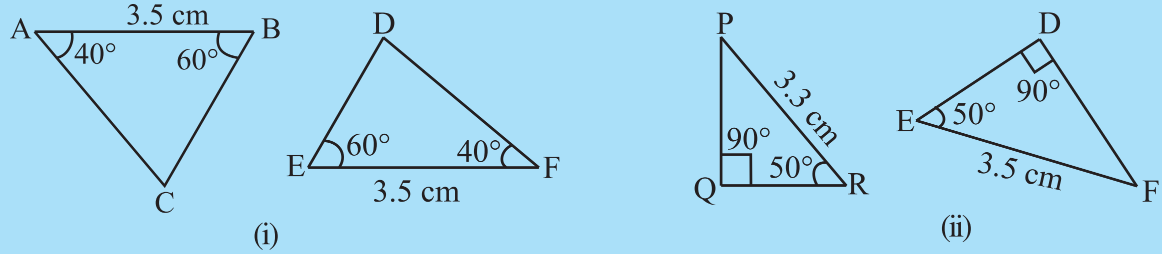
So, by ASA congruence rule, $\Delta AOC \cong \Delta BOD$.

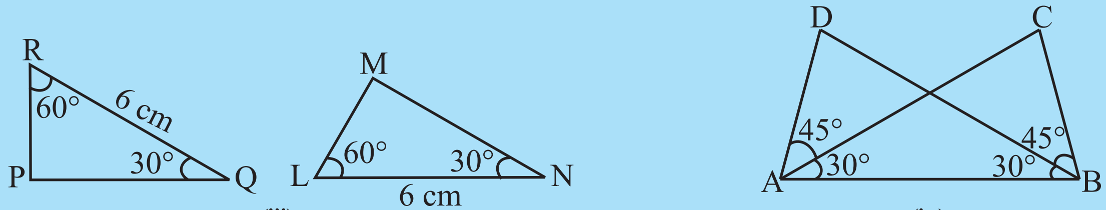
Remark

Given two angles of a triangle, you can always find the third angle of the triangle. So, whenever, two angles and one side of one triangle are equal to the corresponding two angles and one side of another triangle, you may convert it into 'two angles and the included side' form of congruence and then apply the ASA congruence rule.

TRY THESE

1. What is the side included between the angles M and N of ΔMNP?
2. You want to establish $\Delta DEF \cong \Delta MNP$, using the ASA congruence rule. You are given that $\angle D = \angle M$ and $\angle F = \angle P$. What information is needed to establish the congruence? (Draw a rough figure and then try!)
3. In Fig 7.27, measures of some parts are indicated. By applying ASA congruence rule, state which pairs of triangles are congruent. In case of congruence, write the result in symoblic form.





(iii) (iv)

Fig 7.27

4. Given below are measurements of some parts of two triangles. Examine whether the two triangles are congruent or not, by ASA congruence rule. In case of congruence, write it in symbolic form.

	ΔDEF	**ΔPQR**
(i)	$\angle D = 60°$, $\angle F = 80°$, DF = 5 cm	$\angle Q = 60°$, $\angle R = 80°$, QR = 5 cm
(ii)	$\angle D = 60°$, $\angle F = 80°$, DF = 6 cm	$\angle Q = 60°$, $\angle R = 80°$, QP = 6 cm
(iii)	$\angle E = 80°$, $\angle F = 30°$, EF = 5 cm	$\angle P = 80°$, PQ = 5 cm, $\angle R = 30°$

5. In Fig 7.28, ray AZ bisects $\angle DAB$ as well as $\angle DCB$.

(i) State the three pairs of equal parts in triangles BAC and DAC.

(ii) Is $\Delta BAC \cong \Delta DAC$? Give reasons.

(iii) Is AB = AD? Justify your answer.

(iv) Is CD = CB? Give reasons.

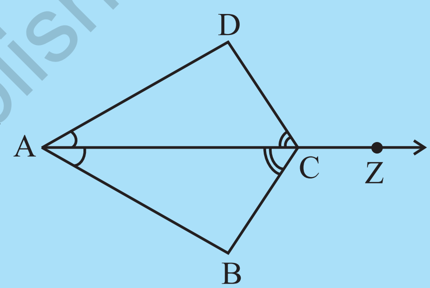


Fig 7.28

7.7 Congruence Among Right-angled Triangles

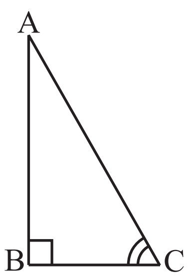


Fig 7.29

Congruence in the case of two right triangles deserves special attention. In such triangles, obviously, the right angles are equal. So, the congruence criterion becomes easy.

Can you draw ΔABC (shown in Fig 7.29) with $\angle B = 90°$, if

(i) only BC is known? (ii) only $\angle C$ is known?

(iii) $\angle A$ and $\angle C$ are known? (iv) AB and BC are known?

(v) AC and one of AB or BC are known?

Try these with rough sketches. You will find that (iv) and (v) help you to draw the triangle. But case (iv) is simply the SAS condition. Case (v) is something new. This leads to the following criterion:

> **RHS Congruence criterion:**
>
> If under a correspondence, the hypotenuse and one side of a right-angled triangle are respectively equal to the hypotenuse and one side of another right-angled triangle, then the triangles are congruent.

Why do we call this 'RHS' congruence? Think about it.

EXAMPLE 8 Given below are measurements of some parts of two triangles. Examine whether the two triangles are congruent or not, using RHS congruence rule. In case of congruent triangles, write the result in symbolic form:

	ΔABC	ΔPQR
(i)	$\angle B = 90°$, AC = 8 cm, AB = 3 cm	$\angle P = 90°$, PR = 3 cm, QR = 8 cm
(ii)	$\angle A = 90°$, AC = 5 cm, BC = 9 cm	$\angle Q = 90°$, PR = 8 cm, PQ = 5 cm

SOLUTION

(i) Here, $\angle B = \angle P = 90°$,
hypotenuse, AC = hypotenuse, RQ (= 8 cm) and
side AB = side RP (= 3 cm)
So, $\Delta ABC \cong \Delta RPQ$ (By RHS Congruence rule). [Fig 7.30(i)]

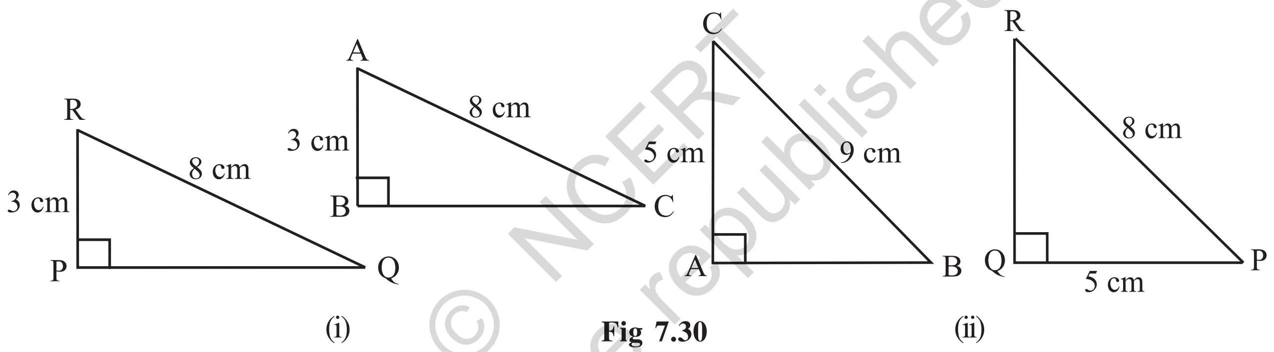


Fig 7.30

(ii) Here, $\angle A = \angle Q$ (= 90°) and
side AC = side PQ (= 5 cm).
But hypotenuse BC $\neq$ hypotenuse PR [Fig 7.30(ii)]
So, the triangles are not congruent.

EXAMPLE 9 In Fig 7.31, DA $\perp$ AB, CB $\perp$ AB and AC = BD.
State the three pairs of equal parts in ΔABC and ΔDAB.
Which of the following statements is meaningful?

(i) $\Delta ABC \cong \Delta BAD$ (ii) $\Delta ABC \cong \Delta ABD$

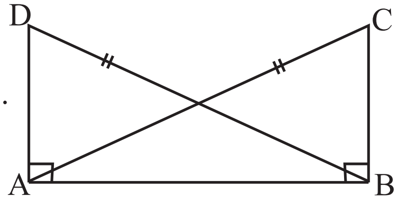


Fig 7.31

SOLUTION The three pairs of equal parts are:

$$\angle ABC = \angle BAD \ (= 90°)$$
$$AC = BD \ \text{(Given)}$$
$$AB = BA \ \text{(Common side)}$$

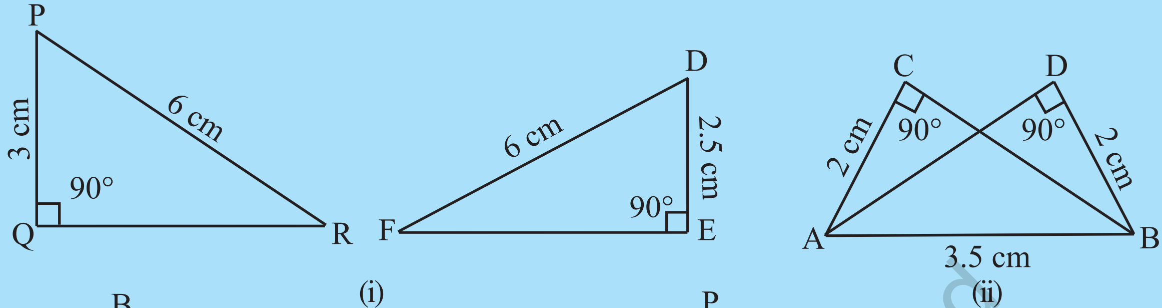
From the above, $\Delta ABC \cong \Delta BAD$ (By RHS congruence rule).

So, statement (i) is true

Statement (ii) is not meaningful, in the sense that the correspondence among the vertices is not satisfied.

Try These

1. In Fig 7.32, measures of some parts of triangles are given.By applying RHS congruence rule, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form.



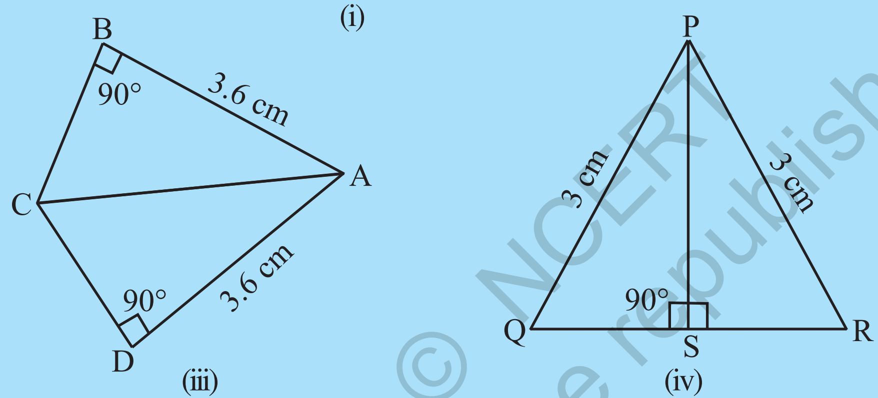


Fig 7.32

2. It is to be established by RHS congruence rule that $\Delta ABC \cong \Delta RPQ$. What additional information is needed, if it is given that

$\angle B = \angle P = 90^\circ$ and $AB = RP$?

3. In Fig 7.33, BD and CE are altitudes of ΔABC such that $BD = CE$.

(i) State the three pairs of equal parts in ΔCBD and ΔBCE.

(ii) Is $\Delta CBD \cong \Delta BCE$? Why or why not?

(iii) Is $\angle DCB = \angle EBC$? Why or why not?

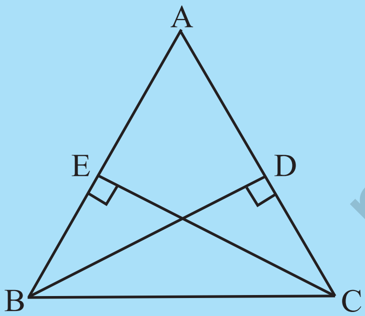


Fig 7.33

4. ABC is an isosceles triangle with $AB = AC$ and AD is one of its altitudes (Fig 7.34).

(i) State the three pairs of equal parts in ΔADB and ΔADC.

(ii) Is $\Delta ADB \cong \Delta ADC$? Why or why not?

(iii) Is $\angle B = \angle C$? Why or why not?

(iv) Is $BD = CD$? Why or why not?

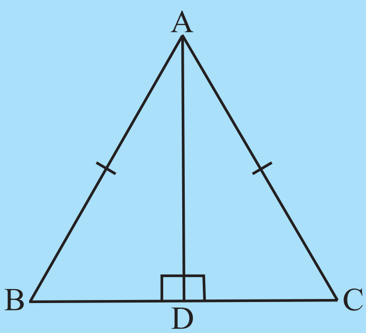


Fig 7.34

We now turn to examples and problems based on the criteria seen so far.

Exercise 7.2

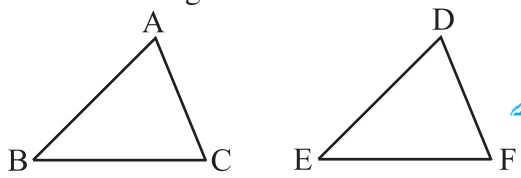
1. Which congruence criterion do you use in the following?

(a) **Given:** AC = DF

AB = DE

BC = EF

So, $\Delta ABC \cong \Delta DEF$

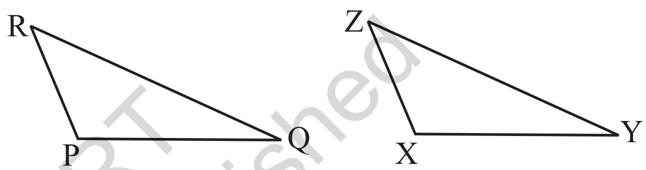


(b) **Given:** ZX = RP

RQ = ZY

$\angle PRQ = \angle XZY$

So, $\Delta PQR \cong \Delta XYZ$

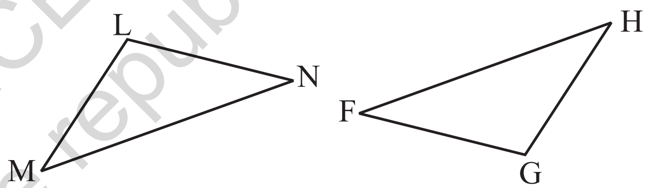


(c) **Given:** $\angle MLN = \angle FGH$

$\angle NML = \angle GFH$

ML = FG

So, $\Delta LMN \cong \Delta GFH$

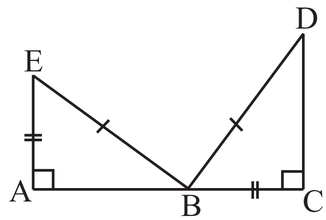


(d) **Given:** EB = DB

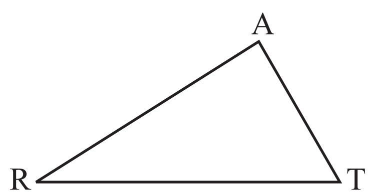
AE = BC

$\angle A = \angle C = 90°$

So, $\Delta ABE \cong \Delta CDB$



2. You want to show that $\Delta ART \cong \Delta PEN$,

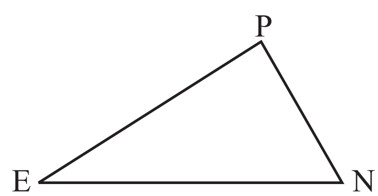


(a) If you have to use SSS criterion, then you need to show

(i) AR = (ii) RT = (iii) AT =

(b) If it is given that $\angle T = \angle N$ and you are to use SAS criterion, you need to have

(i) RT = and (ii) PN =

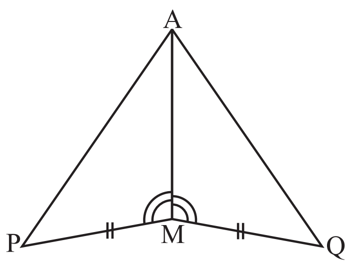


(c) If it is given that AT = PN and you are to use ASA criterion, you need to have

(i) ? (ii) ?

3. You have to show that $\Delta AMP \cong \Delta AMQ$.

In the following proof, supply the missing reasons.



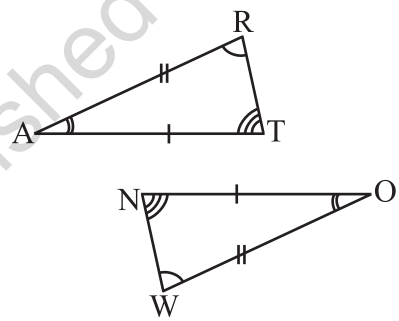
Steps	Reasons
(i) PM = QM	(i) ...
(ii) $\angle PMA = \angle QMA$	(ii) ...
(iii) AM = AM	(iii) ...
(iv) $\Delta AMP \cong \Delta AMQ$	(iv) ...

4. In ΔABC, $\angle A = 30°$, $\angle B = 40°$ and $\angle C = 110°$

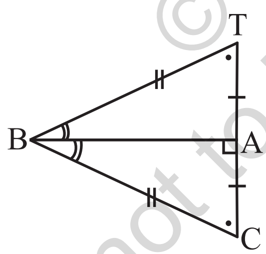
In ΔPQR, $\angle P = 30°$, $\angle Q = 40°$ and $\angle R = 110°$

A student says that $\Delta ABC \cong \Delta PQR$ by AAA congruence criterion. Is he justified? Why or why not?

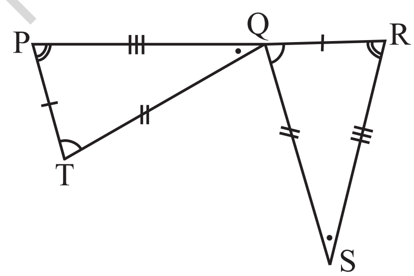
5. In the figure, the two triangles are congruent. The corresponding parts are marked. We can write $\Delta RAT \cong$?



6. Complete the congruence statement:



$\Delta BCA \cong$?



$\Delta QRS \cong$?

7. In a squared sheet, draw two triangles of equal areas such that

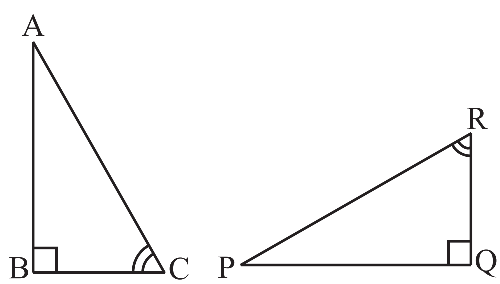
(i) the triangles are congruent.

(ii) the triangles are not congruent.

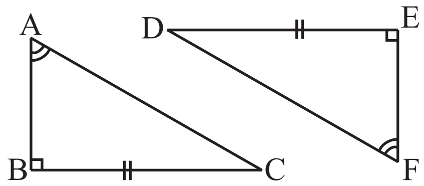
What can you say about their perimeters?

8. Draw a rough sketch of two triangles such that they have five pairs of congruent parts but still the triangles are not congruent.

9. If ΔABC and ΔPQR are to be congruent, name one additional pair of corresponding parts. What criterion did you use?



10. Explain, why
$\Delta ABC \cong \Delta FED$.



Enrichment activity

We saw that superposition is a useful method to test congruence of plane figures. We discussed conditions for congruence of line segments, angles and triangles. You can now try to extend this idea to other plane figures as well.

1. Consider cut-outs of different sizes of squares. Use the method of superposition to find out the condition for congruence of squares. How does the idea of 'corresponding parts' under congruence apply? Are there corresponding sides? Are there corresponding diagonals?
2. What happens if you take circles? What is the condition for congruence of two circles? Again, you can use the method of superposition. Investigate.
3. Try to extend this idea to other plane figures like regular hexagons, etc.
4. Take two congruent copies of a triangle. By paper folding, investigate if they have equal altitudes. Do they have equal medians? What can you say about their perimeters and areas?

WHAT HAVE WE DISCUSSED?

1. Congruent objects are exact copies of one another.

2. The method of superposition examines the congruence of plane figures.

3. Two plane figures, say, F_1 and F_2 are congruent if the trace-copy of F_1 fits exactly on that of F_2. We write this as $F_1 \cong F_2$.

4. Two line segments, say, $\overline{AB}$ and $\overline{CD}$, are congruent if they have equal lengths. We write this as $\overline{AB} \cong \overline{CD}$. However, it is common to write it as $\overline{AB} = \overline{CD}$.

5. Two angles, say, $\angle ABC$ and $\angle PQR$, are congruent if their measures are equal. We write this as $\angle ABC \cong \angle PQR$ or as $m\angle ABC = m\angle PQR$. However, in practice, it is common to write it as $\angle ABC = \angle PQR$.

6. SSS Congruence of two triangles:

Under a given correspondence, two triangles are congruent if the three sides of the one are equal to the three corresponding sides of the other.

7. SAS Congruence of two triangles:

Under a given correspondence, two triangles are congruent if two sides and the angle included between them in one of the triangles are equal to the corresponding sides and the angle included between them of the other triangle.

8. ASA Congruence of two triangles:

Under a given correspondence, two triangles are congruent if two angles and the side included between them in one of the triangles are equal to the corresponding angles and the side included between them of the other triangle.

9. RHS Congruence of two right-angled triangles:

Under a given correspondence, two right-angled triangles are congruent if the hypotenuse and a leg of one of the triangles are equal to the hypotenuse and the corresponding leg of the other triangle.

10. There is no such thing as AAA Congruence of two triangles:

Two triangles with equal corresponding angles need not be congruent. In such a correspondence, one of them can be an enlarged copy of the other. (They would be congruent only if they are exact copies of one another).

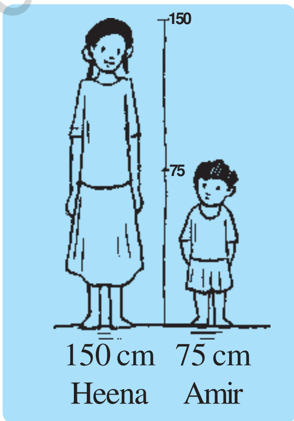
Chapter 8

Comparing Quantities

0756CH08

8.1 Introduction

In our daily life, there are many occasions when we compare two quantities. Suppose we are comparing heights of Heena and Amir. We find that



1. Heena is two times taller than Amir.

   Or

2. Amir's height is $\frac{1}{2}$ of Heena's height.

Consider another example, where 20 marbles are divided between Rita and Amit such that Rita has 12 marbles and Amit has 8 marbles. We say,

1. Rita has $\frac{3}{2}$ times the marbles that Amit has.

   Or

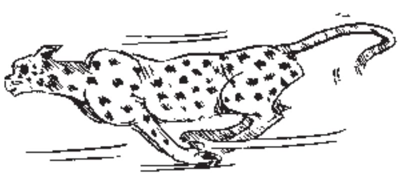
2. Amit has $\frac{2}{3}$ part of what Rita has.

Yet another example is where we compare speeds of a Cheetah and a Man.

The speed of a Cheetah is 6 times the speed of a Man.

Or

The speed of a Man is $\frac{1}{6}$ of the speed of the Cheetah.



Speed of Cheetah
120 km per hour

Speed of Man
20 km per hour

Do you remember comparisons like this? In Class VI, we have learnt to make comparisons by saying how many times one quantity is of the other. Here, we see that it can also be inverted and written as what part one quantity is of the other.

In the given cases, we write the ratio of the heights as :

Heena's height : Amir's height is 150 : 75 or 2 : 1.

Can you now write the ratios for the other comparisons?

These are relative comparisons and could be same for two different situations.

If Heena's height was 150 cm and Amir's was 100 cm, then the ratio of their heights would be,

Heena's height : Amir's height = 150 : 100 = $\frac{150}{100} = \frac{3}{2}$ or 3 : 2.

This is same as the ratio for Rita's to Amit's share of marbles.

Thus, we see that the ratio for two different comparisons may be the same. Remember that *to compare two quantities, the units must be the same.*

A ratio has no units.

EXAMPLE 1 Find the ratio of 3 km to 300 m.

SOLUTION First convert both the distances to the same unit.

So, 3 km = 3 × 1000 m = 3000 m.

Thus, the required ratio, 3 km : 300 m is 3000 : 300 = 10 : 1.

8.2 EQUIVALENT RATIOS

Different ratios can also be compared with each other to know whether they are equivalent or not. To do this, we need to write the ratios in the form of fractions and then compare them by converting them to like fractions. If these like fractions are equal, we say the given ratios are equivalent.

EXAMPLE 2 Are the ratios 1:2 and 2:3 equivalent?

SOLUTION To check this, we need to know whether $\frac{1}{2} = \frac{2}{3}$.

We have, $\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$; $\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$

We find that $\frac{3}{6} < \frac{4}{6}$, which means that $\frac{1}{2} < \frac{2}{3}$.

Therefore, the ratio 1:2 is not equivalent to the ratio 2:3.

Use of such comparisons can be seen by the following example.

EXAMPLE 3 Following is the performance of a cricket team in the matches it played:

Year	Wins	Losses
Last year	8	2
This year	4	2

In which year was the record better?

How can you say so?

SOLUTION Last year, Wins: Losses = 8 : 2 = 4 : 1

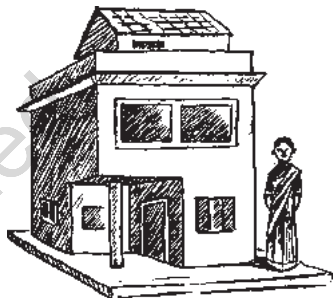
This year, Wins: Losses = 4 : 2 = 2 : 1

Obviously, 4 : 1 > 2 : 1 (In fractional form, $\frac{4}{1} > \frac{2}{1}$)

Hence, we can say that the team performed better last year.

In Class VI, we have also seen the importance of equivalent ratios. The ratios which are equivalent are said to be in proportion. Let us recall the use of proportions.

Keeping things in proportion and getting solutions



Aruna made a sketch of the building she lives in and drew sketch of her mother standing beside the building.

Mona said, "There seems to be something wrong with the drawing"

Can you say what is wrong? How can you say this?

In this case, the ratio of heights in the drawing should be the same as the ratio of actual heights. That is

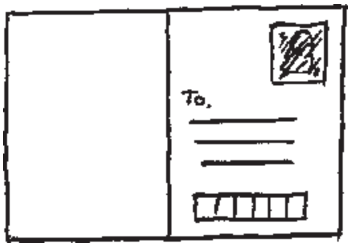
$$\frac{\text{Actual height of building}}{\text{Actual height of mother}} = \frac{\text{Height of building in drawing}}{\text{Height of mother in the drawing}}.$$

Only then would these be in proportion. Often when proportions are maintained, the drawing seems pleasing to the eye.

Another example where proportions are used is in the making of national flags.

Do you know that the flags are always made in a fixed ratio of length to its breadth? These may be different for different countries but are mostly around 1.5 : 1 or 1.7 : 1.

We can take an approximate value of this ratio as 3 : 2. Even the Indian post card is around the same ratio.



Now, can you say whether a card with length 4.5 cm and breadth 3.0 cm is near to this ratio. That is we need to ask, is 4.5 : 3.0 equivalent to 3 : 2?

We note that $4.5 : 3.0 = \frac{4.5}{3.0} = \frac{45}{30} = \frac{3}{2}$

Hence, we see that 4.5 : 3.0 is equivalent to 3 : 2.

We see a wide use of such proportions in real life. Can you think of some more situations?

We have also learnt a method in the earlier classes known as *Unitary Method* in which we first find the value of one unit and then the value of the required number of units.

Let us see how both the above methods help us to achieve the same thing.

EXAMPLE 4 A map is given with a scale of 2 cm = 1000 km. What is the actual distance between the two places in kms, if the distance in the map is 2.5 cm?

SOLUTION

Arun does it like this	**Meera does it like this**
Let distance $= x$ km	2 cm means 1000 km.
then, $1000 : x = 2 : 2.5$	So, 1 cm means $\frac{1000}{2}$ km
$\frac{1000}{x} = \frac{2}{2.5}$	Hence, 2.5 cm means $\frac{1000}{2} \times 2.5$ km
$\frac{1000 \times x \times 2.5}{x} = \frac{2}{2.5} \times x \times 2.5$	$= 1250$ km
$1000 \times 2.5 = x \times 2$	
$x = 1250$	

Arun has solved it by equating ratios to make proportions and then by solving the equation. Meera has first found the distance that corresponds to 1 cm and then used that to find what 2.5 cm would correspond to. She used the unitary method.

Let us solve some more examples using the unitary method.

EXAMPLE 5 6 bowls cost ₹ 90. What would be the cost of 10 such bowls?

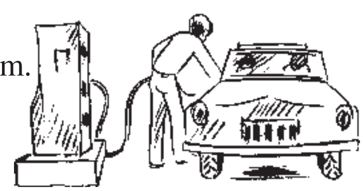
SOLUTION Cost of 6 bowls is ₹ 90.

Therefore, cost of 1 bowl $= ₹\ \frac{90}{6}$

Hence, cost of 10 bowls $= ₹\ \frac{90}{6} \times 10 = ₹\ 150$

EXAMPLE 6 The car that I own can go 150 km with 25 litres of petrol. How far can it go with 30 litres of petrol?

SOLUTION With 25 litres of petrol, the car goes 150 km.



With 1 litre the car will go $\frac{150}{25}$ km.

Hence, with 30 litres of petrol it would go $\frac{150}{25} \times 30$ km $= 180$ km

In this method, we first found the value for one unit or the unit rate. This is done by the comparison of two different properties. For example, when you compare total cost to number of items, we get cost per item or if you take distance travelled to time taken, we get distance per unit time.

Thus, you can see that we often use per to mean for each.

For example, km per hour, children per teacher etc., denote unit rates.

Think, Discuss and Write

An ant can carry 50 times its weight. If a person can do the same, how much would you be able to carry?

Exercise 8.1

1. Find the ratio of:
   (a) ₹ 5 to 50 paise (b) 15 kg to 210 g
   (c) 9 m to 27 cm (d) 30 days to 36 hours

2. In a computer lab, there are 3 computers for every 6 students. How many computers will be needed for 24 students?
3. Population of Rajasthan = 570 lakhs and population of UP = 1660 lakhs.
   Area of Rajasthan = 3 lakh km^2 and area of UP = 2 lakh km^2.
   (i) How many people are there per km^2 in both these States?
   (ii) Which State is less populated?

8.3 Percentage – Another Way of Comparing Quantities

Anita's Report
Total 320/400
Percentage: 80

Rita's Report
Total 300/360
Percentage: 83.3

Anita said that she has done better as she got 320 marks whereas Rita got only 300. Do you agree with her? Who do you think has done better?

Mansi told them that they cannot decide who has done better by just comparing the total marks obtained because the maximum marks out of which they got the marks are not the same.

She said why don't you see the Percentages given in your report cards?

Anita's Percentage was 80 and Rita's was 83.3. So, this shows Rita has done better.

Do you agree?

Percentages are numerators of fractions with denominator 100 and have been used in comparing results. Let us try to understand in detail about it.

8.3.1 Meaning of Percentage

Per cent is derived from Latin word 'per centum' meaning 'per hundred'.

Per cent is represented by the symbol % and means hundredths too. That is 1% means 1 out of hundred or one hundredth. It can be written as: $1\% = \frac{1}{100} = 0.01$

To understand this, let us consider the following example.

Rina made a table top of 100 different coloured tiles. She counted yellow, green, red and blue tiles separately and filled the table below. Can you help her complete the table?

Colour	Number of Tiles	Rate per Hundred	Fraction	Written as	Read as
Yellow	14	14	$\frac{14}{100}$	14%	14 per cent
Green	26	26	$\frac{26}{100}$	26%	26 per cent
Red	35	35	----	----	----
Blue	25	--------	----	----	----
Total	**100**				

TRY THESE

1. Find the Percentage of children of different heights for the following data.

Height	Number of Children	In Fraction	In Percentage
110 cm	22		
120 cm	25		
128 cm	32		
130 cm	21		
Total	**100**		

2. A shop has the following number of shoe pairs of different sizes.

Size 2 : 20 Size 3 : 30 Size 4 : 28

Size 5 : 14 Size 6 : 8

Write this information in tabular form as done earlier and find the Percentage of each shoe size available in the shop.

Percentages when total is not hundred

In all these examples, the total number of items add up to 100. For example, Rina had 100 tiles in all, there were 100 children and 100 shoe pairs. How do we calculate Percentage of an item if the total number of items do not add up to 100? In such cases, we need to convert the fraction to an equivalent fraction with denominator 100. Consider the following example. You have a necklace with twenty beads in two colours.

Colour	Number of Beads	Fraction	Denominator Hundred	In Percentage
Red	8	$\frac{8}{20}$	$\frac{8}{20}\times\frac{100}{100}=\frac{40}{100}$	40%
Blue	12	$\frac{12}{20}$	$\frac{12}{20}\times\frac{100}{100}=\frac{60}{100}$	60%
Total	**20**			

Anwar found the Percentage of red beads like this

Out of 20 beads, the number of red beads is 8.

Hence, out of 100, the number of red beads is

$\frac{8}{20}\times 100 = 40$ (out of hundred) = 40%

Asha does it like this

$\frac{8}{20}=\frac{8\times5}{20\times5}$

$=\frac{40}{100}=40\%$

We see that these three methods can be used to find the Percentage when the total does not add to give 100. In the method shown in the table, we multiply the fraction by $\frac{100}{100}$. This does not change the value of the fraction. Subsequently, only 100 remains in the denominator.

Anwar has used the unitary method. Asha has multiplied by $\frac{5}{5}$ to get 100 in the denominator. You can use whichever method you find suitable. May be, you can make your own method too.

The method used by Anwar can work for all ratios. Can the method used by Asha also work for all ratios? Anwar says Asha's method can be used only if you can find a natural number which on multiplication with the denominator gives 100. Since denominator was 20, she could multiply it by 5 to get 100. If the denominator was 6, she would not have been able to use this method. Do you agree?

TRY THESE

1. A collection of 10 chips with different colours is given .

G G G G
B B B
R R R

Colour	Number	Fraction	Denominator Hundred	In Percentage
Green				
Blue				
Red				
Total				

Fill the table and find the percentage of chips of each colour.

2. Mala has a collection of bangles. She has 20 gold bangles and 10 silver bangles. What is the percentage of bangles of each type? Can you put it in the tabular form as done in the above example?

Think, Discuss and Write

1. Look at the examples below and in each of them, discuss which is better for comparison.

   In the atmosphere, 1 g of air contains:

   .78 g Nitrogen
   .21 g Oxygen
   .01 g Other gas

   or

   78% Nitrogen
   21% Oxygen
   1% Other gas

2. A shirt has:

   $\frac{3}{5}$ Cotton
   $\frac{2}{5}$ Polyster

   or

   60% Cotton
   40% Polyster

8.3.2 Converting Fractional Numbers to Percentage

Fractional numbers can have different denominator. To compare fractional numbers, we need a common denominator and we have seen that it is more convenient to compare if our denominator is 100. That is, we are converting the fractions to Percentages. Let us try converting different fractional numbers to Percentages.

Example 7 Write $\frac{1}{3}$ as per cent.

Solution We have, $\frac{1}{3} = \frac{1}{3} \times \frac{100}{100} = \frac{1}{3} \times 100\%$

$$= \frac{100}{3}\% = 33\frac{1}{3}\%$$

Example 8 Out of 25 children in a class, 15 are girls. What is the percentage of girls?

Solution Out of 25 children, there are 15 girls.

Therefore, percentage of girls = $\frac{15}{25} \times 100 = 60$. There are 60% girls in the class.

Example 9 Convert $\frac{5}{4}$ to per cent.

Solution We have, $\frac{5}{4} = \frac{5}{4} \times 100\% = 125\%$

From these examples, we find that the percentages related to proper fractions are less than 100 whereas percentages related to improper fractions are more than 100.

Think, Discuss and Write

(i) Can you eat 50% of a cake? Can you eat 100% of a cake?
Can you eat 150% of a cake?

(ii) Can a price of an item go up by 50%? Can a price of an item go up by 100%?
Can a price of an item go up by 150%?

8.3.3 Converting Decimals to Percentage

We have seen how fractions can be converted to per cents. Let us now find how decimals can be converted to per cents.

Example 10 Convert the given decimals to per cents:

(a) 0.75 (b) 0.09 (c) 0.2

Solution

(a) $0.75 = 0.75 \times 100\ \%$

$= \frac{75}{100} \times 100\ \% = 75\%$

(b) $0.09 = \frac{9}{100} = 9\ \%$

(c) $0.2 = \frac{2}{10} \times 100\% = 20\ \%$

Try These

1. Convert the following to per cents:

(a) $\frac{12}{16}$ (b) 3.5 (c) $\frac{49}{50}$ (d) $\frac{2}{2}$ (e) 0.05

2. (i) Out of 32 students, 8 are absent. What per cent of the students are absent?
(ii) There are 25 radios, 16 of them are out of order. What per cent of radios are out of order?
(iii) A shop has 500 items, out of which 5 are defective. What per cent are defective?
(iv) There are 120 voters, 90 of them voted yes. What per cent voted yes?

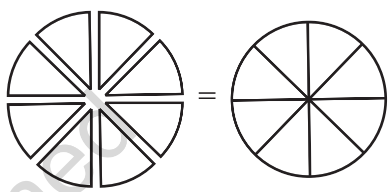
8.3.4 Converting Percentages to Fractions or Decimals

We have so far converted fractions and decimals to percentages. We can also do the reverse. That is, given per cents, we can convert them to decimals or fractions. Look at the table, observe and complete it:

Make some more such examples and solve them.

Per cent	1%	10%	25%	50%	90%	125%	250%
Fraction	$\frac{1}{100}$	$\frac{10}{100} = \frac{1}{10}$					
Decimal	0.01	0.10					

Parts always add to give a whole

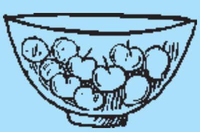


In the examples for coloured tiles, for the heights of children and for gases in the air, we find that when we add the Percentages we get 100. All the parts that form the whole when added together gives the whole or 100%. So, if we are given one part, we can always find out the other part. Suppose, 30% of a given number of students are boys.

This means that if there were 100 students, 30 out of them would be boys and the remaining would be girls.

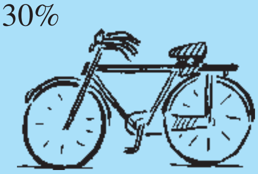
Then girls would obviously be (100 – 30)% = 70%.

Try These



1. 35% + _______% = 100%, 64% + 20% +________ % = 100%

   45% = 100% – _________ %, 70% = ______% – 30%



2. If 65% of students in a class have a bicycle, what per cent of the student do not have bicycles?
3. We have a basket full of apples, oranges and mangoes. If 50% are apples, 30% are oranges, then what per cent are mangoes?

Think, Discuss and Write

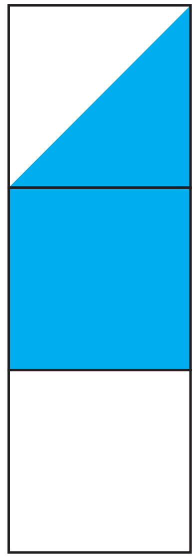
Consider the expenditure made on a dress

20% on embroidery, 50% on cloth, 30% on stitching.

Can you think of more such examples?

8.3.5 Fun with Estimation

Percentages help us to estimate the parts of an area.



EXAMPLE 11 What per cent of the adjoining figure is shaded?

SOLUTION We first find the fraction of the figure that is shaded. From this fraction, the percentage of the shaded part can be found.

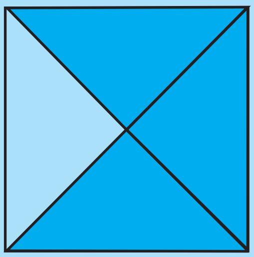
You will find that half of the figure is shaded. And, $\frac{1}{2} = \frac{1}{2} \times 100\ \% = 50\ \%$

Thus, 50 % of the figure is shaded.

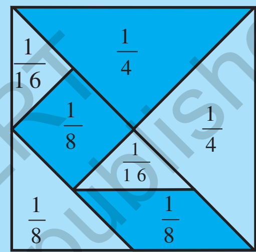
TRY THESE

What per cent of these figures are shaded?

(i)



(ii)



Tangram

You can make some more figures yourself and ask your friends to estimate the shaded parts.

8.4 Use of Percentages

8.4.1 Interpreting Percentages

We saw how percentages were helpful in comparison. We have also learnt to convert fractional numbers and decimals to percentages. Now, we shall learn how percentages can be used in real life. For this, we start with interpreting the following statements:

— 5 % of the income is saved by Ravi. — 20 % of Meera's dresses are blue in colour.

— Rekha gets 10 % on every book sold by her.

What can you infer from each of these statements?

By 5 % we mean 5 parts out of 100 or we write it as $\frac{5}{100}$. It means Ravi is saving ₹ 5 out of every ₹ 100 that he earns. In the same way, interpret the rest of the statements given above.

8.4.2 Converting Percentages to "How Many"

Consider the following examples:

EXAMPLE 12 A survey of 40 children showed that 25% liked playing football. How many children liked playing football?

SOLUTION Here, the total number of children are 40. Out of these, 25% like playing football. Meena and Arun used the following methods to find the number. You can choose either method.

Arun does it like this

Out of 100, 25 like playing football

So out of 40, number of children who like playing football $= \frac{25}{100} \times 40 = 10$

Meena does it like this

25% of $40 = \frac{25}{100} \times 40$

$= 10$

Hence, 10 children out of 40 like playing football.

Try These

1. Find:

(a) 50% of 164 (b) 75% of 12 (c) $12\frac{1}{2}\%$ of 64

2. 8 % children of a class of 25 like getting wet in the rain. How many children like getting wet in the rain.

Example 13 Rahul bought a sweater and saved ₹ 200 when a discount of 25% was given. What was the price of the sweater before the discount?

Solution Rahul has saved ₹ 200 when price of sweater is reduced by 25%. This means that 25% reduction in price is the amount saved by Rahul. Let us see how Mohan and Abdul have found the original cost of the sweater.

Mohan's solution

25% of the original price = ₹ 200

Let the price (in ₹) be P

So, 25% of $P = 200$ or $\frac{25}{100} \times P = 200$

or, $\frac{P}{4} = 200$ or $P = 200 \times 4$

Therefore, $P = 800$

Abdul's solution

₹ 25 is saved for every ₹ 100

Amount for which ₹ 200 is saved

$= \frac{100}{25} \times 200 =$ ₹ 800

Thus both obtained the original price of sweater as ₹ 800.

Try These

1. 9 is 25% of what number? 2. 75% of what number is 15?

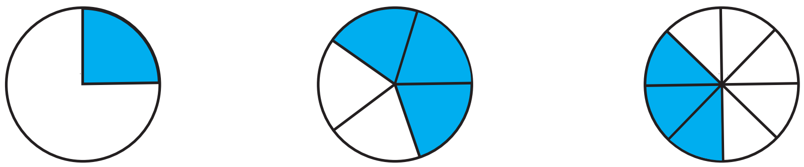
Exercise 8.2

1. Convert the given fractional numbers to per cents.

(a) $\frac{1}{8}$ (b) $\frac{5}{4}$ (c) $\frac{3}{40}$ (d) $\frac{2}{7}$

2. Convert the given decimal fractions to per cents.
(a) 0.65 (b) 2.1 (c) 0.02 (d) 12.35

3. Estimate what part of the figures is coloured and hence find the per cent which is coloured.



(i) (ii) (iii)

4. Find:
(a) 15% of 250 (b) 1% of 1 hour (c) 20% of ₹ 2500 (d) 75% of 1 kg

5. Find the whole quantity if
(a) 5% of it is 600. (b) 12% of it is ₹ 1080. (c) 40% of it is 500 km.
(d) 70% of it is 14 minutes. (e) 8% of it is 40 litres.

6. Convert given per cents to decimal fractions and also to fractions in simplest forms:
(a) 25% (b) 150% (c) 20% (d) 5%

7. In a city, 30% are females, 40% are males and remaining are children. What per cent are children?

8. Out of 15,000 voters in a constituency, 60% voted. Find the percentage of voters who did not vote. Can you now find how many actually did not vote?

9. Meeta saves ₹ 4000 from her salary. If this is 10% of her salary. What is her salary?

10. A local cricket team played 20 matches in one season. It won 25% of them. How many matches did they win?

8.4.3 Ratios to Percents

Sometimes, parts are given to us in the form of ratios and we need to convert those to percentages. Consider the following example:

EXAMPLE 14 Reena's mother said, to make *idlis*, you must take two parts rice and one part *urad dal*. What percentage of such a mixture would be rice and what percentage would be *urad dal*?

SOLUTION In terms of ratio we would write this as Rice : *Urad dal* = 2 : 1.

Now, 2 + 1=3 is the total of all parts. This means $\frac{2}{3}$ part is rice and $\frac{1}{3}$ part is *urad dal*.

Then, percentage of rice would be $\frac{2}{3} \times 100\,\% = \frac{200}{3} = 66\frac{2}{3}\,\%$.

Percentage of *urad dal* would be $\frac{1}{3} \times 100\,\% = \frac{100}{3} = 33\frac{1}{3}\,\%$.

EXAMPLE 15 If ₹ 250 is to be divided amongst Ravi, Raju and Roy, so that Ravi gets two parts, Raju three parts and Roy five parts. How much money will each get? What will it be in percentages?

SOLUTION The parts which the three boys are getting can be written in terms of ratios as 2 : 3 : 5. Total of the parts is 2 + 3 + 5 = 10.

Amounts received by each	Percentages of money for each
$\frac{2}{10} \times ₹\ 250 = ₹\ 50$	Ravi gets $\frac{2}{10} \times 100\ \% = 20\ \%$
$\frac{3}{10} \times ₹\ 250 = ₹\ 75$	Raju gets $\frac{3}{10} \times 100\ \% = 30\ \%$
$\frac{5}{10} \times ₹\ 250 = ₹\ 125$	Roy gets $\frac{5}{10} \times 100\ \% = 50\ \%$

TRY THESE

1. Divide 15 sweets between Manu and Sonu so that they get 20 % and 80 % of them respectively.
2. If angles of a triangle are in the ratio 2 : 3 : 4. Find the value of each angle.

8.4.4 Increase or Decrease as Per Cent

There are times when we need to know the increase or decrease in a certain quantity as percentage. For example, if the population of a state increased from 5,50,000 to 6,05,000. Then the increase in population can be understood better if we say, the population increased by 10 %.

How do we convert the increase or decrease in a quantity as a percentage of the initial amount? Consider the following example.

EXAMPLE 16 A school team won 6 games this year against 4 games won last year. What is the per cent increase?

SOLUTION The increase in the number of wins (or amount of change) = 6 – 4 = 2.

$$\text{Percentage increase} = \frac{\text{amount of change}}{\text{original amount or base}} \times 100$$

$$= \frac{\text{increase in the number of wins}}{\text{original number of wins}} \times 100 = \frac{2}{4} \times 100 = 50$$

EXAMPLE 17 The number of illiterate persons in a country decreased from 150 lakhs to 100 lakhs in 10 years. What is the percentage of decrease?

SOLUTION Original amount = the number of illiterate persons initially = 150 lakhs.

Amount of change = decrease in the number of illiterate persons = 150 – 100 = 50 lakhs
Therefore, the percentage of decrease

$$= \frac{\text{amount of change}}{\text{original amount}} \times 100 = \frac{50}{150} \times 100 = 33\frac{1}{3}$$

Try These

1. Find Percentage of increase or decrease:

– Price of shirt decreased from ₹ 280 to ₹ 210.

– Marks in a test increased from 20 to 30.

2. My mother says, in her childhood petrol was ₹ 1 a litre. It is ₹ 52 per litre today. By what Percentage has the price gone up?

8.5 Prices Related to an Item or Buying and Selling

I bought it for ₹ 600

and will sell it for ₹ 610

The buying price of any item is known as its **cost price**. It is written in short as CP.

The price at which you sell is known as the **selling price** or in short SP.

What would you say is better, to you sell the item at a lower price, same price or higher price than your buying price? You can decide whether the sale was profitable or not depending on the CP and SP. If CP < SP then you made a profit = SP – CP.

If CP = SP then you are in a no profit no loss situation.

If CP > SP then you have a loss = CP – SP.

Let us try to interpret the statements related to prices of items.

- A toy bought for ₹ 72 is sold at ₹ 80.
- A T-shirt bought for ₹ 120 is sold at ₹ 100.
- A cycle bought for ₹ 800 is sold for ₹ 940.

Let us consider the first statement.

The buying price (or CP) is ₹ 72 and the selling price (or SP) is ₹ 80. This means SP is more than CP. Hence profit made = SP – CP = ₹ 80 – ₹ 72 = ₹ 8

Now try interpreting the remaining statements in a similar way.

8.5.1 Profit or Loss as a Percentage

The profit or loss can be converted to a percentage. It is always calculated on the CP.

For the above examples, we can find the profit % or loss %.

Let us consider the example related to the toy. We have CP = ₹ 72, SP = ₹ 80, Profit = ₹ 8. To find the percentage of profit, Neha and Shekhar have used the following methods.

Neha does it this way

Profit per cent = $\frac{\text{Profit}}{\text{CP}} \times 100 = \frac{8}{72} \times 100$

$= \frac{1}{9} \times 100 = 11\frac{1}{9}$

Thus, the profit is ₹ 8 and

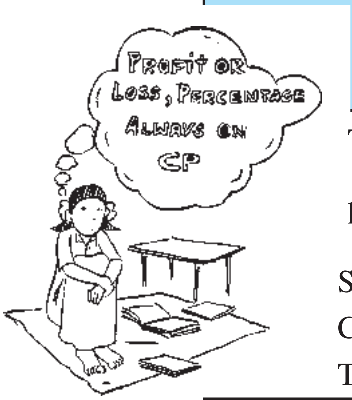
profit Per cent is $11\frac{1}{9}$.

Shekhar does it this way

On ₹ 72 the profit is ₹ 8

On ₹ 100, profit = $\frac{8}{72} \times 100$

$= 11\frac{1}{9}$. Thus, profit per cent = $11\frac{1}{9}$



Similarly you can find the loss per cent in the second situation. Here,

CP = ₹ 120, SP = ₹ 100.

Therefore, Loss = ₹ 120 – ₹ 100 = ₹ 20

Loss per cent = $\frac{\text{Loss}}{\text{CP}} \times 100$

$= \frac{20}{120} \times 100$

$= \frac{50}{3} = 16\frac{2}{3}$

On ₹ 120, the loss is ₹ 20
So on ₹ 100, the loss

$= \frac{20}{120} \times 100 = \frac{50}{3} = 16\frac{2}{3}$

Thus, loss per cent is $16\frac{2}{3}$

Try the last case.

Now we see that given any two out of the three quantities related to prices that is, CP, SP, amount of Profit or Loss or their percentage, we can find the rest.

EXAMPLE 18 The cost of a flower vase is ₹ 120. If the shopkeeper sells it at a loss of 10%, find the price at which it is sold.

SOLUTION We are given that CP = ₹ 120 and Loss per cent = 10. We have to find the SP.

Sohan does it like this

Loss of 10% means if CP is ₹ 100, Loss is ₹ 10

Therefore, SP would be

₹ (100 – 10) = ₹ 90

When CP is ₹ 100, SP is ₹ 90.
Therefore, if CP were ₹ 120 then

SP = $\frac{90}{100} \times 120$ = ₹ 108

Anandi does it like this

Loss is 10% of the cost price
= 10% of ₹ 120

$= \frac{10}{100} \times 120$ = ₹ 12

Therefore

SP = CP – Loss
= ₹ 120 – ₹ 12 = ₹ 108

Thus, by both methods we get the SP as ₹ 108.

EXAMPLE 19 Selling price of a toy car is ₹ 540. If the profit made by shopkeeper is 20%, what is the cost price of this toy?

SOLUTION We are given that SP = ₹ 540 and the Profit = 20%. We need to find the CP.

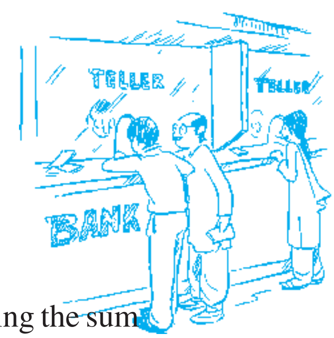
Amina does it like this	Arun does it like this
20% profit will mean if CP is ₹ 100, profit is ₹ 20 Therefore, SP = 100 + 20 = 120 Now, when SP is ₹ 120, then CP is ₹ 100. Therefore, when SP is ₹ 540, then CP = $\frac{100}{120} \times 540$ = ₹ 450	Profit = 20% of CP and SP = CP + Profit So, 540 = CP + 20% of CP $= CP + \frac{20}{100} \times CP = \left[1 + \frac{1}{5}\right] CP$ $= \frac{6}{5} CP$. Therefore, $540 \times \frac{5}{6} = CP$ or ₹ 450 = CP

Thus, by both methods, the cost price is ₹ 450.

TRY THESE

1. A shopkeeper bought a chair for ₹ 375 and sold it for ₹ 400. Find the gain Percentage.
2. Cost of an item is ₹ 50. It was sold with a profit of 12%. Find the selling price.
3. An article was sold for ₹ 250 with a profit of 5%. What was its cost price?
4. An item was sold for ₹ 540 at a loss of 5%. What was its cost price?

8.6 CHARGE GIVEN ON BORROWED MONEY OR SIMPLE INTEREST



Sohini said that they were going to buy a new scooter. Mohan asked her whether they had the money to buy it. Sohini said her father was going to take a loan from a bank. The money you borrow is known as **sum** borrowed or **principal**.

This money would be used by the borrower for some time before it is returned. For keeping this money for some time the borrower has to pay some extra money to the bank. This is known as **Interest.**

You can find the amount you have to pay at the end of the year by adding the sum borrowed and the interest. That is, **Amount = Principal + Interest.**

Interest is generally given in per cent for a period of one year. It is written as say 10% per year or per annum or in short as 10% p.a. (per annum).

10% p.a. means on every ₹ 100 borrowed, ₹ 10 is the interest you have to pay for one year. Let us take an example and see how this works.

EXAMPLE 20 Anita takes a loan of ₹ 5,000 at 15% per year as rate of interest. Find the interest she has to pay at the end of one year.

SOLUTION The sum borrowed = ₹ 5,000, Rate of interest = 15% per year.

This means if ₹ 100 is borrowed, she has to pay ₹ 15 as interest for one year. If she has borrowed ₹ 5,000, then the interest she has to pay for one year

$$= ₹\ \frac{15}{100} \times 5000 = ₹\ 750$$

So, at the end of the year she has to give an amount of ₹ 5,000 + ₹ 750 = ₹ 5,750.

We can write a general relation to find interest for one year. Take P as the principal or sum and R % as Rate per cent per annum.

Now on every ₹ 100 borrowed, the interest paid is ₹ R

Therefore, on ₹ P borrowed, the interest paid for one year would be $\frac{R \times P}{100} = \frac{P \times R}{100}$.

8.6.1 Interest for Multiple Years

If the amount is borrowed for more than one year the interest is calculated for the period the money is kept for. For example, if Anita returns the money at the end of two years and the rate of interest is the same then she would have to pay twice the interest i.e., ₹ 750 for the first year and ₹ 750 for the second. This way of calculating interest where principal is not changed is known as **simple interest**. As the number of years increase the interest also increases. For ₹ 100 borrowed for 3 years at 18%, the interest to be paid at the end of 3 years is 18 + 18 + 18 = 3 × 18 = ₹ 54.

We can find the general form for simple interest for more than one year.

We know that on a principal of ₹ P at R% rate of interest per year, the interest paid for one year is $\frac{R \times P}{100}$. Therefore, interest I paid for T years would be

$$\frac{T \times R \times P}{100} = \frac{P \times R \times T}{100} \text{ or } \frac{PRT}{100}$$

And amount you have to pay at the end of T years is $A = P + I$

TRY THESE

1. ₹ 10,000 is invested at 5% interest rate p.a. Find the interest at the end of one year.
2. ₹ 3,500 is given at 7% p.a. rate of interest. Find the interest which will be received at the end of two years.
3. ₹ 6,050 is borrowed at 6.5% rate of interest p.a.. Find the interest and the amount to be paid at the end of 3 years.
4. ₹ 7,000 is borrowed at 3.5% rate of interest p.a. borrowed for 2 years. Find the amount to be paid at the end of the second year.

Just as in the case of prices related to items, if you are given any two of the three quantities in the relation $I = \frac{P \times T \times R}{100}$, you could find the remaining quantity.

EXAMPLE 21 If Manohar pays an interest of ₹ 750 for 2 years on a sum of ₹ 4,500, find the rate of interest.

Solution 1	**Solution 2**
$\text{I} = \dfrac{P \times T \times R}{100}$ Therefore, $750 = \dfrac{4500 \times 2 \times R}{100}$ or $\dfrac{750}{45 \times 2} = R$ Therefore, Rate $= 8\frac{1}{3}\%$	For 2 years, interest paid is ₹ 750 Therefore, for 1 year, interest paid ₹ $\dfrac{750}{2}$ = ₹ 375 On ₹ 4,500, interest paid is ₹ 375 Therefore, on ₹ 100, rate of interest paid $= \dfrac{375 \times 100}{4500} = 8\frac{1}{3}\%$

TRY THESE

1. You have ₹ 2,400 in your account and the interest rate is 5%. After how many years would you earn ₹ 240 as interest.
2. On a certain sum the interest paid after 3 years is ₹ 450 at 5% rate of interest per annum. Find the sum.

EXERCISE 8.3

1. Tell what is the profit or loss in the following transactions. Also find profit per cent or loss per cent in each case.
   (a) Gardening shears bought for ₹ 250 and sold for ₹ 325.
   (b) A refrigerater bought for ₹ 12,000 and sold at ₹ 13,500.
   (c) A cupboard bought for ₹ 2,500 and sold at ₹ 3,000.
   (d) A skirt bought for ₹ 250 and sold at ₹ 150.
2. Convert each part of the ratio to percentage:
   (a) 3 : 1 (b) 2 : 3 : 5 (c) 1:4 (d) 1 : 2 : 5
3. The population of a city decreased from 25,000 to 24,500. Find the percentage decrease.
4. Arun bought a car for ₹ 3,50,000. The next year, the price went upto ₹ 3,70,000. What was the Percentage of price increase?
5. I buy a T.V. for ₹ 10,000 and sell it at a profit of 20%. How much money do I get for it?
6. Juhi sells a washing machine for ₹ 13,500. She loses 20% in the bargain. What was the price at which she bought it?
7. (i) Chalk contains calcium, carbon and oxygen in the ratio 10:3:12. Find the percentage of carbon in chalk.
   (ii) If in a stick of chalk, carbon is 3g, what is the weight of the chalk stick?

8. Amina buys a book for ₹ 275 and sells it at a loss of 15%. How much does she sell it for?

9. Find the amount to be paid at the end of 3 years in each case:

(a) Principal = ₹ 1,200 at 12% p.a. (b) Principal = ₹ 7,500 at 5% p.a.

10. What rate gives ₹ 280 as interest on a sum of ₹ 56,000 in 2 years?

11. If Meena gives an interest of ₹ 45 for one year at 9% rate p.a.. What is the sum she has borrowed?

What have We Discussed?

1. We are often required to compare two quantities in our daily life. They may be heights, weights, salaries, marks etc.

2. While comparing heights of two persons with heights150 cm and 75 cm, we write it as the ratio 150 : 75 or 2 : 1.

3. Two ratios can be compared by converting them to like fractions. If the two fractions are equal, we say the two given ratios are equivalent.

4. If two ratios are equivalent then the four quantities are said to be in proportion. For example, the ratios 8 : 2 and 16 : 4 are equivalent therefore 8, 2, 16 and 4 are in proportion.

5. A way of comparing quantities is percentage. Percentages are numerators of fractions with denominator 100. Per cent means per hundred.

For example 82% marks means 82 marks out of hundred.

6. Fractions can be converted to percentages and vice-versa.

For example, $\frac{1}{4} = \frac{1}{4} \times 100\,\%$ whereas, $75\% = \frac{75}{100} = \frac{3}{4}$

7. Decimals too can be converted to percentages and vice-versa.

For example, 0.25 = 0.25 × 100% = = 25%

8. Percentages are widely used in our daily life,

(a) We have learnt to find exact number when a certain per cent of the total quantity is given.

(b) When parts of a quantity are given to us as ratios, we have seen how to convert them to percentages.

(c) The increase or decrease in a certain quantity can also be expressed as percentage.

(d) The profit or loss incurred in a certain transaction can be expressed in terms of percentages.

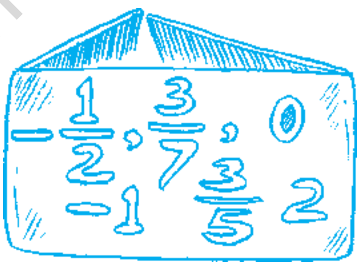
(e) While computing interest on an amount borrowed, the rate of interest is given in terms of per cents. For example, ₹ 800 borrowed for 3 years at 12% per annum.

Rational Numbers

Chapter 9

0756CH09

9.1 Introduction



You began your study of numbers by counting objects around you. The numbers used for this purpose were called counting numbers or natural numbers. They are 1, 2, 3, 4, ... By including 0 to natural numbers, we got the whole numbers, i.e., 0, 1, 2, 3, ... The negatives of natural numbers were then put together with whole numbers to make up integers. Integers are ..., –3, –2, –1, 0, 1, 2, 3, ... We, thus, extended the number system, from natural numbers to whole numbers and from whole numbers to integers.

You were also introduced to fractions. These are numbers of the form $\frac{\text{numerator}}{\text{denominator}}$, where the numerator is either 0 or a positive integer and the denominator, a positive integer. You compared two fractions, found their equivalent forms and studied all the four basic operations of addition, subtraction, multiplication and division on them.

In this Chapter, we shall extend the number system further. We shall introduce the concept of rational numbers alongwith their addition, subtraction, multiplication and division operations.

9.2 Need for Rational Numbers

Earlier, we have seen how integers could be used to denote opposite situations involving numbers. For example, if the distance of 3 km to the right of a place was denoted by 3, then the distance of 5 km to the left of the same place could be denoted by –5. If a profit of ₹ 150 was represented by 150 then a loss of ₹ 100 could be written as –100.

There are many situations similar to the above situations that involve fractional numbers. You can represent a distance of 750m above sea level as $\frac{3}{4}$ km. Can we represent 750m below sea level in km? Can we denote the distance of $\frac{3}{4}$ km below sea level by $\frac{-3}{4}$? We can see $\frac{-3}{4}$ is neither an integer, nor a fractional number. We need to extend our number system to include such numbers.

9.3 What are Rational Numbers?

The word 'rational' arises from the term 'ratio'. You know that a ratio like 3:2 can also be written as $\frac{3}{2}$. Here, 3 and 2 are natural numbers.

Similarly, the ratio of two integers p and q ($q \neq 0$), i.e., p:q can be written in the form $\frac{p}{q}$. This is the form in which rational numbers are expressed.

A rational number is defined as a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Thus, $\frac{4}{5}$ is a rational number. Here, $p = 4$ and $q = 5$.

Is $\frac{-3}{4}$ also a rational number? Yes, because $p = -3$ and $q = 4$ are integers.

- You have seen many fractions like $\frac{3}{8}, \frac{4}{8}, 1\frac{2}{3}$ etc. All fractions are rational numbers. Can you say why?

How about the decimal numbers like 0.5, 2.3, etc.? Each of such numbers can be written as an ordinary fraction and, hence, are rational numbers. For example, $0.5 = \frac{5}{10}$, $0.333 = \frac{333}{1000}$ etc.

Try These

1. Is the number $\frac{2}{-3}$ rational? Think about it.
2. List ten rational numbers.

Numerator and Denominator

In $\frac{p}{q}$, the integer p is the numerator, and the integer q ($\neq 0$) is the denominator.

Thus, in $\frac{-3}{7}$, the numerator is -3 and the denominator is 7.

Mention five rational numbers each of whose

(a) Numerator is a negative integer and denominator is a positive integer.
(b) Numerator is a positive integer and denominator is a negative integer.
(c) Numerator and denominator both are negative integers.
(d) Numerator and denominator both are positive integers.

- Are integers also rational numbers?

Any integer can be thought of as a rational number. For example, the integer -5 is a rational number, because you can write it as $\frac{-5}{1}$. The integer 0 can also be written as $0 = \frac{0}{2}$ or $\frac{0}{7}$ etc. Hence, it is also a rational number.

Thus, rational numbers include integers and fractions.

Equivalent rational numbers

A rational number can be written with different numerators and denominators. For example, consider the rational number $\frac{-2}{3}$.

$$\frac{-2}{3} = \frac{-2\times 2}{3\times 2} = \frac{-4}{6}$$. We see that $\frac{-2}{3}$ is the same as $\frac{-4}{5}$.

Also, $$\frac{-2}{3} = \frac{(-2)\times(-5)}{3\times(-5)} = \frac{10}{-15}$$. So, $\frac{-2}{3}$ is also the same as $\frac{10}{-15}$.

Thus, $\frac{-2}{3} = \frac{-4}{6} = \frac{10}{-15}$. Such rational numbers that are equal to each other are said to be equivalent to each other.

Again, $$\frac{10}{-15} = \frac{-10}{15} \text{ (How?)}$$

By multiplying the numerator and denominator of a rational number by the same non zero integer, we obtain another rational number equivalent to the given rational number. This is exactly like obtaining equivalent fractions.

Just as multiplication, the division of the numerator and denominator by the same non zero integer, also gives equivalent rational numbers. For example,

$$\frac{10}{-15} = \frac{10\div(-5)}{-15\div(-5)} = \frac{-2}{3}, \qquad \frac{-12}{24} = \frac{-12\div 12}{24\div 12} = \frac{-1}{2}$$

> We write $\frac{-2}{3}$ as $-\frac{2}{3}$, $\frac{-10}{15}$ as $-\frac{10}{15}$, etc.

TRY THESE

Fill in the boxes:

(i) $\frac{5}{4} = \frac{\square}{16} = \frac{25}{\square} = \frac{-15}{\square}$

(ii) $\frac{-3}{7} = \frac{\square}{14} = \frac{9}{\square} = \frac{-6}{\square}$

9.4 Positive and Negative Rational Numbers

Consider the rational number $\frac{2}{3}$. Both the numerator and denominator of this number are positive integers. Such a rational number is called a **positive rational number**. So, $\frac{3}{8}, \frac{5}{7}, \frac{2}{9}$ etc. are positive rational numbers.

The numerator of $\frac{-3}{5}$ is a negative integer, whereas the denominator is a positive integer. Such a rational number is called a **negative rational number**. So, $\frac{-5}{7}, \frac{-3}{8}, \frac{-9}{5}$ etc. are negative rational numbers.

TRY THESE

1. Is 5 a positive rational number?
2. List five more positive rational numbers.

Try These

1. Is – 8 a negative rational number?
2. List five more negative rational numbers.

- Is $\frac{8}{-3}$ a negative rational number? We know that $\frac{8}{-3} = \frac{8 \times -1}{-3 \times -1} = \frac{-8}{3}$,

  and $\frac{-8}{3}$ is a negative rational number. So, $\frac{8}{-3}$ is a negative rational number.

  Similarly, $\frac{5}{-7}, \frac{6}{-5}, \frac{2}{-9}$ etc. are all negative rational numbers. Note that their numerators are positive and their denominators negative.

- The number 0 is neither a positive nor a negative rational number.

- What about $\frac{-3}{-5}$?

  You will see that $\frac{-3}{-5} = \frac{-3 \times (-1)}{-5 \times (-1)} = \frac{3}{5}$. So, $\frac{-3}{-5}$ is a positive rational number.

  Thus, $\frac{-2}{-5}, \frac{-5}{-3}$ etc. are positive rational numbers.

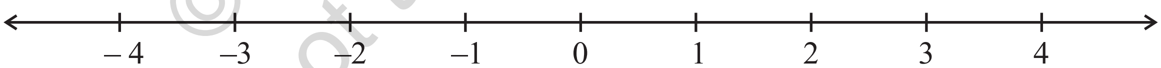
Try These

Which of these are negative rational numbers?

(i) $\frac{-2}{3}$ (ii) $\frac{5}{7}$ (iii) $\frac{3}{-5}$ (iv) 0 (v) $\frac{6}{11}$ (vi) $\frac{-2}{-9}$

9.5 Rational Numbers on a Number Line

You know how to represent integers on a number line. Let us draw one such number line.



The points to the right of 0 are denoted by + sign and are positive integers. The points to the left of 0 are denoted by – sign and are negative integers.

Representation of fractions on a number line is also known to you.

Let us see how the rational numbers can be represented on a number line.

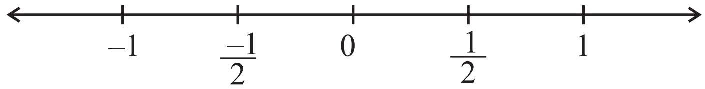
Let us represent the number $-\frac{1}{2}$ on the number line.

As done in the case of positive integers, the positive rational numbers would be marked on the right of 0 and the negative rational numbers would be marked on the left of 0.

To which side of 0 will you mark $-\frac{1}{2}$? Being a negative rational number, it would be marked to the left of 0.

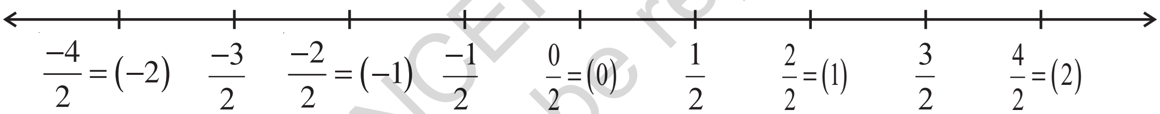
You know that while marking integers on the number line, successive integers are marked at equal intervels. Also, from 0, the pair 1 and –1 is equidistant. So are the pairs 2 and –2, 3 and –3.

In the same way, the rational numbers $\frac{1}{2}$ and $-\frac{1}{2}$ would be at equal distance from 0. We know how to mark the rational number $\frac{1}{2}$. It is marked at a point which is half the distance between 0 and 1. So, $-\frac{1}{2}$ would be marked at a point half the distance between 0 and –1.



We know how to mark $\frac{3}{2}$ on the number line. It is marked on the right of 0 and lies halfway between 1 and 2. Let us now mark $\frac{-3}{2}$ on the number line. It lies on the left of 0 and is at the same distance as $\frac{3}{2}$ from 0.

In decreasing order, we have, $\frac{-1}{2}, \frac{-2}{2}(=-1), \frac{-3}{2}, \frac{-4}{2}(=-2)$. This shows that $\frac{-3}{2}$ lies between – 1 and – 2. Thus, $\frac{-3}{2}$ lies halfway between – 1 and – 2.



Mark $\frac{-5}{2}$ and $\frac{-7}{2}$ in a similar way.

Similarly, $-\frac{1}{3}$ is to the left of zero and at the same distance from zero as $\frac{1}{3}$ is to the right. So as done above, $-\frac{1}{3}$ can be represented on the number line. Once we know how to represent $-\frac{1}{3}$ on the number line, we can go on representing $-\frac{2}{3}, -\frac{4}{3}, -\frac{5}{3}$ and so on. All other rational numbers with different denominators can be represented in a similar way.

9.6 Rational Numbers in Standard Form

Observe the rational numbers $\frac{3}{5}, \frac{-5}{8}, \frac{2}{7}, \frac{-7}{11}$.

The denominators of these rational numbers are positive integers and 1 is the only common factor between the numerators and denominators. Further, the negative sign occurs only in the numerator.

Such rational numbers are said to be in **standard form**.

A rational number is said to be in the standard form if its denominator is a positive integer and the numerator and denominator have no common factor other than 1.

If a rational number is not in the standard form, then it can be reduced to the standard form.

Recall that for reducing fractions to their lowest forms, we divided the numerator and the denominator of the fraction by the same non zero positive integer. We shall use the same method for reducing rational numbers to their standard form.

EXAMPLE 1 Reduce $\frac{-45}{30}$ to the standard form.

SOLUTION We have, $\frac{-45}{30} = \frac{-45 \div 3}{30 \div 3} = \frac{-15}{10} = \frac{-15 \div 5}{10 \div 5} = \frac{-3}{2}$

We had to divide twice. First time by 3 and then by 5. This could also be done as

$$\frac{-45}{30} = \frac{-45 \div 15}{30 \div 15} = \frac{-3}{2}$$

In this example, note that 15 is the HCF of 45 and 30.

Thus, to reduce the rational number to its standard form, we divide its numerator and denominator by their HCF ignoring the negative sign, if any. (The reason for ignoring the negative sign will be studied in Higher Classes)

If there is negative sign in the denominator, divide by '– HCF'.

EXAMPLE 2 Reduce to standard form:

(i) $\frac{36}{-24}$ (ii) $\frac{-3}{-15}$

SOLUTION

(i) The HCF of 36 and 24 is 12.

Thus, its standard form would be obtained by dividing by –12.

$$\frac{36}{-24} = \frac{36 \div (-12)}{-24 \div (-12)} = \frac{-3}{2}$$

(ii) The HCF of 3 and 15 is 3.

Thus, $\frac{-3}{-15} = \frac{-3 \div (-2)}{-15 \div (-3)} = \frac{1}{5}$

TRY THESE

Find the standard form of (i) $\frac{-18}{45}$ (ii) $\frac{-12}{18}$

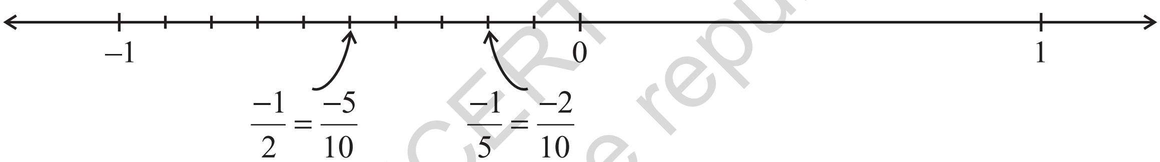
9.7 Comparison of Rational Numbers

We know how to compare two integers or two fractions and tell which is smaller or which is greater among them. Let us now see how we can compare two rational numbers.

- Two positive rational numbers, like $\frac{2}{3}$ and $\frac{5}{7}$ can be compared as studied earlier in the case of fractions.
- Mary compared two negative rational numbers $-\frac{1}{2}$ and $-\frac{1}{5}$ using number line. She knew that the integer which was on the right side of the other integer, was the greater integer.

For example, 5 is to the right of 2 on the number line and $5 > 2$. The integer -2 is on the right of -5 on the number line and $-2 > -5$.

She used this method for rational numbers also. She knew how to mark rational numbers on the number line. She marked $-\frac{1}{2}$ and $-\frac{1}{5}$ as follows:



Has she correctly marked the two points? How and why did she convert $-\frac{1}{2}$ to $-\frac{5}{10}$ and $-\frac{1}{5}$ to $-\frac{2}{10}$? She found that $-\frac{1}{5}$ is to the right of $-\frac{1}{2}$. Thus, $-\frac{1}{5} > -\frac{1}{2}$ or $-\frac{1}{2} < -\frac{1}{5}$.

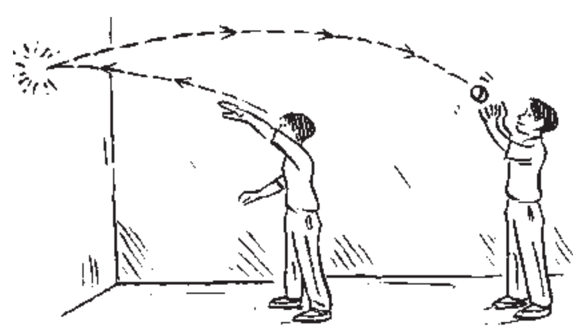
Can you compare $-\frac{3}{4}$ and $-\frac{2}{3}$? $-\frac{1}{3}$ and $-\frac{1}{5}$?

We know from our study of fractions that $\frac{1}{5} < \frac{1}{2}$. And what did Mary get for $-\frac{1}{2}$ and $-\frac{1}{5}$? Was it not exactly the opposite?

You will find that, $\frac{1}{2} > \frac{1}{5}$ but $-\frac{1}{2} < -\frac{1}{5}$.

Do you observe the same for $-\frac{3}{4}, -\frac{2}{3}$ and $-\frac{1}{3}, -\frac{1}{5}$?

Mary remembered that in integers she had studied $4 > 3$ but $-4 < -3$, $5 > 2$ but $-5 < -2$ etc.



- The case of pairs of negative rational numbers is similar. *To compare two negative rational numbers, we compare them ignoring their negative signs and then reverse the order.*

  For example, to compare $-\frac{7}{5}$ and $-\frac{5}{3}$, we first compare $\frac{7}{5}$ and $\frac{5}{3}$.

  We get $\frac{7}{5} < \frac{5}{3}$ and conclude that $\frac{-7}{5} > \frac{-5}{3}$.

  Take five more such pairs and compare them.

  Which is greater $-\frac{3}{8}$ or $-\frac{2}{7}$?; $-\frac{4}{3}$ or $-\frac{3}{2}$?

- Comparison of a negative and a positive rational number is obvious. A negative rational number is to the left of zero whereas a positive rational number is to the right of zero on a number line. So, a negative rational number will always be less than a positive rational number.

  Thus, $-\frac{2}{7} < \frac{1}{2}$.

- To compare rational numbers $\frac{-3}{-5}$ and $\frac{-2}{-7}$ reduce them to their standard forms and then compare them.

Example 3 Do $\frac{4}{-9}$ and $\frac{-16}{36}$ represent the same rational number?

Solution Yes, because $\frac{4}{-9} = \frac{4\times(-4)}{9\times(-4)} = \frac{-16}{36}$ or $\frac{-16}{36} = \frac{-16 \div -4}{36 \div -4} = \frac{-4}{-9}$.

9.8 Rational Numbers Between Two Rational Numbers

Reshma wanted to count the whole numbers between 3 and 10. From her earlier classes, she knew there would be exactly 6 whole numbers between 3 and 10. Similarly, she wanted to know the total number of integers between –3 and 3. The integers between –3 and 3 are –2, –1, 0, 1, 2. Thus, there are exactly 5 integers between –3 and 3.

Are there any integers between –3 and –2? No, there is no integer between –3 and –2. Between two successive integers the number of integers is 0.

Thus, we find that number of integers between two integers are limited (finite). Will the same happen in the case of rational numbers also?

Reshma took two rational numbers $\frac{-3}{5}$ and $\frac{-1}{3}$.

She converted them to rational numbers with same denominators.

So $\qquad \frac{-3}{5} = \frac{-9}{15}$ and $\frac{-1}{3} = \frac{-5}{15}$

We have $\qquad \frac{-9}{15} < \frac{-8}{15} < \frac{-7}{15} < \frac{-6}{15} < \frac{-5}{15}$ or $\frac{-3}{5} < \frac{-8}{15} < \frac{-7}{15} < \frac{-6}{15} < \frac{-1}{3}$

She could find rational numbers $\frac{-8}{15} < \frac{-7}{15} < \frac{-6}{15}$ between $\frac{-3}{5}$ and $\frac{-1}{3}$.

Are the numbers $\frac{-8}{15}, \frac{-7}{15}, \frac{-6}{15}$ the only rational numbers between $-\frac{3}{5}$ and $-\frac{1}{3}$?

We have $\qquad \frac{-3}{5} < \frac{-18}{30}$ and $\frac{-8}{15} < \frac{-16}{30}$

And $\qquad \frac{-18}{30} < \frac{-17}{30} < \frac{-16}{30}$. i.e., $\frac{-3}{5} < \frac{-17}{30} < \frac{-8}{15}$

Hence $\qquad \frac{-3}{5} < \frac{-17}{30} < \frac{-8}{15} < \frac{-7}{15} < \frac{-6}{15} < \frac{-1}{3}$

So, we could find one more rational number between $\frac{-3}{5}$ and $\frac{-1}{3}$.

By using this method, you can insert as many rational numbers as you want between two different rational numbers.

For example, $\qquad \frac{-3}{5} = \frac{-3\times 30}{5\times 30} = \frac{-90}{150}$ and $\frac{-1}{3} = \frac{-1\times 50}{3\times 50} = \frac{-50}{150}$

We get 39 rational numbers $\left(\frac{-89}{150}, \ldots, \frac{-51}{150}\right)$ between $\frac{-90}{150}$ and $\frac{-50}{150}$ i.e., between $\frac{-3}{5}$ and $\frac{-1}{3}$. You will find that the list is unending.

TRY THESE

Find five rational numbers between $\frac{-5}{7}$ and $\frac{-3}{8}$.

Can you list five rational numbers between $\frac{-5}{3}$ and $\frac{-8}{7}$?

We can find unlimited number of rational numbers between any two rational numbers.

EXAMPLE 4 List three rational numbers between – 2 and – 1.

SOLUTION Let us write –1 and –2 as rational numbers with denominator 5. (Why?)

We have, $-1 = \frac{-5}{5}$ and $-2 = \frac{-10}{5}$

So, $\frac{-10}{5} < \frac{-9}{5} < \frac{-8}{5} < \frac{-7}{5} < \frac{-6}{5} < \frac{-5}{5}$ or $-2 < \frac{-9}{5} < \frac{-8}{5} < \frac{-7}{5} < \frac{-6}{5} < -1$

The three rational numbers between –2 and –1 would be, $\frac{-9}{5}, \frac{-8}{5}, \frac{-7}{5}$

(You can take any three of $\frac{-9}{5}, \frac{-8}{5}, \frac{-7}{5}, \frac{-6}{5}$)

EXAMPLE 5 Write four more numbers in the following pattern:

$$\frac{-1}{3}, \frac{-2}{6}, \frac{-3}{9}, \frac{-4}{12}, \ldots$$

SOLUTION We have,

$$\frac{-2}{6} = \frac{-1\times 2}{3\times 2}, \frac{-3}{9} = \frac{-1\times 3}{3\times 3}, \frac{-4}{12} = \frac{-1\times 4}{3\times 4}$$

or $\frac{-1\times 1}{3\times 1} = \frac{-1}{3}, \frac{-1\times 2}{3\times 2} = \frac{-2}{6}, \frac{-1\times 3}{3\times 3} = \frac{-3}{9}, \frac{-1\times 4}{3\times 4} = \frac{-4}{12}$

Thus, we observe a pattern in these numbers.

The other numbers would be $\frac{-1\times 5}{3\times 5} = \frac{-5}{15}, \frac{-1\times 6}{3\times 6} = \frac{-6}{18}, \frac{-1\times 7}{3\times 7} = \frac{-7}{21}$.

EXERCISE 9.1

1. List five rational numbers between:

   (i) –1 and 0 (ii) –2 and –1 (iii) $\frac{-4}{5}$ and $\frac{-2}{3}$ (iv) $-\frac{1}{2}$ and $\frac{2}{3}$

2. Write four more rational numbers in each of the following patterns:

   (i) $\frac{-3}{5}, \frac{-6}{10}, \frac{-9}{15}, \frac{-12}{20}, \ldots\ldots$ (ii) $\frac{-1}{4}, \frac{-2}{8}, \frac{-3}{12}, \ldots\ldots$

(iii) $\frac{-1}{6}, \frac{2}{-12}, \frac{3}{-18}, \frac{4}{-24}, \ldots$ (iv) $\frac{-2}{3}, \frac{2}{-3}, \frac{4}{-6}, \frac{6}{-9}, \ldots$

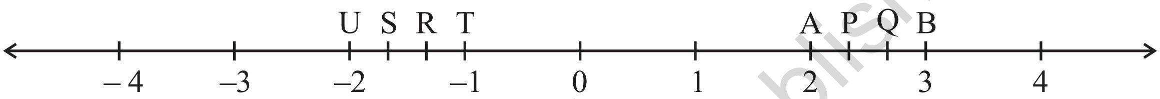
3. Give four rational numbers equivalent to:

(i) $\frac{-2}{7}$ (ii) $\frac{5}{-3}$ (iii) $\frac{4}{9}$

4. Draw the number line and represent the following rational numbers on it:

(i) $\frac{3}{4}$ (ii) $\frac{-5}{8}$ (iii) $\frac{-7}{4}$ (iv) $\frac{7}{8}$

5. The points P, Q, R, S, T, U, A and B on the number line are such that, TR = RS = SU and AP = PQ = QB. Name the rational numbers represented by P, Q, R and S.



6. Which of the following pairs represent the same rational number?

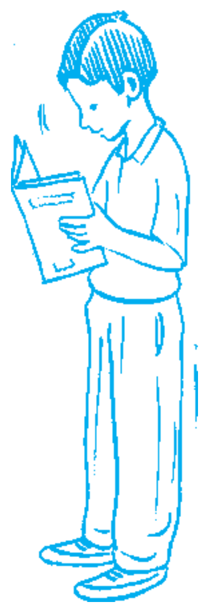
(i) $\frac{-7}{21}$ and $\frac{3}{9}$ (ii) $\frac{-16}{20}$ and $\frac{20}{-25}$ (iii) $\frac{-2}{-3}$ and $\frac{2}{3}$

(iv) $\frac{-3}{5}$ and $\frac{-12}{20}$ (v) $\frac{8}{-5}$ and $\frac{-24}{15}$ (vi) $\frac{1}{3}$ and $\frac{-1}{9}$

(vii) $\frac{-5}{-9}$ and $\frac{5}{-9}$

7. Rewrite the following rational numbers in the simplest form:

(i) $\frac{-8}{6}$ (ii) $\frac{25}{45}$ (iii) $\frac{-44}{72}$ (iv) $\frac{-8}{10}$



8. Fill in the boxes with the correct symbol out of >, <, and =.

(i) $\frac{-5}{7} \square \frac{2}{3}$ (ii) $\frac{-4}{5} \square \frac{-5}{7}$ (iii) $\frac{-7}{8} \square \frac{14}{-16}$

(iv) $\frac{-8}{5} \square \frac{-7}{4}$ (v) $\frac{1}{-3} \square \frac{-1}{4}$ (vi) $\frac{5}{-11} \square \frac{-5}{11}$

(vii) $0 \square \frac{-7}{6}$

9. Which is greater in each of the following:

(i) $\frac{2}{3}, \frac{5}{2}$ (ii) $\frac{-5}{6}, \frac{-4}{3}$ (iii) $\frac{-3}{4}, \frac{2}{-3}$

(iv) $\frac{-1}{4}, \frac{1}{4}$ (v) $-3\frac{2}{7}, -3\frac{4}{5}$

10. Write the following rational numbers in ascending order:

(i) $\frac{-3}{5}, \frac{-2}{5}, \frac{-1}{5}$ (ii) $\frac{-1}{3}, \frac{-2}{9}, \frac{-4}{3}$ (iii) $\frac{-3}{7}, \frac{-3}{2}, \frac{-3}{4}$

9.9 Operations on Rational Numbers

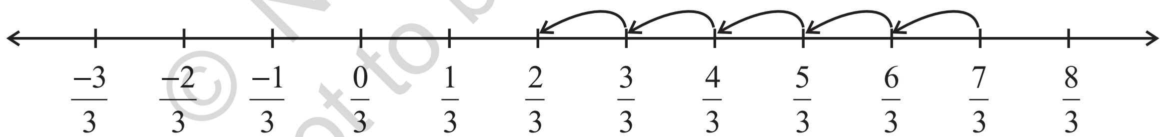
You know how to add, subtract, multiply and divide integers as well as fractions. Let us now study these basic operations on rational numbers.

9.9.1 Addition

- Let us add two rational numbers with same denominators, say $\frac{7}{3}$ and $\frac{-5}{3}$.

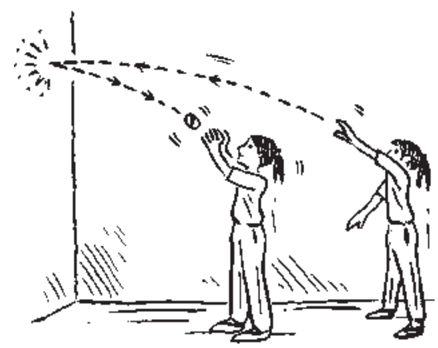
  We find $\frac{7}{3} + \left(\frac{-5}{3}\right)$

  On the number line, we have:



The distance between two consecutive points is $\frac{1}{3}$. So adding $\frac{-5}{3}$ to $\frac{7}{3}$ will mean, moving to the left of $\frac{7}{3}$, making 5 jumps. Where do we reach? We reach at $\frac{2}{3}$.

So, $$\frac{7}{3} + \left(\frac{-5}{3}\right) = \frac{2}{3}.$$



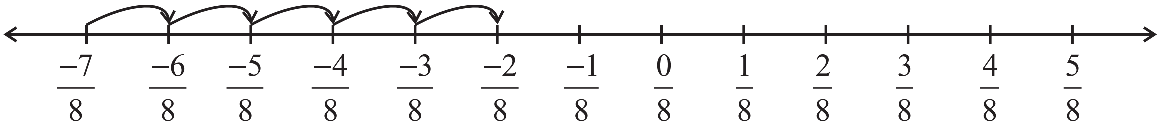
Let us now try this way:

$$\frac{7}{3} + \frac{(-5)}{3} = \frac{7 + (-5)}{3} = \frac{2}{3}$$

We get the same answer.

Find $\frac{6}{5} + \frac{(-2)}{5}$, $\frac{3}{7} + \frac{(-5)}{7}$ in both ways and check if you get the same answers.

Similarly, $\frac{-7}{8}+\frac{5}{8}$ would be



What do you get?

Also, $\frac{-7}{8}+\frac{5}{8}=\frac{-7+5}{8}=$? Are the two values same?

TRY THESE

Find: $\frac{-13}{7}+\frac{6}{7}$, $\frac{19}{5}+\left(\frac{-7}{5}\right)$

So, we find that *while adding rational numbers with same denominators, we add the numerators keeping the denominators same.*

Thus, $\frac{-11}{5}+\frac{7}{5}=\frac{-11+7}{5}=\frac{-4}{5}$

- How do we add rational numbers with different denominators? As in the case of fractions, we first find the LCM of the two denominators. Then, we find the equivalent rational numbers of the given rational numbers with this LCM as the denominator. Then, add the two rational numbers.

For example, let us add $\frac{-7}{5}$ and $\frac{-2}{3}$.

LCM of 5 and 3 is 15.

So, $\frac{-7}{5}=\frac{-21}{15}$ and $\frac{-2}{3}=\frac{-10}{15}$

Thus, $\frac{-7}{5}+\frac{(-2)}{3}=\frac{-21}{15}+\frac{(-10)}{15}=\frac{-31}{15}$

TRY THESE

Find:

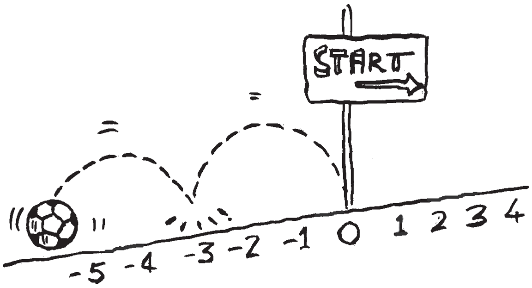
(i) $\frac{-3}{7}+\frac{2}{3}$

(ii) $\frac{-5}{6}+\frac{-3}{11}$

Additive Inverse

What will be $\frac{-4}{7}+\frac{4}{7}=?$

$$\frac{-4}{7}+\frac{4}{7}=\frac{-4+4}{7}=0.\ \text{Also, } \frac{4}{7}+\left(\frac{-4}{7}\right)=0.$$



Similarly, $\frac{-2}{3}+\frac{2}{3}=0=\frac{2}{3}+\left(\frac{-2}{3}\right)$.

In the case of integers, we call – 2 as the additive inverse of 2 and 2 as the additive inverse of – 2.

For rational numbers also, we call $\frac{-4}{7}$ as the **additive inverse** of $\frac{4}{7}$ and $\frac{4}{7}$ as the additive inverse of $\frac{-4}{7}$. Similarly, $\frac{-2}{3}$ is the additive inverse of $\frac{2}{3}$ and $\frac{2}{3}$ is the additive inverse of $\frac{-2}{3}$.

Try These

What will be the additive inverse of $\frac{-3}{9}$?, $\frac{-9}{11}$?, $\frac{5}{7}$?

Example 6 Satpal walks $\frac{2}{3}$ km from a place P, towards east and then from there $1\frac{5}{7}$ km towards west. Where will he be now from P?

Solution Let us denote the distance travelled towards east by positive sign. So, the distances towards west would be denoted by negative sign.

Thus, distance of Satpal from the point P would be

$$\frac{2}{3}+\left(-1\frac{5}{7}\right)=\frac{2}{3}+\frac{(-12)}{7}=\frac{2\times 7}{3\times 7}+\frac{(-12)\times 3}{7\times 3}$$

$$=\frac{14-36}{21}=\frac{-22}{21}=-1\frac{1}{21}$$


P

Since it is negative, it means Satpal is at a distance $1\frac{1}{21}$ km towards west of P.

9.9.2 Subtraction

Savita found the difference of two rational numbers $\frac{5}{7}$ and $\frac{3}{8}$ in this way:

$$\frac{5}{7}-\frac{3}{8}=\frac{40-21}{56}=\frac{19}{56}$$

Farida knew that for two integers a and b she could write $a-b=a+(-b)$

She tried this for rational numbers also and found, $\frac{5}{7}-\frac{3}{8}=\frac{5}{7}+\frac{(-3)}{8}=\frac{19}{56}$.

Both obtained the same difference.

Try to find $\frac{7}{8}-\frac{5}{9}$, $\frac{3}{11}-\frac{8}{7}$ in both ways. Did you get the same answer?

So, we say *while subtracting two rational numbers, we add the additive inverse of the rational number that is being subtracted, to the other rational number.*

Thus, $1\frac{2}{3}-2\frac{4}{5}=\frac{5}{3}-\frac{14}{5}=\frac{5}{3}+\text{additive inverse of }\frac{14}{5}=\frac{5}{3}+\frac{(-14)}{5}$

$=\frac{-17}{15}=-1\frac{2}{15}$.

TRY THESE

Find:

(i) $\frac{7}{9}-\frac{2}{5}$ (ii) $2\frac{1}{5}-\frac{(-1)}{3}$

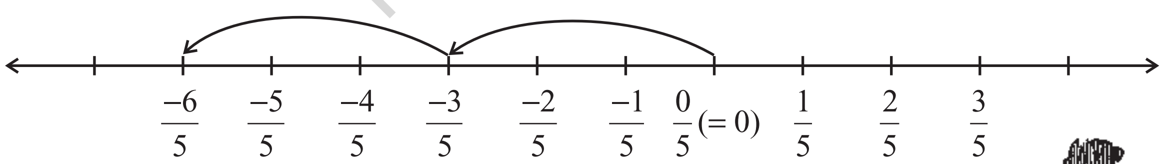
What will be $\frac{2}{7}-\left(\frac{-5}{6}\right)$?

$$\frac{2}{7}-\left(\frac{-5}{6}\right)=\frac{2}{7}+\text{ additive inverse of }\left(\frac{-5}{6}\right)=\frac{2}{7}+\frac{5}{6}=\frac{47}{42}=1\frac{5}{42}$$

9.9.3 Multiplication

Let us multiply the rational number $\frac{-3}{5}$ by 2, i.e., we find $\frac{-3}{5}\times 2$.

On the number line, it will mean two jumps of $\frac{3}{5}$ to the left.



Where do we reach? We reach at $\frac{-6}{5}$. Let us find it as we did in fractions.

$$\frac{-3}{5}\times 2=\frac{-3\times 2}{5}=\frac{-6}{5}$$

We arrive at the same rational number.

Find $\frac{-4}{7}\times 3, \frac{-6}{5}\times 4$ using both ways. What do you observe?

So, *we find that while multiplying a rational number by a positive integer, we multiply the numerator by that integer, keeping the denominator unchanged.*

Let us now multiply a rational number by a negative integer,

$$\frac{-2}{9} \times (-5) = \frac{-2 \times (-5)}{9} = \frac{10}{9}$$

TRY THESE

What will be

(i) $\frac{-3}{5} \times 7$? (ii) $\frac{-6}{5} \times (-2)$?

Remember, –5 can be written as $\frac{-5}{1}$.

So, $\frac{-2}{9} \times \frac{-5}{1} = \frac{10}{9} = \frac{-2 \times (-5)}{9 \times 1}$

Similarly, $\frac{3}{11} \times (-2) = \frac{3 \times (-2)}{11 \times 1} = \frac{-6}{11}$

Based on these observations, we find that, $\frac{-3}{8} \times \frac{5}{7} = \frac{-3 \times 5}{8 \times 7} = \frac{-15}{56}$

So, as we did in the case of fractions, we multiply two rational numbers in the following way:

TRY THESE

Find:

(i) $\frac{-3}{4} \times \frac{1}{7}$

(ii) $\frac{2}{3} \times \frac{-5}{9}$

Step 1 Multiply the numerators of the two rational numbers.

Step 2 Multiply the denominators of the two rational numbers.

Step 3 Write the product as $\frac{\text{Result of Step 1}}{\text{Result of Step 2}}$

Thus, $\frac{-3}{5} \times \frac{2}{7} = \frac{-3 \times 2}{5 \times 7} = \frac{-6}{35}$.

Also, $\frac{-5}{8} \times \frac{-9}{7} = \frac{-5 \times (-9)}{8 \times 7} = \frac{45}{56}$

9.9.4 Division

We have studied reciprocals of a fraction earlier. What is the reciprocal of $\frac{2}{7}$? It will be $\frac{7}{2}$. We extend this idea of reciprocals to non-zero rational numbers also.

The reciprocal of $\frac{-2}{7}$ will be $\frac{7}{-2}$ i.e., $\frac{-7}{2}$; that of $\frac{-3}{5}$ would be $\frac{-5}{3}$.

TRY THESE

What will be the reciprocal of $\frac{-6}{11}$? and $\frac{-8}{5}$?

Product of reciprocals

The product of a rational number with its reciprocal is always 1.

For example, $\frac{-4}{9} \times \left(\text{reciprocal of } \frac{-4}{9}\right)$

$$= \frac{-4}{9} \times \frac{-9}{4} = 1$$

Similarly, $\frac{-6}{13} \times \frac{-13}{6} = 1$

Try some more examples and confirm this observation.

Savita divided a rational number $\frac{4}{9}$ by another rational number $\frac{-5}{7}$ as,

$$\frac{4}{9} \div \frac{-5}{7} = \frac{4}{9} \times \frac{7}{-5} = \frac{-28}{45}.$$

She used the idea of reciprocal as done in fractions.

Arpit first divided $\frac{4}{9}$ by $\frac{5}{7}$ and got $\frac{28}{45}$.

He finally said $\frac{4}{9} \div \frac{-5}{7} = \frac{-28}{45}$. How did he get that?

He divided them as fractions, ignoring the negative sign and then put the negative sign in the value so obtained.

Both of them got the same value $\frac{-28}{45}$. Try dividing $\frac{2}{3}$ by $\frac{-5}{7}$ both ways and see if you get the same answer.

This shows, *to divide one rational number by the other non-zero rational number we multiply the rational number by the reciprocal of the other.*

Thus, $\frac{-6}{5} \div \frac{-2}{3} = \frac{6}{-5} \times \text{reciprocal of } \left(\frac{-2}{3}\right) = \frac{6}{-5} \times \frac{3}{-2} = \frac{18}{10}$

Try These

Find: (i) $\frac{2}{3}\times\frac{-7}{8}$ (ii) $\frac{-6}{7}\times\frac{5}{7}$

Exercise 9.2

1. Find the sum:

(i) $\frac{5}{4}+\left(\frac{-11}{4}\right)$ (ii) $\frac{5}{3}+\frac{3}{5}$ (iii) $\frac{-9}{10}+\frac{22}{15}$

(iv) $\frac{-3}{-11}+\frac{5}{9}$ (v) $\frac{-8}{19}+\frac{(-2)}{57}$ (vi) $\frac{-2}{3}+0$

(vii) $-2\frac{1}{3}+4\frac{3}{5}$

2. Find

(i) $\frac{7}{24}-\frac{17}{36}$ (ii) $\frac{5}{63}-\left(\frac{-6}{21}\right)$ (iii) $\frac{-6}{13}-\left(\frac{-7}{15}\right)$

(iv) $\frac{-3}{8}-\frac{7}{11}$ (v) $-2\frac{1}{9}-6$

3. Find the product:

(i) $\frac{9}{2}\times\left(\frac{-7}{4}\right)$ (ii) $\frac{3}{10}\times(-9)$ (iii) $\frac{-6}{5}\times\frac{9}{11}$

(iv) $\frac{3}{7}\times\left(\frac{-2}{5}\right)$ (v) $\frac{3}{11}\times\frac{2}{5}$ (vi) $\frac{3}{-5}\times\frac{-5}{3}$

4. Find the value of:

(i) $(-4)\div\frac{2}{3}$ (ii) $\frac{-3}{5}\div 2$ (iii) $\frac{-4}{5}\div(-3)$

(iv) $\frac{-1}{8}\div\frac{3}{4}$ (v) $\frac{-2}{13}\div\frac{1}{7}$ (vi) $\frac{-7}{12}\div\left(\frac{-2}{13}\right)$

(vii) $\frac{3}{13}\div\left(\frac{-4}{65}\right)$

What have We Discussed?

1. A number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number. The numbers $\frac{-2}{7}, \frac{3}{8}, 3$ etc. are rational numbers.
2. All integers and fractions are rational numbers.
3. If the numerator and denominator of a rational number are multiplied or divided by a non-zero integer, we get a rational number which is said to be equivalent to the given rational number. For example $\frac{-3}{7} = \frac{-3 \times 2}{7 \times 2} = \frac{-6}{14}$. So, we say $\frac{-6}{14}$ is the equivalent form of $\frac{-3}{7}$. Also note that $\frac{-6}{14} = \frac{-6 \div 2}{14 \div 2} = \frac{-3}{7}$.
4. Rational numbers are classified as Positive and Negative rational numbers. When the numerator and denominator, both, are positive integers, it is a positive rational number. When either the numerator or the denominator is a negative integer, it is a negative rational number. For example, $\frac{3}{8}$ is a positive rational number whereas $\frac{-8}{9}$ is a negative rational number.
5. The number 0 is neither a positive nor a negative rational number.
6. A rational number is said to be in the standard form if its denominator is a positive integer and the numerator and denominator have no common factor other than 1. The numbers $\frac{-1}{3}, \frac{2}{7}$ etc. are in standard form.
7. There are unlimited number of rational numbers between two rational numbers.
8. Two rational numbers with the same denominator can be added by adding their numerators, keeping the denominator same. Two rational numbers with different denominators are added by first taking the LCM of the two denominators and then converting both the rational numbers to their equivalent forms having the LCM as the denominator. For example, $\frac{-2}{3} + \frac{3}{8} = \frac{-16}{24} + \frac{9}{24} = \frac{-16+9}{24} = \frac{-7}{24}$. Here, LCM of 3 and 8 is 24.
9. While subtracting two rational numbers, we add the additive inverse of the rational number to be subtracted to the other rational number.

   Thus, $\frac{7}{8} - \frac{2}{3} = \frac{7}{8} + \text{additive inverse of } \frac{2}{3} = \frac{7}{8} + \frac{(-2)}{3} = \frac{21+(-16)}{24} = \frac{5}{24}$.

10. To multiply two rational numbers, we multiply their numerators and denominators separately, and write the product as $\frac{\text{product of numerators}}{\text{product of denominators}}$.

11. To divide one rational number by the other non-zero rational number, we multiply the rational number by the reciprocal of the other. Thus,

$$\frac{-7}{2} \div \frac{4}{3} = \frac{-7}{2} \times (\text{reciprocal of } \frac{4}{3}) = \frac{-7}{2} \times \frac{3}{4} = \frac{-21}{8}.$$

Chapter 10

Practical Geometry

0756CH10

10.1 Introduction

You are familiar with a number of shapes. You learnt how to draw some of them in the earlier classes. For example, you can draw a line segment of given length, a line perpendicular to a given line segment, an angle, an angle bisector, a circle etc.

Now, you will learn how to draw parallel lines and some types of triangles.

10.2 Construction of a Line Parallel to a Given Line, Through a Point not on the Line

Let us begin with an activity (Fig 10.1)

(i) Take a sheet of paper. Make a fold. This fold represents a line *l*.

(ii) Unfold the paper. Mark a point A on the paper outside *l*.

(iii) Fold the paper perpendicular to the line such that this perpendicular passes through A. Name the perpendicular AN.

(iv) Make a fold perpendicular to this perpendicular through the point A. Name the new perpendicular line as *m*. Now, $l \parallel m$. Do you see 'why'?

Which property or properties of parallel lines can help you here to say that lines *l* and *m* are parallel.

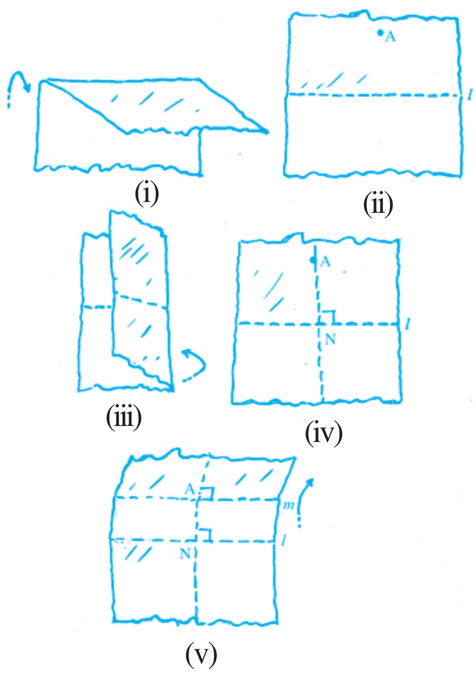
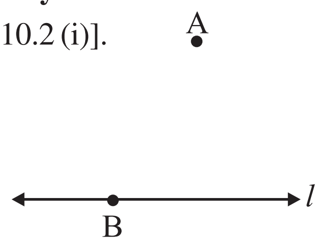


Fig 10.1

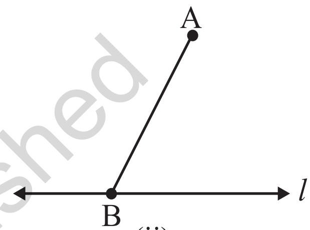
You can use any one of the properties regarding the transversal and parallel lines to make this **construction using ruler and compasses only.**

Step 1 Take a line 'l' and a point 'A' outside 'l' [Fig10.2 (i)].



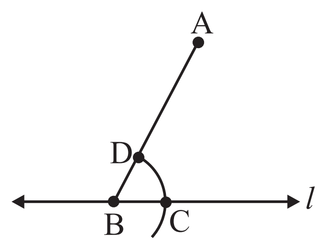
(i)

Step 2 Take any point B on l and join B to A [Fig 10.2(ii)].



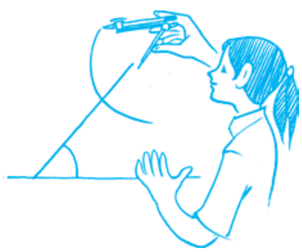
(ii)

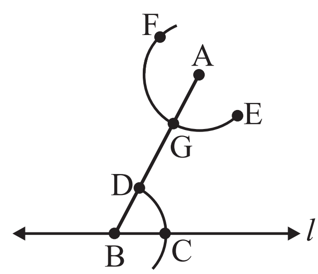
Step 3 With B as centre and a convenient radius, draw an arc cutting l at C and BA at D [Fig 10.2(iii)].



(iii)

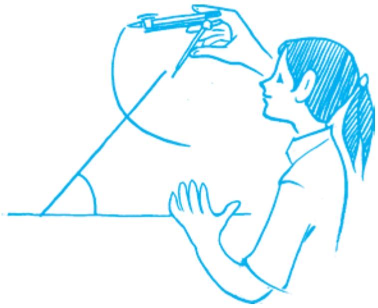
Step 4 Now with A as centre and the same radius as in Step 3, draw an arc EF cutting AB at G [Fig 10.2 (iv)].

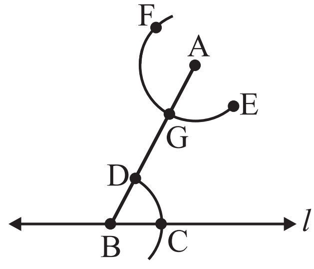




(iv)

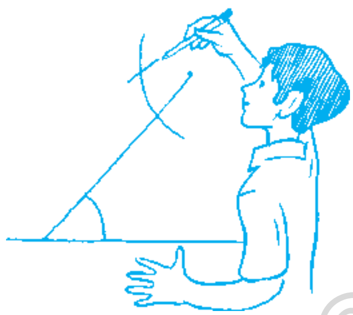
Step 5 Place the pointed tip of the compasses at C and adjust the opening so that the pencil tip is at D [Fig 10.2 (v)].

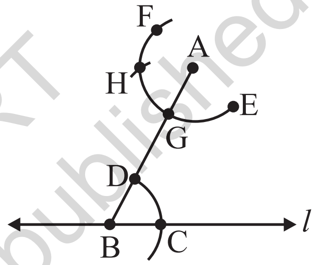




(v)

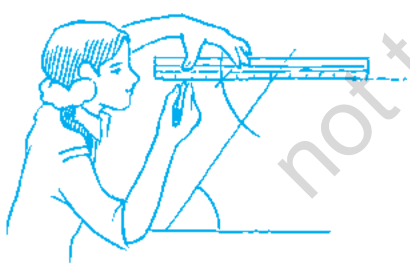
Step 6 With the same opening as in Step 5 and with G as centre, draw an arc cutting the arc EF at H [Fig 10.2 (vi)].

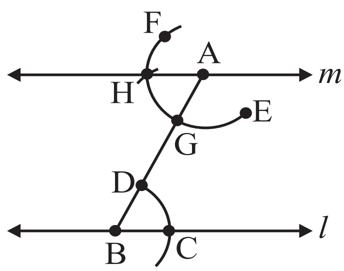




(vi)

Step 7 Now, join AH to draw a line '*m*' [Fig 10.2 (vii)].





(vii)

Note that $\angle$ABC and $\angle$BAH are alternate interior angles.

Therefore $m \parallel l$

Fig 10.2 (i)–(vii)

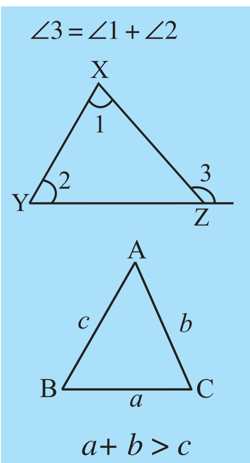
THINK, DISCUSS AND WRITE

1. In the above construction, can you draw any other line through A that would be also parallel to the line *l* ?
2. Can you slightly modify the above construction to use the idea of equal corresponding angles instead of equal alternate angles?

Exercise 10.1

1. Draw a line, say AB, take a point C outside it. Through C, draw a line parallel to AB using ruler and compasses only.
2. Draw a line l. Draw a perpendicular to l at any point on l. On this perpendicular choose a point X, 4 cm away from l. Through X, draw a line m parallel to l.
3. Let l be a line and P be a point not on l. Through P, draw a line m parallel to l. Now join P to any point Q on l. Choose any other point R on m. Through R, draw a line parallel to PQ. Let this meet l at S. What shape do the two sets of parallel lines enclose?

10.3 Construction of Triangles



It is better for you to go through this section after recalling ideas on triangles, in particular, the chapters on properties of triangles and congruence of triangles.

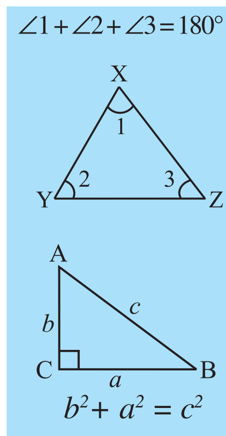
You know how triangles are classified based on sides or angles and the following important properties concerning triangles:

(i) The exterior angle of a triangle is equal in measure to the sum of interior opposite angles.

(ii) The total measure of the three angles of a triangle is 180°.

(iii) Sum of the lengths of any two sides of a triangle is greater than the length of the third side.

(iv) In any right-angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



In the chapter on 'Congruence of Triangles', we saw that a triangle can be drawn if any one of the following sets of measurements are given:

(i) Three sides.

(ii) Two sides and the angle between them.

(iii) Two angles and the side between them.

(iv) The hypotenuse and a leg in the case of a right-angled triangle.

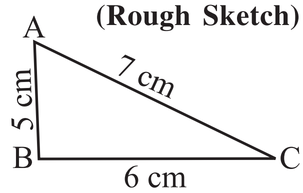
We will now attempt to use these ideas to construct triangles.

10.4 Constructing a Triangle when the Lengths of its Three Sides are Known (SSS Criterion)

In this section, we would construct triangles when all its sides are known. We draw first a rough sketch to give an idea of where the sides are and then begin by drawing any one of the three lines. See the following example:

EXAMPLE 1 Construct a triangle ABC, given that AB = 5 cm, BC = 6 cm and AC = 7 cm.

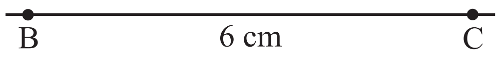
SOLUTION



(i)

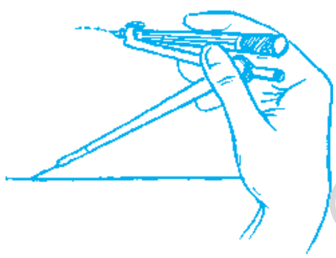
Step 1 First, we draw a rough sketch with given measure, (This will help us in deciding how to proceed) [Fig 10.3(i)].

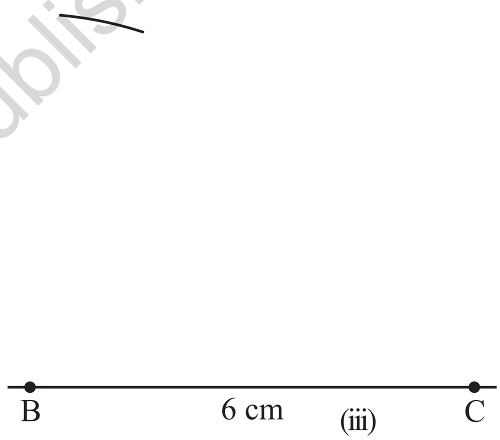
Step 2 Draw a line segment BC of length 6 cm [Fig 10.3(ii)].



(ii)

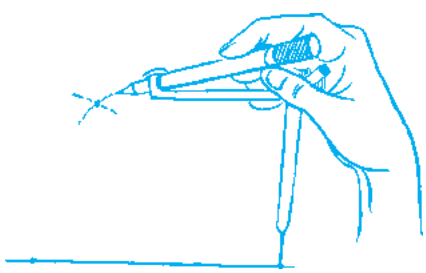
Step 3 From B, point A is at a distance of 5 cm. So, with B as centre, draw an arc of radius 5 cm. (Now A will be somewhere on this arc. Our job is to find where exactly A is) [Fig 10.3(iii)].

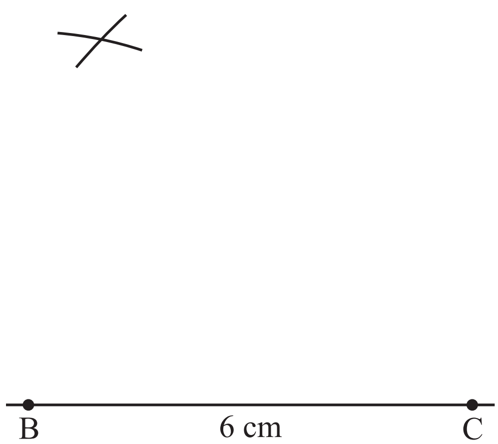




(iii)

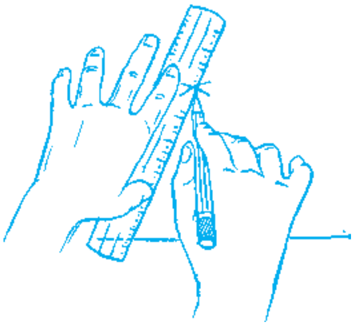
Step 4 From C, point A is at a distance of 7 cm. So, with C as centre, draw an arc of radius 7 cm. (A will be somewhere on this arc, we have to fix it) [Fig 10.3(iv)].





(iv)

Step 5 A has to be on both the arcs drawn. So, it is the point of intersection of arcs. Mark the point of intersection of arcs as A. Join AB and AC. ΔABC is now ready [Fig 10.3(v)].



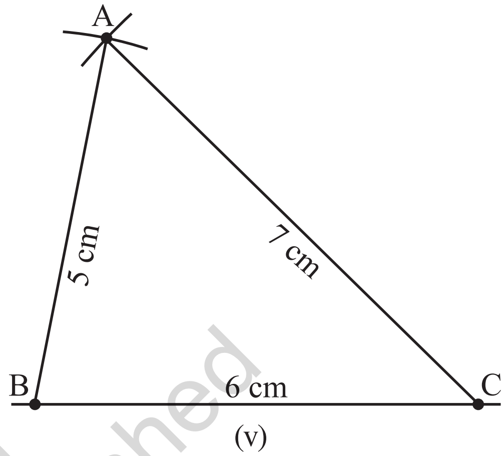


Fig 10.3 (i) – (v)

Do This

Now, let us construct another triangle DEF such that DE = 5 cm, EF = 6 cm, and DF = 7 cm. Take a cutout of ΔDEF and place it on ΔABC. What do we observe?

We observe that ΔDEF exactly coincides with ΔABC. (Note that the triangles have been constructed when their three sides are given.) Thus, if three sides of one triangle are equal to the corresponding three sides of another triangle, then the two triangles are congruent. This is SSS congruency rule which we have learnt in our earlier chapter.

Think, Discuss and Write

A student attempted to draw a triangle whose rough figure is given here. He drew QR first. Then with Q as centre, he drew an arc of 3 cm and with R as centre, he drew an arc of 2 cm. But he could not get P. What is the reason? What property of triangle do you know in connection with this problem?

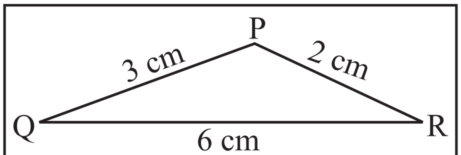
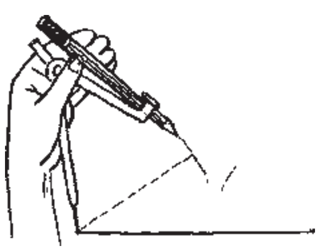
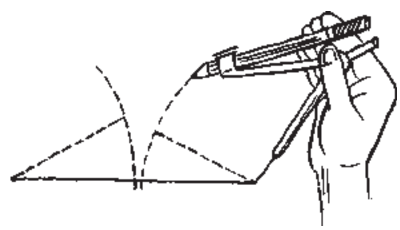


Fig 10.4 Think: Is this right?

Can such a triangle exist? (Remember the property of triangles 'The sum of any two sides of a traingle is always greater than the third side'!)





Exercise 10.2

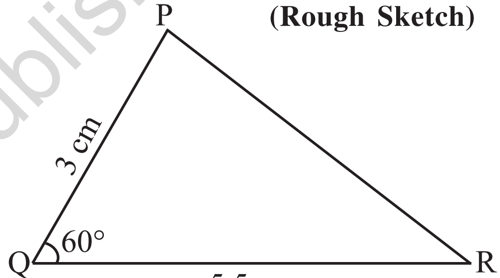
1. Construct ΔXYZ in which $XY = 4.5$ cm, $YZ = 5$ cm and $ZX = 6$ cm.
2. Construct an equilateral triangle of side 5.5 cm.
3. Draw ΔPQR with $PQ = 4$ cm, $QR = 3.5$ cm and $PR = 4$ cm. What type of triangle is this?
4. Construct ΔABC such that $AB = 2.5$ cm, $BC = 6$ cm and $AC = 6.5$ cm. Measure $\angle B$.

10.5 Constructing a Triangle when the Lengths of Two Sides and the Measure of the Angle Between them are Known. (SAS Criterion)

Here, we have two sides given and the one angle between them. We first draw a sketch and then draw one of the given line segments. The other steps follow. See Example 2.

Example 2 Construct a triangle PQR, given that $PQ = 3$ cm, $QR = 5.5$ cm and $\angle PQR = 60°$.

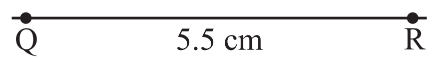
(Rough Sketch)



(i)

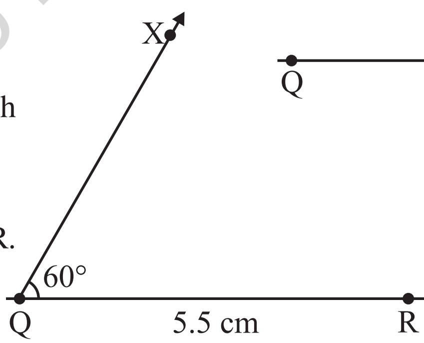
Solution

Step 1 First, we draw a rough sketch with given measures. (This helps us to determine the procedure in construction) [Fig 10.5(i)].



(ii)

Step 2 Draw a line segment QR of length 5.5 cm [Fig 10.5(ii)].

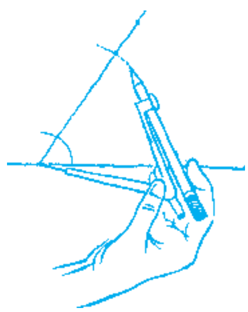


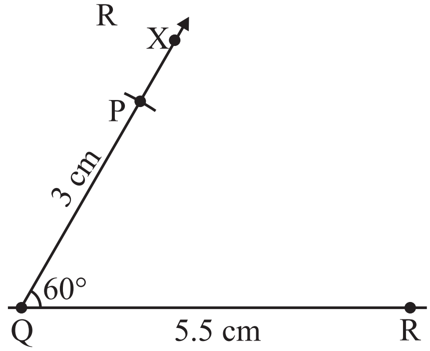
(iii)

Step 3 At Q, draw QX making 60° with QR. (The point P must be somewhere on this ray of the angle) [Fig 10.5(iii)].

Step 4 (To fix P, the distance QP has been given).

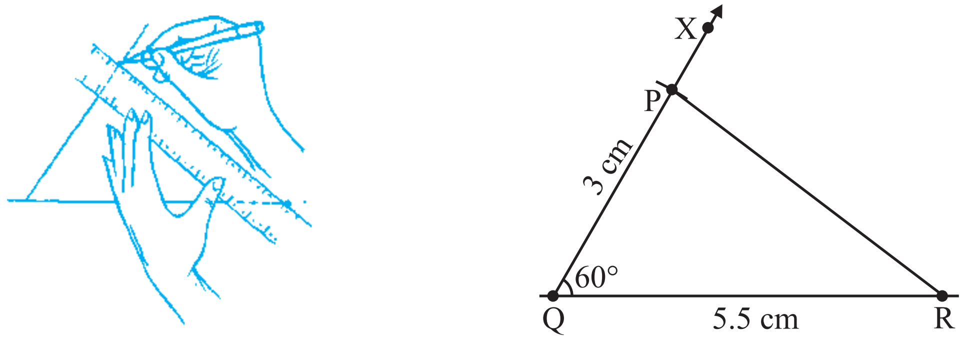
With Q as centre, draw an arc of radius 3 cm. It cuts QX at the point P [Fig 10.5(iv)].





(iv)

Step 5 Join PR. ΔPQR is now obtained (Fig 10.5(v)).



(v)

Fig 10.5 (i) – (v)

Do This

Let us now construct another triangle ABC such that AB = 3 cm, BC = 5.5 cm and $m\angle ABC = 60°$. Take a cut out of ΔABC and place it on ΔPQR. What do we observe? We observe that ΔABC exactly coincides with ΔPQR. Thus, if two sides and the included angle of one triangle are equal to the corresponding two sides and the included angle of another triangle, then the two triangles are congruent. This is SAS congruency rule which we have learnt in our earlier chapter. (Note that the triangles have been constructed when their two sides and the angle included between these two sides are given.)

Think, Discuss and Write

In the above construction, lengths of two sides and measure of one angle were given. Now study the following problems:

In ΔABC, if AB = 3cm, AC = 5 cm and $m\angle C = 30°$. Can we draw this triangle? We may draw AC = 5 cm and draw $\angle C$ of measure 30°. CA is one arm of $\angle C$. Point B should be lying on the other arm of $\angle C$. But, observe that point B cannot be located uniquely. Therefore, the given data is not sufficient for construction of ΔABC.

Now, try to construct ΔABC if AB = 3cm, AC = 5 cm and $m\angle B = 30°$. What do we observe? Again, ΔABC cannot be constructed uniquely. Thus, we can conclude that a unique triangle can be constructed only if the lengths of its two sides and the measure of the included angle between them is given.

Exercise 10.3

1. Construct ΔDEF such that DE = 5 cm, DF = 3 cm and $m\angle EDF = 90°$.
2. Construct an isosceles triangle in which the lengths of each of its equal sides is 6.5 cm and the angle between them is 110°.
3. Construct ΔABC with BC = 7.5 cm, AC = 5 cm and $m\angle C = 60°$.

10.6 Constructing a Triangle when the Measures of two of its Angles and the Length of the Side included between Them is Given. (ASA Criterion)

As before, draw a rough sketch. Now, draw the given line segment. Make angles on the two ends. See the Example 3.

Example 3 Construct ΔXYZ if it is given that XY = 6 cm, m∠ZXY = 30° and m∠XYZ = 100°.

Solution

Step 1 Before actual construction, we draw a rough sketch with measures marked on it. (This is just to get an idea as how to proceed) [Fig 10.6(i)].

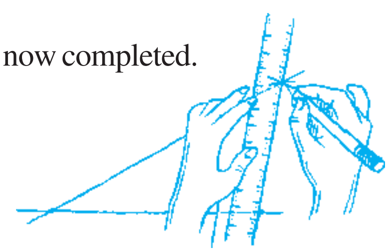
Step 2 Draw XY of length 6 cm.

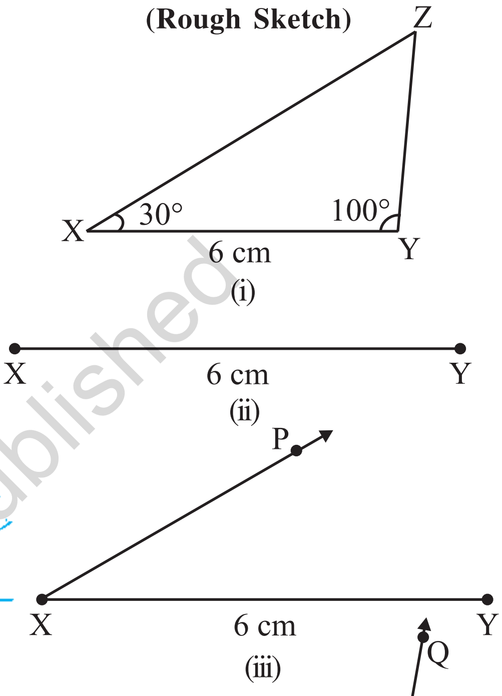
Step 3 At X, draw a ray XP making an angle of 30° with XY. By the given condition Z must be somewhere on the XP.

Step 4 At Y, draw a ray YQ making an angle of 100° with YX. By the given condition, Z must be on the ray YQ also.

Step 5 Z has to lie on both the rays XP and YQ. So, the point of intersection of the two rays is Z.

ΔXYZ is now completed.





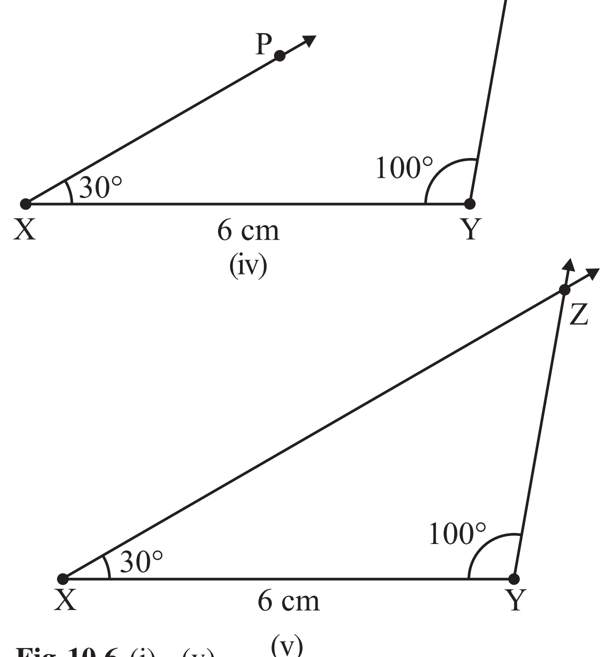


Fig 10.6 (i) – (v)

Do This

Now, draw another ΔLMN, where m∠NLM = 30°, LM = 6 cm and m∠NML = 100°. Take a cutout of ΔLMN and place it on the ΔXYZ. We observe that ΔLMN exactly coincides with ΔXYZ. Thus, if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of another triangle, then the two triangles are congruent. This is ASA congruency rule which you have learnt in the earlier chapter. (Note that the triangles have been constructed when two angles and the included side between these angles are given.)

Think, Discuss and Write

In the above example, length of a side and measures of two angles were given. Now study the following problem:

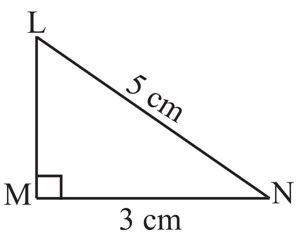
In ΔABC, if AC = 7 cm, m∠A = 60° and m∠B = 50°, can you draw the triangle? (Angle-sum property of a triangle may help you!)

Exercise 10.4

1. Construct ΔABC, given m∠A = 60°, m∠B = 30° and AB = 5.8 cm.
2. Construct ΔPQR if PQ = 5 cm, m∠PQR = 105° and m∠QRP = 40°.
   (Hint**:** Recall angle-sum property of a triangle).
3. Examine whether you can construct ΔDEF such that EF = 7.2 cm, m∠E = 110° and m∠F = 80°. Justify your answer.

10.7 Constructing a Right-Angled Triangle when the Length of One Leg and its Hypotenuse are Given (RHS Criterion)

(Rough Sketch)



(i)

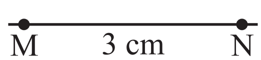
Here it is easy to make the rough sketch. Now, draw a line as per the given side. Make a right angle on one of its points. Use compasses to mark length of side and hypotenuse of the triangle. Complete the triangle. Consider the following:

Example 4 Construct ΔLMN, right-angled at M, given that LN = 5 cm and MN = 3 cm.

Solution

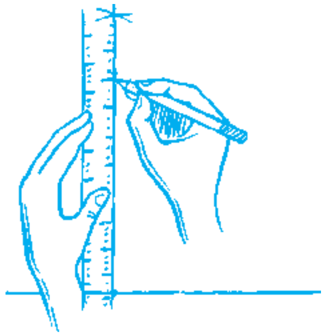
Step 1 Draw a rough sketch and mark the measures. Remember to mark the right angle [Fig 10.7(i)].

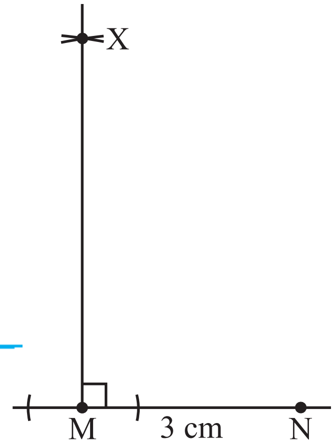
Step 2 Draw MN of length 3 cm. [Fig 10.7(ii)].



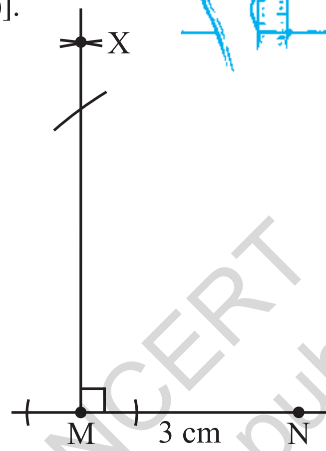
(ii)

Step 3 At M, draw MX ⊥ MN. (L should be somewhere on this perpendicular) [Fig 10.7(iii)].





Step 4 With N as centre, draw an arc of radius 5 cm. (L must be on this arc, since it is at a distance of 5 cm from N) [Fig 10.7(iv)].



Step 5 L has to be on the perpendicular line MX as well as on the arc drawn with centre N. Therefore, L is the meeting point of these two.

ΔLMN is now obtained.

[Fig 10.7 (v)]

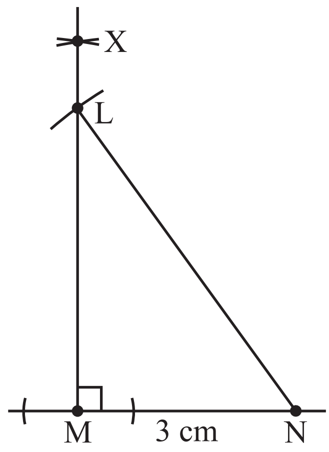


Fig 10.7 (i) – (v)

Exercise 10.5

1. Construct the right angled ΔPQR, where m∠Q = 90°, QR = 8cm and PR = 10 cm.
2. Construct a right-angled triangle whose hypotenuse is 6 cm long and one of the legs is 4 cm long.
3. Construct an isosceles right-angled triangle ABC, where m∠ACB = 90° and AC = 6 cm.

Miscellaneous questions

Below are given the measures of certain sides and angles of triangles. Identify those which cannot be constructed and, say why you cannot construct them. Construct rest of the triangles.

	Triangle	**Given measurements**		
1.	ΔABC	$m\angle A = 85°$;	$m\angle B = 115°$;	$AB = 5$ cm.
2.	ΔPQR	$m\angle Q = 30°$;	$m\angle R = 60°$;	$QR = 4.7$ cm.
3.	ΔABC	$m\angle A = 70°$;	$m\angle B = 50°$;	$AC = 3$ cm.
4.	ΔLMN	$m\angle L = 60°$;	$m\angle N = 120°$;	$LM = 5$ cm.
5.	ΔABC	$BC = 2$ cm;	$AB = 4$ cm;	$AC = 2$ cm.
6.	ΔPQR	$PQ = 3.5$ cm.;	$QR = 4$ cm.;	$PR = 3.5$ cm.
7.	ΔXYZ	$XY = 3$ cm;	$YZ = 4$ cm;	$XZ = 5$ cm
8.	ΔDEF	$DE = 4.5$cm;	$EF = 5.5$cm;	$DF = 4$ cm.

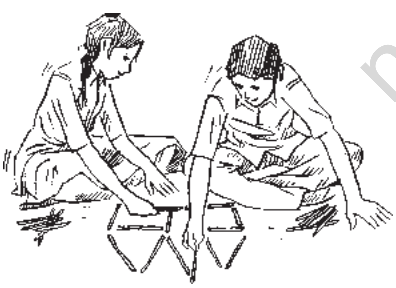
What have We Discussed?

In this Chapter, we looked into the methods of some ruler and compasses constructions.

1. Given a line l and a point not on it, we used the idea of 'equal alternate angles' in a transversal diagram to draw a line parallel to l.

We could also have used the idea of 'equal corresponding angles' to do the construction.

2. We studied the method of drawing a triangle, using indirectly the concept of congruence of triangles.



The following cases were discussed:

(i) SSS: Given the three side lengths of a triangle.

(ii) SAS: Given the lengths of any two sides and the measure of the angle between these sides.

(iii) ASA: Given the measures of two angles and the length of side included between them.

(iv) RHS: Given the length of hypotenuse of a right-angled triangle and the length of one of its legs.

Perimeter and Area

0756CH11

Chapter 11

11.1 Introduction

In Class VI, you have already learnt perimeters of plane figures and areas of squares and rectangles. Perimeter is the distance around a closed figure while area is the part of plane or region occupied by the closed figure.

In this class, you will learn about perimeters and areas of a few more plane figures.

11.2 Squares and Rectangles

Ayush and Deeksha made pictures. Ayush made his picture on a rectangular sheet of length 60 cm and breadth 20 cm while Deeksha made hers on a rectangular sheet of length 40 cm and breadth 35 cm. Both these pictures have to be separately framed and laminated.

Who has to pay more for framing, if the cost of framing is ₹ 3.00 per cm?

If the cost of lamination is ₹ 2.00 per cm^2, who has to pay more for lamination?

For finding the cost of framing, we need to find perimeter and then multiply it by the rate for framing. For finding the cost of lamination, we need to find area and then multiply it by the rate for lamination.

Try These

What would you need to find, area or perimeter, to answer the following?

1. How much space does a blackboard occupy?
2. What is the length of a wire required to fence a rectangular flower bed?
3. What distance would you cover by taking two rounds of a triangular park?
4. How much plastic sheet do you need to cover a rectangular swimming pool?

Do you remember,

Perimeter of a regular polygon = number of sides × length of one side

Perimeter of a square = 4 × side

Fig 11.1

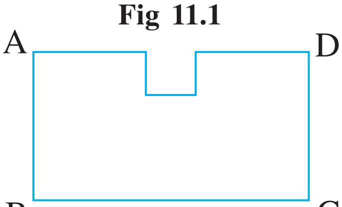


Fig 11.2

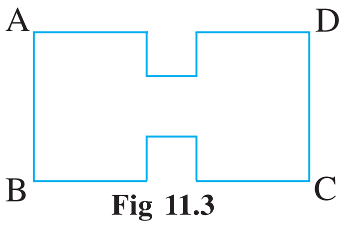


Fig 11.3

Perimeter of a rectangle = $2 \times (l + b)$

Area of a rectangle = $l \times b$, Area of a square = side $\times$ side

Tanya needed a square of side 4 cm for completing a collage. She had a rectangular sheet of length 28 cm and breadth 21 cm (Fig 11. 1). She cuts off a square of side 4 cm from the rectangular sheet. Her friend saw the remaining sheet (Fig 11.2) and asked Tanya, "Has the perimeter of the sheet increased or decreased now?"

Has the total length of side AD increased after cutting off the square?

Has the area increased or decreased?

Tanya cuts off one more square from the opposite side (Fig 11.3).

Will the perimeter of the remaining sheet increase further?

Will the area increase or decrease further?

So, what can we infer from this?

It is clear that the increase of perimeter need not lead to increase in area.

Try These

1. Experiment with several such shapes and cut-outs. You might find it useful to draw these shapes on squared sheets and compute their areas and perimeters.

   You have seen that increase in perimeter does not mean that area will also increase.

2. Give two examples where the area increases as the perimeter increases.
3. Give two examples where the area does not increase when perimeter increases.

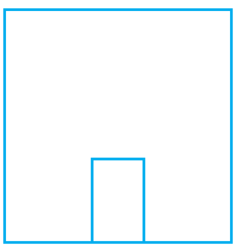


Fig 11. 4

Example 1 A door-frame of dimensions 3 m × 2 m is fixed on the wall of dimension 10 m × 10 m. Find the total labour charges for painting the wall if the labour charges for painting 1 m^2 of the wall is ₹ 2.50.

Solution Painting of the wall has to be done excluding the area of the door.

Area of the door = $l \times b$

$= 3 \times 2 \text{ m}^2 = 6 \text{ m}^2$

Area of wall including door = side × side = $10 \text{ m} \times 10 \text{ m} = 100 \text{ m}^2$

Area of wall excluding door = $(100 - 6) \text{ m}^2 = 94 \text{ m}^2$

Total labour charges for painting the wall = ₹ 2.50×94 = ₹ 235

Example 2 The area of a rectangular sheet is 500 cm^2. If the length of the sheet is 25 cm, what is its width? Also find the perimeter of the rectangular sheet.

Solution Area of the rectangular sheet = 500 cm^2

Length (l) = 25 cm

Area of the rectangle = $l \times b$ (where b = width of the sheet)

Therefore, width $b = \frac{\text{Area}}{l} = \frac{500}{25} = 20$ cm

Perimeter of sheet = $2 \times (l + b) = 2 \times (25 + 20)$ cm = 90 cm

So, the width of the rectangular sheet is 20 cm and its perimeter is 90 cm.

Example 3 Anu wants to fence the garden in front of her house (Fig 11.5), on three sides with lengths 20 m, 12 m and 12 m. Find the cost of fencing at the rate of ₹ 150 per metre.

Fig 11.5

Solution The length of the fence required is the perimeter of the garden (excluding one side) which is equal to 20 m + 12 m + 12 m, i.e., 44 m.

Cost of fencing = ₹ 150 × 44 = ₹ 6,600.

Example 4 A wire is in the shape of a square of side 10 cm. If the wire is rebent into a rectangle of length 12 cm, find its breadth. Which encloses more area, the square or the rectangle?

Solution Side of the square = 10 cm

Length of the wire = Perimeter of the square = 4 × side = 4 × 10 cm

= 40 cm

Length of the rectangle, l = 12 cm. Let b be the breadth of the rectangle.

Perimeter of rectangle = Length of wire = 40 cm

Perimeter of the rectangle = $2\,(l + b)$

Thus, $40 = 2\,(12 + b)$

or $\frac{40}{2} = 12 + b$

Therefore, $b = 20 - 12 = 8$ cm

The breadth of the rectangle is 8 cm.

Area of the square = $(\text{side})^2$

= 10 cm × 10 cm = 100 cm^2

Area of the rectangle = $l \times b$

= 12 cm × 8 cm = 96 cm^2

So, the square encloses more area even though its perimeter is the same as that of the rectangle.

Example 5 The area of a square and a rectangle are equal. If the side of the square is 40 cm and the breadth of the rectangle is 25 cm, find the length of the rectangle. Also, find the perimeter of the rectangle.

Solution Area of square = $(\text{side})^2$

= 40 cm × 40 cm = 1600 cm^2

It is given that,

The area of the rectangle = The area of the square

Area of the rectangle = 1600 cm^2, breadth of the rectangle = 25 cm.

Area of the rectangle = $l \times b$

or $1600 = l \times 25$

or $\frac{1600}{25} = l$ or $l = 64$ cm

So, the length of rectangle is 64 cm.

Perimeter of the rectangle = $2\,(l + b) = 2\,(64 + 25)$ cm

$= 2 \times 89$ cm $= 178$ cm

So, the perimeter of the rectangle is 178 cm even though its area is the same as that of the square.

EXERCISE 11.1

1. The length and the breadth of a rectangular piece of land are 500 m and 300 m respectively. Find
   (i) its area (ii) the cost of the land, if 1 m^2 of the land costs ₹ 10,000.
2. Find the area of a square park whose perimeter is 320 m.
3. Find the breadth of a rectangular plot of land, if its area is 440 m^2 and the length is 22 m. Also find its perimeter.
4. The perimeter of a rectangular sheet is 100 cm. If the length is 35 cm, find its breadth. Also find the area.
5. The area of a square park is the same as of a rectangular park. If the side of the square park is 60 m and the length of the rectangular park is 90 m, find the breadth of the rectangular park.
6. A wire is in the shape of a rectangle. Its length is 40 cm and breadth is 22 cm. If the same wire is rebent in the shape of a square, what will be the measure of each side. Also find which shape encloses more area?
7. The perimeter of a rectangle is 130 cm. If the breadth of the rectangle is 30 cm, find its length. Also find the area of the rectangle.
8. A door of length 2 m and breadth 1m is fitted in a wall. The length of the wall is 4.5 m and the breadth is 3.6 m (Fig11.6). Find the cost of white washing the wall, if the rate of white washing the wall is ₹ 20 per m^2.

Fig 11.6

11.2.1 Triangles as Parts of Rectangles

Take a rectangle of sides 8 cm and 5 cm. Cut the rectangle along its diagonal to get two triangles (Fig 11.7).

Superpose one triangle on the other.

Are they exactly the same in size?

Can you say that both the triangles are equal in area?

Are the triangles congruent also?

What is the area of each of these triangles?

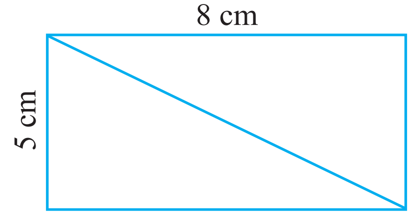


Fig 11.7

You will find that sum of the areas of the two triangles is the same as the area of the rectangle. Both the triangles are equal in area.

$$\text{The area of each triangle} = \frac{1}{2}\,(\text{Area of the rectangle})$$

$$= \frac{1}{2}\times(l\times b) = \frac{1}{2}(8\times 5)$$

$$= \frac{40}{2} = 20\text{ cm}^2$$

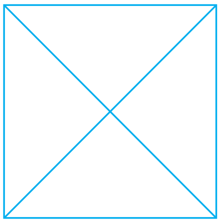


Fig 11.8

Take a square of side 5 cm and divide it into 4 triangles as shown (Fig 11.8).

Are the four triangles equal in area?

Are they congruent to each other? (Superpose the triangles to check).

What is the area of each triangle?

$$\text{The area of each triangle} = \frac{1}{4}(\text{Area of the square})$$

$$= \frac{1}{4}(\text{side})^2 = \frac{1}{4}(5)^2\text{ cm}^2 = 6.25\text{ cm}^2$$

11.2.2 Generalising for other Congruent Parts of Rectangles

A rectangle of length 6 cm and breadth 4 cm is divided into two parts as shown in the Fig 11.9. Trace the rectangle on another paper and cut off the rectangle along EF to divide it into two parts.

Superpose one part on the other, see if they match. (You may have to rotate them).

Are they congruent? The two parts are congruent to each other. So, the area of one part is equal to the area of the other part.

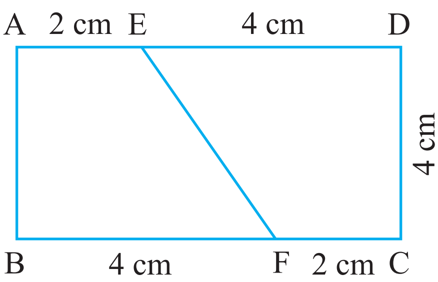


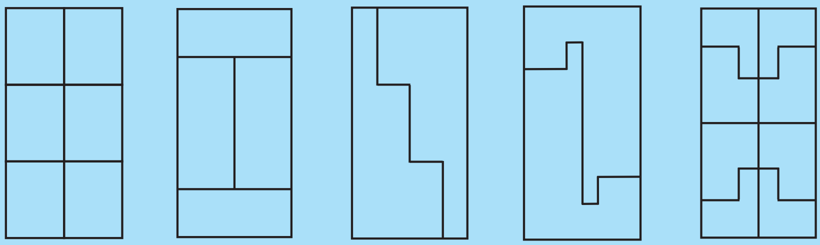
Fig 11.9

$$\text{Therefore, the area of each congruent part} = \frac{1}{2}\,(\text{The area of the rectangle})$$

$$= \frac{1}{2}\times(6\times 4)\text{ cm}^2 = 12\text{ cm}^2$$

Try These

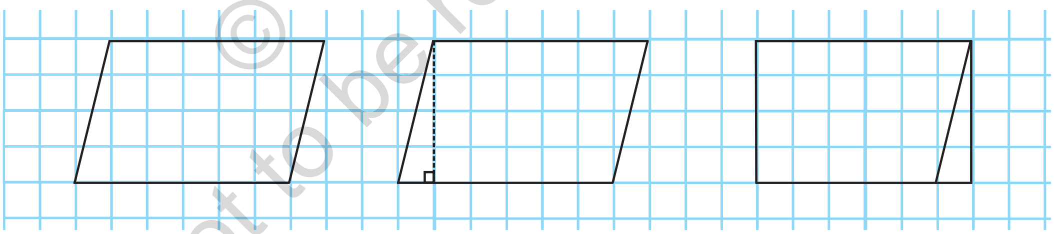
Each of the following rectangles of length 6 cm and breadth 4 cm is composed of congruent polygons. Find the area of each polygon.



11.3 Area of a Parallelogram

We come across many shapes other than squares and rectangles.
How will you find the area of a land which is a parallelogram in shape?
Let us find a method to get the area of a parallelogram.
Can a parallelogram be converted into a rectangle of equal area?

Draw a parallelogram on a graph paper as shown in Fig 11.10(i). Cut out the parallelogram. Draw a line from one vertex of the parallelogram perpendicular to the opposite side [Fig 11.10(ii)]. Cut out the triangle. Move the triangle to the other side of the parallelogram.



(i) (ii) (iii)

Fig 11.10

What shape do you get? You get a rectangle.

Is the area of the parallelogram equal to the area of the rectangle formed?

Yes, area of the parallelogram = area of the rectangle formed

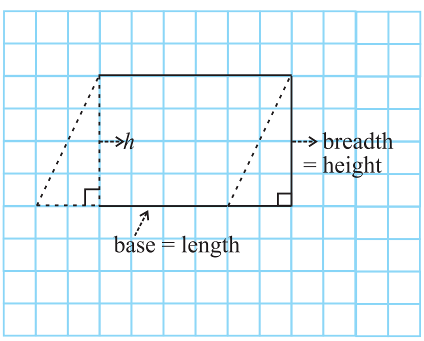


Fig 11.11

What are the length and the breadth of the rectangle?

We find that the length of the rectangle formed is equal to the base of the parallelogram and the breadth of the rectangle is equal to the height of the parallelogram (Fig 11.11).

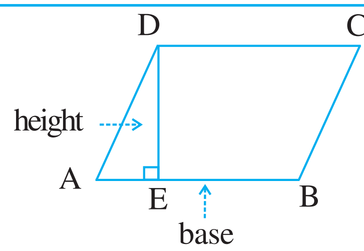
Now, Area of parallelogram = Area of rectangle

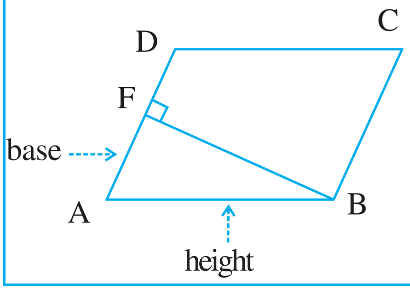
= length × breadth = $l \times b$

But the length l and breadth b of the rectangle are exactly the base b and the height h, respectively of the parallelogram.

Thus, the area of parallelogram = base × height = $b \times h$.

Any side of a parallelogram can be chosen as **base** of the parallelogram. The perpendicular dropped on that side from the opposite vertex is known as **height** (altitude). In the parallelogram ABCD, DE is perpendicular to AB. Here AB is the base and DE is the height of the parallelogram.





In this parallelogram ABCD, BF is the perpendicular to opposite side AD. Here AD is the **base** and BF is the **height**.

Consider the following parallelograms (Fig 11.12).

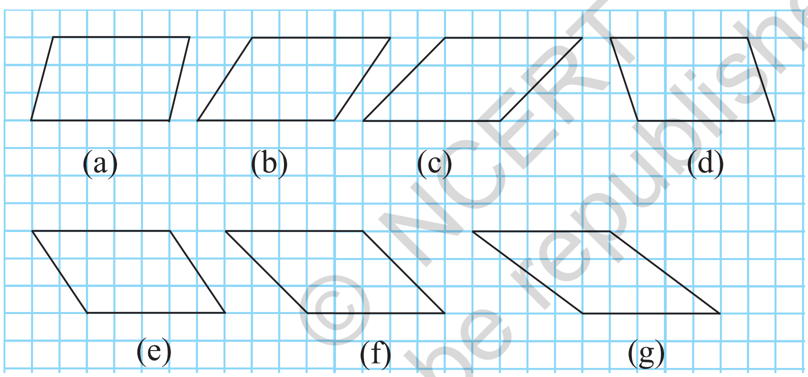


Fig 11.12

Find the areas of the parallelograms by counting the squares enclosed within the figures and also find the perimeters by measuring the sides.

Complete the following table:

Parallelogram	Base	Height	Area	Perimeter
(a)	5 units	3 units	15 sq units	
(b)				
(c)				
(d)				
(e)				
(f)				
(g)				

You will find that all these parallelograms have equal areas but different perimeters. Now, consider the following parallelograms with sides 7 cm and 5 cm (Fig 11.13).

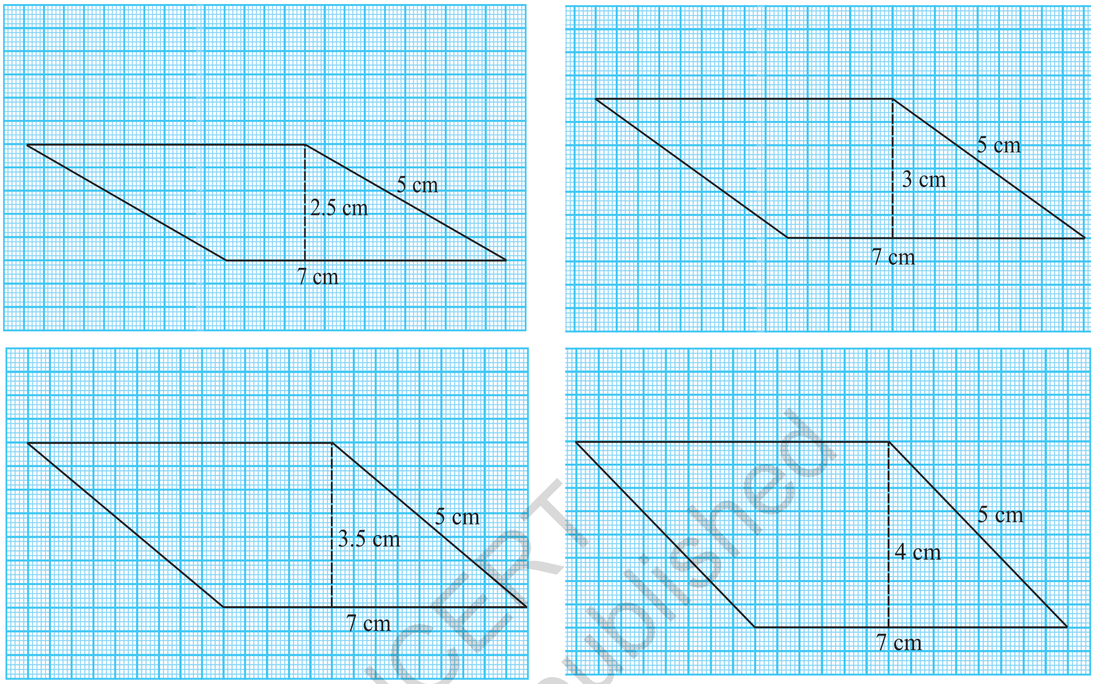


Fig 11.13

Find the perimeter and area of each of these parallelograms. Analyse your results.

You will find that these parallelograms have different areas but equal perimeters.

To find the area of a parallelogram, you need to know only the base and the corresponding height of the parallelogram.

TRY THESE

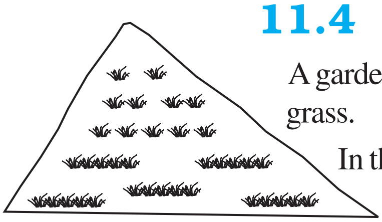
Find the area of following parallelograms:

(i)

(ii)

(iii) In a parallelogram ABCD, AB = 7.2 cm and the perpendicular from C on AB is 4.5 cm.

11.4 AREA OF A TRIANGLE



A gardener wants to know the cost of covering the whole of a triangular garden with grass.

In this case we need to know the area of the triangular region.

Let us find a method to get the area of a triangle.

Draw a scalene triangle on a piece of paper. Cut out the triangle. Place this triangle on another piece of paper and cut out another triangle of the same size.

So now you have two scalene triangles of the same size.

Are both the triangles congruent?

Superpose one triangle on the other so that they match. You may have to rotate one of the two triangles.

Now place both the triangles such that a pair of corresponding sides is joined as shown in Fig 11.14.

Is the figure thus formed a parallelogram?

Compare the area of each triangle to the area of the parallelogram.

Compare the base and height of the triangles with the base and height of the parallelogram.

You will find that the sum of the areas of both the triangles is equal to the area of the parallelogram. The base and the height of the triangle are the same as the base and the height of the parallelogram, respectively.

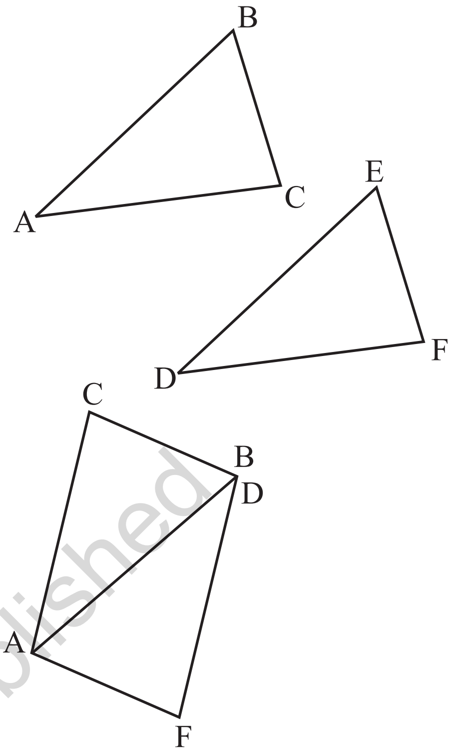


Fig 11.14

$$\text{Area of each triangle} = \frac{1}{2}(\text{Area of parallelogram})$$

$$= \frac{1}{2}(\text{base} \times \text{height}) \text{ (Since area of a parallelogram = base} \times \text{height)}$$

$$= \frac{1}{2}(b \times h) \text{ (or } \frac{1}{2}bh\text{, in short)}$$

TRY THESE

1. Try the above activity with different types of triangles.
2. Take different parallelograms. Divide each of the parallelograms into two triangles by cutting along any of its diagonals. Are the triangles congruent?

In the figure (Fig 11.15) all the triangles are on the base AB = 6 cm.

What can you say about the height of each of the triangles corresponding to the base AB?

Can we say all the triangles are equal in area? Yes.

Are the triangles congruent also? No.

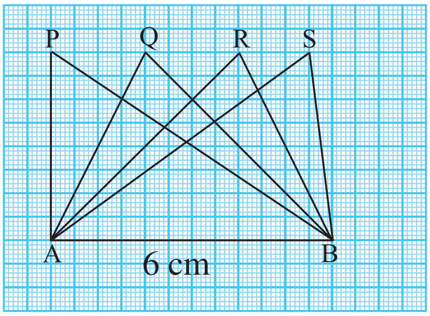


Fig 11.15

We conclude that **all the congruent triangles are equal in area but the triangles equal in area need not be congruent.**

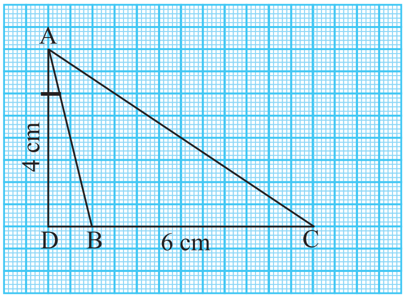


Fig 11.16

Consider the obtuse-angled triangle ABC of base 6 cm (Fig 11.16).

Its height AD which is perpendicular from the vertex A is outside the triangle.

Can you find the area of the triangle?

EXAMPLE 6 One of the sides and the corresponding height of a parallelogram are 4 cm and 3 cm respectively. Find the area of the parallelogram (Fig 11.17).

SOLUTION Given that length of base (b) = 4 cm, height (h) = 3 cm

Area of the parallelogram = $b \times h$

$= 4 \text{ cm} \times 3 \text{ cm} = 12 \text{ cm}^2$

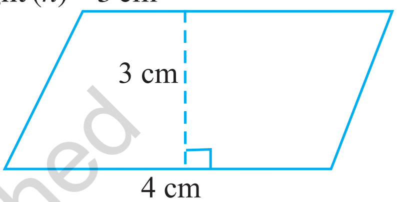


Fig 11.17

EXAMPLE 7 Find the height 'x' if the area of the parallelogram is 24 cm² and the base is 4 cm.

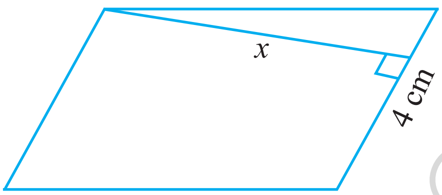


Fig 11.18

SOLUTION Area of parallelogram = $b \times h$

Therefore, $24 = 4 \times x$ (Fig 11.18)

or $\frac{24}{4} = x$ or $x = 6$ cm

So, the height of the parallelogram is 6 cm.

EXAMPLE 8 The two sides of the parallelogram ABCD are 6 cm and 4 cm. The height corresponding to the base CD is 3 cm (Fig 11.19). Find the

(i) area of the parallelogram. (ii) the height corresponding to the base AD.

SOLUTION

(i) Area of parallelogram = $b \times h$

$= 6 \text{ cm} \times 3 \text{ cm} = 18 \text{ cm}^2$

(ii) base (b) = 4 cm, height = x (say),

Area = 18 cm²

Area of parallelogram = $b \times x$

$18 = 4 \times x$

$\frac{18}{4} = x$

Therefore, $x = 4.5$ cm

Thus, the height corresponding to base AD is 4.5 cm.

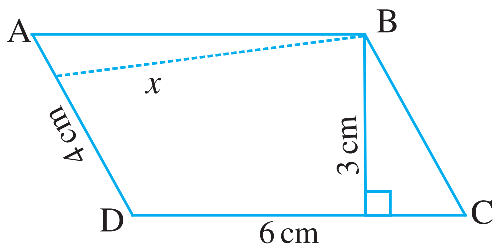


Fig 11.19

EXAMPLE 9 Find the area of the following triangles (Fig 11.20).

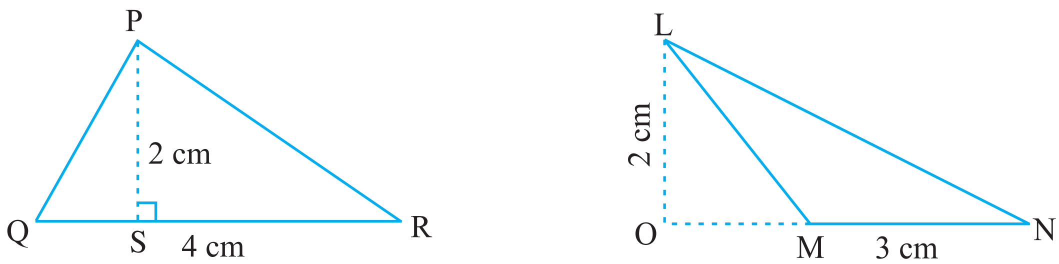


Fig 11.20

SOLUTION

(i) Area of triangle $= \frac{1}{2}bh = \frac{1}{2} \times \text{QR} \times \text{PS}$

$= \frac{1}{2} \times 4 \text{ cm} \times 2 \text{ cm} = 4 \text{ cm}^2$

(ii) Area of triangle $= \frac{1}{2}bh = \frac{1}{2} \times \text{MN} \times \text{LO}$

$= \frac{1}{2} \times 3 \text{ cm} \times 2 \text{ cm} = 3 \text{ cm}^2$

EXAMPLE 10 Find BC, if the area of the triangle ABC is 36 cm^2 and the height AD is 3 cm (Fig 11.21).

SOLUTION Height = 3 cm, Area = 36 cm^2

Area of the triangle ABC $= \frac{1}{2}bh$

or $36 = \frac{1}{2} \times b \times 3$ i.e., $b = \frac{36 \times 2}{3} = 24$ cm

So, BC = 24 cm

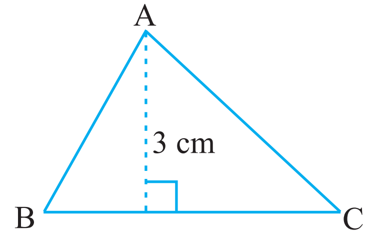


Fig 11.21

EXAMPLE 11 In ΔPQR, PR = 8 cm, QR = 4 cm and PL = 5 cm (Fig 11.22). Find:

(i) the area of the ΔPQR (ii) QM

SOLUTION

(i) QR = base = 4 cm, PL = height = 5 cm

Area of the triangle PQR $= \frac{1}{2}bh$

$= \frac{1}{2} \times 4 \text{ cm} \times 5 \text{ cm} = 10 \text{ cm}^2$

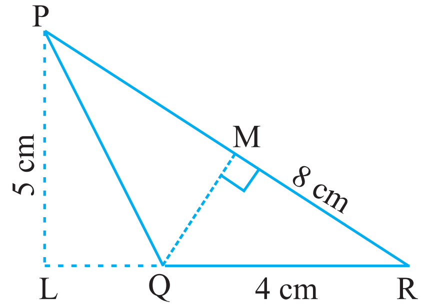


Fig 11.22

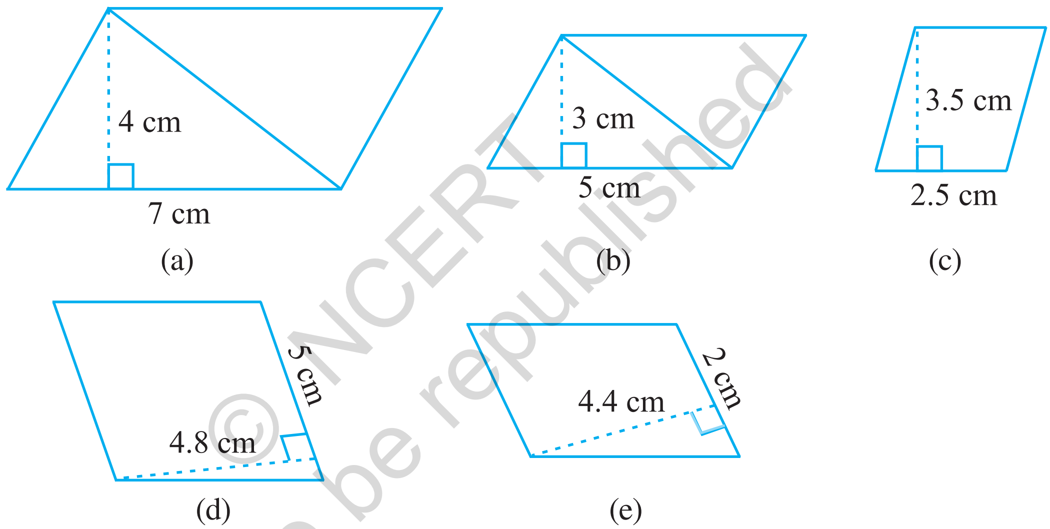
(ii) PR = base = 8 cm QM = height = ? Area = 10 cm²

$$\text{Area of triangle} = \frac{1}{2} \times b \times h \qquad \text{i.e.,} \qquad 10 = \frac{1}{2} \times 8 \times h$$

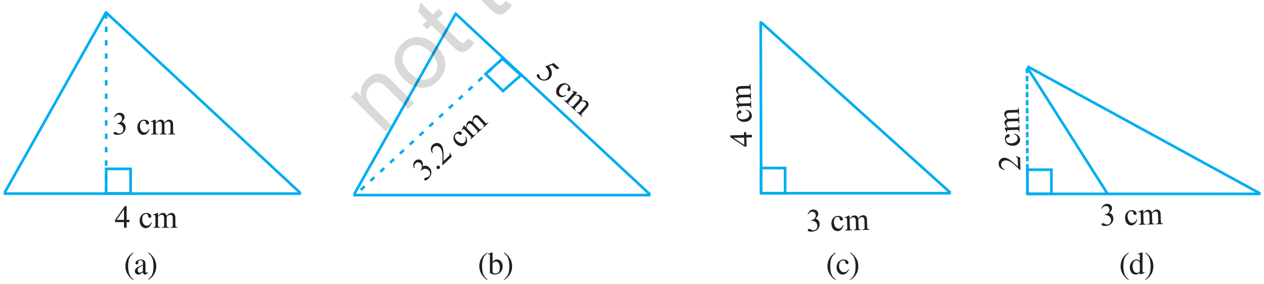
$$h = \frac{10}{4} = \frac{5}{2} = 2.5. \qquad \text{So,} \qquad QM = 2.5 \text{ cm}$$

EXERCISE 11.2

1. Find the area of each of the following parallelograms:



2. Find the area of each of the following triangles:



3. Find the missing values:

S.No.	Base	Height	Area of the Parallelogram
a.	20 cm		246 cm²
b.		15 cm	154.5 cm²
c.		8.4 cm	48.72 cm²
d.	15.6 cm		16.38 cm²

4. Find the missing values:

Base	Height	Area of Triangle
15 cm	______	87 cm^2
______	31.4 mm	1256 mm^2
22 cm	______	170.5 cm^2

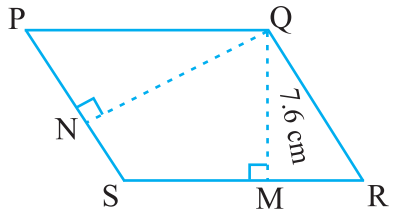


Fig 11.23

5. PQRS is a parallelogram (Fig 11.23). QM is the height from Q to SR and QN is the height from Q to PS. If SR = 12 cm and QM = 7.6 cm. Find:

(a) the area of the parallegram PQRS (b) QN, if PS = 8 cm

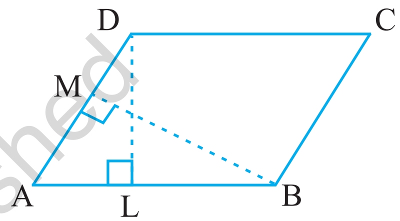


Fig 11.24

6. DL and BM are the heights on sides AB and AD respectively of parallelogram ABCD (Fig 11.24). If the area of the parallelogram is 1470 cm^2, AB = 35 cm and AD = 49 cm, find the length of BM and DL.

7. ΔABC is right angled at A (Fig 11.25). AD is perpendicular to BC. If AB = 5 cm, BC = 13 cm and AC = 12 cm, Find the area of ΔABC. Also find the length of AD.

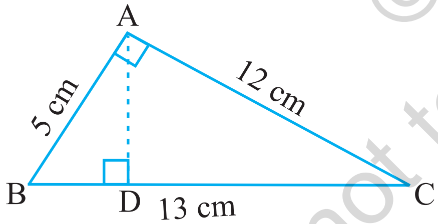


Fig 11.25

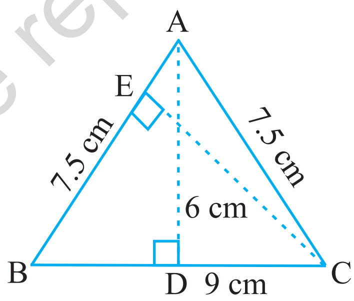


Fig 11.26

8. ΔABC is isosceles with AB = AC = 7.5 cm and BC = 9 cm (Fig 11.26). The height AD from A to BC, is 6 cm. Find the area of ΔABC. What will be the height from C to AB i.e., CE?

11.5 Circles

A racing track is semi-circular at both ends (Fig 11.27).

Can you find the distance covered by an athlete if he takes two rounds of a racing track? We need to find a method to find the distances around when a shape is circular.

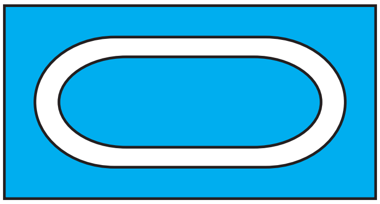
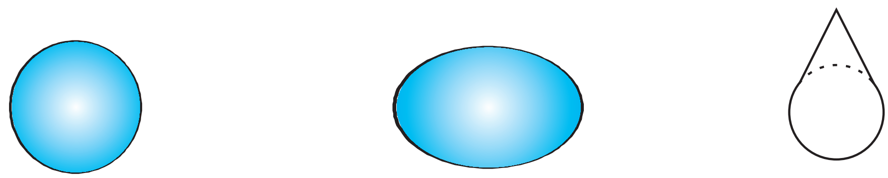


Fig 11.27

11.5.1 Circumference of a Circle

Tanya cut different cards, in curved shape from a cardboard. She wants to put lace around to decorate these cards. What length of the lace does she require for each? (Fig 11.28)



(a) (b) (c)

Fig 11.28

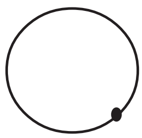


Fig 11.29

You cannot measure the curves with the help of a ruler, as these figures are not "straight". What can you do?

Here is a way to find the length of lace required for shape in Fig 11.28(a). Mark a point on the edge of the card and place the card on the table. Mark the position of the point on the table also (Fig 11. 29).

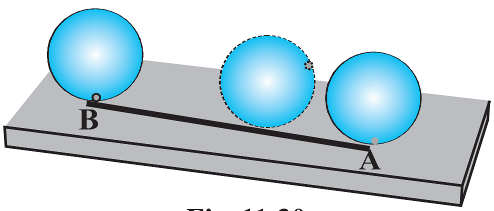


Fig 11.30

Now roll the circular card on the table along a straight line till the marked point again touches the table. Measure the distance along the line. This is the length of the lace required (Fig 11.30). It is also the distance along the edge of the card from the marked point back to the marked point.

You can also find the distance by putting a string on the edge of the circular object and taking all round it.

The distance around a circular region is known as its circumference.

Do This

Take a bottle cap, a bangle or any other circular object and find the circumference.

Now, can you find the distance covered by the athlete on the track by this method?

Still, it will be very difficult to find the distance around the track or any other circular object by measuring through string. Moreover, the measurement will not be accurate.

So, we need some formula for this, as we have for rectilinear figures or shapes.

Let us see if there is any relationship between the diameter and the circumference of the circles.

Consider the following table: Draw six circles of different radii and find their circumference by using string. Also find the ratio of the circumference to the diameter.

Circle	Radius	Diameter	Circumference	Ratio of Circumference to Diameter
1.	3.5 cm	7.0 cm	22.0 cm	$\frac{22}{7} = 3.14$

2.	7.0 cm	14.0 cm	44.0 cm	$\frac{44}{14} = 3.14$
3.	10.5 cm	21.0 cm	66.0 cm	$\frac{66}{21} = 3.14$
4.	21.0 cm	42.0 cm	132.0 cm	$\frac{132}{42} = 3.14$
5.	5.0 cm	10.0 cm	32.0 cm	$\frac{32}{10} = 3.2$
6.	15.0 cm	30.0 cm	94.0 cm	$\frac{94}{30} = 3.13$

What do you infer from the above table? Is this ratio approximately the same? Yes.

Can you say that the circumference of a circle is always more than three times its diameter? Yes.

This ratio is a constant and is denoted by π (pi). Its approximate value is $\frac{22}{7}$ or 3.14.

So, we can say that $\frac{C}{d} = \pi$, where 'C' represents circumference of the circle and 'd' its diameter.

or $\qquad C = \pi d$

We know that diameter (d) of a circle is twice the radius (r) i.e., $d = 2r$

So, $\qquad C = \pi d = \pi \times 2r \qquad$ or $\qquad C = 2\pi r$.

TRY THESE

In Fig 11.31,

(a) Which square has the larger perimeter?

(b) Which is larger, perimeter of smaller square or the circumference of the circle?

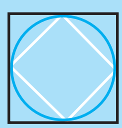


Fig 11.31

DO THIS

Take one each of quarter plate and half plate. Roll once each of these on a table-top. Which plate covers more distance in one complete revolution? Which plate will take less number of revolutions to cover the length of the table-top?

Example 12 What is the circumference of a circle of diameter 10 cm (Take $\pi = 3.14$)?

Solution Diameter of the circle (d) = 10 cm

Circumference of circle = πd

$= 3.14 \times 10$ cm $= 31.4$ cm

So, the circumference of the circle of diameter 10 cm is 31.4 cm.

Example 13 What is the circumference of a circular disc of radius 14 cm?

$$\left(\text{Use } \pi = \frac{22}{7}\right)$$

Solution Radius of circular disc (r) = 14 cm

Circumference of disc = $2\pi r$

$= 2 \times \frac{22}{7} \times 14$ cm $= 88$ cm

So, the circumference of the circular disc is 88 cm.

Example 14 The radius of a circular pipe is 10 cm. What length of a tape is required to wrap once around the pipe ($\pi = 3.14$)?

Solution Radius of the pipe (r) = 10 cm

Length of tape required is equal to the circumference of the pipe.

Circumference of the pipe = $2\pi r$

$= 2 \times 3.14 \times 10$ cm

$= 62.8$ cm

Therefore, length of the tape needed to wrap once around the pipe is 62.8 cm.

Example 15 Find the perimeter of the given shape (Fig 11.32) (Take $\pi = \frac{22}{7}$).

Solution In this shape we need to find the circumference of semicircles on each side of the square. Do you need to find the perimeter of the square also? No. The outer boundary, of this figure is made up of semicircles. Diameter of each semicircle is 14 cm.

We know that:

Circumference of the circle = πd

Circumference of the semicircle = $\frac{1}{2}\pi d$

$= \frac{1}{2} \times \frac{22}{7} \times 14$ cm $= 22$ cm

Circumference of each of the semicircles is 22 cm

Therefore, perimeter of the given figure = 4×22 cm $= 88$ cm

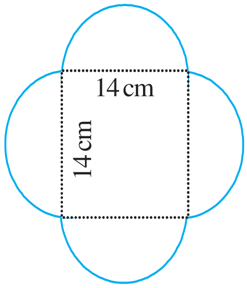


Fig 11.32

EXAMPLE 16 Sudhanshu divides a circular disc of radius 7 cm in two equal parts. What is the perimeter of each semicircular shape disc? (Use $\pi = \frac{22}{7}$)

SOLUTION To find the perimeter of the semicircular disc (Fig 11.33), we need to find

(i) Circumference of semicircular shape (ii) Diameter

Given that radius (r) = 7 cm. We know that the circumference of circle = $2\pi r$

So, the circumference of the semicircle = $\frac{1}{2} \times 2\pi r = \pi r$

$$= \frac{22}{7} \times 7 \text{ cm} = 22 \text{ cm}$$

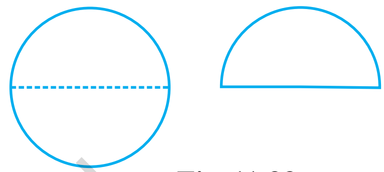


Fig 11.33

So, the diameter of the circle = $2r = 2 \times 7$ cm = 14 cm

Thus, perimeter of each semicircular disc = 22 cm + 14 cm = 36 cm

11.5.2 Area of Circle

Consider the following:

- A farmer dug a flower bed of radius 7 m at the centre of a field. He needs to purchase fertiliser. If 1 kg of fertiliser is required for 1 square metre area, how much fertiliser should he purchase?
- What will be the cost of polishing a circular table-top of radius 2 m at the rate of ₹ 10 per square metre?

Can you tell what we need to find in such cases, Area or Perimeter? In such cases we need to find the area of the circular region. Let us find the area of a circle, using graph paper.

Draw a circle of radius 4 cm on a graph paper (Fig 11.34). Find the area by counting the number of squares enclosed.

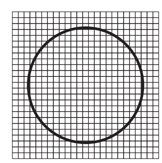
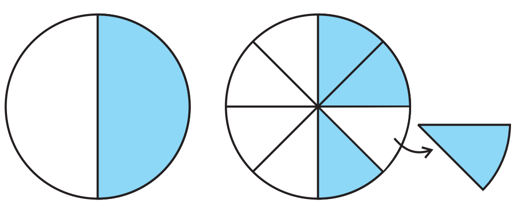


Fig 11.34

As the edges are not straight, we get a rough estimate of the area of circle by this method. There is another way of finding the area of a circle.

Draw a circle and shade one half of the circle [Fig 11.35(i)]. Now fold the circle into **eighths** and cut along the folds [Fig 11.35(ii)].



(i) (ii)

Fig 11.35

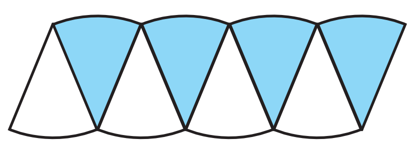


Fig 11.36

Arrange the separate pieces as shown, in Fig 11.36, which is roughly a parallelogram.

The more sectors we have, the nearer we reach an appropriate parallelogram.

As done above if we divide the circle in 64 sectors, and arrange these sectors. It gives nearly a rectangle (Fig 11.37).

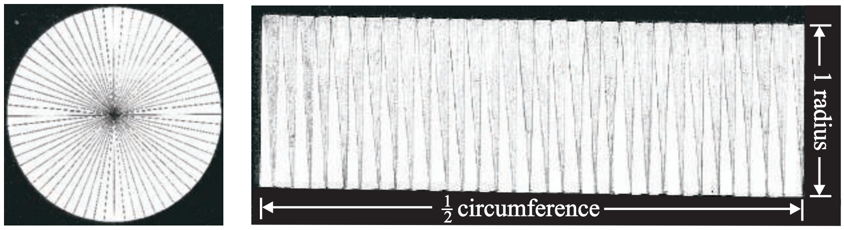


Fig 11.37

What is the breadth of this rectangle? The breadth of this rectangle is the radius of the circle, i.e., 'r'.

As the whole circle is divided into 64 sectors and on each side we have 32 sectors, the length of the rectangle is the length of the 32 sectors, which is half of the circumference. (Fig 11.37)

Area of the circle = Area of rectangle thus formed = $l \times b$

= (Half of circumference) × radius $= \left(\frac{1}{2} \times 2\pi r\right) \times r = \pi r^2$

So, the area of the circle = πr^2

Try These

Draw circles of different radii on a graph paper. Find the area by counting the number of squares. Also find the area by using the formula. Compare the two answers.

Example 17 Find the area of a circle of radius 30 cm (use $\pi = 3.14$).

Solution Radius, $r = 30$ cm

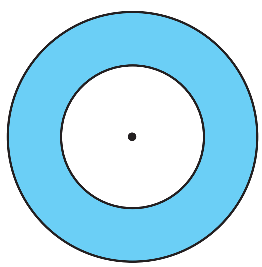
Area of the circle = $\pi r^2 = 3.14 \times 30^2 = 2{,}826$ cm^2

Example 18 Diameter of a circular garden is 9.8 m. Find its area.

Solution Diameter, $d = 9.8$ m. Therefore, radius $r = 9.8 \div 2 = 4.9$ m

Area of the circle = $\pi r^2 = \frac{22}{7} \times (4.9)^2 \text{ m}^2 = \frac{22}{7} \times 4.9 \times 4.9 \text{ m}^2 = 75.46 \text{ m}^2$

Example 19 The adjoining figure shows two circles with the same centre. The radius of the larger circle is 10 cm and the radius of the smaller circle is 4 cm.



Find: (a) the area of the larger circle

(b) the area of the smaller circle

(c) the shaded area between the two circles. ($\pi = 3.14$)

SOLUTION

(a) Radius of the larger circle = 10 cm

So, area of the larger circle = πr^2

$= 3.14 \times 10 \times 10 = 314\text{ cm}^2$

(b) Radius of the smaller circle = 4 cm

Area of the smaller circle = πr^2

$= 3.14 \times 4 \times 4 = 50.24\text{ cm}^2$

(c) Area of the shaded region = $(314 - 50.24)\text{ cm}^2 = 263.76\text{ cm}^2$

EXERCISE 11.3

1. Find the circumference of the circles with the following radius: (Take $\pi = \frac{22}{7}$)

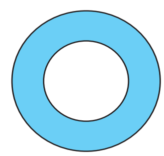
(a) 14 cm (b) 28 mm (c) 21 cm

2. Find the area of the following circles, given that:

(a) radius = 14 mm (Take $\pi = \frac{22}{7}$) (b) diameter = 49 m

(c) radius = 5 cm

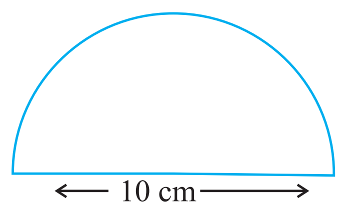
3. If the circumference of a circular sheet is 154 m, find its radius. Also find the area of the sheet. (Take $\pi = \frac{22}{7}$)



4. A gardener wants to fence a circular garden of diameter 21m. Find the length of the rope he needs to purchase, if he makes 2 rounds of fence. Also find the cost of the rope, if it costs ₹ 4 per meter. (Take $\pi = \frac{22}{7}$)

5. From a circular sheet of radius 4 cm, a circle of radius 3 cm is removed. Find the area of the remaining sheet. (Take $\pi = 3.14$)

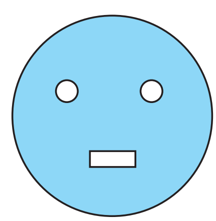
6. Saima wants to put a lace on the edge of a circular table cover of diameter 1.5 m. Find the length of the lace required and also find its cost if one meter of the lace costs ₹ 15. (Take $\pi = 3.14$)



7. Find the perimeter of the adjoining figure, which is a semicircle including its diameter.

8. Find the cost of polishing a circular table-top of diameter 1.6 m, if the rate of polishing is ₹ 15/m^2. (Take $\pi = 3.14$)

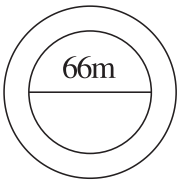
9. Shazli took a wire of length 44 cm and bent it into the shape of a circle. Find the radius of that circle. Also find its area. If the same wire is bent into the shape of a square, what will be the length of each of its sides? Which figure encloses more area, the circle or the square? (Take $\pi = \frac{22}{7}$)



10. From a circular card sheet of radius 14 cm, two circles of radius 3.5 cm and a rectangle of length 3 cm and breadth 1cm are removed. (as shown in the adjoining figure). Find the area of the remaining sheet. (Take $\pi = \frac{22}{7}$)

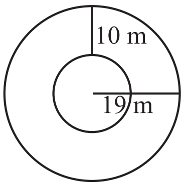
11. A circle of radius 2 cm is cut out from a square piece of an aluminium sheet of side 6 cm. What is the area of the left over aluminium sheet? (Take $\pi = 3.14$)

12. The circumference of a circle is 31.4 cm. Find the radius and the area of the circle? (Take $\pi = 3.14$)



13. A circular flower bed is surrounded by a path 4 m wide. The diameter of the flower bed is 66 m. What is the area of this path? ($\pi = 3.14$)

14. A circular flower garden has an area of 314 m^2. A sprinkler at the centre of the garden can cover an area that has a radius of 12 m. Will the sprinkler water the entire garden? (Take $\pi = 3.14$)



15. Find the circumference of the inner and the outer circles, shown in the adjoining figure? (Take $\pi = 3.14$)

16. How many times a wheel of radius 28 cm must rotate to go 352 m? (Take $\pi = \frac{22}{7}$)

17. The minute hand of a circular clock is 15 cm long. How far does the tip of the minute hand move in 1 hour. (Take $\pi = 3.14$)

11.6 Conversion of Units

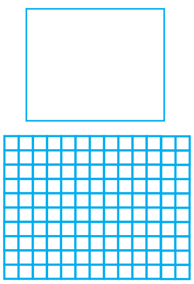


Fig 11.38

We know that 1 cm = 10 mm. Can you tell 1 cm^2 is equal to how many mm^2? Let us explore similar questions and find how to convert units while measuring areas to another unit.

Draw a square of side 1cm (Fig 11.38), on a graph sheet.

You find that this square of side 1 cm will be divided into 100 squares, each of side 1 mm.

Area of a square of side 1cm = Area of 100 squares, of each side 1mm.

Therefore, $1 \text{ cm}^2 = 100 \times 1 \text{ mm}^2$

or $1 \text{ cm}^2 = 100 \text{ mm}^2$

Similarly, $1 \text{ m}^2 = 1 \text{ m} \times 1 \text{ m}$

$= 100 \text{ cm} \times 100 \text{ cm}$ (As 1 m = 100 cm)

$= 10000 \text{ cm}^2$

Now can you convert 1 km^2 into m^2?

In the metric system, areas of land are also measured in hectares [written "ha" in short].

A square of side 100 m has an area of 1 hectare.

So, $1 \text{ hectare} = 100 \times 100 \text{ m}^2 = 10{,}000 \text{ m}^2$

When we convert a unit of area to a smaller unit, the resulting number of units will be bigger.

For example, $1000 \text{ cm}^2 = 1000 \times 100 \text{ mm}^2$

$= 100000 \text{ mm}^2$

But when we convert a unit of area to a larger unit, the number of larger units will be smaller.

For example, $1000 \text{ cm}^2 = \frac{1000}{10000} \text{ m}^2 = 0.1 \text{ m}^2$

Try These

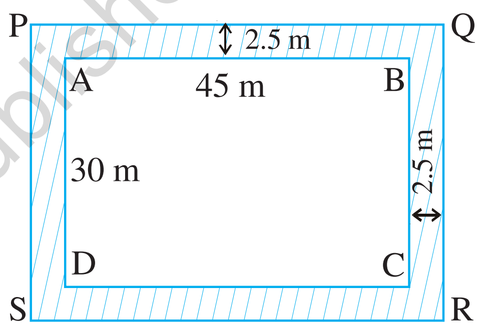
Convert the following:

(i) 50 cm^2 in mm^2 (ii) 2 ha in m^2 (iii) 10 m^2 in cm^2 (iv) 1000 cm^2 in m^2

11.7 Applications

You must have observed that quite often, in gardens or parks, some space is left all around in the form of path or in between as cross paths. A framed picture has some space left all around it.

We need to find the areas of such pathways or borders when we want to find the cost of making them.



Example 20 A rectangular park is 45 m long and 30 m wide. A path 2.5 m wide is constructed outside the park. Find the area of the path.

Solution Let ABCD represent the rectangular park and the shaded region represent the path 2.5 m wide.

To find the area of the path, we need to find (Area of rectangle PQRS – Area of rectangle ABCD).

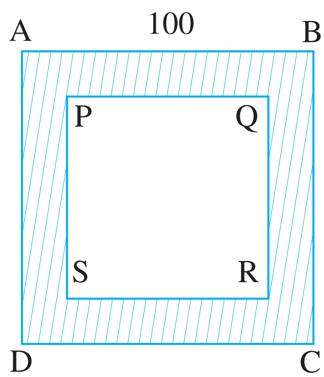
We have, PQ = (45 + 2.5 + 2.5) m = 50 m

PS = (30 + 2.5 + 2.5) m = 35 m

Area of the rectangle ABCD = $l \times b$ = $45 \times 30 \text{ m}^2 = 1350 \text{ m}^2$

Area of the rectangle PQRS = $l \times b$ = $50 \times 35 \text{ m}^2 = 1750 \text{ m}^2$

Area of the path = Area of the rectangle PQRS – Area of the rectangle ABCD = $(1750 - 1350) \text{ m}^2 = 400 \text{ m}^2$



Example 21 A path 5 m wide runs along inside a square park of side 100 m. Find the area of the path. Also find the cost of cementing it at the rate of ₹ 250 per 10 m².

Solution Let ABCD be the square park of side 100 m. The shaded region represents the path 5 m wide.

PQ = 100 – (5 + 5) = 90 m

Area of square ABCD = $(\text{side})^2 = (100)^2 \text{ m}^2 = 10000 \text{ m}^2$

Area of square PQRS = $(\text{side})^2 = (90)^2 \text{ m}^2 = 8100 \text{ m}^2$

Therefore, area of the path = $(10000 - 8100) \text{ m}^2 = 1900 \text{ m}^2$

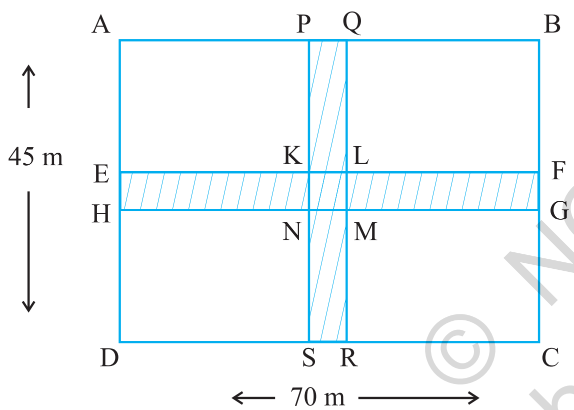
Cost of cementing 10 m^2 = ₹ 250

Therefore, cost of cementing 1 m^2 = ₹ $\dfrac{250}{10}$

So, cost of cementing 1900 m^2 = ₹ $\dfrac{250}{10} \times 1900$ = ₹ 47,500

Example 22 Two cross roads, each of width 5 m, run at right angles through the centre of a rectangular park of length 70 m and breadth 45 m and parallel to its sides. Find the area of the roads. Also find the cost of constructing the roads at the rate of ₹ 105 per m^2.

Solution Area of the cross roads is the area of shaded portion, i.e., the area of the rectangle PQRS and the area of the rectangle EFGH. But while doing this, the area of the square KLMN is taken twice, which is to be subtracted.



Now, PQ = 5 m and PS = 45 m

EH = 5 m and EF = 70 m

KL = 5 m and KN = 5 m

Area of the path = Area of the rectangle PQRS + area of the rectangle EFGH – Area of the square KLMN

$= PS \times PQ + EF \times EH - KL \times KN$

$= (45 \times 5 + 70 \times 5 - 5 \times 5)\ m^2$

$= (225 + 350 - 25)\ m^2 = 550\ m^2$

Cost of constructing the path = ₹ 105 × 550 = ₹ 57,750

Exercise 11.4

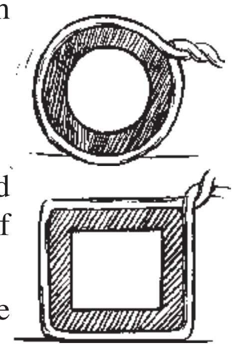
1. A garden is 90 m long and 75 m broad. A path 5 m wide is to be built outside and around it. Find the area of the path. Also find the area of the garden in hectare.
2. A 3 m wide path runs outside and around a rectangular park of length 125 m and breadth 65 m. Find the area of the path.
3. A picture is painted on a cardboard 8 cm long and 5 cm wide such that there is a margin of 1.5 cm along each of its sides. Find the total area of the margin.
4. A *verandah* of width 2.25 m is constructed all along outside a room which is 5.5 m long and 4 m wide. Find:
   (i) the area of the *verandah*.
   (ii) the cost of cementing the floor of the *verandah* at the rate of ₹ 200 per m^2.
5. A path 1 m wide is built along the border and inside a square garden of side 30 m. Find:
   (i) the area of the path
   (ii) the cost of planting grass in the remaining portion of the garden at the rate of ₹ 40 per m^2.

6. Two cross roads, each of width 10 m, cut at right angles through the centre of a rectangular park of length 700 m and breadth 300 m and parallel to its sides. Find the area of the roads. Also find the area of the park excluding cross roads. Give the answer in hectares.

7. Through a rectangular field of length 90 m and breadth 60 m, two roads are constructed which are parallel to the sides and cut each other at right angles through the centre of the fields. If the width of each road is 3 m, find

(i) the area covered by the roads.

(ii) the cost of constructing the roads at the rate of ₹ 110 per m^2.



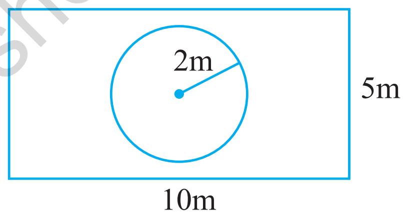
8. Pragya wrapped a cord around a circular pipe of radius 4 cm (adjoining figure) and cut off the length required of the cord. Then she wrapped it around a square box of side 4 cm (also shown). Did she have any cord left? ($\pi = 3.14$)

9. The adjoining figure represents a rectangular lawn with a circular flower bed in the middle. Find:

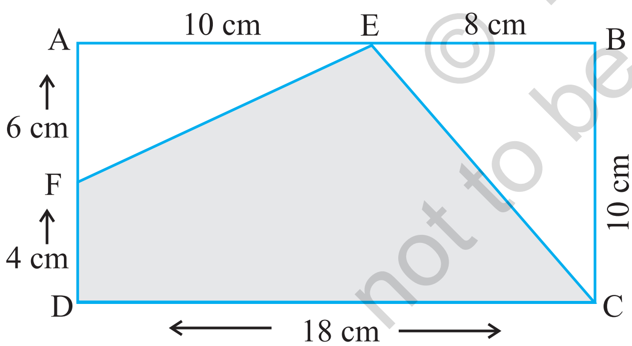
(i) the area of the whole land (ii) the area of the flower bed

(iii) the area of the lawn excluding the area of the flower bed

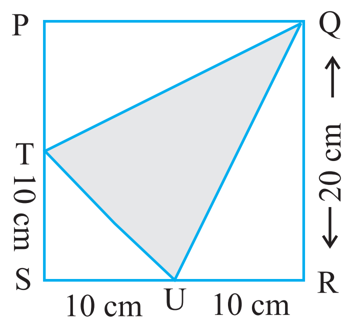
(iv) the circumference of the flower bed.



10. In the following figures, find the area of the shaded portions:



(i)



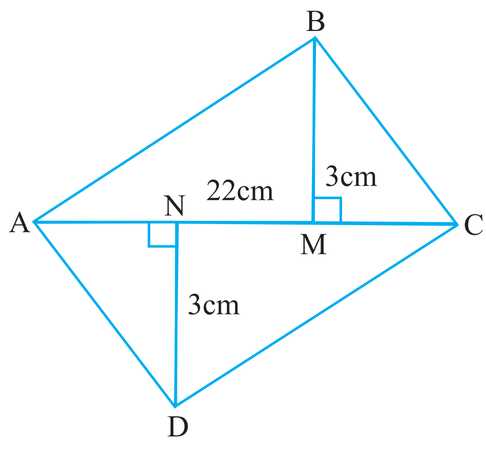
(ii)

11. Find the area of the quadrilateral ABCD.

Here, AC = 22 cm, BM = 3 cm,

DN = 3 cm, and

BM ⊥ AC, DN ⊥ AC



What have We Discussed?

1. Perimeter is the distance around a closed figure whereas area is the part of plane occupied by the closed figure.
2. We have learnt how to find perimeter and area of a square and rectangle in the earlier class. They are:
   (a) Perimeter of a square = 4 × side
   (b) Perimeter of a rectangle = 2 × (length + breadth)
   (c) Area of a square = side × side
   (d) Area of a rectangle = length × breadth
3. Area of a parallelogram = base × height
4. Area of a triangle = $\frac{1}{2}$ (area of the parallelogram generated from it)

   $= \frac{1}{2} \times$ base × height
5. The distance around a circular region is known as its circumference.

   Circumference of a circle = πd, where d is the diameter of a circle and $\pi = \frac{22}{7}$ or 3.14 (approximately).
6. Area of a circle = πr^2, where r is the radius of the circle.
7. Based on the conversion of units for lengths, studied earlier, the units of areas can also be converted:

   1 cm^2 = 100 mm^2, 1 m^2 = 10000 cm^2, 1 hectare = 10000 m^2.

Chapter 12

Algebraic Expressions

0756CH12

12.1 Introduction

We have already come across simple algebraic expressions like $x + 3$, $y - 5$, $4x + 5$, $10y - 5$ and so on. In Class VI, we have seen how these expressions are useful in formulating puzzles and problems. We have also seen examples of several expressions in the chapter on simple equations.

Expressions are a central concept in algebra. This Chapter is devoted to algebraic expressions. When you have studied this Chapter, you will know how algebraic expressions are formed, how they can be combined, how we can find their values and how they can be used.

12.2 How are Expressions Formed?

We now know very well what a variable is. We use letters x, y, l, m, ... etc. to denote variables. A **variable** can take various values. Its value is not fixed. On the other hand, a **constant** has a fixed value. Examples of constants are: 4, 100, –17, etc.

We combine variables and constants to make algebraic expressions. For this, we use the operations of addition, subtraction, multiplication and division. We have already come across expressions like $4x + 5$, $10y - 20$. The expression $4x + 5$ is obtained from the variable x, first by multiplying x by the constant 4 and then adding the constant 5 to the product. Similarly, $10y - 20$ is obtained by first multiplying y by 10 and then subtracting 20 from the product.

The above expressions were obtained by combining variables with constants. We can also obtain expressions by combining variables with themselves or with other variables.

Look at how the following expressions are obtained:

$$x^2,\ 2y^2,\ 3x^2 - 5,\ xy,\ 4xy + 7$$

(i) The expression x^2 is obtained by multiplying the variable x by itself;

$$x \times x = x^2$$

Just as 4×4 is written as 4^2, we write $x \times x = x^2$. It is commonly read as x squared.

(Later, when you study the chapter 'Exponents and Powers' you will realise that x^2 may also be read as x raised to the power 2).

In the same manner, we can write $\quad x \times x \times x = x^3$

Commonly, x^3 is read as 'x cubed'. Later, you will realise that x^3 may also be read as x raised to the power 3.

$x, x^2, x^3, \ldots$ are all algebraic expressions obtained from x.

(ii) The expression $2y^2$ is obtained from y: $\quad 2y^2 = 2 \times y \times y$

Here by multiplying y with y we obtain y^2 and then we multiply y^2 by the constant 2.

(iii) In $(3x^2 - 5)$ we first obtain x^2, and multiply it by 3 to get $3x^2$. From $3x^2$, we subtract 5 to finally arrive at $3x^2 - 5$.

(iv) In xy, we multiply the variable x with another variable y. Thus, $x \times y = xy$.

(v) In $4xy + 7$, we first obtain xy, multiply it by 4 to get $4xy$ and add 7 to $4xy$ to get the expression.

TRY THESE

Describe how the following expressions are obtained:

$7xy + 5,\ x^2y,\ 4x^2 - 5x$

12.3 TERMS OF AN EXPRESSION

We shall now put in a systematic form what we have learnt above about how expressions are formed. For this purpose, we need to understand what **terms** of an expression and their **factors** are.

Consider the expression $(4x + 5)$. In forming this expression, we first formed $4x$ separately as a product of 4 and x and then added 5 to it. Similarly consider the expression $(3x^2 + 7y)$. Here we first formed $3x^2$ separately as a product of 3, x and x. We then formed $7y$ separately as a product of 7 and y. Having formed $3x^2$ and $7y$ separately, we added them to get the expression.

You will find that the expressions we deal with can always be seen this way. They have parts which are formed separately and then added. Such parts of an expression which are formed separately first and then added are known as **terms**. Look at the expression $(4x^2 - 3xy)$. We say that it has two terms, $4x^2$ and $-3xy$. The term $4x^2$ is a product of 4, x and x, and the term $(-3xy)$ is a product of (-3), x and y.

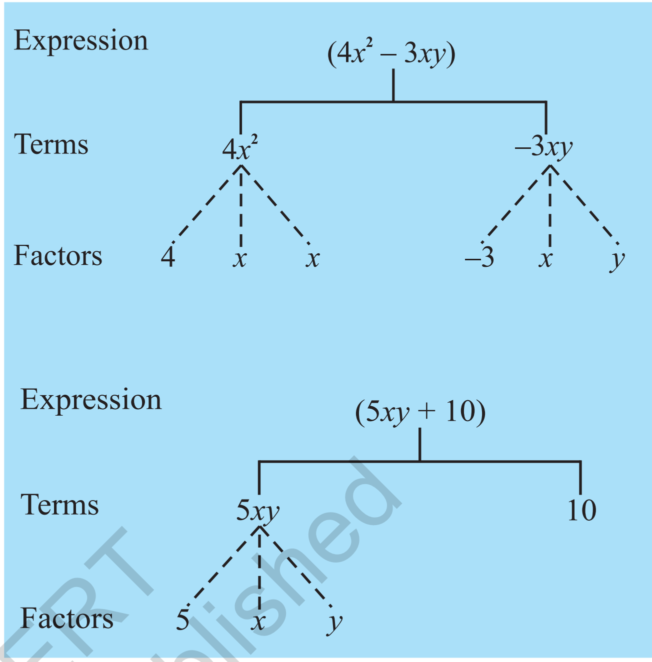
Terms are added to form expressions. Just as the terms $4x$ and 5 are added to form the expression $(4x + 5)$, the terms $4x^2$ and $(-3xy)$ are added to give the expression $(4x^2 - 3xy)$. This is because $4x^2 + (-3xy) = 4x^2 - 3xy$.

Note, the minus sign (–) is included in the term. In the expression $4x^2 - 3xy$, we took the term as $(-3xy)$ and not as $(3xy)$. That is why we do not need to say that terms are 'added or subtracted' to form an expression; just 'added' is enough.

Factors of a term

We saw above that the expression $(4x^2 - 3xy)$ consists of two terms $4x^2$ and $-3xy$. The term $4x^2$ is a product of 4, x and x; we say that 4, x and x are the factors of the term $4x^2$. A term is a product of its factors. The term $-3xy$ is a product of the factors -3, x and y.

We can represent the terms and factors of the terms of an expression conveniently and elegantly by a tree diagram. The tree for the expression ($4x^2$ – $3xy$) is as shown in the adjacent figure.



Note, in the tree diagram, we have used dotted lines for factors and continuous lines for terms. This is to avoid mixing them.

Let us draw a tree diagram for the expression $5xy + 10$.

The factors are such that they cannot be further factorised. Thus we do not write $5xy$ as $5 \times xy$, because xy can be further factorised. Similarly, if x^3 were a term, it would be written as $x \times x \times x$ and not $x^2 \times x$. Also, remember that 1 is not taken as a separate factor.

Coefficients

We have learnt how to write a term as a product of factors. One of these factors may be numerical and the others algebraic (i.e., they contain variables). The numerical factor is said to be the numerical coefficient or simply the **coefficient** of the term. It is also said to be the coefficient of the rest of the term (which is obviously the product of algebraic factors of the term). Thus in $5xy$, 5 is the coefficient of the term. It is also the coefficient of xy. In the term $10xyz$, 10 is the coefficient of xyz, in the term $-7x^2y^2$, –7 is the coefficient of x^2y^2.

When the coefficient of a term is +1, it is usually omitted. For example, $1x$ is written as x; $1\,x^2y^2$ is written as x^2y^2 and so on. Also, the coefficient (–1) is indicated only by the minus sign. Thus $(-1)\,x$ is written as $-x$; $(-1)\,x^2y^2$ is written as $-x^2y^2$ and so on.

Try These

1. What are the terms in the following expressions? Show how the terms are formed. Draw a tree diagram for each expression:

   $8y + 3x^2$, $7mn - 4$, $2x^2y$.

2. Write three expression each having 4 terms.

Sometimes, the word 'coefficient' is used in a more general way. Thus we say that in the term $5xy$, 5 is the coefficient of xy, x is the coefficient of $5y$ and y is the coefficient of $5x$. In $10xy^2$, 10 is the coefficient of xy^2, x is the coefficient of $10y^2$ and y^2 is the coefficient of $10x$. Thus, in this more general way, a coefficient may be either a numerical factor or an algebraic factor or a product of two or more factors. It is said to be the coefficient of the product of the remaining factors.

Try These

Identify the coefficients of the terms of following expressions:

$4x - 3y$, $a + b + 5$, $2y + 5$, $2xy$

Example 1 Identify, in the following expressions, terms which are not constants. Give their numerical coefficients:

$$xy + 4,\ 13 - y^2,\ 13 - y + 5y^2,\ 4p^2q - 3pq^2 + 5$$

SOLUTION

S. No.	Expression	Term (which is not a Constant)	Numerical Coefficient
(i)	$xy + 4$	xy	1
(ii)	$13 - y^2$	$-y^2$	–1
(iii)	$13 - y + 5y^2$	$-y$ $5y^2$	–1 5
(iv)	$4p^2q - 3pq^2 + 5$	$4p^2q$ $-3pq^2$	4 –3

EXAMPLE 2

(a) What are the coefficients of x in the following expressions?

$$4x - 3y,\ 8 - x + y,\ y^2x - y,\ 2z - 5xz$$

(b) What are the coefficients of y in the following expressions?

$$4x - 3y,\ 8 + yz,\ yz^2 + 5,\ my + m$$

SOLUTION

(a) In each expression we look for a term with x as a factor. The remaining part of that term is the coefficient of x.

S. No.	Expression	Term with Factor x	Coefficient of x
(i)	$4x - 3y$	$4x$	4
(ii)	$8 - x + y$	$-x$	–1
(iii)	$y^2x - y$	y^2x	y^2
(iv)	$2z - 5xz$	$-5xz$	$-5z$

(b) The method is similar to that in (a) above.

S. No.	Expression	Term with factor y	Coefficient of y
(i)	$4x - 3y$	$-3y$	–3
(ii)	$8 + yz$	yz	z
(iii)	$yz^2 + 5$	yz^2	z^2
(iv)	$my + m$	my	m

12.4 LIKE AND UNLIKE TERMS

When terms have the same algebraic factors, they are **like** terms. When terms have different algebraic factors, they are **unlike** terms. For example, in the expression $2xy - 3x + 5xy - 4$, look at the terms $2xy$ and $5xy$. The factors of $2xy$ are 2, x and y. The factors of $5xy$ are 5, x and y. Thus their algebraic (i.e., those which contain variables) factors are the same and hence they are **like** terms. On the other hand the terms $2xy$ and $-3x$, have different algebraic factors. They are **unlike** terms. Similarly, the terms, $2xy$ and 4, are unlike terms. Also, the terms $-3x$ and 4 are unlike terms.

TRY THESE

Group the like terms together from the following:

$12x$, 12, $-25x$, -25, $-25y$, 1, x, $12y$, y

12.5 MONOMIALS, BINOMIALS, TRINOMIALS AND POLYNOMIALS

An expression with only one term is called a **monomial**; for example, $7xy$, $-5m$, $3z^2$, 4 etc.

An expression which contains two unlike terms is called a **binomial**; for example, $x + y$, $m - 5$, $mn + 4m$, $a^2 - b^2$ are binomials. The expression $10pq$ is not a binomial; it is a monomial. The expression $(a + b + 5)$ is not a binomial. It contains three terms.

TRY THESE

Classify the following expressions as a monomial, a binomial or a trinomial: a, $a + b$, $ab + a + b$, $ab + a + b - 5$, xy, $xy + 5$, $5x^2 - x + 2$, $4pq - 3q + 5p$, 7, $4m - 7n + 10$, $4mn + 7$.

An expression which contains three terms is called a **trinomial**; for example, the expressions $x + y + 7$, $ab + a + b$, $3x^2 - 5x + 2$, $m + n + 10$ are trinomials. The expression $ab + a + b + 5$ is, however not a trinomial; it contains four terms and not three. The expression $x + y + 5x$ is not a trinomial as the terms x and $5x$ are like terms.

In general, an expression with one or more terms is called a **polynomial**. Thus a monomial, a binomial and a trinomial are all polynomials.

EXAMPLE 3 State with reasons, which of the following pairs of terms are of like terms and which are of unlike terms:

(i) $7x$, $12y$ (ii) $15x$, $-21x$ (iii) $-4ab$, $7ba$ (iv) $3xy$, $3x$

(v) $6xy^2$, $9x^2y$ (vi) pq^2, $-4pq^2$ (vii) mn^2, $10mn$

SOLUTION

S. No.	Pair	Factors	Algebraic factors same or different	Like/ Unlike terms	Remarks
(i)	$7x$ $12y$	$7, x$ $12, y$	Different	Unlike	The variables in the terms are different.
(ii)	$15x$ $-21x$	$15, x$ $-21, x$	Same	Like	
(iii)	$-4ab$ $7ba$	$-4, a, b$ $7, a, b$	Same	Like	Remember $ab = ba$

(iv)	$3xy$ $3x$	$\left.\begin{matrix}3, x, y\\ 3, x\end{matrix}\right\}$	Different	Unlike	The variable y is only in one term.
(v)	$6xy^2$ $9x^2y$	$\left.\begin{matrix}6, x, y, y\\ 9, x, x, y\end{matrix}\right\}$	Different	Unlike	The variables in the two terms match, but their powers do not match.
(vi)	pq^2 $-4pq^2$	$\left.\begin{matrix}1, p, q, q\\ -4, p, q, q\end{matrix}\right\}$	Same	Like	Note, numerical factor 1 is not shown

Following simple steps will help you to decide whether the given terms are **like** *or* **unlike** *terms:*

(i) Ignore the numerical coefficients. Concentrate on the algebraic part of the terms.

(ii) Check the variables in the terms. They must be the same.

(iii) Next, check the powers of each variable in the terms. They must be the same.

Note that in deciding like terms, two things do not matter (1) the numerical coefficients of the terms and (2) the order in which the variables are multiplied in the terms.

EXERCISE 12.1

1. Get the algebraic expressions in the following cases using variables, constants and arithmetic operations.

(i) Subtraction of z from y.

(ii) One-half of the sum of numbers x and y.

(iii) The number z multiplied by itself.

(iv) One-fourth of the product of numbers p and q.

(v) Numbers x and y both squared and added.

(vi) Number 5 added to three times the product of numbers m and n.

(vii) Product of numbers y and z subtracted from 10.

(viii) Sum of numbers a and b subtracted from their product.

2. (i) Identify the terms and their factors in the following expressions

Show the terms and factors by tree diagrams.

(a) $x - 3$ (b) $1 + x + x^2$ (c) $y - y^3$

(d) $5xy^2 + 7x^2y$ (e) $-ab + 2b^2 - 3a^2$

(ii) Identify terms and factors in the expressions given below:

(a) $-4x + 5$ (b) $-4x + 5y$ (c) $5y + 3y^2$

(d) $xy + 2x^2y^2$ (e) $pq + q$ (f) $1.2\,ab - 2.4\,b + 3.6\,a$

(g) $\frac{3}{4}x + \frac{1}{4}$ (h) $0.1\ p^2 + 0.2\ q^2$

3. Identify the numerical coefficients of terms (other than constants) in the following expressions:

(i) $5 - 3t^2$ (ii) $1 + t + t^2 + t^3$ (iii) $x + 2xy + 3y$

(iv) $100m + 1000n$ (v) $-p^2q^2 + 7pq$ (vi) $1.2\ a + 0.8\ b$

(vii) $3.14\ r^2$ (viii) $2\ (l + b)$ (ix) $0.1\ y + 0.01\ y^2$

4. (a) Identify terms which contain x and give the coefficient of x.

(i) $y^2x + y$ (ii) $13y^2 - 8yx$ (iii) $x + y + 2$

(iv) $5 + z + zx$ (v) $1 + x + xy$ (vi) $12xy^2 + 25$

(vii) $7x + xy^2$

(b) Identify terms which contain y^2 and give the coefficient of y^2.

(i) $8 - xy^2$ (ii) $5y^2 + 7x$ (iii) $2x^2y - 15xy^2 + 7y^2$

5. Classify into monomials, binomials and trinomials.

(i) $4y - 7z$ (ii) y^2 (iii) $x + y - xy$ (iv) 100

(v) $ab - a - b$ (vi) $5 - 3t$ (vii) $4p^2q - 4pq^2$ (viii) $7mn$

(ix) $z^2 - 3z + 8$ (x) $a^2 + b^2$ (xi) $z^2 + z$

(xii) $1 + x + x^2$

6. State whether a given pair of terms is of like or unlike terms.

(i) 1, 100 (ii) $-7x,\ \frac{5}{2}x$ (iii) $-29x,\ -29y$

(iv) $14xy,\ 42yx$ (v) $4m^2p,\ 4mp^2$ (vi) $12xz,\ 12x^2z^2$

7. Identify like terms in the following:

(a) $-xy^2,\ -4yx^2,\ 8x^2,\ 2xy^2,\ 7y,\ -11x^2,\ -100x,\ -11yx,\ 20x^2y,$

$-6x^2,\ y,\ 2xy,\ 3x$

(b) $10pq,\ 7p,\ 8q,\ -p^2q^2,\ -7qp,\ -100q,\ -23,\ 12q^2p^2,\ -5p^2,\ 41,\ 2405p,\ 78qp,$
$13p^2q,\ qp^2,\ 701p^2$

12.6 Addition and Subtraction of Algebraic Expressions

Consider the following problems:

1. Sarita has some marbles. Ameena has 10 more. Appu says that he has 3 more marbles than the number of marbles Sarita and Ameena together have. How do you get the number of marbles that Appu has?

   Since it is not given how many marbles Sarita has, we shall take it to be x. Ameena then has 10 more, i.e., $x + 10$. Appu says that he has 3 more marbles than what Sarita and Ameena have together. So we take the sum of the numbers of Sarita's marbles and Ameena's marbles, and to this sum add 3, that is, we take the sum of x, $x + 10$ and 3.

2. Ramu's father's present age is 3 times Ramu's age. Ramu's grandfather's age is 13 years more than the sum of Ramu's age and Ramu's father's age. How do you find Ramu's grandfather's age?

   Since Ramu's age is not given, let us take it to be y years. Then his father's age is $3y$ years. To find Ramu's grandfather's age we have to take the sum of Ramu's age (y) and his father's age ($3y$) and to the sum add 13, that is, we have to take the sum of y, $3y$ and 13.

3. In a garden, roses and marigolds are planted in square plots. The length of the square plot in which marigolds are planted is 3 metres greater than the length of the square plot in which roses are planted. How much bigger in area is the marigold plot than the rose plot?

   Let us take l metres to be length of the side of the rose plot. The length of the side of the marigold plot will be $(l + 3)$ metres. Their respective areas will be l^2 and $(l + 3)^2$. The difference between $(l^2 + 3)^2$ and l^2 will decide how much bigger in area the marigold plot is.

In all the three situations, we had to carry out addition or subtraction of algebraic expressions. There are a number of real life problems in which we need to use expressions and do arithmetic operations on them. In this section, we shall see how algebraic expressions are added and subtracted.

TRY THESE

Think of atleast two situations in each of which you need to form two algebraic expressions and add or subtract them

Adding and subtracting like terms

The simplest expressions are monomials. They consist of only one term. To begin with we shall learn how to add or subtract like terms.

- Let us add $3x$ and $4x$. We know x is a number and so also are $3x$ and $4x$.

  Now, $3x + 4x = (3 \times x) + (4 \times x)$

  $= (3 + 4) \times x$ (using distributive law)

  $= 7 \times x = 7x$

  or $3x + 4x = 7x$

  Since variables are numbers, we can use distributive law for them.

- Let us next add $8xy$, $4xy$ and $2xy$

  $8xy + 4xy + 2xy = (8 \times xy) + (4 \times xy) + (2 \times xy)$

  $= (8 + 4 + 2) \times xy$

  $= 14 \times xy = 14xy$

  or $8xy + 4xy + 2xy = 14\ xy$

- Let us subtract $4n$ from $7n$.

$$7n - 4n = (7 \times n) - (4 \times n)$$
$$= (7 - 4) \times n = 3 \times n = 3n$$

or $$7n - 4n = 3n$$

- In the same way, subtract $5ab$ from $11ab$.

$$11ab - 5ab = (11 - 5)\ ab = 6ab$$

Thus, the sum of two or more like terms is a like term with a numerical coefficient equal to the sum of the numerical coefficients of all the like terms.

Similarly, the difference between two like terms is a like term with a numerical coefficient equal to the difference between the numerical coefficients of the two like terms.

Note, unlike terms cannot be added or subtracted the way like terms are added or subtracted. We have already seen examples of this, when 5 is added to x, we write the result as $(x + 5)$. Observe that in $(x + 5)$ both the terms 5 and x are retained.

Similarly, if we add the unlike terms $3xy$ and 7, the sum is $3xy + 7$.

If we subtract 7 from $3xy$, the result is $3xy - 7$

Adding and subtracting general algebraic expressions

Let us take some examples:

- Add $3x + 11$ and $7x - 5$

  The sum = $3x + 11 + 7x - 5$

  Now, we know that the terms $3x$ and $7x$ are like terms and so also are 11 and -5. Further $3x + 7x = 10x$ and $11 + (-5) = 6$. We can, therefore, simplify the sum as:

  The sum = $3x + 11 + 7x - 5$

  $= 3x + 7x + 11 - 5$ (rearranging terms)

  $= 10x + 6$

  Hence, $3x + 11 + 7x - 5 = 10x + 6$

- Add $3x + 11 + 8z$ and $7x - 5$.

  The sum = $3x + 11 + 8z + 7x - 5$

  $= 3x + 7x + 11 - 5 + 8z$ (rearranging terms)

  Note we have put like terms together; the single unlike term $8z$ will remain as it is.

  Therefore, the sum = $10x + 6 + 8z$

- Subtract $a - b$ from $3a - b + 4$

$$\text{The difference} = 3a - b + 4 - (a - b)$$
$$= 3a - b + 4 - a + b$$

Observe how we took $(a - b)$ in brackets and took care of signs in opening the bracket. Rearranging the terms to put like terms together,

$$\text{The difference} = 3a - a + b - b + 4$$
$$= (3 - 1)\,a + (1 - 1)\,b + 4$$
$$\text{The difference} = 2a + (0)\,b + 4 = 2a + 4$$

or $\quad 3a - b + 4 - (a - b) = 2a + 4$

Note, just as
$-(5 - 3) = -5 + 3$,
$-(a - b) = -a + b$.
The signs of algebraic terms are handled in the same way as signs of numbers.

We shall now solve some more examples on addition and subtraction of expression for practice.

EXAMPLE 4 Collect like terms and simplify the expression:

$$12m^2 - 9m + 5m - 4m^2 - 7m + 10$$

SOLUTION Rearranging terms, we have

$$12m^2 - 4m^2 + 5m - 9m - 7m + 10$$
$$= (12 - 4)\,m^2 + (5 - 9 - 7)\,m + 10$$
$$= 8m^2 + (-4 - 7)\,m + 10$$
$$= 8m^2 + (-11)\,m + 10$$
$$= 8m^2 - 11m + 10$$

TRY THESE

Add and subtract
(i) $m - n, m + n$
(ii) $mn + 5 - 2, mn + 3$

EXAMPLE 5 Subtract $24ab - 10b - 18a$ from $30ab + 12b + 14a$.

SOLUTION $30ab + 12b + 14a - (24ab - 10b - 18a)$

$$= 30ab + 12b + 14a - 24ab + 10b + 18a$$
$$= 30ab - 24ab + 12b + 10b + 14a + 18a$$
$$= 6ab + 22b + 32a$$

Note, subtracting a term is the same as adding its inverse. Subtracting $-10b$ is the same as adding $+10b$; Subtracting $-18a$ is the same as adding 18a and subtracting $24ab$ is the same as adding $-24ab$. The signs shown below the expression to be subtracted are a help in carrying out the subtraction properly.

Alternatively, we write the expressions one below the other with the like terms appearing exactly below like terms as:

$$\begin{array}{r} 30ab + 12b + 14a \\ 24ab - 10b - 18a \\ - \quad\quad + \quad\quad + \\ \hline 6ab + 22b + 32a \\ \hline \end{array}$$

EXAMPLE 6 From the sum of $2y^2 + 3yz, -y^2 - yz - z^2$ and $yz + 2z^2$, subtract the sum of $3y^2 - z^2$ and $-y^2 + yz + z^2$.

SOLUTION We first add $2y^2 + 3yz, -y^2 - yz - z^2$ and $yz + 2z^2$.

$$\begin{array}{rrrrrr} & 2y^2 & + & 3yz & & \\ - & y^2 & - & yz & - & z^2 \\ & & + & yz & + & 2z^2 \\ \hline & y^2 & + & 3yz & + & z^2 \\ \hline \end{array} \qquad (1)$$

We then add $3y^2 - z^2$ and $-y^2 + yz + z^2$

$$\begin{array}{rrrrrr} & 3y^2 & & & - & z^2 \\ - & y^2 & + & yz & + & z^2 \\ \hline & 2y^2 & + & yz & & \\ \hline \end{array} \qquad (2)$$

Now we subtract sum (2) from the sum (1):

$$\begin{array}{rrrrrr} & y^2 & + & 3yz & + & z^2 \\ & 2y^2 & + & yz & & \\ - & & - & & & \\ \hline & -y^2 & + & 2yz & + & z^2 \\ \hline \end{array}$$

EXERCISE 12.2

1. Simplify combining like terms:
   (i) $21b - 32 + 7b - 20b$
   (ii) $-z^2 + 13z^2 - 5z + 7z^3 - 15z$
   (iii) $p - (p - q) - q - (q - p)$
   (iv) $3a - 2b - ab - (a - b + ab) + 3ab + b - a$
   (v) $5x^2y - 5x^2 + 3yx^2 - 3y^2 + x^2 - y^2 + 8xy^2 - 3y^2$
   (vi) $(3y^2 + 5y - 4) - (8y - y^2 - 4)$

2. Add:
   (i) $3mn, -5mn, 8mn, -4mn$
   (ii) $t - 8tz, 3tz - z, z - t$
   (iii) $-7mn + 5, 12mn + 2, 9mn - 8, -2mn - 3$
   (iv) $a + b - 3, b - a + 3, a - b + 3$
   (v) $14x + 10y - 12xy - 13, 18 - 7x - 10y + 8xy, 4xy$
   (vi) $5m - 7n, 3n - 4m + 2, 2m - 3mn - 5$
   (vii) $4x^2y, -3xy^2, -5xy^2, 5x^2y$

(viii) $3p^2q^2 - 4pq + 5, - 10\ p^2q^2, 15 + 9pq + 7p^2q^2$

(ix) $ab - 4a, 4b - ab, 4a - 4b$

(x) $x^2 - y^2 - 1, y^2 - 1 - x^2, 1 - x^2 - y^2$

3. Subtract:

(i) $-5y^2$ from y^2

(ii) $6xy$ from $-12xy$

(iii) $(a - b)$ from $(a + b)$

(iv) $a\,(b - 5)$ from $b\,(5 - a)$

(v) $-m^2 + 5mn$ from $4m^2 - 3mn + 8$

(vi) $- x^2 + 10x - 5$ from $5x - 10$

(vii) $5a^2 - 7ab + 5b^2$ from $3ab - 2a^2 - 2b^2$

(viii) $4pq - 5q^2 - 3p^2$ from $5p^2 + 3q^2 - pq$

4. (a) What should be added to $x^2 + xy + y^2$ to obtain $2x^2 + 3xy$?

(b) What should be subtracted from $2a + 8b + 10$ to get $- 3a + 7b + 16$?

5. What should be taken away from $3x^2 - 4y^2 + 5xy + 20$ to obtain $- x^2 - y^2 + 6xy + 20$?

6. (a) From the sum of $3x - y + 11$ and $- y - 11$, subtract $3x - y - 11$.

(b) From the sum of $4 + 3x$ and $5 - 4x + 2x^2$, subtract the sum of $3x^2 - 5x$ and $-x^2 + 2x + 5$.

12.7 Finding the Value of an Expression

We know that the value of an algebraic expression depends on the values of the variables forming the expression. There are a number of situations in which we need to find the value of an expression, such as when we wish to check whether a particular value of a variable satisfies a given equation or not.

We find values of expressions, also, when we use formulas from geometry and from everyday mathematics. For example, the area of a square is l^2, where l is the length of a side of the square. If $l = 5$ cm., the area is $5^2\,\text{cm}^2$ or $25\,\text{cm}^2$; if the side is 10 cm, the area is $10^2\,\text{cm}^2$ or $100\,\text{cm}^2$ and so on. We shall see more such examples in the next section.

Example 7 Find the values of the following expressions for $x = 2$.

(i) $x + 4$ (ii) $4x - 3$ (iii) $19 - 5x^2$

(iv) $100 - 10x^3$

Solution Putting $x = 2$

(i) In $x + 4$, we get the value of $x + 4$, i.e.,

$x + 4 = 2 + 4 = 6$

(ii) In $4x - 3$, we get

$4x - 3 = (4 \times 2) - 3 = 8 - 3 = 5$

(iii) In $19 - 5x^2$, we get

$19 - 5x^2 = 19 - (5 \times 2^2) = 19 - (5 \times 4) = 19 - 20 = -1$

(iv) In $100 - 10x^3$, we get

$100 - 10x^3 = 100 - (10 \times 2^3) = 100 - (10 \times 8)$ (Note $2^3 = 8$)

$= 100 - 80 = 20$

EXAMPLE 8 Find the value of the following expressions when $n = -2$.

(i) $5n - 2$ (ii) $5n^2 + 5n - 2$ (iii) $n^3 + 5n^2 + 5n - 2$

SOLUTION

(i) Putting the value of $n = -2$, in $5n - 2$, we get,

$5(-2) - 2 = -10 - 2 = -12$

(ii) In $5n^2 + 5n - 2$, we have,

for $n = -2$, $5n - 2 = -12$

and $5n^2 = 5 \times (-2)^2 = 5 \times 4 = 20$ [as $(-2)^2 = 4$]

Combining,

$5n^2 + 5n - 2 = 20 - 12 = 8$

(iii) Now, for $n = -2$,

$5n^2 + 5n - 2 = 8$ and

$n^3 = (-2)^3 = (-2) \times (-2) \times (-2) = -8$

Combining,

$n^3 + 5n^2 + 5n - 2 = -8 + 8 = 0$

We shall now consider expressions of two variables, for example, $x + y$, xy. To work out the numerical value of an expression of two variables, we need to give the values of both variables. For example, the value of $(x + y)$, for $x = 3$ and $y = 5$, is $3 + 5 = 8$.

EXAMPLE 9 Find the value of the following expressions for $a = 3$, $b = 2$.

(i) $a + b$ (ii) $7a - 4b$ (iii) $a^2 + 2ab + b^2$

(iv) $a^3 - b^3$

SOLUTION Substituting $a = 3$ and $b = 2$ in

(i) $a + b$, we get

$a + b = 3 + 2 = 5$

(ii) $7a - 4b$, we get

$7a - 4b = 7 \times 3 - 4 \times 2 = 21 - 8 = 13.$

(iii) $a^2 + 2ab + b^2$, we get

$a^2 + 2ab + b^2 = 3^2 + 2 \times 3 \times 2 + 2^2 = 9 + 2 \times 6 + 4 = 9 + 12 + 4 = 25$

(iv) $a^3 - b^3$, we get

$a^3 - b^3 = 3^3 - 2^3 = 3 \times 3 \times 3 - 2 \times 2 \times 2 = 9 \times 3 - 4 \times 2 = 27 - 8 = 19$

EXERCISE 12.3

1. If $m = 2$, find the value of:

(i) $m - 2$ (ii) $3m - 5$ (iii) $9 - 5m$

(iv) $3m^2 - 2m - 7$ (v) $\frac{5m}{2}$ 4

2. If $p = -2$, find the value of:

(i) $4p + 7$ (ii) $-3p^2 + 4p + 7$ (iii) $-2p^3 - 3p^2 + 4p + 7$

3. Find the value of the following expressions, when $x = -1$:

(i) $2x - 7$ (ii) $-x + 2$ (iii) $x^2 + 2x + 1$

(iv) $2x^2 - x - 2$

4. If $a = 2$, $b = -2$, find the value of:

(i) $a^2 + b^2$ (ii) $a^2 + ab + b^2$ (iii) $a^2 - b^2$

5. When $a = 0$, $b = -1$, find the value of the given expressions:

(i) $2a + 2b$ (ii) $2a^2 + b^2 + 1$ (iii) $2a^2b + 2ab^2 + ab$

(iv) $a^2 + ab + 2$

6. Simplify the expressions and find the value if x is equal to 2

(i) $x + 7 + 4(x - 5)$ (ii) $3(x + 2) + 5x - 7$

(iii) $6x + 5(x - 2)$ (iv) $4(2x - 1) + 3x + 11$

7. Simplify these expressions and find their values if $x = 3$, $a = -1$, $b = -2$.

(i) $3x - 5 - x + 9$ (ii) $2 - 8x + 4x + 4$

(iii) $3a + 5 - 8a + 1$ (iv) $10 - 3b - 4 - 5b$

(v) $2a - 2b - 4 - 5 + a$

8. (i) If $z = 10$, find the value of $z^3 - 3(z - 10)$.

(ii) If $p = -10$, find the value of $p^2 - 2p - 100$

9. What should be the value of a if the value of $2x^2 + x - a$ equals to 5, when $x = 0$?

10. Simplify the expression and find its value when $a = 5$ and $b = -3$.

$$2(a^2 + ab) + 3 - ab$$

12.8 Using Algebraic Expressions – Formulas and Rules

We have seen earlier also that formulas and rules in mathematics can be written in a concise and general form using algebraic expressions. We see below several examples.

● Perimeter formulas

1. The perimeter of an equilateral triangle = $3 \times$ the length of its side. If we denote the length of the side of the equilateral triangle by l, then **the perimeter of the equilateral triangle = $3l$**
2. Similarly, **the perimeter of a square = $4l$**
   where l = the length of the side of the square.
3. **Perimeter of a regular pentagon = $5l$**
   where l = the length of the side of the pentagon and so on.

● Area formulas

1. If we denote the length of a square by l, then the area of the square = l^2
2. If we denote the length of a rectangle by l and its breadth by b, then the area of the rectangle = $l \times b = lb$.
3. Similarly, if b stands for the base and h for the height of a triangle, then the area of the triangle = $\dfrac{b \times h}{2} = \dfrac{bh}{2}$.

Once a formula, that is, the algebraic expression for a given quantity is known, the value of the quantity can be computed as required.

For example, for a square of length 3 cm, the perimeter is obtained by putting the value $l = 3$ cm in the expression of the perimeter of a square, i.e., $4l$.

The perimeter of the given square = (4×3) cm = 12 cm.

Similarly, the area of the square is obtained by putting in the value of l (= 3 cm) in the expression for the area of a square, that is, l^2;

Area of the given square = $(3)^2$ cm^2 = 9 cm^2.

● Rules for number patterns

Study the following statements:

1. If a natural number is denoted by n, its successor is $(n + 1)$. We can check this for any natural number. For example, if $n = 10$, its successor is $n + 1 = 11$, which is known.

2. If a natural number is denoted by n, $2n$ is an even number and $(2n + 1)$ an odd number. Let us check it for any number, say, 15; $2n = 2 \times n = 2 \times 15 = 30$ is indeed an even number and $2n + 1 = 2 \times 15 + 1 = 30 + 1 = 31$ is indeed an odd number.

Do This

Take (small) line segments of equal length such as matchsticks, tooth pricks or pieces of straws cut into smaller pieces of equal length. Join them in patterns as shown in the figures given:

1. Observe the pattern in Fig 12.1.

It consists of repetitions of the shape □ made from 4 line segments. As you see for one shape you need 4 segments, for two shapes 7, for three 10 and so on. If n is the number of shapes, then the number of segments required to form n shapes is given by $(3n + 1)$.

You may verify this by taking $n = 1, 2, 3, 4, ..., 10, ...$ etc. For example, if the number of shapes formed is 3, then the number of line segments required is $3 \times 3 + 1 = 9 + 1 = 10$, as seen from the figure.

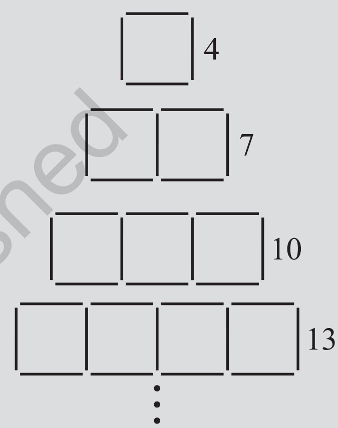


Fig 12.1

2. Now, consider the pattern in Fig 12.2. Here the shape $\sqcup$ is repeated. The number of segments required to form 1, 2, 3, 4, ... shapes are 3, 5, 7, 9, ... respectively. If n stands for the shapes formed, the number of segments required is given by the expression $(2n + 1)$. You may check if the expression is correct by taking any value of n, say $n = 4$. Then $(2n + 1) = (2 \times 4) + 1 = 9$, which is indeed the number of line segments required to make 4 $\sqcup$s.

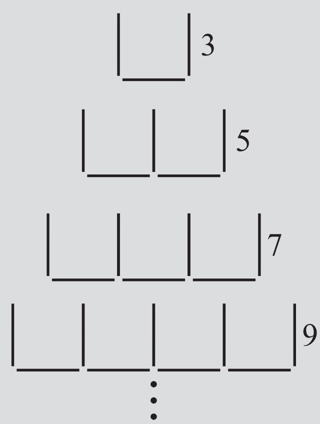
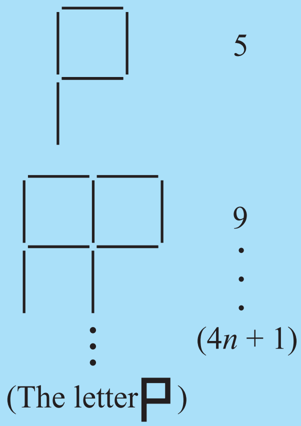


Fig 12.2

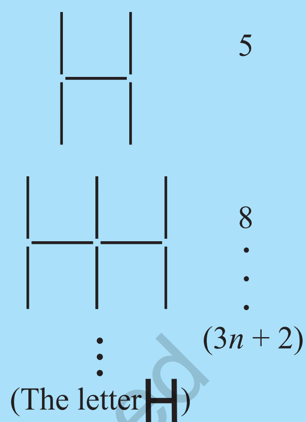
TRY THESE

Make similar pattern with basic figures as shown

(i)



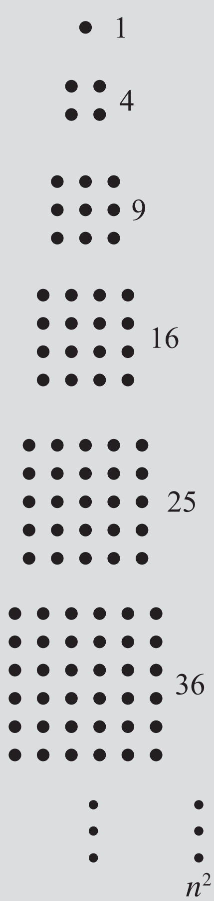
(ii)



(The number of segments required to make the figure is given to the right. Also, the expression for the number of segments required to make n shapes is also given).

Go ahead and discover more such patterns.

DO THIS



Make the following pattern of dots. If you take a graph paper or a dot paper, it will be easier to make the patterns.

Observe how the dots are arranged in a square shape. If the number of dots in a row or a column in a particular figure is taken to be the variable n, then the number of dots in the figure is given by the expression $n \times n = n^2$. For example, take $n = 4$. The number of dots for the figure with 4 dots in a row (or a column) is $4 \times 4 = 16$, as is indeed seen from the figure. You may check this for other values of n. The ancient Greek mathematicians called the number 1, 4, 9, 16, 25, ... square numbers.

● Some more number patterns

Let us now look at another pattern of numbers, this time without any drawing to help us

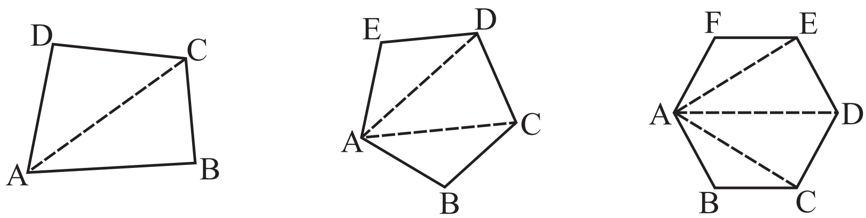
$$3, 6, 9, 12, ..., 3n, ...$$

The numbers are such that they are multiples of 3 arranged in an increasing order, beginning with 3. The term which occurs at the n^{th} position is given by the expression $3n$. You can easily find the term which occurs in the 10^{th} position (which is $3 \times 10 = 30$); 100^{th} position (which is $3 \times 100 = 300$) and so on.

● Pattern in geometry

What is the number of diagonals we can draw from one vertex of a quadrilateral? Check it, it is one.

From one vertex of a pentagon? Check it, it is 2.

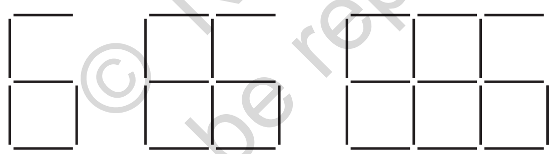


From one vertex of a hexagon? It is 3.

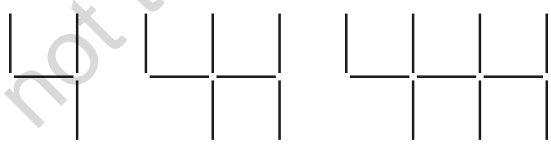
The number of diagonals we can draw from one vertex of a polygon of n sides is $(n - 3)$. Check it for a heptagon (7 sides) and octagon (8 sides) by drawing figures. What is the number for a triangle (3 sides)? Observe that the diagonals drawn from any one vertex divide the polygon in as many non-overlapping triangles as the number of diagonals that can be drawn from the vertex plus one.

EXERCISE 12.4

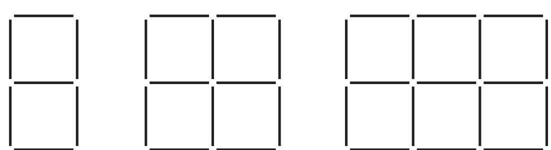
1. Observe the patterns of digits made from line segments of equal length. You will find such segmented digits on the display of electronic watches or calculators.

(a)  ... ...

6 11 16 21 ... $(5n + 1)$ ...

(b)  ... ...

4 7 10 13 ... $(3n + 1)$ ...

(c)  ... ...

7 12 17 22 ... $(5n + 2)$ ...

If the number of digits formed is taken to be n, the number of segments required to form n digits is given by the algebraic expression appearing on the right of each pattern.

How many segments are required to form 5, 10, 100 digits of the kind .

2. Use the given algebraic expression to complete the table of number patterns.

S. No.	Expression	Terms									
		1st	2nd	3rd	4th	5th	...	10th	...	100th	...
(i)	$2n - 1$	1	3	5	7	9	-	19	-	-	-
(ii)	$3n + 2$	5	8	11	14	-	-	-	-	-	-
(iii)	$4n + 1$	5	9	13	17	-	-	-	-	-	-
(iv)	$7n + 20$	27	34	41	48	-	-	-	-	-	-
(v)	$n^2 + 1$	2	5	10	17	-	-	-	-	10,001	-

What have We Discussed?

1. **Algebraic expressions** are formed from **variables** and **constants**. We use the operations of **addition, subtraction, multiplication** and **division** on the variables and constants to form expressions. For example, the expression $4xy + 7$ is formed from the variables x and y and constants 4 and 7. The constant 4 and the variables x and y are multiplied to give the product $4xy$ and the constant 7 is added to this product to give the expression.
2. Expressions are made up of **terms**. Terms are **added** to make an expression. For example, the addition of the terms $4xy$ and 7 gives the expression $4xy + 7$.
3. A term is a **product of factors**. The term $4xy$ in the expression $4xy + 7$ is a product of factors x, y and 4. Factors containing variables are said to be **algebraic factors**.
4. The **coefficient** is the numerical factor in the term. Sometimes anyone factor in a term is called the coefficient of the remaining part of the term.
5. Any expression with one or more terms is called a **polynomial**. Specifically a one term expression is called a **monomial**; a two-term expression is called a **binomial**; and a three-term expression is called a **trinomial**.
6. Terms which have the same algebraic factors are **like** terms. Terms which have different algebraic factors are **unlike** terms. Thus, terms $4xy$ and $-3xy$ are like terms; but terms $4xy$ and $-3x$ are not like terms.
7. The **sum** (or **difference**) **of two like terms** is a **like** term with coefficient equal to the **sum** (or **difference**) **of the coefficients** of the two like terms. Thus, $8xy - 3xy = (8 - 3)xy$, i.e., $5xy$.
8. When we **add** two algebraic expressions, the like terms are added as given above; the **unlike** terms are **left as they are**. Thus, the sum of $4x^2 + 5x$ and $2x + 3$ is $4x^2 + 7x + 3$; the like terms $5x$ and $2x$ add to $7x$; the unlike terms $4x^2$ and 3 are left as they are.

9. In situations such as solving an equation and using a formula, we have to **find the value of an expression**. The value of the expression depends on the value of the variable from which the expression is formed. Thus, the value of $7x - 3$ for $x = 5$ is 32, since $7(5) - 3 = 35 - 3 = 32$.

10. **Rules and formulas** in mathematics are **written** in a concise and general form using algebraic expressions:

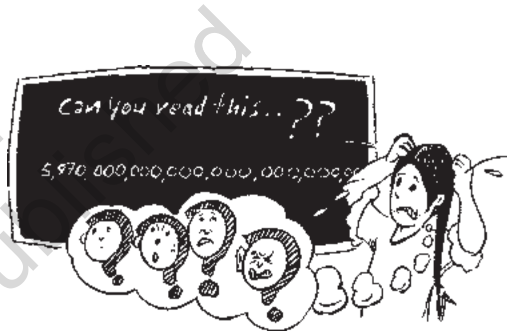
    Thus, the area of rectangle = lb, where l is the length and b is the breadth of the rectangle.

    The general (n^{th}) term of a number pattern (or a sequence) is an expression in n. Thus, the n^{th} term of the number pattern 11, 21, 31, 41, . . . is $(10n + 1)$.

Chapter 13

Exponents and Powers

13.1 Introduction



Do you know what the mass of earth is? It is 5,970,000,000,000,000,000,000,000 kg! Can you read this number?

Mass of Uranus is 86,800,000,000,000,000,000,000,000 kg.

Which has greater mass, Earth or Uranus?

Distance between Sun and Saturn is 1,433,500,000,000 m and distance between Saturn and Uranus is 1,439,000,000,000 m. Can you read these numbers? Which distance is less?

These very large numbers are difficult to read, understand and compare. To make these numbers easy to read, understand and compare, we use exponents. In this Chapter, we shall learn about exponents and also learn how to use them.

13.2 Exponents

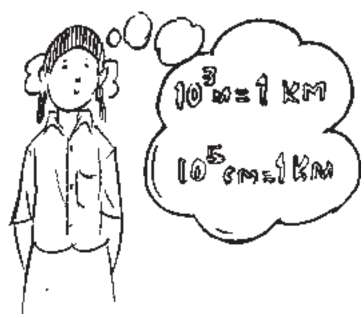
We can write large numbers in a shorter form using exponents.

Observe $\quad 10,000 = 10 \times 10 \times 10 \times 10 = 10^4$

The short notation 10^4 stands for the product $10\times10\times10\times10$. Here '10' is called the **base** and '4' the **exponent**. **The number 10^4 is read as 10 raised to the power of 4** or simply as **fourth power of 10. 10^4** is called the **exponential form** of 10,000.

We can similarly express 1,000 as a power of 10. Note that

$$1000 = 10 \times 10 \times 10 = 10^3$$



Here again, 10^3 is the exponential form of 1,000.

Similarly, $\quad 1,00,000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^5$

10^5 is the exponential form of 1,00,000

In both these examples, the base is 10; in case of 10^3, the exponent is 3 and in case of 10^5 the exponent is 5.

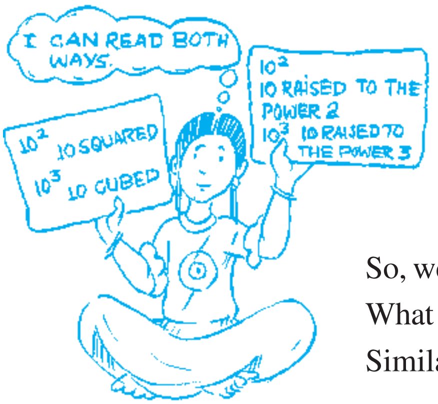
We have used numbers like 10, 100, 1000 etc., while writing numbers in an expanded form. For example, $47561 = 4 \times 10000 + 7 \times 1000 + 5 \times 100 + 6 \times 10 + 1$

This can be written as $4 \times 10^4 + 7 \times 10^3 + 5 \times 10^2 + 6 \times 10 + 1$.

Try writing these numbers in the same way 172, 5642, 6374.

In all the above given examples, we have seen numbers whose base is 10. However the base can be any other number also. For example:

$81 = 3 \times 3 \times 3 \times 3$ can be written as $81 = 3^4$, here 3 is the base and 4 is the exponent.



Some powers have special names. For example,

10^2, which is 10 raised to the power 2, also read as '10 squared' and

10^3, which is 10 raised to the power 3, also read as '10 cubed'.

Can you tell what 5^3 (5 cubed) means?

$$5^3 = 5 \times 5 \times 5 = 125$$

So, we can say 125 is the third power of 5.

What is the exponent and the base in 5^3?

Similarly, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$, which is the fifth power of 2.

In 2^5, 2 is the base and 5 is the exponent.

In the same way,
$$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$$
$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$$
$$625 = 5 \times 5 \times 5 \times 5 = 5^4$$

Try These

Find five more such examples, where a number is expressed in exponential form. Also identify the base and the exponent in each case.

You can also extend this way of writing when the base is a negative integer.

What does $(-2)^3$ mean?

It is $(-2)^3 = (-2) \times (-2) \times (-2) = -8$

Is $(-2)^4 = 16$? Check it.

Instead of taking a fixed number let us take any integer a as the base, and write the numbers as,

$a \times a = a^2$ (read as 'a squared' or 'a raised to the power 2')

$a \times a \times a = a^3$ (read as 'a cubed' or 'a raised to the power 3')

$a \times a \times a \times a = a^4$ (read as a raised to the power 4 or the 4th power of a)

....

$a \times a \times a \times a \times a \times a \times a = a^7$ (read as a raised to the power 7 or the 7th power of a) and so on.

$a \times a \times a \times b \times b$ can be expressed as a^3b^2 (read as a cubed b squared)

$a \times a \times b \times b \times b \times b$ can be expressed as a^2b^4 (read as a squared into b raised to the power of 4).

TRY THESE

Express:

(i) 729 as a power of 3

(ii) 128 as a power of 2

(iii) 343 as a power of 7

EXAMPLE 1 Express 256 as a power 2.

SOLUTION We have $256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.

So we can say that $256 = 2^8$

EXAMPLE 2 Which one is greater 2^3 or 3^2?

SOLUTION We have, $2^3 = 2 \times 2 \times 2 = 8$ and $3^2 = 3 \times 3 = 9$.

Since $9 > 8$, so, 3^2 is greater than 2^3

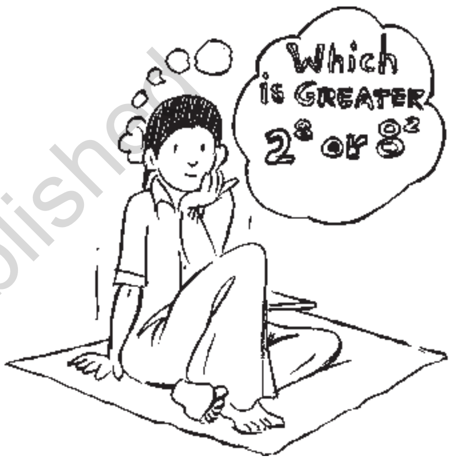
EXAMPLE 3 Which one is greater 8^2 or 2^8?

SOLUTION

$$8^2 = 8 \times 8 = 64$$

$$2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$$

Clearly, $2^8 > 8^2$



EXAMPLE 4 Expand $a^3 b^2$, $a^2 b^3$, $b^2 a^3$, $b^3 a^2$. Are they all same?

SOLUTION

$$a^3 b^2 = a^3 \times b^2 = (a \times a \times a) \times (b \times b) = a \times a \times a \times b \times b$$

$$a^2 b^3 = a^2 \times b^3 = a \times a \times b \times b \times b$$

$$b^2 a^3 = b^2 \times a^3 = b \times b \times a \times a \times a$$

$$b^3 a^2 = b^3 \times a^2 = b \times b \times b \times a \times a$$

Note that in the case of terms $a^3 b^2$ and $a^2 b^3$ the powers of a and b are different. Thus $a^3 b^2$ and $a^2 b^3$ are different.

On the other hand, $a^3 b^2$ and $b^2 a^3$ are the same, since the powers of a and b in these two terms are the same. The order of factors does not matter.

Thus, $a^3 b^2 = a^3 \times b^2 = b^2 \times a^3 = b^2 a^3$. Similarly, $a^2 b^3$ and $b^3 a^2$ are the same.

EXAMPLE 5 Express the following numbers as a product of powers of prime factors:

(i) 72 (ii) 432 (iii) 1000 (iv) 16000

SOLUTION

(i) $72 = 2 \times 36 = 2 \times 2 \times 18$

$= 2 \times 2 \times 2 \times 9$

$= 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

Thus, $72 = 2^3 \times 3^2$ (required prime factor product form)

2	72
2	36
2	18
3	9
	3

(ii) $432 = 2 \times 216 = 2 \times 2 \times 108 = 2 \times 2 \times 2 \times 54$

$= 2 \times 2 \times 2 \times 2 \times 27 = 2 \times 2 \times 2 \times 2 \times 3 \times 9$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

or $432 = 2^4 \times 3^3$ (required form)

(iii) $1000 = 2 \times 500 = 2 \times 2 \times 250 = 2 \times 2 \times 2 \times 125$

$= 2 \times 2 \times 2 \times 5 \times 25 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$

or $1000 = 2^3 \times 5^3$

> Atul wants to solve this example in another way:
>
> $1000 = 10 \times 100 = 10 \times 10 \times 10$
>
> $= (2 \times 5) \times (2 \times 5) \times (2 \times 5)$ (Since $10 = 2 \times 5$)
>
> $= 2 \times 5 \times 2 \times 5 \times 2 \times 5 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$
>
> or $1000 = 2^3 \times 5^3$
>
> Is Atul's method correct?

(iv) $16{,}000 = 16 \times 1000 = (2 \times 2 \times 2 \times 2) \times 1000 = 2^4 \times 10^3$ (as $16 = 2 \times 2 \times 2 \times 2$)

$= (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 5 \times 5 \times 5) = 2^4 \times 2^3 \times 5^3$

(Since $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$)

$= (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2) \times (5 \times 5 \times 5)$

or, $16{,}000 = 2^7 \times 5^3$

EXAMPLE 6 Work out $(1)^5$, $(-1)^3$, $(-1)^4$, $(-10)^3$, $(-5)^4$.

SOLUTION

(i) We have $(1)^5 = 1 \times 1 \times 1 \times 1 \times 1 = 1$

In fact, you will realise that 1 raised to any power is 1.

(ii) $(-1)^3 = (-1) \times (-1) \times (-1) = 1 \times (-1) = -1$

(iii) $(-1)^4 = (-1) \times (-1) \times (-1) \times (-1) = 1 \times 1 = 1$

You may check that (-1) raised to any **odd** power is (-1), and (-1) raised to any **even** power is $(+1)$.

> $(-1)^{\text{odd number}} = -1$
>
> $(-1)^{\text{even number}} = +1$

(iv) $(-10)^3 = (-10) \times (-10) \times (-10) = 100 \times (-10) = -1000$

(v) $(-5)^4 = (-5) \times (-5) \times (-5) \times (-5) = 25 \times 25 = 625$

EXERCISE 13.1

1. Find the value of:

(i) 2^6 (ii) 9^3 (iii) 11^2 (iv) 5^4

2. Express the following in exponential form:

(i) $6 \times 6 \times 6 \times 6$ (ii) $t \times t$ (iii) $b \times b \times b \times b$

(iv) $5 \times 5 \times 7 \times 7 \times 7$ (v) $2 \times 2 \times a \times a$ (vi) $a \times a \times a \times c \times c \times c \times c \times d$

3. Express each of the following numbers using exponential notation:

(i) 512 (ii) 343 (iii) 729 (iv) 3125

4. Identify the greater number, wherever possible, in each of the following?

(i) 4^3 or 3^4 (ii) 5^3 or 3^5 (iii) 2^8 or 8^2

(iv) 100^2 or 2^{100} (v) 2^{10} or 10^2

5. Express each of the following as product of powers of their prime factors:

(i) 648 (ii) 405 (iii) 540 (iv) 3,600

6. Simplify :

(i) 2×10^3 (ii) $7^2 \times 2^2$ (iii) $2^3 \times 5$ (iv) 3×4^4

(v) 0×10^2 (vi) $5^2 \times 3^3$ (vii) $2^4 \times 3^2$ (viii) $3^2 \times 10^4$

7. Simplify :

(i) $(-4)^3$ (ii) $(-3) \times (-2)^3$ (iii) $(-3)^2 \times (-5)^2$

(iv) $(-2)^3 \times (-10)^3$

8. Compare the following numbers:

(i) 2.7×10^{12} ; 1.5×10^8 (ii) 4×10^{14} ; 3×10^{17}

13.3 Laws of Exponents

13.3.1 Multiplying Powers with the Same Base

(i) Let us calculate $2^2 \times 2^3$

$$2^2 \times 2^3 = (2 \times 2) \times (2 \times 2 \times 2)$$
$$= 2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 2^{2+3}$$

Note that the base in 2^2 and 2^3 is same and the sum of the exponents, i.e., 2 and 3 is 5

(ii) $(-3)^4 \times (-3)^3 = [(-3) \times (-3) \times (-3) \times (-3)] \times [(-3) \times (-3) \times (-3)]$
$$= (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3)$$
$$= (-3)^7$$
$$= (-3)^{4+3}$$

Again, note that the base is same and the sum of exponents, i.e., 4 and 3, is 7

(iii) $a^2 \times a^4 = (a \times a) \times (a \times a \times a \times a)$
$$= a \times a \times a \times a \times a \times a = a^6$$

(Note: the base is the same and the sum of the exponents is $2 + 4 = 6$)

Similarly, verify :

$$4^2 \times 4^2 = 4^{2+2}$$
$$3^2 \times 3^3 = 3^{2+3}$$

TRY THESE

Simplify and write in exponential form:

(i) $2^5 \times 2^3$

(ii) $p^3 \times p^2$

(iii) $4^3 \times 4^2$

(iv) $a^3 \times a^2 \times a^7$

(v) $5^3 \times 5^7 \times 5^{12}$

(vi) $(-4)^{100} \times (-4)^{20}$

Can you write the appropriate number in the box.

$$(-11)^2 \times (-11)^6 = (-11)^{\square}$$

$$b^2 \times b^3 = b^{\square} \text{ (Remember, base is same; } b \text{ is any integer).}$$

$$c^3 \times c^4 = c^{\square} \ (c \text{ is any integer})$$

$$d^{10} \times d^{20} = d^{\square}$$

From this we can generalise that for any non-zero integer a, where m and n are whole numbers,

$$a^m \times a^n = a^{m+n}$$

Caution!

Consider $2^3 \times 3^2$

Can you add the exponents? No! Do you see 'why'? The base of 2^3 is 2 and base of 3^2 is 3. The bases are not same.

13.3.2 Dividing Powers with the Same Base

Let us simplify $3^7 \div 3^4$?

$$3^7 \div 3^4 = \frac{3^7}{3^4} = \frac{3\times3\times3\times3\times3\times3\times3}{3\times3\times3\times3}$$

$$= 3 \times 3 \times 3 = 3^3 = 3^{7-4}$$

Thus $\quad 3^7 \div 3^4 = 3^{7-4}$

(Note, in 3^7 and 3^4 the base is same and $3^7 \div 3^4$ becomes 3^{7-4})

Similarly,

$$5^6 \div 5^2 = \frac{5^6}{5^2} = \frac{5\times5\times5\times5\times5\times5}{5\times5}$$

$$= 5 \times 5 \times 5 \times 5 = 5^4 = 5^{6-2}$$

or $\quad 5^6 \div 5^2 = 5^{6-2}$

Let a be a non-zero integer, then,

$$a^4 \div a^2 = \frac{a^4}{a^2} = \frac{a\times a\times a\times a}{a\times a} = a\times a = a^2 = a^{4\ \ 2}$$

or $\quad a^4 \div a^2 = a^{4-2}$

Now can you answer quickly?

$$10^8 \div 10^3 = 10^{8-3} = 10^5$$

$$7^9 \div 7^6 = 7^{\square}$$

$$a^8 \div a^5 = a^{\square}$$

For non-zero integers b and c,

$$b^{10} \div b^{5} = b^{\square}$$

$$c^{100} \div c^{90} = c^{\square}$$

In general, for any non-zero integer a,

$$a^{m} \div a^{n} = a^{m-n}$$

where m and n are whole numbers and $m > n$.

TRY THESE

Simplify and write in exponential form: (eg., $11^6 \div 11^2 = 11^4$)

(i) $2^9 \div 2^3$ (ii) $10^8 \div 10^4$
(iii) $9^{11} \div 9^7$ (iv) $20^{15} \div 20^{13}$
(v) $7^{13} \div 7^{10}$

13.3.3 Taking Power of a Power

Consider the following

Simplify $\left(2^3\right)^2$; $\left(3^2\right)^4$

Now, $\left(2^3\right)^2$ means 2^3 is multiplied two times with itself.

$$\left(2^3\right)^2 = 2^3 \times 2^3$$

$$= 2^{3+3} \text{ (Since } a^m \times a^n = a^{m+n}\text{)}$$

$$= 2^6 = 2^{3\times 2}$$

Thus $\left(2^3\right)^2 = 2^{3\times 2}$

Similarly $\left(3^2\right)^4 = 3^2 \times 3^2 \times 3^2 \times 3^2$

$$= 3^{2+2+2+2}$$

$$= 3^8 \quad \text{(Observe 8 is the product of 2 and 4).}$$

$$= 3^{2\times 4}$$

Can you tell what would $\left(7^2\right)^{10}$ would be equal to?

So $\left(2^3\right)^2 = 2^{3\times 2} = 2^6$

$$\left(3^2\right)^4 = 3^{2\times 4} = 3^8$$

$$\left(7^2\right)^{10} = 7^{2\times 10} = 7^{20}$$

$$\left(a^2\right)^3 = a^{2\times 3} = a^6$$

$$\left(a^m\right)^3 = a^{m\times 3} = a^{3m}$$

From this we can generalise for any non-zero integer 'a', where 'm' and 'n' are whole numbers,

$$\left(a^m\right)^n = a^{mn}$$

TRY THESE

Simplify and write the answer in exponential form:

(i) $\left(6^2\right)^4$ (ii) $\left(2^2\right)^{100}$

(iii) $\left(7^{50}\right)^2$ (iv) $\left(5^3\right)^7$

EXAMPLE 7 Can you tell which one is greater $(5^2) \times 3$ or $\left(5^2\right)^3$?

SOLUTION $(5^2) \times 3$ means 5^2 is multiplied by 3 i.e., $5 \times 5 \times 3 = 75$

but $\left(5^2\right)^3$ means 5^2 is multiplied by itself three times i.e. ,

$$5^2 \times 5^2 \times 5^2 = 5^6 = 15{,}625$$

Therefore $\qquad (5^2)^3 > (5^2) \times 3$

13.3.4 Multiplying Powers with the Same Exponents

Can you simplify $2^3 \times 3^3$? Notice that here the two terms 2^3 and 3^3 have different bases, but the same exponents.

Now, $\quad 2^3 \times 3^3 = (2 \times 2 \times 2) \times (3 \times 3 \times 3)$

$= (2 \times 3) \times (2 \times 3) \times (2 \times 3)$

$= 6 \times 6 \times 6$

$= 6^3$ (Observe 6 is the product of bases 2 and 3)

Consider $4^4 \times 3^4$ $= (4 \times 4 \times 4 \times 4) \times (3 \times 3 \times 3 \times 3)$

$= (4 \times 3) \times (4 \times 3) \times (4 \times 3) \times (4 \times 3)$

$= 12 \times 12 \times 12 \times 12$

$= 12^4$

Consider, also, $3^2 \times a^2$ $= (3 \times 3) \times (a \times a)$

$= (3 \times a) \times (3 \times a)$

$= (3 \times a)^2$

$= (3a)^2$ (Note: $3 \times a = 3a$)

Similarly, $a^4 \times b^4$ $= (a \times a \times a \times a) \times (b \times b \times b \times b)$

$= (a \times b) \times (a \times b) \times (a \times b) \times (a \times b)$

$= (a \times b)^4$

$= (ab)^4$ (Note $a \times b = ab$)

TRY THESE

Put into another form using $a^m \times b^m = (ab)^m$:

(i) $4^3 \times 2^3$ (ii) $2^5 \times b^5$

(iii) $a^2 \times t^2$ (iv) $5^6 \times (-2)^6$

(v) $(-2)^4 \times (-3)^4$

In general, for any non-zero integer a

$$a^m \times b^m = (ab)^m \qquad \text{(where } m \text{ is any whole number)}$$

EXAMPLE 8 Express the following terms in the exponential form:

(i) $(2 \times 3)^5$ (ii) $(2a)^4$ (iii) $(-4m)^3$

SOLUTION

(i) $(2 \times 3)^5 = (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3) \times (2 \times 3)$

$= (2 \times 2 \times 2 \times 2 \times 2) \times (3 \times 3 \times 3 \times 3 \times 3)$

$= 2^5 \times 3^5$

(ii) $(2a)^4 = 2a \times 2a \times 2a \times 2a$

$= (2 \times 2 \times 2 \times 2) \times (a \times a \times a \times a)$

$= 2^4 \times a^4$

(iii) $(-4m)^3 = (-4 \times m)^3$

$= (-4 \times m) \times (-4 \times m) \times (-4 \times m)$

$= (-4) \times (-4) \times (-4) \times (m \times m \times m) = (-4)^3 \times (m)^3$

13.3.5 Dividing Powers with the Same Exponents

Observe the following simplifications:

(i) $$\frac{2^4}{3^4} = \frac{2\times2\times2\times2}{3\times3\times3\times3} = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \left(\frac{2}{3}\right)^4$$

(ii) $$\frac{a^3}{b^3} = \frac{a\times a\times a}{b\times b\times b} = \frac{a}{b}\times\frac{a}{b}\times\frac{a}{b} = \left(\frac{a}{b}\right)^3$$

From these examples we may generalise

$a^m \div b^m = \dfrac{a^m}{b^m} = \left(\dfrac{a}{b}\right)^m$ where a and b are any non-zero integers and m is a whole number.

TRY THESE

Put into another form using $a^m \div b^m = \left(\dfrac{a}{b}\right)^m$:

(i) $4^5 \div 3^5$
(ii) $2^5 \div b^5$
(iii) $(-2)^3 \div b^3$
(iv) $p^4 \div q^4$
(v) $5^6 \div (-2)^6$

EXAMPLE 9 Expand: (i) $\left(\dfrac{3}{5}\right)^4$ (ii) $\left(\dfrac{-4}{7}\right)^5$

SOLUTION

(i) $$\left(\frac{3}{5}\right)^4 = \frac{3^4}{5^4} = \frac{3\times3\times3\times3}{5\times5\times5\times5}$$

(ii) $$\left(\frac{-4}{7}\right)^5 = \frac{(-4)^5}{7^5} = \frac{(-4)\times(-4)\times(-4)\times(-4)\times(-4)}{7\times7\times7\times7\times7}$$

- **Numbers with exponent zero**

Can you tell what $\dfrac{3^5}{3^5}$ equals to?

$$\frac{3^5}{3^5} = \frac{3\times3\times3\times3\times3}{3\times3\times3\times3\times3} = 1$$

by using laws of exponents

What is a^0?
Obeserve the following pattern:

$2^6 = 64$
$2^5 = 32$
$2^4 = 16$
$2^3 = 8$
$2^2 = ?$
$2^1 = ?$
$2^0 = ?$

You can guess the value of 2^0 by just studying the pattern!

You find that $2^0 = 1$

If you start from $3^6 = 729$, and proceed as shown above finding $3^5, 3^4, 3^3$,... etc, what will be $3^0 = ?$

$$3^5 \div 3^5 = 3^{5-5} = 3^0$$

So $\quad 3^0 = 1$

Can you tell what 7^0 is equal to?

$$7^3 \div 7^3 = 7^{3-3} = 7^0$$

And $\quad \dfrac{7^3}{7^3} = \dfrac{7 \times 7 \times 7}{7 \times 7 \times 7} = 1$

Therefore $\quad 7^0 = 1$

Similarly $\quad a^3 \div a^3 = a^{3-3} = a^0$

And $\quad a^3 \div a^3 = \dfrac{a^3}{a^3} = \dfrac{a \times a \times a}{a \times a \times a} = 1$

Thus $\quad a^0 = 1$ (for any non-zero integer a)

So, we can say that any number (except 0) raised to the power (or exponent) 0 is 1.

13.4 Miscellaneous Examples using the Laws of Exponents

Let us solve some examples using rules of exponents developed.

Example 10 Write exponential form for $8 \times 8 \times 8 \times 8$ taking base as 2.

Solution We have, $8 \times 8 \times 8 \times 8 = 8^4$

But we know that $\quad 8 = 2 \times 2 \times 2 = 2^3$

Therefore $\quad 8^4 = (2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$

$\quad = 2^{3 \times 4} \qquad$ [You may also use $(a^m)^n = a^{mn}$]

$\quad = 2^{12}$

Example 11 Simplify and write the answer in the exponential form.

(i) $\left(\dfrac{3^7}{3^2}\right) \times 3^5$ $\qquad$ (ii) $2^3 \times 2^2 \times 5^5$ $\qquad$ (iii) $(6^2 \times 6^4) \div 6^3$

(iv) $[(2^2)^3 \times 3^6] \times 5^6$ $\qquad$ (v) $8^2 \div 2^3$

Solution

(i) $\left(\dfrac{3^7}{3^2}\right) \times 3^5 = \left(3^{7-2}\right) \times 3^5$

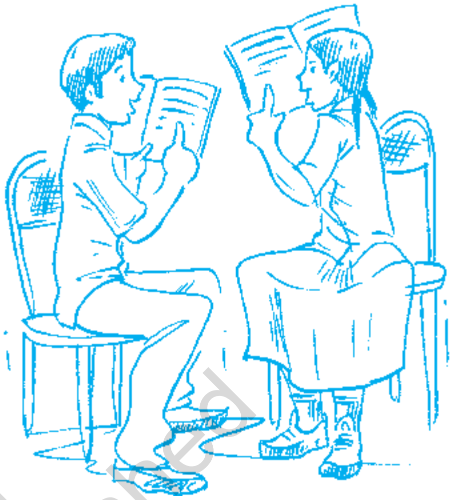
$$= 3^5 \times 3^5 = 3^{5+5} = 3^{10}$$

(ii) $2^3 \times 2^2 \times 5^5 = 2^{3+2} \times 5^5$

$= 2^5 \times 5^5 = (2 \times 5)^5 = 10^5$

(iii) $\left(6^2 \times 6^4\right) \div 6^3 = 6^{2+4} \div 6^3$

$= \dfrac{6^6}{6^3} = 6^{6-3} = 6^3$



(iv) $\left[\left(2^2\right)^3 \times 3^6\right] \times 5^6 = [2^6 \times 3^6] \times 5^6$

$= \left(2 \times 3\right)^6 \times 5^6$

$= \left(2 \times 3 \times 5\right)^6 = 30^6$

(v) $8 = 2 \times 2 \times 2 = 2^3$

Therefore $8^2 \div 2^3 = (2^3)^2 \div 2^3$

$= 2^6 \div 2^3 = 2^{6-3} = 2^3$

EXAMPLE 12 Simplify:

(i) $\dfrac{12^4 \times 9^3 \times 4}{6^3 \times 8^2 \times 27}$ (ii) $2^3 \times a^3 \times 5a^4$ (iii) $\dfrac{2 \times 3^4 \times 2^5}{9 \times 4^2}$

SOLUTION

(i) We have

$$\frac{12^4 \times 9^3 \times 4}{6^3 \times 8^2 \times 27} = \frac{(2^2 \times 3)^4 \times (3^2)^3 \times 2^2}{(2 \times 3)^3 \times (2^3)^2 \times 3^3}$$

$$= \frac{\left(2^2\right)^4 \times (3)^4 \times 3^{2\times 3} \times 2^2}{2^3 \times 3^3 \times 2^{2\times 3} \times 3^3} = \frac{2^8 \times 2^2 \times 3^4 \times 3^6}{2^3 \times 2^6 \times 3^3 \times 3^3}$$

$$= \frac{2^{8+2} \times 3^{4+6}}{2^{3+6} \times 3^{3+3}} = \frac{2^{10} \times 3^{10}}{2^9 \times 3^6}$$

$= 2^{10-9} \times 3^{10-6} = 2^1 \times 3^4$

$= 2 \times 81 = 162$

(ii) $2^3 \times a^3 \times 5a^4 = 2^3 \times a^3 \times 5 \times a^4$

$= 2^3 \times 5 \times a^3 \times a^4 = 8 \times 5 \times a^{3+4}$

$= 40\, a^7$

(ii) $\dfrac{2\times 3^4\times 2^5}{9\times 4^2} = \dfrac{2\times 3^4\times 2^5}{3^2\times\left(2^2\right)^2} = \dfrac{2\times 2^5\times 3^4}{3^2\times 2^{2\times 2}}$

$= \dfrac{2^{1+5}\times 3^4}{2^4\times 3^2} = \dfrac{2^6\times 3^4}{2^4\times 3^2} = 2^{6-4}\times 3^{4-2}$

$= 2^2\times 3^2 = 4\times 9 = 36$

Note: In most of the examples that we have taken in this Chapter, the base of a power was taken an integer. But all the results of the chapter apply equally well to a base which is a rational number.

EXERCISE 13.2

1. Using laws of exponents, simplify and write the answer in exponential form:

(i) $3^2\times 3^4\times 3^8$ (ii) $6^{15}\div 6^{10}$ (iii) $a^3\times a^2$

(iv) $7^x\times 7^2$ (v) $\left(5^2\right)^3\div 5^3$ (vi) $2^5\times 5^5$

(vii) $a^4\times b^4$ (viii) $\left(3^4\right)^3$ (ix) $\left(2^{20}\div 2^{15}\right)\times 2^3$

(x) $8^t\div 8^2$

2. Simplify and express each of the following in exponential form:

(i) $\dfrac{2^3\times 3^4\times 4}{3\times 32}$ (ii) $\left(\left(5^2\right)^3\times 5^4\right)\div 5^7$ (iii) $25^4\div 5^3$

(iv) $\dfrac{3\times 7^2\times 11^8}{21\times 11^3}$ (v) $\dfrac{3^7}{3^4\times 3^3}$ (vi) $2^0+3^0+4^0$

(vii) $2^0\times 3^0\times 4^0$ (viii) $(3^0+2^0)\times 5^0$ (ix) $\dfrac{2^8\times a^5}{4^3\times a^3}$

(x) $\left(\dfrac{a^5}{a^3}\right)\times a^8$ (xi) $\dfrac{4^5\times a^8b^3}{4^5\times a^5b^2}$ (xii) $\left(2^3\times 2\right)^2$

3. Say true or false and justify your answer:

(i) $10\times 10^{11}=100^{11}$ (ii) $2^3>5^2$ (iii) $2^3\times 3^2=6^5$

(iv) $3^0=(1000)^0$

4. Express each of the following as a product of prime factors only in exponential form:

(i) 108×192 (ii) 270 (iii) 729×64

(iv) 768

5. Simplify:

(i) $\dfrac{\left(2^5\right)^2 \times 7^3}{8^3 \times 7}$ (ii) $\dfrac{25 \times 5^2 \times t^8}{10^3 \times t^4}$ (iii) $\dfrac{3^5 \times 10^5 \times 25}{5^7 \times 6^5}$

13.5 Decimal Number System

Let us look at the expansion of 47561, which we already know:

$$47561 = 4 \times 10000 + 7 \times 1000 + 5 \times 100 + 6 \times 10 + 1$$

We can express it using powers of 10 in the exponent form:

Therefore, $47561 = 4 \times 10^4 + 7 \times 10^3 + 5 \times 10^2 + 6 \times 10^1 + 1 \times 10^0$

(Note $10,000 = 10^4$, $1000 = 10^3$, $100 = 10^2$, $10 = 10^1$ and $1 = 10^0$)

Let us expand another number:

$$\begin{aligned} 104278 &= 1 \times 100,000 + 0 \times 10,000 + 4 \times 1000 + 2 \times 100 + 7 \times 10 + 8 \times 1 \\ &= 1 \times 10^5 + 0 \times 10^4 + 4 \times 10^3 + 2 \times 10^2 + 7 \times 10^1 + 8 \times 10^0 \\ &= 1 \times 10^5 + 4 \times 10^3 + 2 \times 10^2 + 7 \times 10^1 + 8 \times 10^0 \end{aligned}$$

Notice how the exponents of 10 start from a maximum value of 5 and go on decreasing by 1 at a step from the left to the right upto 0.

13.6 Expressing Large Numbers in the Standard Form

Let us now go back to the beginning of the chapter. We said that large numbers can be conveniently expressed using exponents. We have not as yet shown this. We shall do so now.

1. Sun is located 300,000,000,000,000,000,000 m from the centre of our Milky Way Galaxy.
2. Number of stars in our Galaxy is 100,000,000,000.
3. Mass of the Earth is 5,976,000,000,000,000,000,000,000 kg.

These numbers are not convenient to write and read. To make it convenient we use powers.

Observe the following:

$$59 = 5.9 \times 10 = 5.9 \times 10^1$$
$$590 = 5.9 \times 100 = 5.9 \times 10^2$$
$$5900 = 5.9 \times 1000 = 5.9 \times 10^3$$
$$5900 = 5.9 \times 10000 = 5.9 \times 10^4 \text{ and so on.}$$

Try These

Expand by expressing powers of 10 in the exponential form:

(i) 172
(ii) 5,643
(iii) 56,439
(iv) 1,76,428

We have expressed all these numbers in the **standard form**. Any number can be expressed as a decimal number between 1.0 and 10.0 including 1.0 multiplied by a power of 10. Such a form of a number is called its **standard form**. Thus,

$5,985 = 5.985 \times 1,000 = 5.985 \times 10^3$ is the standard form of 5,985.

Note, 5,985 can also be expressed as 59.85×100 or 59.85×10^2. But these are not the standard forms, of 5,985. Similarly, $5,985 = 0.5985 \times 10,000 = 0.5985 \times 10^4$ is also not the standard form of 5,985.

We are now ready to express the large numbers we came across at the beginning of the chapter in this form.

The**,** distance of Sun from the centre of our Galaxy i.e.,

300,000,000,000,000,000,000 m can be written as

$3.0 \times 100,000,000,000,000,000,000 = 3.0 \times 10^{20}$ m

Now, can you express 40,000,000,000 in the similar way?

Count the number of zeros in it. It is 10.

So, $40,000,000,000 = 4.0 \times 10^{10}$

Mass of the Earth = 5,976,000,000,000,000,000,000,000 kg

$= 5.976 \times 10^{24}$ kg

Do you agree with the fact, that the number when written in the standard form is much easier to read, understand and compare than when the number is written with 25 digits?

Now,

Mass of Uranus = 86,800,000,000,000,000,000,000,000 kg

$= 8.68 \times 10^{25}$ kg

Simply by comparing the powers of 10 in the above two, you can tell that the mass of Uranus is greater than that of the Earth.

The distance between Sun and Saturn is 1,433,500,000,000 m or 1.4335×10^{12} m. The distance betwen Saturn and Uranus is 1,439,000,000,000 m or 1.439×10^{12} m. The distance between Sun and Earth is 149, 600,000,000 m or 1.496×10^{11} m.

Can you tell which of the three distances is smallest?

EXAMPLE 13 Express the following numbers in the standard form:

(i) 5985.3 (ii) 65,950

(iii) 3,430,000 (iv) 70,040,000,000

SOLUTION

(i) $5985.3 = 5.9853 \times 1000 = 5.9853 \times 10^3$

(ii) $65,950 = 6.595 \times 10,000 = 6.595 \times 10^4$

(iii) $3,430,000 = 3.43 \times 1,000,000 = 3.43 \times 10^6$

(iv) $70,040,000,000 = 7.004 \times 10,000,000,000 = 7.004 \times 10^{10}$

A point to remember is that one less than the digit count (number of digits) to the left of the decimal point in a given number is the exponent of 10 in the standard form. Thus, in 70,040,000,000 there is no decimal point shown; we assume it to be at the (right) end. From there, the count of the places (digits) to the left is 11. The exponent of 10 in the standard form is 11 – 1 = 10. In 5985.3 there are 4 digits to the left of the decimal point and hence the exponent of 10 in the standard form is 4 – 1 = 3.

EXERCISE 13.3

1. Write the following numbers in the expanded forms:

279404, 3006194, 2806196, 120719, 20068

2. Find the number from each of the following expanded forms:

(a) $8 \times 10^4 + 6 \times 10^3 + 0\times10^2 + 4\times10^1 + 5\times10^0$

(b) $4 \times 10^5 + 5\times10^3 + 3\times10^2 + 2\times10^0$

(c) $3 \times 10^4 + 7\times10^2 + 5\times10^0$

(d) $9 \times 10^5 + 2\times10^2 + 3\times10^1$

3. Express the following numbers in standard form:

(i) 5,00,00,000 (ii) 70,00,000 (iii) 3,18,65,00,000

(iv) 3,90,878 (v) 39087.8 (vi) 3908.78

4. Express the number appearing in the following statements in standard form.

(a) The distance between Earth and Moon is 384,000,000 m.

(b) Speed of light in vacuum is 300,000,000 m/s.

(c) Diameter of the Earth is 1,27,56,000 m.

(d) Diameter of the Sun is 1,400,000,000 m.

(e) In a galaxy there are on an average 100,000,000,000 stars.

(f) The universe is estimated to be about 12,000,000,000 years old.

(g) The distance of the Sun from the centre of the Milky Way Galaxy is estimated to be 300,000,000,000,000,000,000 m.

(h) 60,230,000,000,000,000,000,000 molecules are contained in a drop of water weighing 1.8 gm.

(i) The earth has 1,353,000,000 cubic km of sea water.

(j) The population of India was about 1,027,000,000 in March, 2001.

What have We Discussed?

1. Very large numbers are difficult to read, understand, compare and operate upon. To make all these easier, we use exponents, converting many of the large numbers in a shorter form.
2. The following are exponential forms of some numbers?

   $10{,}000 = 10^4$ (read as 10 raised to 4)

   $243 = 3^5, \quad 128 = 2^7.$

   Here, 10, 3 and 2 are the bases, whereas 4, 5 and 7 are their respective exponents. We also say, 10,000 is the 4th power of 10, 243 is the 5th power of 3, etc.
3. Numbers in exponential form obey certain laws, which are:

   For any non-zero integers a and b and whole numbers m and n,

   (a) $a^m \times a^n = a^{m+n}$

   (b) $a^m \div a^n = a^{m-n}, \quad m > n$

   (c) $(a^m)^n = a^{mn}$

   (d) $a^m \times b^m = (ab)^m$

   (e) $a^m \div b^n = \left(\dfrac{a}{b}\right)^m$

   (f) $a^0 = 1$

   (g) $(-1)^{\text{even number}} = 1$

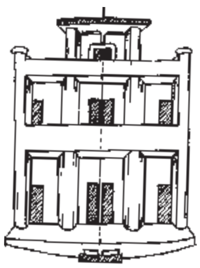
   $(-1)^{\text{odd number}} = -1$

Chapter 14

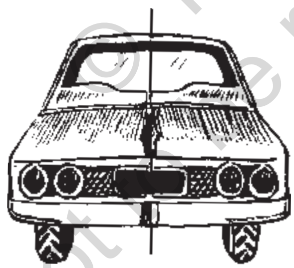
Symmetry

14.1 Introduction

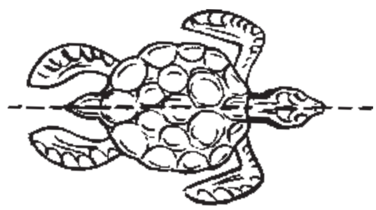
Symmetry is an important geometrical concept, commonly exhibited in nature and is used almost in every field of activity. Artists, professionals, designers of clothing or jewellery, car manufacturers, architects and many others make use of the idea of symmetry. The beehives, the flowers, the tree-leaves, religious symbols, rugs, and handkerchiefs — everywhere you find symmetrical designs.



Architecture



Engineering

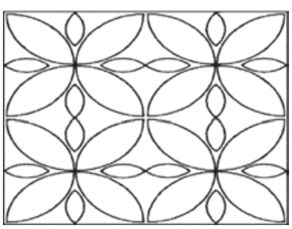


Nature

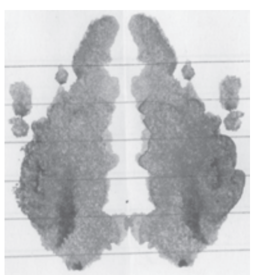
You have already had a 'feel' of **line symmetry** in your previous class.

A figure has a line symmetry, if there is a line about which the figure may be folded so that the two parts of the figure will coincide.

You might like to recall these ideas. Here are some activities to help you.



Compose a picture-album showing symmetry.

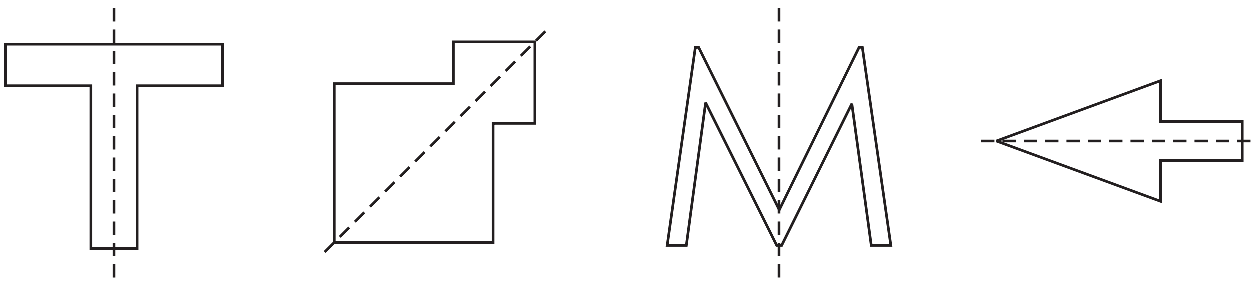


Create some colourful Ink-dot devils

Make some symmetrical paper-cut designs.

Enjoy identifying lines (also called axes) of symmetry in the designs you collect.

Let us now strengthen our ideas on symmetry further. Study the following figures in which the lines of symmetry are marked with dotted lines. [Fig 14.1 (i) to (iv)]



(i) (ii) (iii) (iv)

Fig 14.1

14.2 Lines of Symmetry for Regular Polygons

You know that a polygon is a closed figure made of several line segments. The polygon made up of the least number of line segments is the triangle. (Can there be a polygon that you can draw with still fewer line segments? Think about it).

A polygon is said to be regular if all its sides are of equal length and all its angles are of equal measure. Thus, an equilateral triangle is a regular polygon of three sides. Can you name the regular polygon of four sides?

An equilateral triangle is regular because each of its sides has same length and each of its angles measures 60° (Fig 14.2).

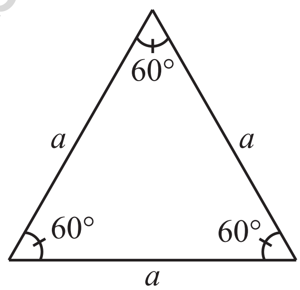


Fig 14.2

A square is also regular because all its sides are of equal length and each of its angles is a right angle (i.e., 90°). Its diagonals are seen to be perpendicular bisectors of one another (Fig 14.3).

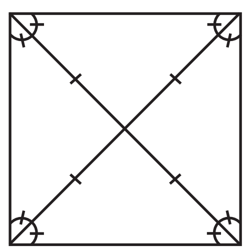


Fig 14.3

If a pentagon is regular, naturally, its sides should have equal length. You will, later on, learn that the measure of each of its angles is 108° (Fig 14.4).

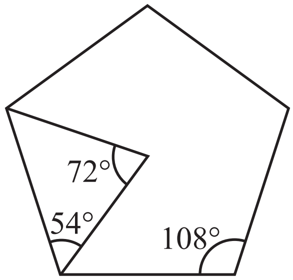


Fig 14.4

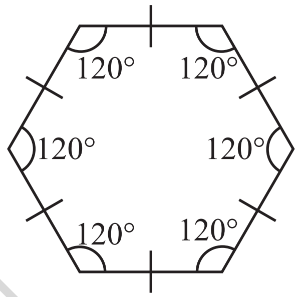


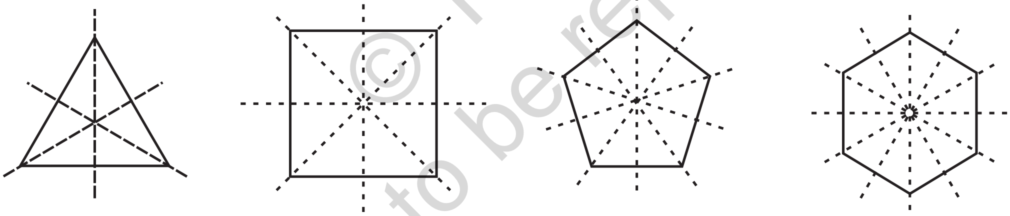
Fig 14.5

A regular hexagon has all its sides equal and each of its angles measures 120°. You will learn more of these figures later (Fig 14.5).

The regular polygons are symmetrical figures and hence their lines of symmetry are quite interesting,

Each regular polygon has as many lines of symmetry as it has sides [Fig 14.6 (i) - (iv)]. We say, they have multiple lines of symmetry.

three lines of symmetry | four lines of symmetry | five lines of symmetry | six lines of symmetry



Equilateral Triangle (i) | Square (ii) | Regular Pentagon (iii) | Regular Hexagon (iv)

Fig 14.6

Perhaps, you might like to investigate this by paper folding. Go ahead!

The concept of line symmetry is closely related to mirror reflection. A shape has line symmetry when one half of it is the mirror image of the other half (Fig 14.7). A mirror line, thus, helps to visualise a line of symmetry (Fig 14.8).

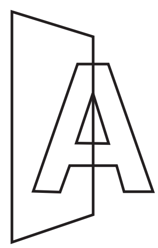
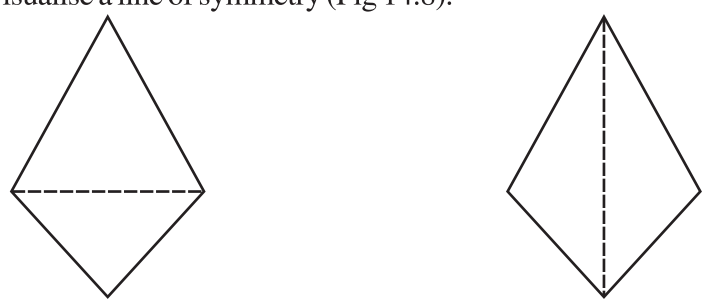


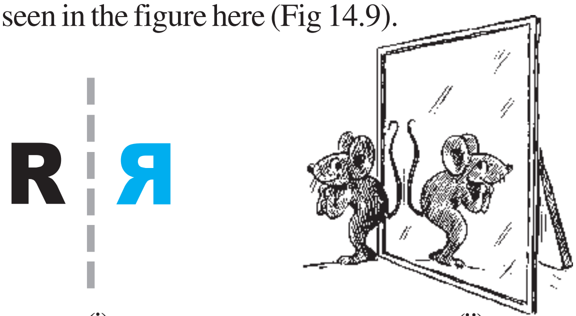
Fig 14.7



Is the dotted line a mirror line? No. Is the dotted line a mirror line? Yes.

Fig 14.8

While dealing with mirror reflection, care is needed to note down the left-right changes in the orientation, as seen in the figure here (Fig 14.9).

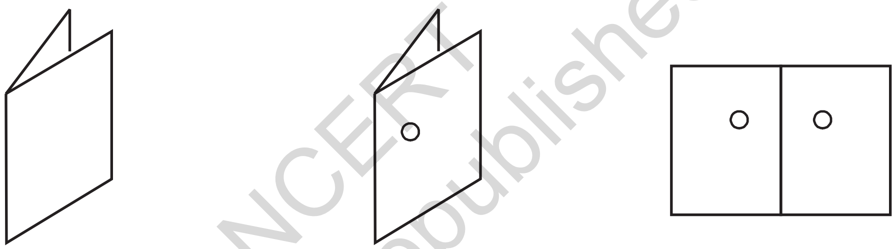


(i) (ii)

Fig 14.9

The shape is same, but the other way round!

Play this punching game!



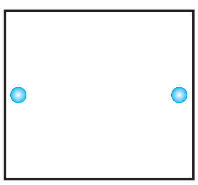
Fold a sheet into two halves | Punch a hole | two holes about the symmetric fold.

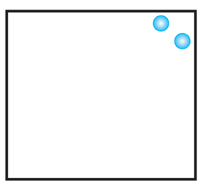
Fig 14.10

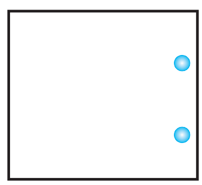
The fold is a line (or axis) of symmetry. Study about punches at different locations on the folded paper and the corresponding lines of symmetry (Fig 14.10).

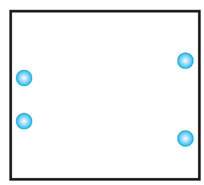
EXERCISE 14.1

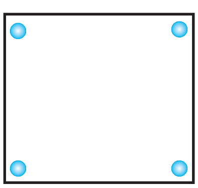
1. Copy the figures with punched holes and find the axes of symmetry for the following:

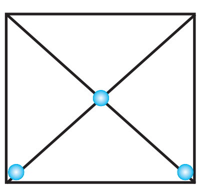

(a)

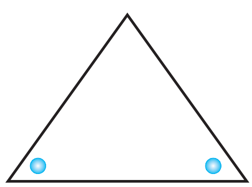

(b)

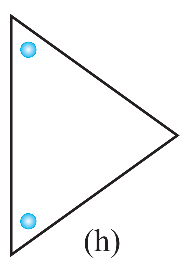

(c)

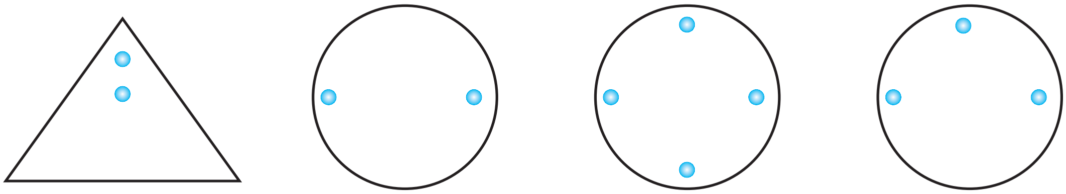

(d)


(e)


(f)

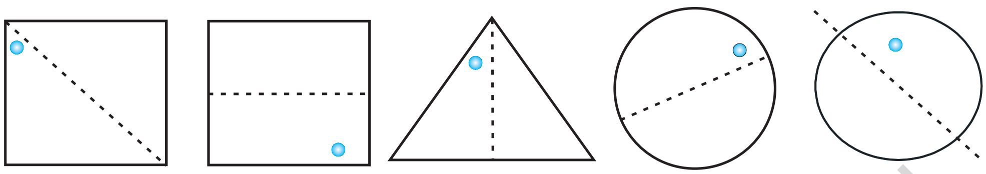

(g)


(h)



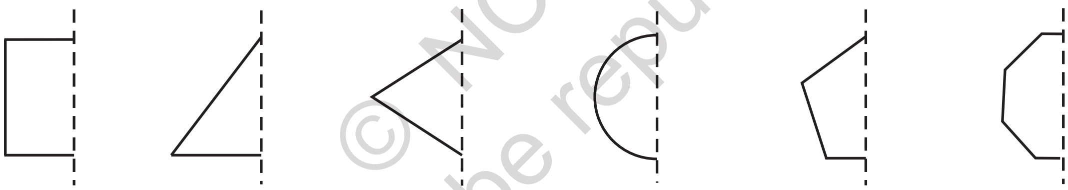
(i) (j) (k) (l)

2. Given the line(s) of symmetry, find the other hole(s):



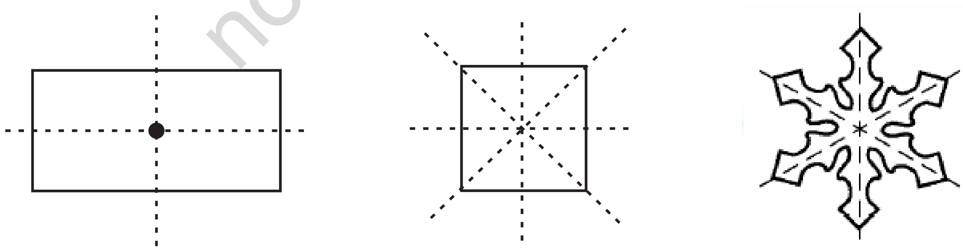
(a) (b) (c) (d) (e)

3. In the following figures, the mirror line (i.e., the line of symmetry) is given as a dotted line. Complete each figure performing reflection in the dotted (mirror) line. (You might perhaps place a mirror along the dotted line and look into the mirror for the image). Are you able to recall the name of the figure you complete?



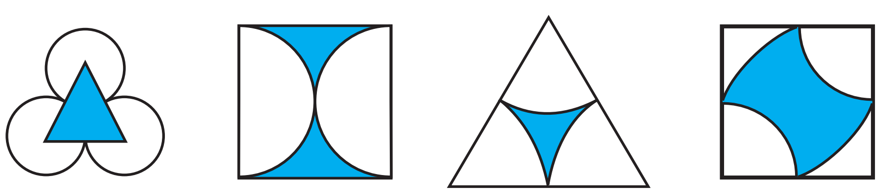
(a) (b) (c) (d) (e) (f)

4. The following figures have more than one line of symmetry. Such figures are said to have multiple lines of symmetry.

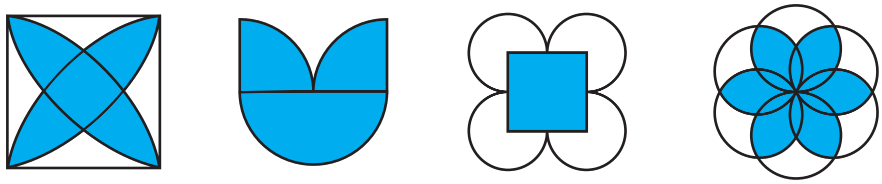


(a) (b) (c)

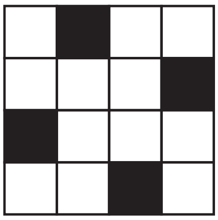
Identify multiple lines of symmetry, if any, in each of the following figures:



(a) (b) (c) (d)



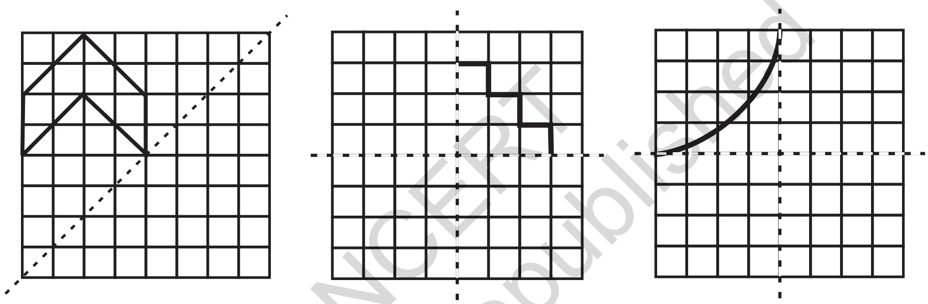
(e) (f) (g) (h)



5. Copy the figure given here.

Take any one diagonal as a line of symmetry and shade a few more squares to make the figure symmetric about a diagonal. Is there more than one way to do that? Will the figure be symmetric about both the diagonals?

6. Copy the diagram and complete each shape to be symmetric about the mirror line(s):



(a) (b) (c)

7. State the number of lines of symmetry for the following figures:

(a) An equilateral triangle (b) An isosceles triangle (c) A scalene triangle
(d) A square (e) A rectangle (f) A rhombus
(g) A parallelogram (h) A quadrilateral (i) A regular hexagon
(j) A circle

8. What letters of the English alphabet have reflectional symmetry (i.e., symmetry related to mirror reflection) about.

(a) a vertical mirror (b) a horizontal mirror
(c) both horizontal and vertical mirrors

9. Give three examples of shapes with no line of symmetry.

10. What other name can you give to the line of symmetry of

(a) an isosceles triangle? (b) a circle?

14.3 Rotational Symmetry

What do you say when the hands of a clock go round?

You say that they rotate. The hands of a clock rotate in only one direction, about a fixed point, the centre of the clock-face.

Rotation, like movement of the hands of a clock, is called a clockwise rotation; otherwise it is said to be anticlockwise.

What can you say about the rotation of the blades of a ceiling fan? Do they rotate clockwise or anticlockwise? Or do they rotate both ways?

If you spin the wheel of a bicycle, it rotates. It can rotate in either way: both clockwise and anticlockwise. Give three examples each for (i) a clockwise rotation and (ii) anticlockwise rotation.

When an object rotates, its shape and size do not change. The rotation turns an object about a fixed point. This fixed point is the **centre of rotation**. What is the centre of rotation of the hands of a clock? Think about it.

The angle of turning during rotation is called the **angle of rotation**. A full turn, you know, means a rotation of 360°. What is the degree measure of the angle of rotation for (i) a half-turn? (ii) a quarter-turn?

A half-turn means rotation by 180°; a quarter-turn is rotation by 90°.

When it is 12 O'clock, the hands of a clock are together. By 3 O'clock, the minute hand would have made three complete turns; but the hour hand would have made only a quarter-turn. What can you say about their positions at 6 O'clock?

Have you ever made a paper windmill? The Paper windmill in the picture looks symmetrical (Fig 14.11); but you do not find any line of symmetry. No folding can help you to have coincident halves. However if you rotate it by 90° about the fixed point, the windmill will look exactly the same. We say the windmill has a **rotational symmetry.**

Fig 14.11

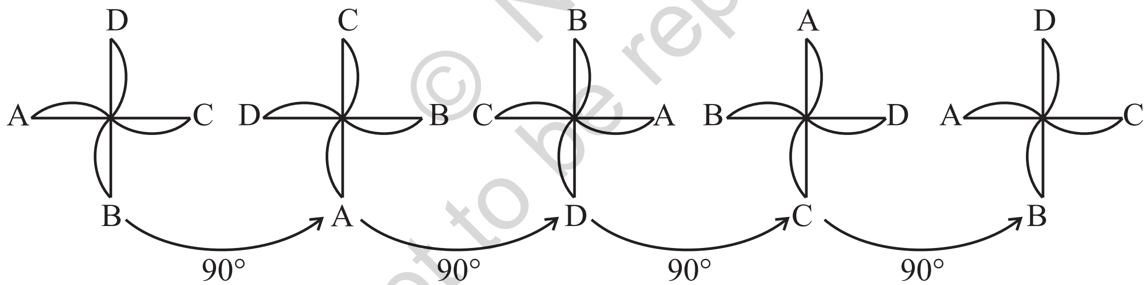


Fig 14.12

In a full turn, there are precisely **four positions** (on rotation through the angles 90°, 180°, 270° and 360°) when the windmill looks exactly the same. Because of this, we say it has a rotational symmetry of order 4.

Here is one more example for rotational symmetry.

Consider a square with P as one of its corners (Fig 14.13).

Let us perform quarter-turns about the centre of the square marked ×.

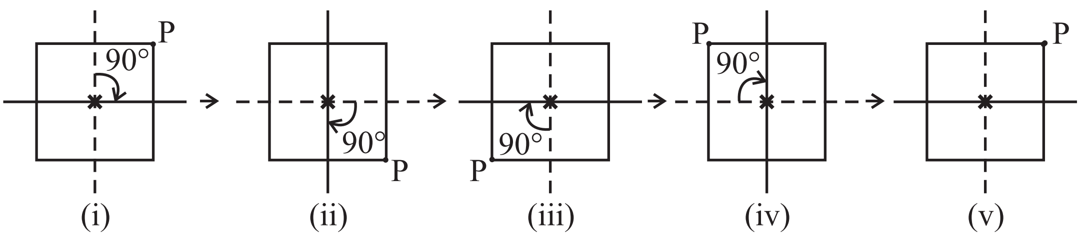


Fig 14.13

Fig 14.13 (i) is the initial position. Rotation by 90° about the centre leads to Fig 14.13 (ii). Note the position of P now. Rotate again through 90° and you get Fig 14.13 (iii). In this way, when you complete four quarter-turns, the square reaches its original position. It now looks the same as Fig14.13 (i). This can be seen with the help of the positions taken by P.

Thus a square has a **rotational symmetry of order 4** about its centre. Observe that in this case,

(i) The centre of rotation is the centre of the square.

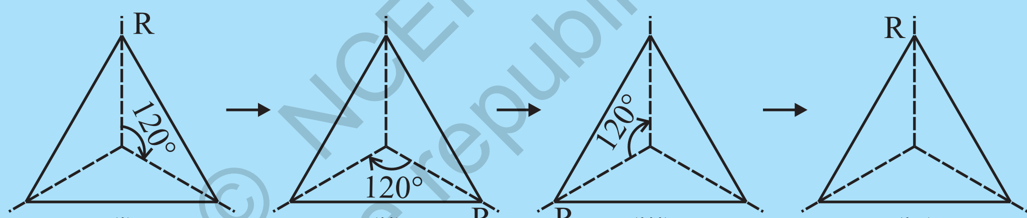
(ii) The angle of rotation is 90°.

(iii) The direction of rotation is clockwise.

(iv) The order of rotational symmetry is 4.

Try These

1. (a) Can you now tell the order of the rotational symmetry for an equilateral triangle? (Fig 14.14)

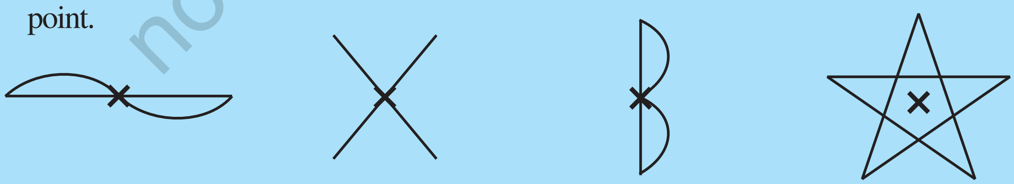


(i) (ii) (iii) (iv)

Fig 14.14

(b) How many positions are there at which the triangle looks exactly the same, when rotated about its centre by 120°?

2. Which of the following shapes (Fig 14.15) have rotational symmetry about the marked point.



(i) (ii) (iii) (iv)

Fig 14.15

Do This

Draw two identical parallelograms, one-ABCD on a piece of paper and the other A' B' C' D' on a transparent sheet. Mark the points of intersection of their diagonals, O and O' respectively (Fig 14.16).

Place the parallelograms such that A' lies on A, B' lies on B and so on. O' then falls on O.

Stick a pin into the shapes at the point O.

Now turn the transparent shape in the clockwise direction.

How many times do the shapes coincide in one full round?

What is the order of rotational symmetry?

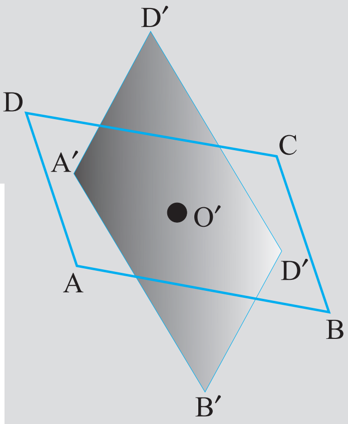
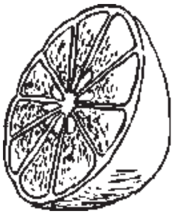


Fig 14.16

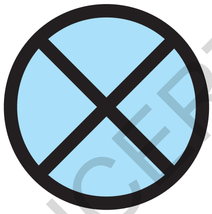
The point where we have the pin is the centre of rotation. It is the intersecting point of the diagonals in this case.

Every object has a rotational symmetry of order 1, as it occupies same position after a rotation of 360° (i.e., one complete revolution). Such cases have no interest for us.

You have around you many shapes, which possess rotational symmetry (Fig 14.17).


Fruit

(i)


Road sign

(ii)


Wheel

(iii)

Fig 14.17

For example, when you slice certain fruits, the cross-sections are shapes with rotational symmetry. This might surprise you when you notice them [Fig 14.17(i)].

Then there are many road signs that exhibit rotational symmetry. Next time when you walk along a busy road, try to identify such road signs and find about the order of rotational symmetry [Fig 14.17(ii)].

Think of some more examples for rotational symmetry. Discuss in each case:

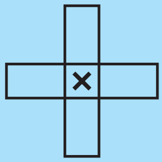
(i) the centre of rotation (ii) the angle of rotation

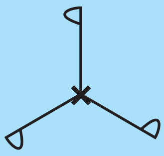
(iii) the direction in which the rotation is affected and

(iv) the order of the rotational symmetry.

TRY THESE

Give the order of the rotational symmetry of the given figures about the point marked × (Fig 14.17).


(i)


(ii)

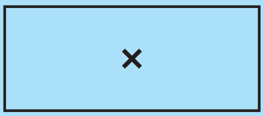
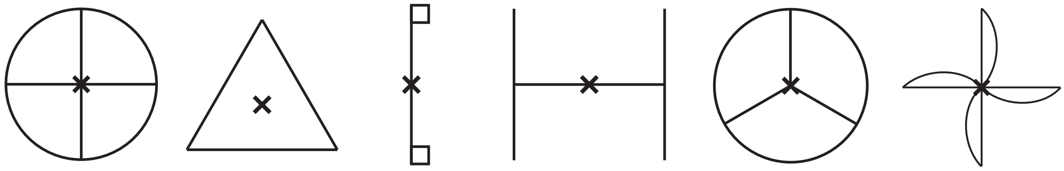

(iii)

Fig 14.18

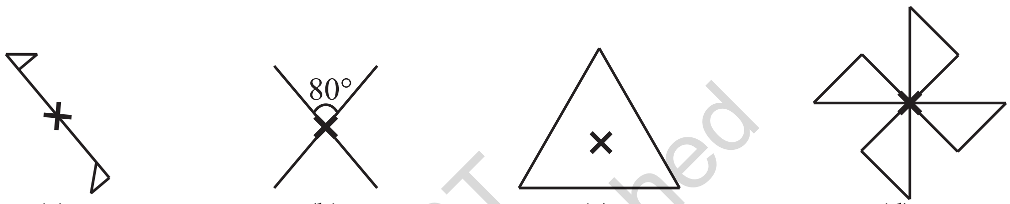
EXERCISE 14.2

1. Which of the following figures have rotational symmetry of order more than 1:

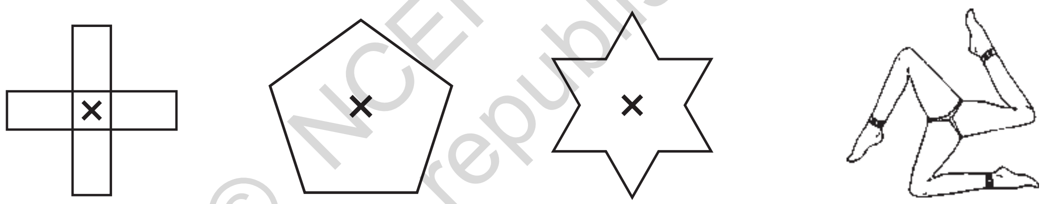


(a) (b) (c) (d) (e) (f)

2. Give the order of rotational symmetry for each figure:



(a) (b) (c) (d)



(e) (f) (g) (h)

14.4 LINE SYMMETRY AND ROTATIONAL SYMMETRY

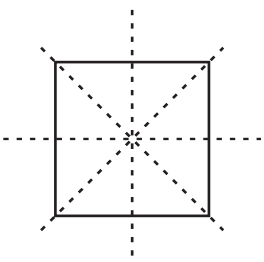


Fig 14.19

You have been observing many shapes and their symmetries so far. By now you would have understood that some shapes have only line symmetry, some have only rotational symmetry and some have both line symmetry and rotational symmetry.

For example, consider the square shape (Fig 14.19).

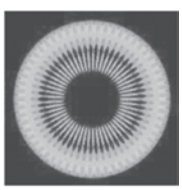
How many lines of symmetry does it have?

Does it have any rotational symmetry?

If 'yes', what is the order of the rotational symmetry?

Think about it.

The circle is the most perfect symmetrical figure, because it can be rotated around its centre through any angle and at the same time it has unlimited number of lines of symmetry. Observe any circle pattern. Every line through the centre (that is every diameter) forms a line of (reflectional) symmetry and it has rotational symmetry around the centre for every angle.



Do This

Some of the English alphabets have fascinating symmetrical structures. Which capital letters have just one line of symmetry (like **E**)? Which capital letters have a rotational symmetry of order 2 (like **I**)?

By attempting to think on such lines, you will be able to fill in the following table:

Alphabet Letters	Line Symmetry	Number of Lines of Symmetry	Rotational Symmetry	Order of Rotational Symmetry
Z	No	0	Yes	2
S				
H	Yes		Yes	
O	Yes		Yes	
E	Yes			
N			Yes	
C				

Exercise 14.3

1. Name any two figures that have both line symmetry and rotational symmetry.

2. Draw, wherever possible, a rough sketch of
   (i) a triangle with both line and rotational symmetries of order more than 1.
   (ii) a triangle with only line symmetry and no rotational symmetry of order more than 1.
   (iii) a quadrilateral with a rotational symmetry of order more than 1 but not a line symmetry.
   (iv) a quadrilateral with line symmetry but not a rotational symmetry of order more than 1.

3. If a figure has two or more lines of symmetry, should it have rotational symmetry of order more than 1?

4. Fill in the blanks:

Shape	Centre of Rotation	Order of Rotation	Angle of Rotation
Square			
Rectangle			
Rhombus			
Equilateral Triangle			
Regular Hexagon			
Circle			
Semi-circle			

5. Name the quadrilaterals which have both line and rotational symmetry of order more than 1.
6. After rotating by 60° about a centre, a figure looks exactly the same as its original position. At what other angles will this happen for the figure?
7. Can we have a rotational symmetry of order more than 1 whose angle of rotation is
   (i) 45°? (ii) 17°?

WHAT HAVE WE DISCUSSED?

1. A figure has **line symmetry**, if there is a line about which the figure may be folded so that the two parts of the figure will coincide.
2. Regular polygons have equal sides and equal angles. They have multiple (i.e., more than one) lines of symmetry.
3. Each regular polygon has as many lines of symmetry as it has sides.

Regular Polygon	Regular hexagon	Regular pentagon	Square	Equilateral triangle
Number of lines of symmetry	6	5	4	3

4. Mirror reflection leads to symmetry, under which the left-right orientation have to be taken care of.
5. Rotation turns an object about a fixed point.

   This fixed point is the **centre of rotation**.

   The angle by which the object rotates is the **angle of rotation**.

   A half-turn means rotation by 180°; a quarter-turn means rotation by 90°. Rotation may be clockwise or anticlockwise.
6. If, after a rotation, an object looks exactly the same, we say that it has a **rotational symmetry**.
7. In a complete turn (of 360°), the number of times an object looks exactly the same is called the **order of rotational symmetry**. The order of symmetry of a square, for example, is 4 while, for an equilateral triangle, it is 3.
8. Some shapes have only one line of symmetry, like the letter E; some have only rotational symmetry, like the letter S; and some have both symmetries like the letter H.

   The study of symmetry is important because of its frequent use in day-to-day life and more because of the beautiful designs it can provide us.

Chapter 15

Visualising Solid Shapes

0756CH15

15.1 Introduction: Plane Figures and Solid Shapes

In this chapter, you will classify figures you have seen in terms of what is known as *dimension*.

In our day to day life, we see several objects like books, balls, ice-cream cones etc., around us which have different shapes. One thing common about most of these objects is that they all have some length, breadth and height or depth.

That is, they all occupy space and have three dimensions.

Hence, they are called three dimensional shapes.

Do you remember some of the three dimensional shapes (i.e., solid shapes) we have seen in earlier classes?

Try These

Match the shape with the name:

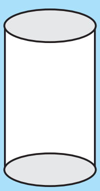
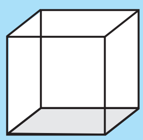
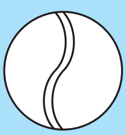
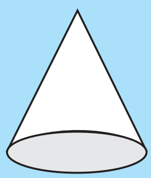
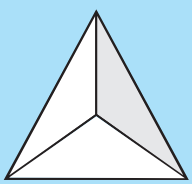
(i)		(a) Cuboid	(iv)		(d) Sphere
(ii)		(b) Cylinder	(v)		(e) Pyramid
(iii)		(c) Cube	(vi)		(f) Cone

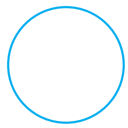
Fig 15.1

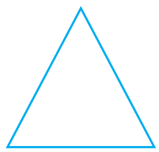
Try to identify some objects shaped like each of these.

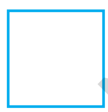
By a similar argument, we can say figures drawn on paper which have only length and breadth are called two dimensional (i.e., plane) figures. We have also seen some two dimensional figures in the earlier classes.

Match the 2 dimensional figures with the names (Fig 15.2):

(i)  (a) Circle

(ii)  (b) Rectangle

(iii)  (c) Square

(iv)  (d) Quadrilateral

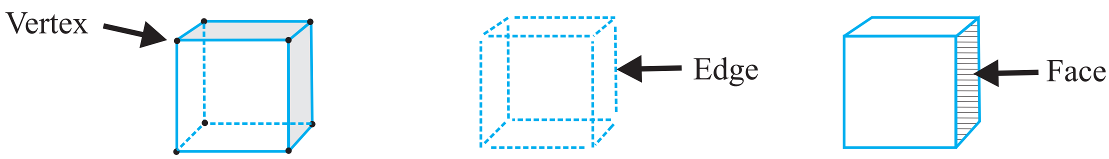
(v)  (e) Triangle

Fig 15.2

Note: We can write 2-D in short for 2-dimension and 3-D in short for 3-dimension.

15.2 Faces, Edges and Vertices

Do you remember the Faces, Vertices and Edges of solid shapes, which you studied earlier? Here you see them for a cube:



(i) (ii) (iii)

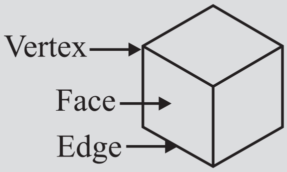
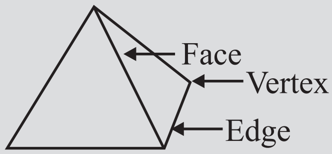
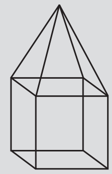
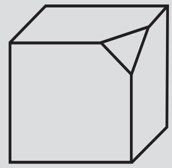
Fig 15.3

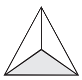
The 8 corners of the cube are its **vertices**. The 12 line segments that form the skeleton of the cube are its **edges**. The 6 flat square surfaces that are the skin of the cube are its **faces**.

Do This

Complete the following table:

Table 15.1

				
Faces **(F)**	6	4		
Edges **(E)**	12			
Vertices **(V)**	8	4		

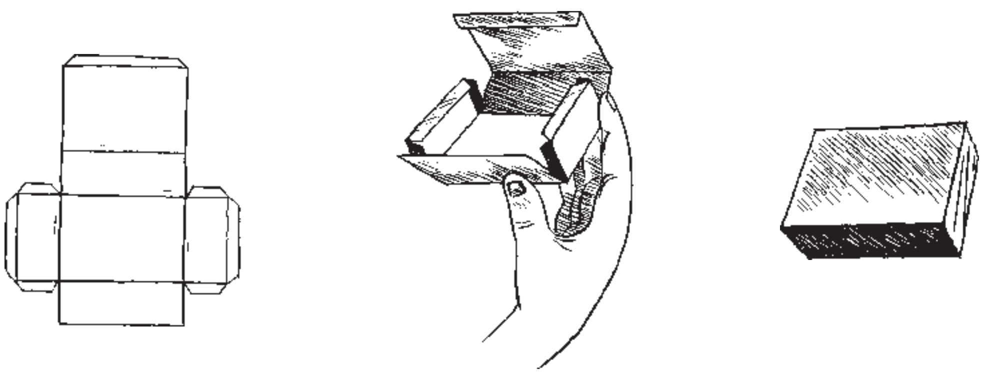
Can you see that, the two dimensional figures can be identified as the faces of the three dimensional shapes? For example a cylinder  has two faces which are circles, and a pyramid, shaped like this  has triangles as its faces.

We will now try to see how some of these 3-D shapes can be visualised on a 2-D surface, that is, on paper.

In order to do this, we would like to get familiar with three dimensional objects closely. Let us try forming these objects by making what are called nets.

15.3 Nets for Building 3-D Shapes

Take a cardboard box. Cut the edges to lay the box flat. You have now a **net** for that box. A net is a sort of skeleton-outline in 2-D [Fig154 (i)], which, when folded [Fig154 (ii)], results in a 3-D shape [Fig154 (iii)].



(i) (ii) (iii)

Fig 15.4

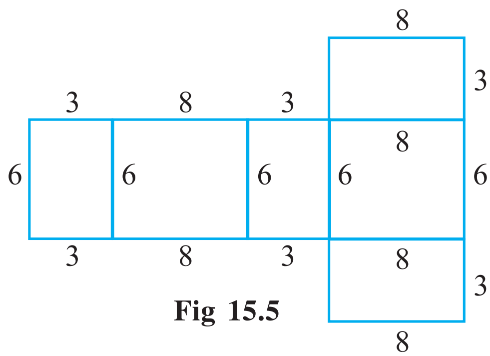


Fig 15.5

Here you got a **net** by suitably separating the edges. Is the reverse process possible?

Here is a net pattern for a box (Fig 15.5). Copy an enlarged version of the net and try to make the box by suitably folding and gluing together. (You may use suitable units). The box is a solid. It is a 3-D object with the shape of a cuboid.

Similarly, you can get a net for a cone by cutting a slit along its slant surface (Fig 15.6).

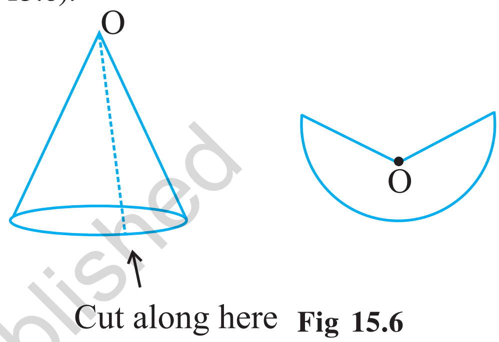
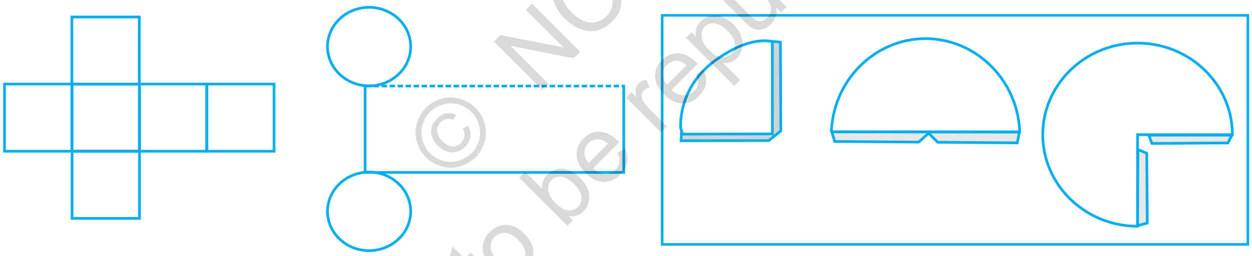


Fig 15.6

You have different nets for different shapes. Copy enlarged versions of the nets given (Fig 15.7) and try to make the 3-D shapes indicated. (You may also like to prepare skeleton models using strips of cardboard fastened with paper clips).



Cube
(i)

Cylinder
(ii)

Cone
(iii)

Fig 15.7

We could also try to make a net for making a pyramid like the Great Pyramid in Giza (Egypt) (Fig 15.8). That pyramid has a square base and triangles on the four sides.

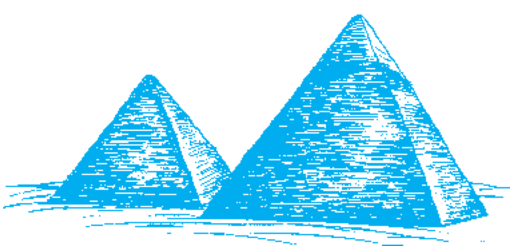


Fig 15.8

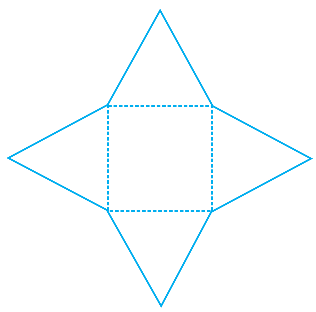


Fig 15.9

See if you can make it with the given net (Fig 15.9).

Try These

Here you find four nets (Fig 15.10). There are two *correct* nets among them to make a tetrahedron. See if you can work out which nets will make a tetrahedron.

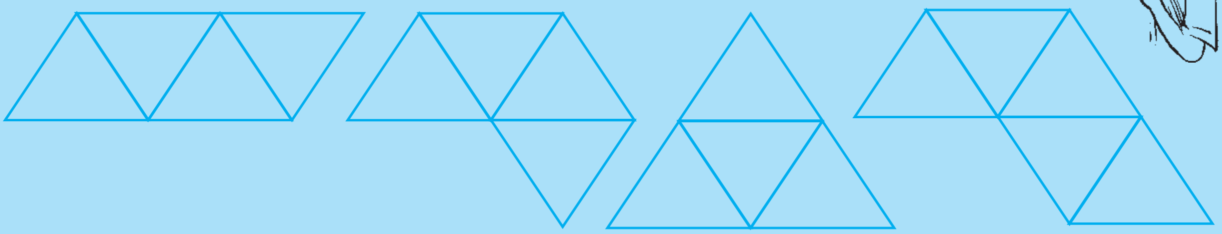
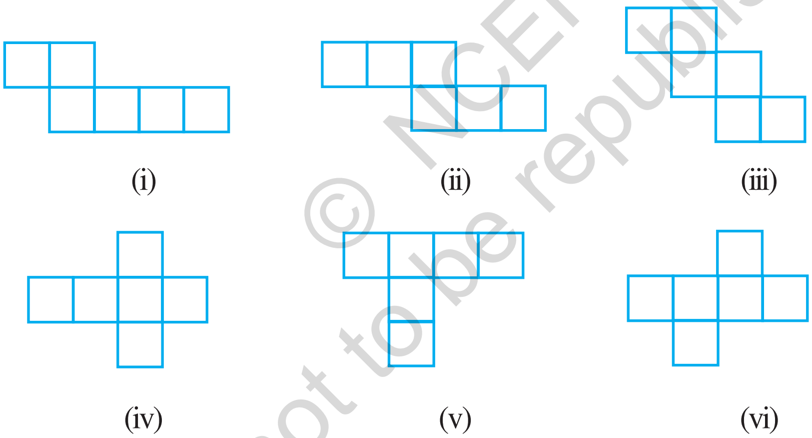


Fig 15.10

Exercise 15.1

1. Identify the nets which can be used to make cubes (cut out copies of the nets and try it):



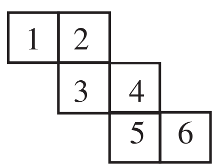
2. Dice are cubes with dots on each face. Opposite faces of a die always have a total of seven dots on them.

Here are two nets to make dice (cubes); the numbers inserted in each square indicate the number of dots in that box.

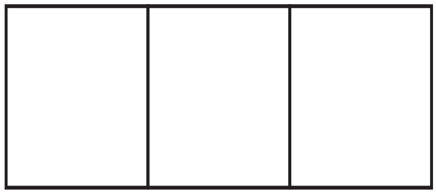
Insert suitable numbers in the blanks, remembering that the number on the opposite faces should total to 7.

3. Can this be a net for a die?

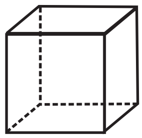
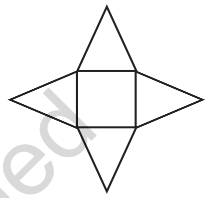
Explain your answer.

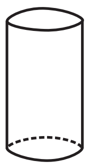
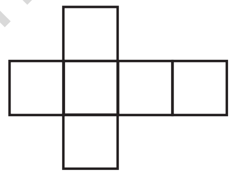


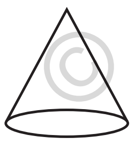
4. Here is an incomplete net for making a cube. Complete it in at least two different ways. Remember that a cube has six faces. How many are there in the net here? (Give two separate diagrams. If you like, you may use a squared sheet for easy manipulation.)

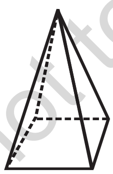
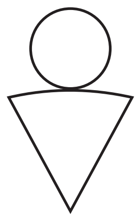


5. Match the nets with appropriate solids:

(a)  (i) 

(b)  (ii) 

(c)  (iii) 

(d)  (iv) 

Play this game

You and your friend sit back-to-back. One of you reads out a net to make a 3-D shape, while the other attempts to copy it and sketch or build the described 3-D object.

15.4 Drawing Solids on a Flat Surface

Your drawing surface is paper, which is flat. When you draw a solid shape, the images are somewhat distorted to make them appear three-dimensional. It is a visual illusion. You will find here two techniques to help you.

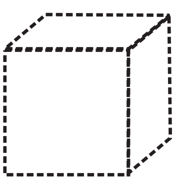


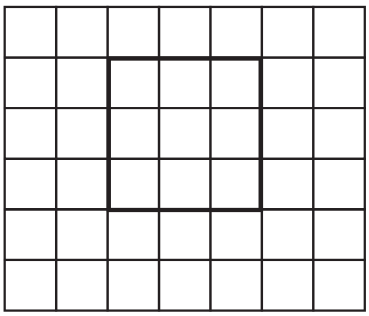
Fig 15.11

15.4.1 Oblique Sketches

Here is a picture of a cube (Fig 15.11). It gives a clear idea of how the cube looks like, when seen from the front. You do not see certain faces. In the drawn picture, the lengths are not equal, as they should be in a cube. Still, you are able to recognise it as a cube. Such a sketch of a solid is called an **oblique sketch**.

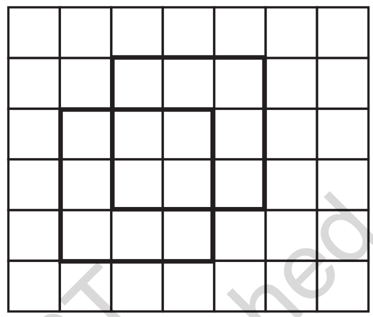
How can you draw such sketches? Let us attempt to learn the technique.

You need a squared (lines or dots) paper. Initially practising to draw on these sheets will later make it easy to sketch them on a plain sheet (without the aid of squared lines or dots!) Let us attempt to draw an oblique sketch of a $3 \times 3 \times 3$ (each edge is 3 units) cube (Fig 15.12).



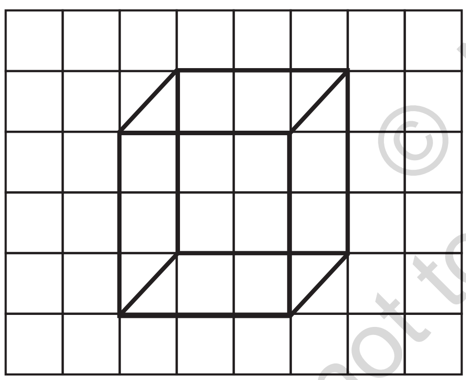
Step 1

Draw the **front** face.



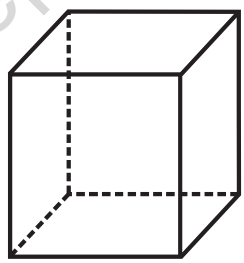
Step 2

Draw the **opposite** face. Sizes of the faces have to be same, but the sketch is somewhat off-set from step 1.



Step 3

Join the corresponding corners



Step 4

Redraw using dotted lines for hidden edges. (It is a convention) The sketch is ready now.

Fig 15.12

In the oblique sketch above, did you note the following?

(i) The sizes of the front faces and its opposite are same; and

(ii) The edges, which are all equal in a cube, appear so in the sketch, though the actual measures of edges are not taken so.

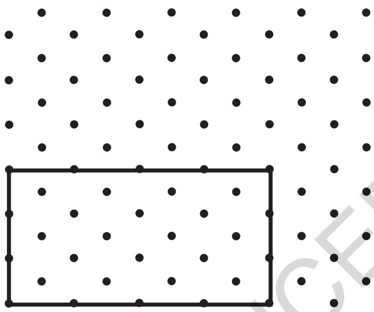
You could now try to make an oblique sketch of a cuboid (remember the faces in this case are rectangles)

Note: You can draw sketches in which measurements also agree with those of a given solid. To do this we need what is known as an **isometric sheet**. Let us try to make a cuboid with dimensions 4 cm length, 3 cm breadth and 3 cm height on given isometric sheet.

15.4.2 Isometric Sketches

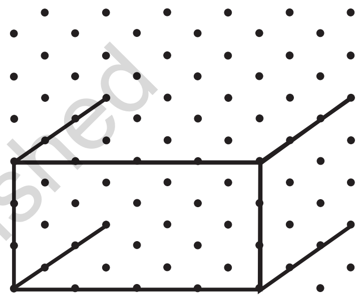
Have you seen an isometric dot sheet? (A sample is given at the end of the book). Such a sheet divides the paper into small equilateral triangles made up of dots or lines. *To draw sketches in which measurements also agree with those of the solid,* we can use isometric dot sheets. [Given on inside of the back cover (3rd cover page).]

Let us attempt to draw an isometric sketch of a cuboid of dimensions $4 \times 3 \times 3$ (which means the edges forming length, breadth and height are 4, 3, 3 units respectively) (Fig 15.13).



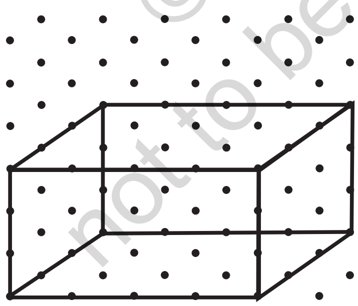
Step 1

Draw a rectangle to show the front face.



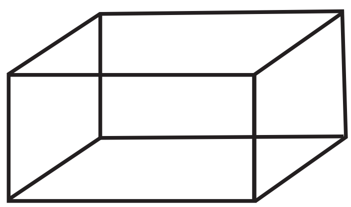
Step 2

Draw four parallel line segments of length 3 starting from the four corners of the rectangle.



Step 3

Connect the matching corners with appropriate line segments.



Step 4

This is an isometric sketch of the cuboid.

Fig 15.13

Note that the measurements are of exact size in an isometric sketch; this is not so in the case of an oblique sketch.

EXAMPLE 1 Here is an oblique sketch of a cuboid [Fig 15.14(i)]. Draw an isometric sketch that matches this drawing.

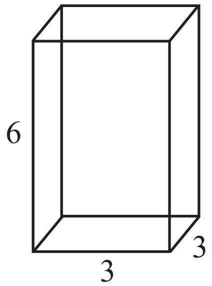


Fig 15.14 (i)

SOLUTION Here is the solution [Fig 15.14(ii)]. Note how the measurements are taken care of.

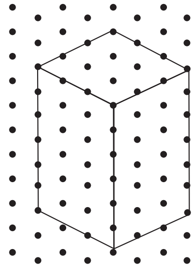


Fig 15.14 (ii)

How many units have you taken along (i) 'length'? (ii) 'breadth'? (iii) 'height'? Do they match with the units mentioned in the oblique sketch?

EXERCISE 15.2

1. Use isometric dot paper and make an isometric sketch for each one of the given shapes:

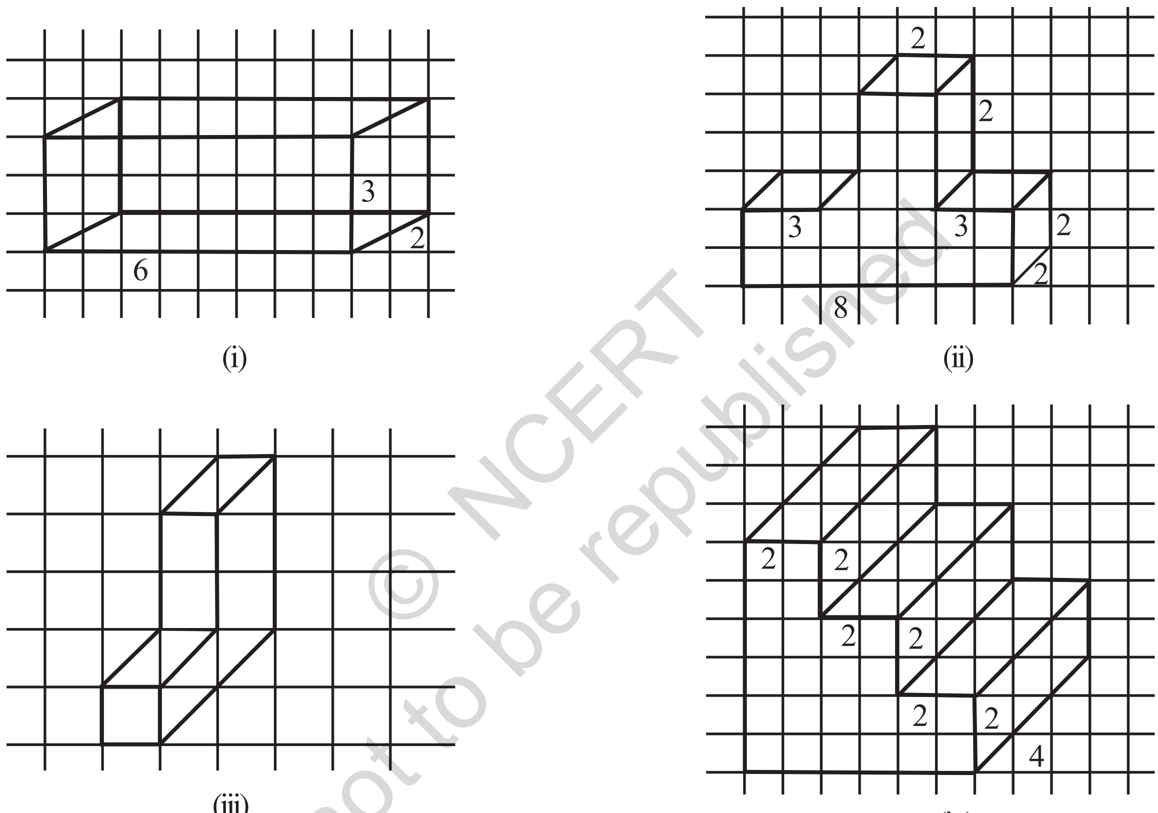
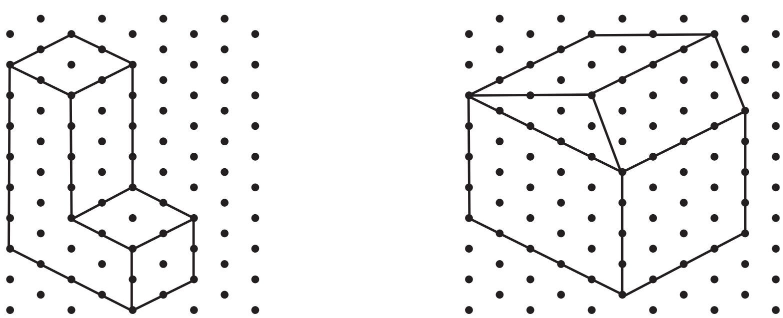


Fig 15.15

2. The dimensions of a cuboid are 5 cm, 3 cm and 2 cm. Draw three different isometric sketches of this cuboid.
3. Three cubes each with 2 cm edge are placed side by side to form a cuboid. Sketch an oblique or isometric sketch of this cuboid.
4. Make an oblique sketch for each one of the given isometric shapes:



5. Give (i) an oblique sketch and (ii) an isometric sketch for each of the following:

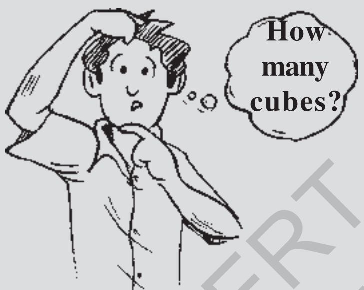
(a) A cuboid of dimensions 5 cm, 3 cm and 2 cm. (Is your sketch unique?)

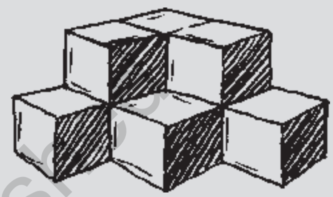
(b) A cube with an edge 4 cm long.

An isometric sheet is attached at the end of the book. You could try to make on it some cubes or cuboids of dimensions specified by your friend.

15.4.3 Visualising Solid Objects

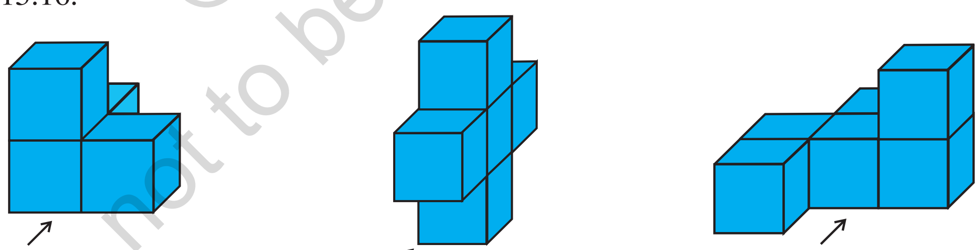
DO THIS





Sometimes when you look at combined shapes, some of them may be hidden from your view.

Here are some activities you could try in your free time to help you visualise some solid objects and how they look. Take some cubes and arrange them as shown in Fig 15.16.



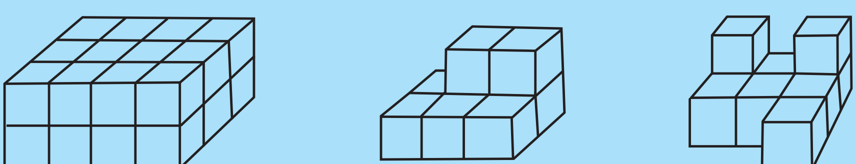
(i) (ii) (iii)

Fig 15.16

Now ask your friend to guess how many cubes there are when observed from the view shown by the arrow mark.

TRY THESE

Try to guess the number of cubes in the following arrangements (Fig 15.17).



(i) (ii) (iii)

Fig 15.17

Such visualisation is very helpful. Suppose you form a cuboid by joining such cubes. You will be able to guess what the length, breadth and height of the cuboid would be.

EXAMPLE 2 If two cubes of dimensions 2 cm by 2cm by 2cm are placed side by side, what would the dimensions of the resulting cuboid be?

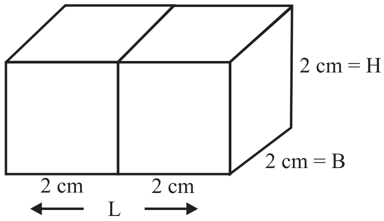


Fig 15.18

SOLUTION As you can see (Fig 15.18) when kept side by side, the length is the only measurement which increases, it becomes $2 + 2 = 4$ cm.

The breadth = 2 cm and the height = 2 cm.

TRY THESE

1. Two dice are placed side by side as shown: Can you say what the total would be on the face opposite to

   (a) $5 + 6$ (b) $4 + 3$

   (Remember that in a die sum of numbers on opposite faces is 7)

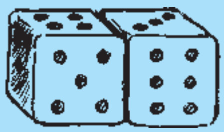


Fig 15.19

2. Three cubes each with 2 cm edge are placed side by side to form a cuboid. Try to make an oblique sketch and say what could be its length, breadth and height.

15.5 VIEWING DIFFERENT SECTIONS OF A SOLID

Now let us see how an object which is in 3-D can be viewed in different ways.

15.5.1 One Way to View an Object is by Cutting or Slicing

Slicing game

Here is a loaf of bread (Fig 15.20). It is like a cuboid with a square face. You 'slice' it with a knife.

When you give a 'vertical' cut, you get several pieces, as shown in the Figure 15.20. Each face of the piece is a square! We call this face a **'cross-section'** of the whole bread. The cross section is nearly a square in this case.

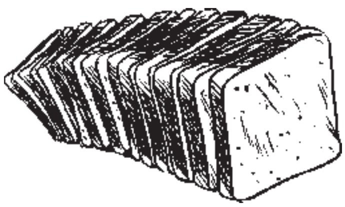


Fig 15.20

Beware! If your cut is not 'vertical' you may get a different cross section! Think about it. The boundary of the cross-section you obtain is a plane curve. Do you notice it?

A kitchen play

Have you noticed cross-sections of some vegetables when they are cut for the purposes of cooking in the kitchen? Observe the various slices and get aware of the shapes that result as cross-sections.

Play this

Make clay (or plasticine) models of the following solids and make vertical or horizontal cuts. Draw rough sketches of the cross-sections you obtain. Name them wherever you can.

Fig 15.21

EXERCISE 15.3

1. What cross-sections do you get when you give a
   (i) vertical cut (ii) horizontal cut

   to the following solids?

   (a) A brick (b) A round apple (c) A die
   (d) A circular pipe (e) An ice cream cone

15.5.2 Another Way is by Shadow Play

Fig 15.22

A shadow play

Shadows are a good way to illustrate how three-dimensional objects can be viewed in two dimensions. Have you seen a **shadow play**? It is a form of entertainment using solid articulated figures in front of an illuminated back-drop to create the illusion of moving images. It makes some indirect use of ideas in Mathematics.

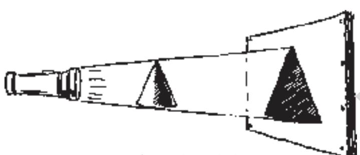


Fig 15.23

You will need a source of light and a few solid shapes for this activity. (If you have an overhead projector, place the solid under the lamp and do these investigations.)

Keep a torchlight, *right in front of* a Cone. What type of shadow does it cast on the screen? (Fig 15.23)

The solid is three-dimensional; what is the dimension of the shadow?

If, instead of a cone, you place a cube in the above game, what type of shadow will you get?

Experiment with different positions of the source of light and with different positions of the solid object. Study their effects on the shapes and sizes of the shadows you get.

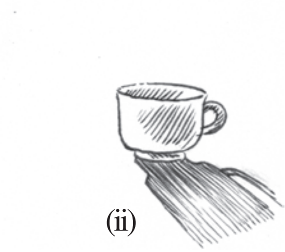
Here is another funny experiment that you might have tried already: Place a circular plate in the open when the Sun at the noon time is just *right above* it as shown in Fig 15.24 (i). What is the shadow that you obtain?

(i)

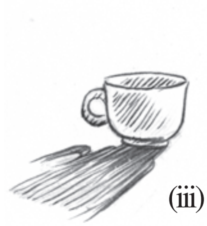
Will it be same during

(a) forenoons?

(b) evenings?



(ii)



(iii)

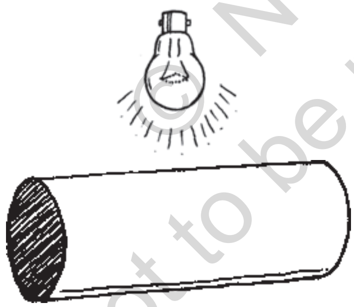
Fig 15.24 (i) - (iii)

Study the shadows in relation to the position of the Sun and the time of observation.

EXERCISE 15.4

1. A bulb is kept burning just right above the following solids. Name the shape of the shadows obtained in each case. Attempt to give a rough sketch of the shadow. (You may try to experiment first and then answer these questions).

A ball
(i)

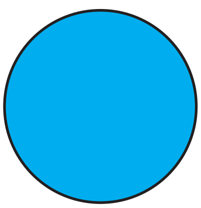


A cylindrical pipe
(ii)

A book
(iii)

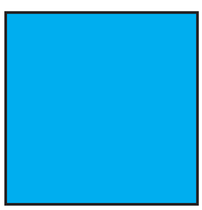
2. Here are the shadows of some 3-D objects, when seen under the lamp of an overhead projector. Identify the solid(s) that match each shadow. (There may be multiple answers for these!)

A circle



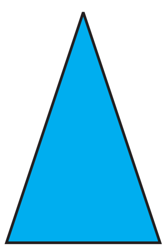
(i)

A square



(ii)

A triangle



(iii)

A rectangle

(iv)

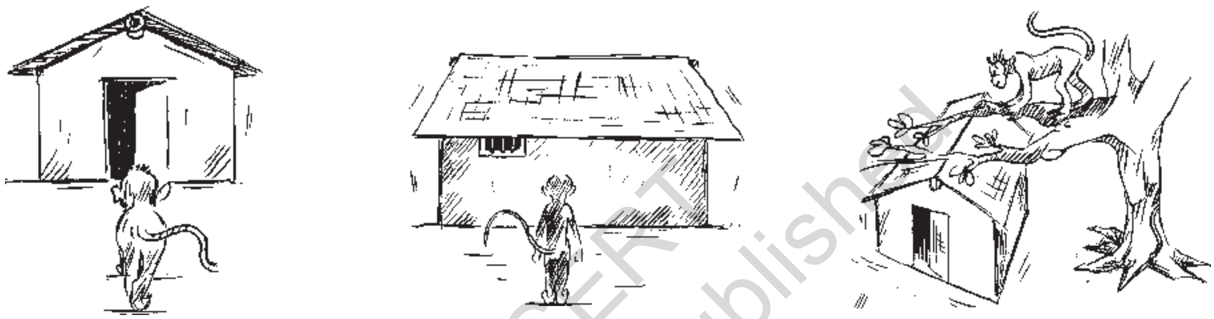
3. Examine if the following are true statements:

(i) The cube can cast a shadow in the shape of a rectangle.

(ii) The cube can cast a shadow in the shape of a hexagon.

15.5.3 A Third Way is by Looking at it from Certain Angles to Get Different Views

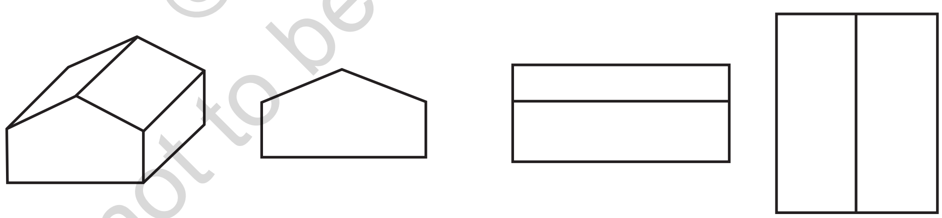
One can look at an object standing in front of it or by the side of it or from above. Each time one will get a different view (Fig 15.25).



Front view Side view Top view

Fig 15.25

Here is an example of how one gets different views of a given building. (Fig 15.26)



Building Front view Side view Top view

Fig 15.26

You could do this for figures made by joining cubes.

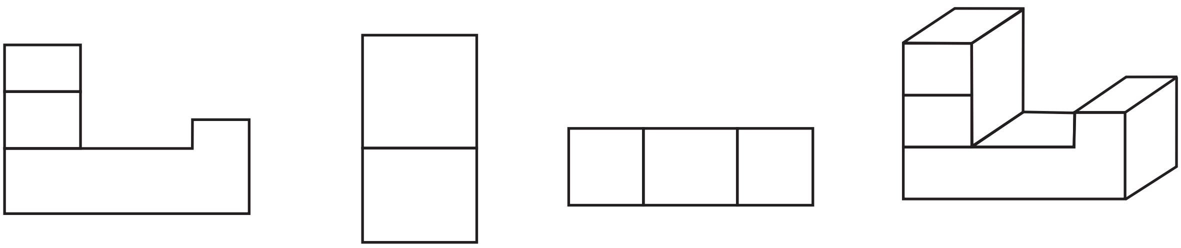


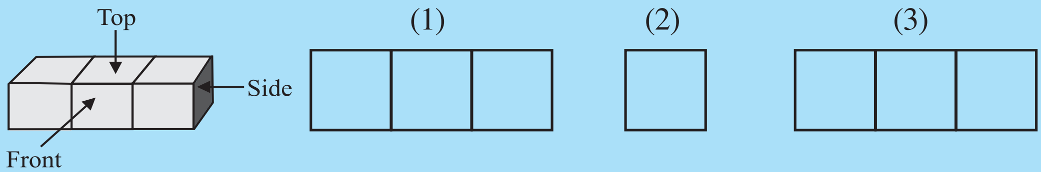
Fig 15.27

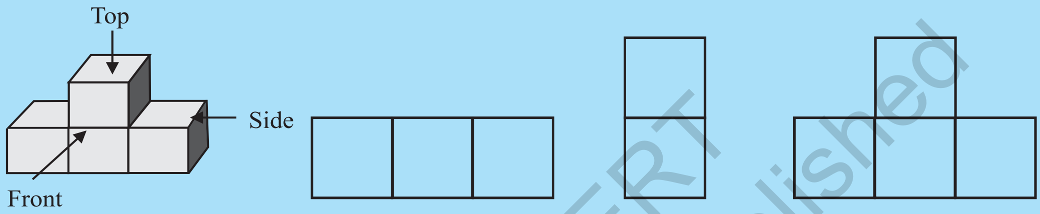
Try putting cubes together and then making such sketches from different sides.

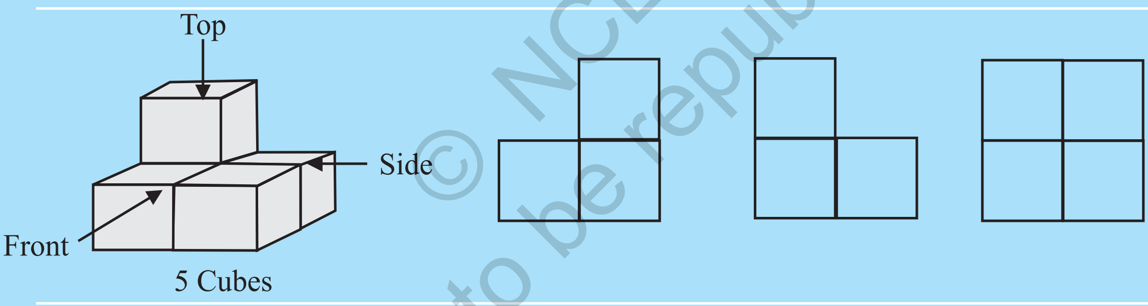
Try These

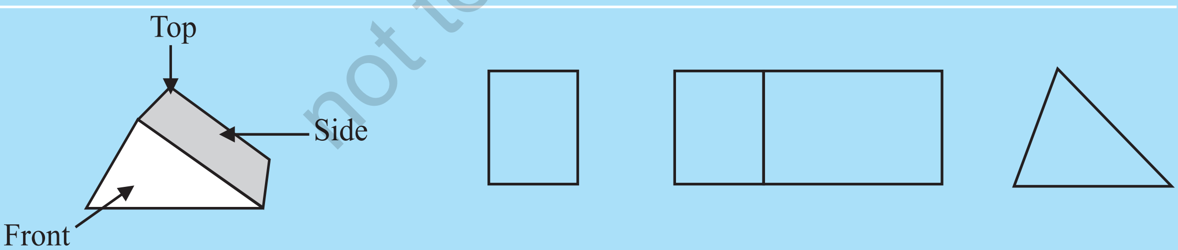
1. For each solid, the three views (1), (2), (3) are given. Identify for each solid the corresponding top, front and side views.

Solid **Its views**

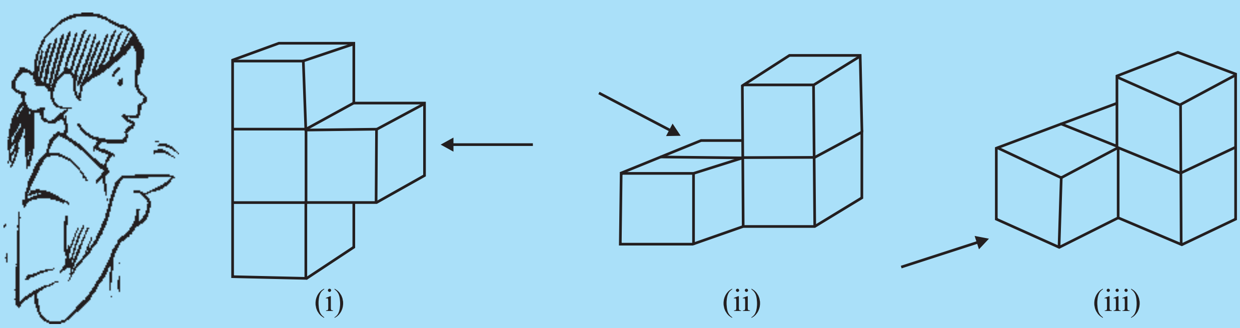








2. Draw a view of each solid as seen from the direction indicated by the arrow.



What have We Discussed?

1. The circle, the square, the rectangle, the quadrilateral and the triangle are examples of **plane figures**; the cube, the cuboid, the sphere, the cylinder, the cone and the pyramid are examples of **solid shapes**.
2. Plane figures are of **two-dimensions** (**2-D**) and the solid shapes are of three-dimensions (**3-D**).
3. The corners of a solid shape are called its **vertices**; the line segments of its skeleton are its **edges**; and its flat surfaces are its **faces**.
4. A **net** is a skeleton-outline of a solid that can be folded to make it. The same solid can have several types of nets.
5. Solid shapes can be drawn on a flat surface (like paper) realistically. We call this **2-D representation of a 3-D solid**.
6. Two types of sketches of a solid are possible:
   (a) An **oblique sketch** does not have proportional lengths. Still it conveys all important aspects of the appearance of the solid.
   (b) An **isometric sketch** is drawn on an isometric dot paper, a sample of which is given at the end of this book. In an isometric sketch of the solid the measurements kept proportional.
7. **Visualising solid shapes** is a very useful skill. You should be able to see 'hidden' parts of the solid shape.
8. Different sections of a solid can be viewed in many ways:
   (a) One way is to view by cutting or **slicing** the shape, which would result in the cross-section of the solid.
   (b) Another way is by observing a 2-D **shadow** of a 3-D shape.
   (c) A third way is to look at the shape from different angles; the **front-view**, the **side-view** and the **top-view** can provide a lot of information about the shape observed.