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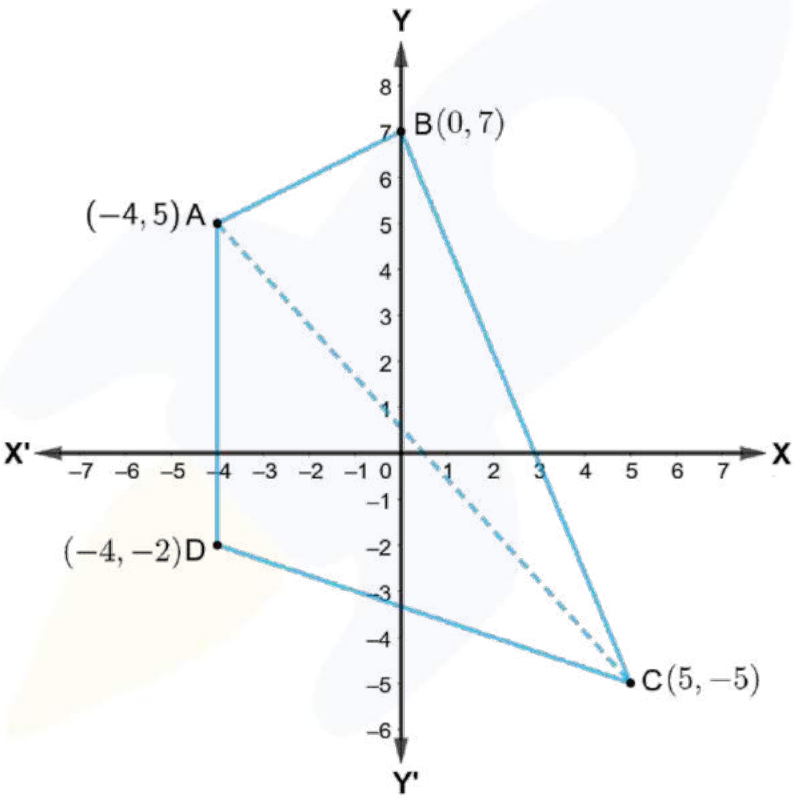
NCERT Solutions Class 11 Maths Chapter 10 Straight lines

Question 1:

Draw a quadrilateral in the Cartesian plane, whose vertices are $(-4,5),(0,7),(5,-5)$ and $(-4,-2)$. Also, find its area.

Solution:

Let ABCD be the given quadrilateral with vertices $A(-4,5)$, $B(0,7)$, $C(5,-5)$ and $D(-4,-2)$. Then, by plotting A, B, C and D on the Cartesian plane and joining AB, BC, CD and DA, the given quadrilateral can be drawn as



To find the area of quadrilateral ABCD, we draw one diagonal, say AC.

Accordingly, $ar(\square ABCD) = ar(\Delta ABC) + ar(\Delta ACD)$

We know that the area of a triangles whose vertices are $(x_1, y_1),(x_2, y_2)$ and (x_3, y_3) is
$\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

Therefore,

$$ar(\Delta ABC)=\frac{1}{2}\left|-4(7+5)+0(-5+5)+5(5-7)\right|$$
$$=\frac{1}{2}\left|-48+0-10\right|$$
$$=\frac{1}{2}\left|-58\right|$$
$$=\frac{1}{2}\times 58$$
$$=29$$

$$ar(\Delta ACD)=\frac{1}{2}\left|(-4)(-5+2)+5(-2-5)+(-4)(5+5)\right|$$
$$=\frac{1}{2}\left|12-35-40\right|$$
$$=\frac{1}{2}\left|-63\right|$$
$$=\frac{1}{2}\times 63$$
$$=\frac{63}{2}$$

Thus,

$$ar(\square ABCD)=ar(\Delta ABC)+ar(\Delta ACD)$$
$$=\left(29+\frac{63}{2}\right)sq.\,unit$$
$$=\left(\frac{58+63}{2}\right)sq.\,unit$$
$$=\frac{121}{2}sq.\,unit$$

Question 2:

The base of an equilateral triangle with side $2a$ lies along the y-axis such that the midpoint of the base is at the origin. Find vertices of the triangle.

Solution:

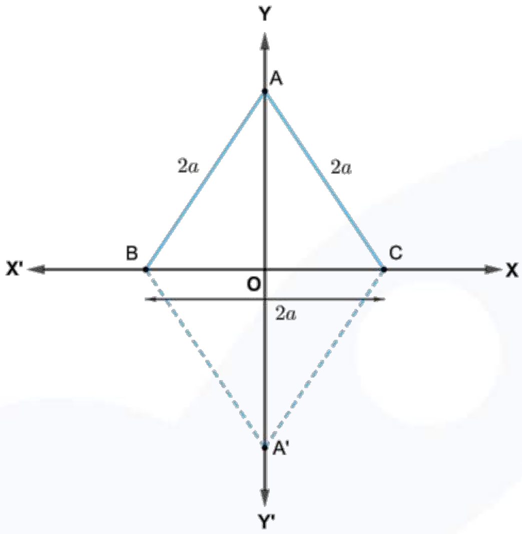
Let ABC be the given equilateral triangle with side $2a$.

Accordingly, $AB = BC = CA = 2a$

Assume that base BC lies along the x-axis such that the mid-point of BC is at the origin. i.e., $BO = OC = a$, where O is the origin.

Now, it is clear that the coordinates of point $C(0,a)$, while the coordinates of point $B(0,-a)$.

It is known that the line joining a vertex of an equilateral triangle with the mid-point of its opposite side is perpendicular.
Hence, vertex A lies on the y-axis.



On applying Pythagoras theorem to ΔAOC, we obtain

$$
\begin{aligned}
AC^2 &= OA^2 + OC^2 \\
OA^2 &= AC^2 - OC^2 \\
&= (2a)^2 + (a)^2 \\
&= 4a^2 - a^2 \\
&= 3a^2 \\
OA &= \pm\sqrt{3}a
\end{aligned}
$$

Therefore, coordinates of point $A(\pm\sqrt{3}a, 0)$

Thus, the vertices of the given equilateral triangle are $(0,a), (0,-a)$ and $(\sqrt{3}a, 0)$ or $(0,a), (0,-a)$ and $(-\sqrt{3}a, 0)$.

Question 3:

Find the distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ when: (i) PQ is parallel to the y-axis, (ii) PQ is parallel to the x-axis

Solution:

The given points are $P(x_1, y_1)$ and $Q(x_2, y_2)$

(i) When PQ is parallel to the y-axis, $x_1 = x_2$.

In this case, distance between P and Q $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$PQ = \sqrt{(y_2 - y_1)^2}$$
$$= |y_2 - y_1|$$

(ii) When PQ is parallel to the x-axis $y_1 = y_2$

In this case, distance between P and Q $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$PQ = \sqrt{(x_2 - x_1)^2}$$
$$= |x_2 - x_1|$$

Question 4:

Find a point on the x-axis, which is equidistant from the points (7, 6) and (3, 4).

Solution:

Let (a, 0) be the point on the X- axis that is equidistance from the points (7, 6) and (3, 4). Accordingly,

$$\sqrt{(7-a)^2 + (6-0)^2} = \sqrt{(3-a)^2 + (4-0)^2}$$
$$\Rightarrow \sqrt{49 + a^2 - 14a + 36} = \sqrt{9 + a^2 - 6a + 16}$$
$$\Rightarrow \sqrt{a^2 - 14a + 85} = \sqrt{a^2 - 6a + 25}$$

On squaring on both sides, we obtain

$$a^2 - 14a + 85 = a^2 - 6a + 25$$
$$\Rightarrow -14a + 6a = 25 - 85$$
$$\Rightarrow -8a = 60$$
$$\Rightarrow a = \frac{60}{8}$$
$$\Rightarrow a = \frac{15}{2}$$

Thus, the required point on the x-axis is $\left(\frac{15}{2}, 0\right)$

Question 5:

Find the slope of a line, which passes through the origin, and the mid-point of the segment joining the points $P(0,-4)$ and $B(8,0)$.

Solution:

The coordinates of the mid-point of the line segment joining the points

$P(0,-4)$ and $B(8,0)$ are $\left(\frac{0+8}{2},\frac{-4+0}{2}\right)=(4,-2)$

It is known that the slope (m) of a non-vertical line passing through the points (x_1,y_1) and (x_2,y_2) is given by $m=\frac{y_2-y_1}{x_2-x_1},x_2\neq x_1$

Therefore, the slope of the line passing through $(0,0)$ and $(4,-2)$ is

$$\frac{-2-0}{4-0}=-\frac{2}{4}=-\frac{1}{2}$$

Hence, the required slope of the line is $-\frac{1}{2}$

Question 6:

Without using the Pythagoras theorem, show that the points $(4,4),(3,5)$ and $(-1,-1)$ are vertices of a right-angled triangle.

Solution:

The vertices of the given triangles are $A(4,4),B(3,5)$ and $C(-1,-1)$.

It is known that the slope (m) of a non-vertical line passing through the points (x_1,y_1) and (x_2,y_2) is given by $m=\frac{y_2-y_1}{x_2-x_1},x_2\neq x_1$

Therefore, slope of AB $(m_1)=\frac{5-4}{3-4}=-1$

Slope of BC $(m_2)=\frac{-1-5}{-1-3}=\frac{-6}{-4}=\frac{3}{2}$

Slope of CA $(m_3)=\frac{4+1}{4+1}=\frac{5}{5}=1$

It is observed that $m_1m_3=-1$

This shows that line segments AB and CA are perpendicular to each other i.e., the given triangle is right-angled at $A(4,4)$.

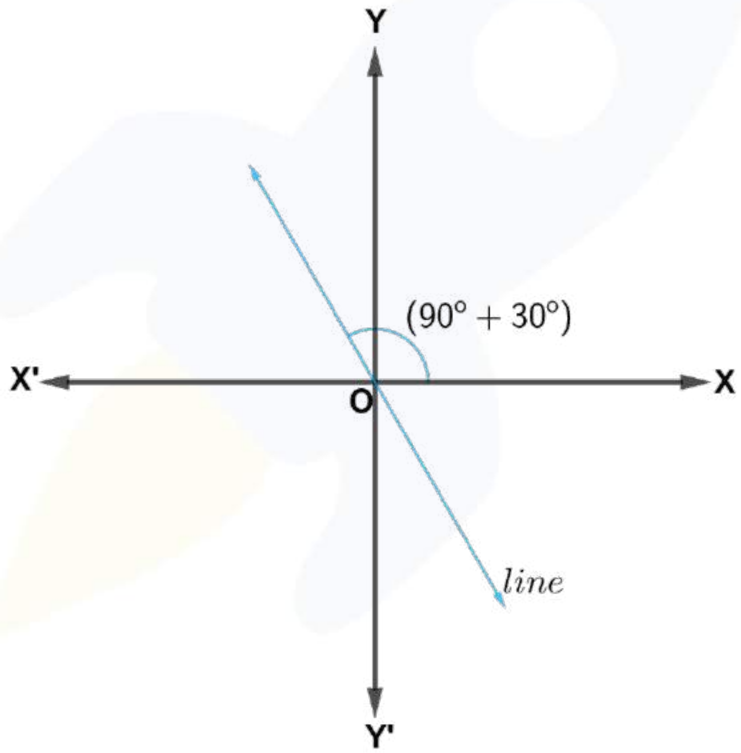
Thus, the points $(4,4),(3,5)$ and $(-1,-1)$ are the vertices of a right-angled triangle.

Question 7:

Find the slope of the line, which makes an angle of 30° with the positive direction of y-axis measured anticlockwise.

Solution:

If a line makes an angle of $30°$ with positive direction of the y-axis measured anticlockwise, then the angle made by the line with the positive direction of the x-axis measured anticlockwise is $90°+30°=120°$.



Thus, the slope of the given line is

$$\begin{aligned}\tan 120° &= \tan(180°-60°)\\ &= -\tan 60°\\ &= -\sqrt{3}\end{aligned}$$

Question 8:

Find the value of x for which the points $(x,-1),(2,1)$ and $(4,5)$ are collinear.

Solution:

If points $A(x,-1)$, $B(2,1)$ and $C(4,5)$ are collinear,
Then, Slope of AB = Slope of BC

$$\Rightarrow \frac{1-(-1)}{2-x}=\frac{5-1}{4-2}$$
$$\Rightarrow \frac{1+1}{2-x}=\frac{4}{2}$$
$$\Rightarrow \frac{2}{2-x}=2$$
$$\Rightarrow 2=4-2x$$
$$\Rightarrow 2x=4-2$$
$$\Rightarrow x=1$$

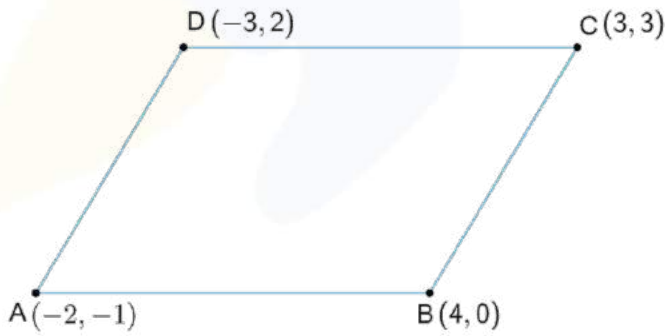
Thus, the required value of $x=1$

Question 9:

Without using distance formula, show that points $(-2,-1),(4,0),(3,3)$ and $(-3,\ 2)$ are vertices of a parallelogram.

Solution:

Let points $(-2,-1),(4,0),(3,3)$ and $(-3,\ 2)$ be respectively denoted by A, B, C and D.



Slope of AB $=\frac{0+1}{4+2}=\frac{1}{6}$

Slope of CD $=\frac{2-3}{-3-3}=\frac{-1}{-6}=\frac{1}{6}$

Therefore, Slope of AB = Slope of CD

Hence, AB and CD are parallel to each other.

Now,

Slope of BC $=\frac{3-0}{3-4}=\frac{3}{-1}=-3$

Slope of AD $=\frac{2+1}{-3+2}=\frac{3}{-1}=-3$

Therefore, Slope of BC = Slope of AD

Hence, BC and AD are parallel to each other.

Therefore, both pairs of opposite side of quadrilateral ABCD are parallel. Hence, ABCD is a parallelogram.

Thus, points $(-2,-1),(4,0),(3,3)$ and $(-3, 2)$ are the vertices of a parallelogram.

Question 10:

Find the angle between the x-axis and the line joining the points $(3,-1)$ and $(4,-2)$

Solution:

The slope of the line joining the points $(3,-1)$ and $(4,-2)$ is

$$\begin{aligned} m &= \frac{-2-(-1)}{4-3} \\ &= -2+1 \\ &= -1 \end{aligned}$$

Now, the inclination (θ) of the line joining the points $(3,-1)$ and $(4,-2)$ is given by

$$\begin{aligned} \tan\theta &= -1 \\ \theta &= (90^\circ + 45^\circ) \\ \theta &= 135^\circ \end{aligned}$$

Thus, the angle between the *x*-axis and the line joining the points $(3,-1)$ and $(4,-2)$ is 135°.

Question 11:

The slope of a line is double of the slope of another line. If tangent of the angle between them is $\frac{1}{3}$, find the slopes of the lines.

Solution:

We know that if θ is the angle between the lines l_1 and l_2 with slopes m_1 and m_2 then

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

It is given that the tangent of the angle between the two lines is $\frac{1}{3}$ and slope of a line is double of the slope of another line.

Let m and $2m$ be the slopes of the given lines

Therefore,

$$\frac{1}{3} = \left|\frac{2m - m}{1 + m(2m)}\right| \qquad \frac{1}{3} = \left|\frac{m - 2m}{1 + m(2m)}\right|$$

$$\frac{1}{3} = \left|\frac{m}{1 + 2m^2}\right| \qquad \frac{1}{3} = \left|\frac{-m}{1 + 2m^2}\right|$$

$$\frac{1}{3} = \frac{m}{1 + 2m^2} \quad \text{or} \quad \frac{1}{3} = \frac{-m}{1 + 2m^2}$$

Case I

$$\frac{1}{3} = \frac{-m}{1 + 2m^2}$$

$$1 + 2m^2 = -3m$$

$$2m^2 + 2m + m + 1 = 0$$

$$2m(m+1) + 1(m+1) = 0$$

$$(m+1)(2m+1) = 0$$

$$\Rightarrow m = -1 \text{ or } m = -\frac{1}{2}$$

If $m = -1$, then the slopes of the lines are -1 and -2.

If $m = -\frac{1}{2}$, then the slopes of the lines are $-\frac{1}{2}$ and -1.

Case II

$$\frac{1}{3}=\frac{m}{1+2m^2}$$
$$2m^2+1=3m$$
$$2m^2-3m+1=0$$
$$2m^2-2m-m+1=0$$
$$2m(m-1)-1(m-1)=0$$
$$(2m-1)(m-1)=0$$
$$\Rightarrow m=1 \text{ or } m=\frac{1}{2}$$

If $m=1$, then the slopes of the lines are 1 and 2.

If $m=\frac{1}{2}$, then the slopes of the lines are $\frac{1}{2}$ and 1.

Hence, the slopes of the lines are -1 and -2, or $-\frac{1}{2}$ and -1, or 1 and 2, or $\frac{1}{2}$ and 1.

Question 12:

A line passes through (x_1, y_1) and (h,k). If slope of the line is m, show that $k-y_1=m(h-x_1)$

Solution:

The slope of the line passing through (x_1, y_1) and (h,k) is $\frac{k-y_1}{h-x_1}$
It is given that the slope of the line is m
Therefore,

$$\frac{k-y_1}{h-x_1}=m$$
$$k-y_1=m(h-x_1)$$

Hence, $k-y_1=m(h-x_1)$ proved.

Question 13:

If three points $(h,0),(a,b)$ and $(0,k)$ lie on a line, show that $\frac{a}{h}+\frac{b}{k}=1$

Solution:

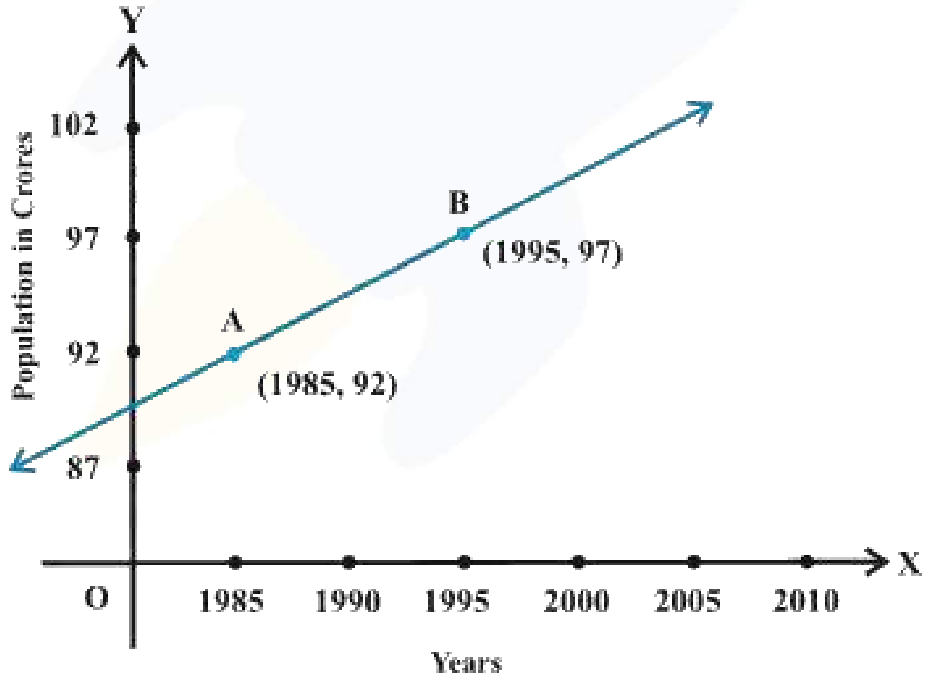
If three points $A(h,0), B(a,b)$ and $C(0,k)$ lie on a line, then
Slope of AB = Slope of BC

$$\frac{b-0}{a-h}=\frac{k-b}{0-a}$$
$$\frac{b}{a-h}=\frac{k-b}{-a}$$
$$-ab=(k-b)(a-h)$$
$$-ab=ka-kh-ab+bh$$
$$ka+bh=kh$$

On dividing both sides by kh, we obtain

$$\frac{ka}{kh}+\frac{bh}{kh}=\frac{kh}{kh}$$
$$\frac{a}{h}+\frac{b}{k}=1$$

Question 14:
Consider the given population and year graph. Find the slope of the line AB and using it, find what will be the population in the year 2010?



Solution:

Since line AB passes through points $A(1985,92)$ and $B(1995,97)$, its slope is

$$\frac{97-92}{1995-1985}=\frac{5}{10}$$
$$=\frac{1}{2}$$

Let y be the population in the year 2010.

Then, according to the given graph, line AB must pass through point $C(2010, y)$.
Therefore, Slope of AB = Slope of BC

$$\frac{1}{2} = \frac{y-97}{2010-1995}$$
$$\frac{1}{2} = \frac{y-97}{15}$$
$$\frac{15}{2} = y-97$$
$$y = 7.5+97$$
$$y = 104.5$$

Thus, the slope of line AB is $\frac{1}{2}$, while in the year 2010, the population will be 104.5 crores.

EXERCISE 10.2

Question 1:

Write the equation for the x and y-axes.

Solution:

The y-coordinate of every point on the x-axis is 0.

Therefore, the equation of the x-axis is $y=0$.

The x-coordinate of every point on the y-axis is 0.

Therefore, the equation of the y-axis is $x=0$.

Question 2:

Find the equation of the line which passes through the point $(-4,3)$ with slope.

Solution:

We know that the equation of the line passing through point (x_0, y_0), whose slope is m, is $(y-y_0)=m(x-x_0)$

Thus, the equation of the line passing through point $(-4,3)$, whose slope is $\frac{1}{2}$,

$$(y-3)=\frac{1}{2}(x+4)$$
$$2(y-3)=x+4$$
$$2y-6=x+4$$
$$x-2y+10=0$$

Hence the equation is $x-2y+10=0$.

Question 3:

Find the equation of the line which passes through $(0,0)$ with slope m.

Solution:

We know that the equation of the line passing through point (x_0, y_0), whose slope is m,

$(y-y_0)=m(x-x_0)$

Thus, the equation of the line passing through point $(0,0)$, whose slope is m, is

$$(y-0)=m(x-0)$$
$$y=mx$$

Hence the equation is $y = mx$.

Question 4:

Find the equation of the line which passes through $\left(2, 2\sqrt{3}\right)$ and is inclined with the *x*-axis at an angle of 75°.

Solution:

The slope of the line that inclines with the *x*-axis at an angle of 75° is $m = \tan 75^\circ$

$$\begin{aligned} m &= \tan\left(45^\circ + 30^\circ\right) \\ &= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ)} \\ &= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}} \\ &= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} \\ &= \frac{\sqrt{3}+1}{\sqrt{3}-1} \end{aligned}$$

We know that the equation of the line passing through point $\left(x_0, y_0\right)$, whose slope is m, $\left(y - y_0\right) = m\left(x - x_0\right)$

Thus, if a line passes through $\left(2, 2\sqrt{3}\right)$ and inclines with the *x*-axis at an angle of 75°, then the equation of the line is given as

$$\begin{aligned} \left(y - 2\sqrt{3}\right) &= \frac{\sqrt{3}+1}{\sqrt{3}-1}(x-2) \\ \left(y - 2\sqrt{3}\right)\left(\sqrt{3}-1\right) &= \left(\sqrt{3}+1\right)(x-2) \\ y\left(\sqrt{3}-1\right) - 2\sqrt{3}\left(\sqrt{3}-1\right) &= x\left(\sqrt{3}+1\right) - 2\left(\sqrt{3}+1\right) \\ x\left(\sqrt{3}+1\right) - y\left(\sqrt{3}-1\right) &= 2\sqrt{3} + 2 - 6 + 2\sqrt{3} \\ x\left(\sqrt{3}+1\right) - y\left(\sqrt{3}-1\right) &= 4\sqrt{3} - 4 \\ \left(\sqrt{3}+1\right)x - \left(\sqrt{3}-1\right)y &= 4\left(\sqrt{3}-1\right) \end{aligned}$$

Hence the equation is $\left(\sqrt{3}+1\right)x-\left(\sqrt{3}-1\right)y=4\left(\sqrt{3}-1\right)$.

Question 5:
Find the equation of the line which intersects the x-axis at a distance of 3 units to the left of origin with slope -2.

Solution:
It is known that if a line with slope m makes x-intercept d, then the equation of the time is given as $y=m(x-d)$

For the line intersecting the x-axis at a distance of 3 units to the left of the origin, $d=-3$.

The slope of the line is given as $m=-2$

Thus, the required equation of the given line is

$$y=-2\left[x-(-3)\right]$$
$$y=-2x-6$$
$$2x+y+6=0$$

Hence the equation is $2x+y+6=0$

Question 6:
Find the equation of the line which intersects the y-axis at a distance of 2 units above the origin and makes an angle of 30° with the positive direction of the x-axis.

Solution:
It is known that if a line with slope m makes y-intercept c, then the equation of the line is given as $y=mx+c$

Here, $c=2$ and $m=\tan 30^\circ=\frac{1}{\sqrt{3}}$
Thus, the required equations of the given line is

$$y=\frac{1}{\sqrt{3}}x+2$$
$$y=\frac{x+2\sqrt{3}}{\sqrt{3}}$$
$$\sqrt{3}y=x+2\sqrt{3}$$
$$x-\sqrt{3}y+2\sqrt{3}=0$$

Hence, the equation of the line is $x-\sqrt{3}y+2\sqrt{3}=0$

Question 7:

Find the slope of the line, which makes an angle of 30° with the positive direction of y-axis measured anticlockwise.

Solution:

It is known that the equation of the line passes through points (x_1,y_1) and (x_2,y_2) is

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

Therefore, the equation of the line passing through the points $(-1,1)$ and $(2,-4)$ is

$$(y-1)=\frac{-4-1}{2+1}(x+1)$$
$$(y-1)=-\frac{5}{3}(x+1)$$
$$3(y-1)=-5(x+1)$$
$$3y-3=-5x-5$$
$$5x+3y+2=0$$

Hence, the equation of the line is $5x+3y+2=0$

Question 8:

Find the equation of the line which is at a perpendicular distance of 5 units from the origin and the angle made by the perpendicular with the positive x-axis is 30°.

Solution:

If p is the length of the normal from the origin to a line and ω is the angle made by the normal with the positive direction of the x-axis, then the equation of the line given by

$$x\cos\omega+y\sin\omega=p$$

Here, $p=5$ units and $\omega=30^\circ$

Thus, the required equation of the given line is

$$x\cos 30° + y\sin 30° = 5$$
$$x.\frac{\sqrt{3}}{2} + y.\frac{1}{2} = 5$$
$$\sqrt{3}x + y - 10 = 0$$

Hence, the equation of the line is $\sqrt{3}x + y - 10 = 0$

Question 9:

The vertices of ΔPQR are $P(2,1), Q(-2,3)$ and $R(4,5)$. Find equation of the median through the vertex R.

Solution:

It is given that the vertices of ΔPQR are $P(2,1), Q(-2,3)$ and $R(4,5)$. Let RL be the median through vertex R.

Accordingly, L be the mid-point of PQ.

By mid-point formula, the coordinates of point L are given by $\left(\frac{2-2}{2}, \frac{1+3}{2}\right) = (0,2)$

Therefore, the equation of the line passing through points $(4,5)$ and $(0,2)$ is

$$y - 5 = \frac{2-5}{0-4}(x-4)$$
$$y - 5 = \frac{-3}{-4}(x-4)$$
$$4(y-5) = 3(x-4)$$
$$4y - 20 = 3x - 12$$
$$3x - 4y + 8 = 0$$

Thus, the equation of the median through vertex R is $3x - 4y + 8 = 0$.

Question 10:

Find the equation of the line passing through $(-3,5)$ and perpendicular to the line through the points $(2,5)$ and $(-3,6)$.

Solution:

The slope of the line joining the points $(2,5)$ and $(-3,6)$ is $m = \frac{6-5}{-3-2} = -\frac{1}{5}$

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

Therefore, slope of the line perpendicular to the line through the points $(2,5)$ and $(-3,6)$ is $-\frac{1}{m}=-\frac{1}{\left(-\frac{1}{5}\right)}=5$

Now, the equation of the line passing through point $(-3,5)$, whose slope is 5, is

$$\begin{aligned}(y-5)&=5(x+3)\\ y-5&=5x+15\\ 5x-y+20&=0\end{aligned}$$

Hence, the equation of the line is $5x-y+20=0$

Question 11:

A line perpendicular to the line segment joining the points $(1,0)$ and $(2,3)$ divides it in the ratio $1:n$. Find the equation of the line.

Solution:

According to the section formula, the coordinates of the point that divides the line segment joining the points $(1,0)$ and $(2,3)$ in the ratio $1:n$ is given by

$$\left(\frac{n(1)+1(2)}{1+n},\frac{n(0)+1(3)}{1+n}\right)=\left(\frac{n+2}{n+1},\frac{3}{n+1}\right)$$

The slope of the line joining the points $(1,0)$ and $(2,3)$ is $m=\frac{3-0}{2-1}=3$

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

Therefore, slope of the line that is perpendicular to the line joining the points $(1,0)$ and $(2,3)$ is $-\frac{1}{m}=-\frac{1}{3}$

Now the equation of the line passing through $\left(\frac{n+2}{n+1},\frac{3}{n+1}\right)$ and whose slope is $-\frac{1}{3}$, given by

$$\left(y-\frac{3}{n+1}\right)=-\frac{1}{3}\left(x-\frac{(n+2)}{(n+1)}\right)$$
$$3\left[(n+1)y-3\right]=-\left[x(n+1)-(n-2)\right]$$
$$3(n+1)y-9=-(n+1)x+n+2$$
$$(1+n)x+3(1+n)y=n+11$$

Hence, the equation of the line is $(1+n)x+3(1+n)y=n+11$

Question 12:
Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the points $(2,3)$.

Solution:
The equation of the line in the intercept form is

$$\frac{x}{a}+\frac{y}{b}=1 \qquad \ldots(1)$$

Here, a and b are the intercepts on x and y axes respectively.

It is given that the line cuts off equal intercepts on both the axes. This means that $a=b$.
Accordingly, equation (1) reduces to

$$x+y=a \qquad \ldots(2)$$

Since the given line passes through point $(2,3)$, equation (2) reduces to

$$2+3=a$$
$$a=5$$

On substituting the value of a in equation (2), we obtain

$x+y=5$, which is the required equation of the line.

Question 13:

Find the equation of the line passing through the points $(2,2)$ and cutting off intercepts on the axis whose sum is 9.

Solution:
The equation of a line in the intercept form is

$$\frac{x}{a}+\frac{y}{b}=1 \qquad \ldots(1)$$

Here, a and b are the intercepts on x and y axis respectively.

It is given that

$$a+b=9$$

$$b=9-a \qquad \ldots(2)$$

From equation (1) and (2), we obtain

$$\frac{x}{a}+\frac{y}{9-a}=1 \qquad \ldots(3)$$

It is given that the line passes through point $(2,2)$.
Therefore, equation (3) reduces to

$$\Rightarrow \frac{2}{a}+\frac{2}{9-a}=1$$

$$\Rightarrow \frac{2(9-a)+2a}{a(9-a)}=1$$

$$\Rightarrow \frac{18-2a+2a}{(9a-a^2)}=1$$

$$\Rightarrow \frac{18}{9a-a^2}=1$$

$$\Rightarrow 18=9a-a^2$$

$$\Rightarrow a^2-9a+18=0$$

$$\Rightarrow a^2-6a-3a+18=0$$

$$\Rightarrow a(a-6)-3(a-6)=0$$

$$\Rightarrow (a-6)(a-3)=0$$

$$\Rightarrow a=6 \text{ or } a=3$$

If $a=6$ then $b=9-6=3$,

Hence, the equation of the line is

$$\Rightarrow \frac{x}{6}+\frac{y}{3}=1$$

$$\Rightarrow x+2y-6=0$$

If $a=3$ then $b=9-3=6$

Hence, the equation of the line is

$$\Rightarrow \frac{x}{3}+\frac{y}{6}=1$$

$$\Rightarrow 2x+y-6=0$$

Thus, the equation of the line is $x+2y-6=0$ or $2x+y-6=0$.

Question 14:

Find equation of the line through the points $(0,2)$ making an angle $\frac{2\pi}{3}$ with the positive x-axis. Also, find the equation of the line parallel to it and crossing the y-axis at a distance of 2 units below the origin.

Solution:

The slope of the line making an angle $\frac{2\pi}{3}$ with the positive x-axis is $m=\tan\left(\frac{2\pi}{3}\right)=-\sqrt{3}$

Now, the equation of the line passing through points $(0,2)$ and having a slope $-\sqrt{3}$ is

$$(y-2)=-\sqrt{3}(x-0)$$
$$\sqrt{3}x+y-2=0$$

The slope of line parallel to line $\sqrt{3}x+y-2=0$ is $-\sqrt{3}$

It is given that the line parallel to line $\sqrt{3}x+y-2=0$ crosses the y-axis 2 units below the origin, i.e., it passes through point $(0,2)$.

Hence, the equation of the line passing through points $(0,2)$ and having a slope $-\sqrt{3}$ is

$$y-(-2)=-\sqrt{3}(x-0)$$
$$y+2=-\sqrt{3}x$$
$$\sqrt{3}x+y+2=0$$

Thus, the equation of the line is $\sqrt{3}x+y+2=0$

Question 15:

The perpendicular from the origin to a line meets it at the point $(-2,9)$, find the equation of the line.

Solution:

The slope of the line joining the origin $(0,0)$ and point $(-2,9)$, $m=\frac{9-0}{-2-0}=-\frac{9}{2}$

Accordingly, the slope of the line perpendicular to the line joining the origin and point $(-2,9)$ is

$$m_2 = \frac{1}{m_1} = -\frac{1}{\left(-\frac{9}{2}\right)} = \frac{2}{9}$$

Now, the equation of the line passing through point $(-2,9)$ and having a slope m_2 is

$$(y-9) = \frac{2}{9}(x+2)$$
$$9y-81 = 2x+4$$
$$2x-9y+85=0$$

Thus, the equation of the line is $2x-9y+85=0$

Question 16:

The length L (in centimeter) of a copper rod is a linear function of its Celsius temperature C. In an experiment, if $L=124.942$ when $C=20$ and $L=125.134$ when $C=110$, express L in terms of C.

Solution:

It is given that when $C=20$, the value of $L=124.942$, whereas when $C=110$, the value of $L=125.134$.

Accordingly, points $(20,124.942)$ and $(110,125.134)$ satisfy the linear relation between L and C.

Now, assuming C along the x-axis and L along the y-axis, we have two points i.e., $(20,124.942)$ and $(110,125.134)$ in the XY plane.

Therefore, the linear relation between L and C is the equation of the line passing through points $(20,124.942)$ and $(110,125.134)$.

$$(L-124.942) = \frac{125.134-124.942}{110-20}(C-20)$$
$$(L-124.942) = \frac{0.192}{90}(C-20)$$
$$L = \frac{0.192}{90}(C-20)+124.942$$

Thus, the required linear relation is $L = \frac{0.192}{90}(C-20)+124.942$

Question 17:

The owner of a milk store finds that, he can sell 980 liters of milk each week at ₹ 14/litre and 1220 liters of milk each week at ₹ 16/litre. Assuming a linear relationship between selling price and demand, how many liters could he sell weekly at ₹ 17/litre?

Solution:

The relationship between selling price and demand is linear.
Assuming selling price per liter along the *x*-axis and demand along the *y*-axis, we have two points i.e., $(14,980)$ and $(16,1220)$ in the XY plane that satisfy the linear relationship between selling price and demand.

Therefore, the line passing through points $(14,980)$ and $(16,1220)$.

$$y-980=\frac{1220-980}{16-14}(x-14)$$
$$y-980=\frac{240}{2}(x-14)$$
$$y-980=120(x-14)$$
$$y=120(x-14)+980$$

When $x=17$,

$$\begin{aligned} y &= 120(17-14)+980 \\ &= 120\times3+980 \\ &= 360+980 \\ &= 1340 \end{aligned}$$

Thus, the owner of the milk store could sell 1340 litres of milk weekly at ₹ 17/litre.

Question 18:

$P(a,b)$ is the mod-point of a line segment between axes. Show that the equation of the line is $\frac{x}{a}+\frac{y}{b}=2$.

Solution:

Let AB be the line segment between the axis and let $P(a,b)$ be its mid-point.

Let the coordinates of A and B be $(0,y)$ and $(x,0)$ respectively.
Since $P(a,b)$ is the mid-point of AB,

$$\left(\frac{0+x}{2},\frac{y+0}{2}\right)=(a,b)$$
$$\Rightarrow\left(\frac{x}{2},\frac{y}{2}\right)=(a,b)$$
$$\Rightarrow\frac{x}{2}=a \text{ and } \frac{y}{2}=b$$
$$\Rightarrow x=2a \text{ and } y=2b$$

Thus, the respective coordinates of A and B are $(0,2b)$ and $(2a,0)$. The equation of the line passing through points $(0,2b)$ and $(2a,0)$ is

$$(y-2b)=\frac{(0-2b)}{(2a-0)}(x-0)$$
$$(y-2b)=-\frac{2b}{2a}(x)$$
$$a(y-2b)=-bx$$
$$ay-2ab=-bx$$
$$bx+ay=2ab$$

On dividing both sides by ab, we obtain

$$\frac{bx}{ab}+\frac{ay}{ab}=\frac{2ab}{ab}$$
$$\Rightarrow\frac{x}{a}+\frac{y}{b}=2$$

Hence, the equation of the line is $\frac{x}{a}+\frac{y}{b}=2$ proved.

Question 19:

Point $R(h,k)$ divides a line segment between the axes in the ratio $1:2$. Find equation of the line.

Solution:

Let AB be the line segment between the axes such that point $R(h,k)$ divides AB in the ratio $1:2$.

Let the respective coordinates of A and B be $(x,0)$ and $(0,y)$.

Since point $R(h,k)$ divides AB in the ratio $1:2$, according to the section formula,

$$(h,k)=\left(\frac{1(0)+2(x)}{1+2},\frac{1(y)+2(0)}{1+2}\right)$$
$$\Rightarrow (h,k)=\left(\frac{2x}{3},\frac{y}{3}\right)$$
$$\Rightarrow h=\frac{2x}{3} \text{ and } k=\frac{y}{3}$$
$$\Rightarrow x=\frac{3h}{2} \text{ and } y=3k$$

Therefore, the respective coordinates of A and B are $\left(\frac{3h}{2},0\right)$ and $(0,3k)$

Now, the equation of the line AB passing through points $\left(\frac{3h}{2},0\right)$ and $(0,3k)$ is

$$(y-0)=\frac{3k-0}{0-\frac{3h}{2}}\left(x-\frac{3h}{2}\right)$$
$$y=-\frac{2k}{h}\left(x-\frac{3h}{2}\right)$$
$$hy=-2k\left(\frac{2x-3h}{2}\right)$$
$$hy=-k(2x-3h)$$
$$hy=-2kx+3kh$$
$$2kx+hy=3kh$$

Thus, the required equation of a line is $2kx+hy=3kh$

Question 20:

By using the concept of equation of a line, prove that the three points $(3,0),(-2,-2)$ and $(8,2)$ are collinear.

Solution:

In order to show that the points $(3,0),(-2,-2)$ and $(8,2)$ are collinear, it suffices to show that the line passing through points $(3,0)$ and $(-2,-2)$ also passes through point $(8,2)$.

The equation of the line passing through points $(3,0)$ and $(-2,-2)$ is

$$(y-0)=\frac{(-2-0)}{(-2-3)}(x-3)$$
$$y=\frac{-2}{-5}(x-3)$$
$$5y=2x-6$$
$$2x-5y-6=0$$

It is observed that at $x=8,\ y=2$

$$\begin{aligned}LHS &= 2\times 8-5\times 2-6\\ &=16-10-6\\ &=0\\ &=RHS\end{aligned}$$

Therefore, the line passing through points $(3,0)$ and $(-2,-2)$ also passes through point $(8,2)$.

Hence, points $(3,0),(-2,-2)$ and $(8,2)$ are collinear, proved.

EXERCISE 10.3

Question 1:

Reduce the following equation into slope-intercept form and find their slopes and the y-intercepts.

(i) $x+7y=0$ (ii) $6x+3y-5=0$ (iii) $y=0$

Solution:

(i) The given equation is $x+7y=0$

It can be written as

$$y=-\frac{1}{7}x+0$$

This equation is of the form $y=mx+c$, where $m=-\frac{1}{7}$ and $c=0$

Therefore, equation $x+7y=0$ is the slope-intercept form, where the slope and the y-intercept are $-\frac{1}{7}$ and 0 respectively.

(ii) The given equation is $6x+3y-5=0$

It can be written as

$$y=\frac{1}{3}(-6x+5)$$

$$y=-2x+\frac{5}{3}$$

This equation is of the form $y=mx+c$, where $m=-2$ and $c=\frac{5}{3}$

Therefore, equation $6x+3y-5=0$ is the slope-intercept form, where the slope and the y-intercept are -2 and $\frac{5}{3}$ respectively.

(iii) The given equation is $y=0$.

It can be written as $y=0.x+0$

This equation is of the form $y=mx+c$, where $m=0$ and $c=0$.

Therefore, equation $y=0$ is in the slope-intercept form, where the slope and the y-intercept are 0 and 0 respectively.

Question 2:

Reduce the following equations into intercept form and find their intercepts on the axis.

(i) $3x+2y-12=0$ (ii) $4x-3y=6$ (iii) $3y+2=0$

Solution:

(i) The given equation is $3x+2y-12=0$

It can be written as

$$3x+2y=12$$

$$\frac{3x}{12}+\frac{2y}{12}=1$$

$$\frac{x}{4}+\frac{y}{6}=1 \qquad \ldots(1)$$

This equation is of the form $\frac{x}{a}+\frac{y}{b}=1$, where $a=4$ and $b=6$.

Therefore, equation (1) is in the intercept form, where the intercepts on the x and y axes are 4 and 6 respectively.

(ii) The given equation is $4x-3y=6$.

It can be written as

$$\frac{4x}{6}-\frac{3y}{6}=1$$

$$\frac{2x}{3}-\frac{y}{2}=1$$

$$\frac{x}{\frac{3}{2}}+\frac{y}{(-2)}=1 \qquad \ldots(2)$$

This equation is of the form $\frac{x}{a}+\frac{y}{b}=1$, where $a=\frac{3}{2}$ and $b=-2$.

Therefore, equation (2) is in the intercept form, where the intercepts on x and y-axes are $\frac{3}{2}$ and -2 respectively.

(iii) The given equation is $3y+2=0$

It can be written as

$$3y=-2$$

$$\frac{y}{\left(-\frac{2}{3}\right)}=1 \qquad \ldots(3)$$

This equation is of the form $\frac{x}{a}+\frac{y}{b}=1$, where $a=0$ and $b=-\frac{2}{3}$.

Therefore, equation (3) is in the intercept form, where the intercepts on the y-axis is $-\frac{2}{3}$ and it has no intercept on the x-axis.

Question 3:

Reduce the following equations into normal form. Find their perpendicular distance from the origin and angle between perpendicular and the positive x-axis.

(i) $x-3\sqrt{y}+8=0$ (ii) $y-2=0$ (iii) $x-y=4$

Solution:

(i) The given equation is $x-3\sqrt{y}+8=0$
It can be written as

$$x-3\sqrt{y}=-8$$
$$-x+3\sqrt{y}=8$$

On dividing both sides by $\sqrt{(-1)^2+\left(\sqrt{3}\right)^2}=\sqrt{4}=2$, we obtain

$$-\frac{x}{2}+\frac{\sqrt{3}}{2}y=\frac{8}{2}$$
$$\left(-\frac{1}{2}\right)x+\left(\frac{\sqrt{3}}{2}\right)y=4$$
$$x\cos 120°+y\sin 120°=4 \qquad \ldots(1)$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of the line

$x\cos\omega+y\sin\omega=p$, we obtain $\omega=120°$ and $p=4$.

Thus, the perpendicular distance of the line from the origin is 4, while the angle between the perpendicular and the positive *x*-axis is 120°.

(ii) The given equation is $y-2=0$
It can be represented as $0.x+1.y=2$

On dividing both sides by $\sqrt{0^2+1^2}=1$, we obtain

$$0.x+1.y=2$$
$$\Rightarrow x\cos 90°+y\sin 90°=2 \qquad \ldots(2)$$

Equation (2) is in the normal form.

On comparing equation (2) with the normal form of equation of line

$x\cos\omega + y\sin\omega = p$, we obtain $\omega = 90°$ and $p = 2$.

Thus, the perpendicular distance of the line from the origin is 2, while the angle between the perpendicular and the positive *x*-axis is $90°$.

(iii) The given equation is $x - y = 4$.

It can be reduced as $1.x + (-1)y = 4$

On dividing both sides by $\sqrt{1^2 + (-1)^2} = \sqrt{2}$, we obtain

$$\frac{1}{\sqrt{2}}x + \left(-\frac{1}{\sqrt{2}}\right)y = \frac{4}{\sqrt{2}}$$

$$\Rightarrow x\cos\left(2\pi - \frac{\pi}{4}\right) + y\sin\left(2\pi - \frac{\pi}{4}\right) = 2\sqrt{2}$$

$$\Rightarrow x\cos 315° + y\sin 315° = 2\sqrt{2} \qquad \ldots(3)$$

Equation (3) is in the normal form.

On comparing equation (3) with the normal form of the equation of the line $x\cos\omega + y\sin\omega = p$, we obtain $\omega = 315°$ and $p = 2\sqrt{2}$.

Thus, the perpendicular distance of the line from the origin is $2\sqrt{2}$, while the angle between the perpendicular and the positive *x*-axis is $315°$.

Question 4:

Find the distance of the points $(-1,1)$ from the line $12(x+6) = 5(y-2)$.

Solution:

The given equation of the line is $12(x+6) = 5(y-2)$

$$\Rightarrow 12x + 72 = 5y - 10$$

$$\Rightarrow 12x - 5y + 82 = 0 \qquad \ldots(1)$$

On comparing equation (1) with general equation of line $Ax + By + C = 0$ we obtain $A = 12, B = -5$ and $C = 82$.

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point

(x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

The given point is $(x_1, y_1) = (-1, 1)$.

Therefore, the distance of point $(-1,1)$ from the given line is

$$\frac{|12(-1)+(-5)(1)+82|}{\sqrt{12^2+(-5)^2}} = \frac{|-12-5+82|}{\sqrt{169}}$$
$$= \frac{|65|}{13}$$
$$= 5$$

Hence, the distance of point $(-1,1)$ from the given line is 5 units.

Question 5:

Find the points on the x-axis whose distance from the line $\frac{x}{3} + \frac{y}{4} = 1$ are 4 units.

Solution:

The given equation of line is

$$\frac{x}{3} + \frac{y}{4} = 1$$
$$4x + 3y - 12 = 0 \qquad \ldots(1)$$

On comparing equation (1) with general equation of line $Ax + By + C = 0$ we obtain $A = 4$, $B = 3$, and $C = -12$.

Let $(a, 0)$ be the point on the *x*-axis whose distance from the given line is 4 units.

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$
Therefore,

$$\Rightarrow (4a - 12) = 20 \text{ or } -(4a - 12) = 20$$
$$\Rightarrow 4a = 20 + 12 \text{ or } 4a = -20 + 12$$
$$\Rightarrow a = 8 \text{ or } a = -2$$

Thus, the required points on x-axis are $(-2,0)$ and $(8,0)$.

Question 6:

Find the distance between parallel lines

(i) $15x+8y-34=0$ and $15x+8y+31=0$

(ii) $l(x+y)+p=0$ and $l(x+y)-r=0$

Solution:

It is known that the distance (d) between parallel lines $Ax+By+C_1=0$ and $Ax+By+C_2=0$ is given by $d=\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$

(i) The given parallel lines are $15x+8y-34=0$ and $15x+8y+31=0$

Here, $A=15, B=8, C_1=-34$ and $C_2=-31$

Therefore, the distance between the parallel lines is

$$\begin{aligned} d &= \frac{|C_1-C_2|}{\sqrt{A^2+B^2}} \\ &= \frac{|34-31|}{\sqrt{(15)^2+(8)^2}} \textit{units} \\ &= \frac{|-65|}{\sqrt{289}} \textit{units} \\ &= \frac{65}{17} \textit{units} \end{aligned}$$

(ii) The given parallel lines are $l(x+y)+p=0$ and $l(x+y)-r=0$

Here, $A=B=l, C_1=p$ and $C_2=-r$

Therefore, the distance between the parallel lines is

$$
\begin{aligned}
d &= \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} \\
&= \frac{|p+r|}{\sqrt{l^2 + l^2}} units \\
&= \frac{|p+r|}{\sqrt{2l^2}} units \\
&= \frac{|p+r|}{l\sqrt{2}} units \\
&= \frac{1}{\sqrt{2}} \frac{|p+r|}{l} units
\end{aligned}
$$

Question7:

Find equation of the line parallel to the line $3x - 4y + 2 = 0$ and passing through the point $(-2, 3)$.

Solution:

The equation of the given line is

$$3x - 4y + 2 = 0$$

$$y = \frac{3x}{4} + \frac{2}{4}$$

$y = \frac{3}{4}x + \frac{2}{4}$, which is of the form $y = mx + c$

Therefore, slope of the given line is $\frac{3}{4}$

It is known that parallel lines have the same slope.

Slope of the other line is $m = \frac{3}{4}$

Now, the equation of the line that has a slope of $\frac{3}{4}$ and passes through the points $(-2, 3)$ is

$$
\begin{aligned}
(y-3) &= \frac{3}{4}\{x - (-2)\} \\
4y - 12 &= 3x + 6 \\
3x - 4y + 18 &= 0
\end{aligned}
$$

Question 8:

Find the equation of the line perpendicular to the line $x - 7y + 5 = 0$ and having x intercept 3.

Solution:

The given equation of the line is $x-7y+5=0$

Or $y=\frac{1}{7}x+\frac{5}{7}$, which is of the form $y=mx+c$

Therefore, slope of the given line is $\frac{1}{7}$

The slope of the line perpendicular to the line having a slope is $m=-\frac{1}{\left(\frac{1}{7}\right)}=-7$

The equation of the line with slope -7 and x-intercept 3 is given by

$$y=m(x-d)$$
$$y=-7(x-3)$$
$$y=-7x+21$$
$$7x+y-21=0$$

Hence, the required equation of the line is $7x+y-21=0$

Question 9:

Find the angles between the lines $\sqrt{3}x+y=1$ and $x+\sqrt{3}y=1$

Solution:

The given lines are $\sqrt{3}x+y=1$ and $x+\sqrt{3}y=1$

$y=-\sqrt{3}x+1 \quad \ldots(1)$ and $y=-\frac{1}{\sqrt{3}}x+\frac{1}{\sqrt{3}} \quad \ldots(2)$

The slope of line (1) is $m_1=-3$, while the slope of the line (2) is $m_2=-\frac{1}{\sqrt{3}}$

The actual angle i.e., θ between the two lines is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-\sqrt{3} + \frac{1}{\sqrt{3}}}{1 + \left(-\sqrt{3}\left(-\frac{1}{\sqrt{3}}\right)\right)}\right|$$

$$= \left|\frac{\frac{-3+1}{\sqrt{3}}}{1+1}\right| = \left|\frac{-2}{2 \times \sqrt{3}}\right|$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

Thus, the angle between the given lines is either 30° or $180^\circ - 30^\circ = 150^\circ$.

Question 10:

The line through the points $(h, 3)$ and $(4, 1)$ intersects the line $7x - 9y - 19 = 0$. At right angle. Find the value of h.

Solution:

The slope of the line passing through points $(h, 3)$ and $(4, 1)$ is

$$m_1 = \frac{1-3}{4-h} = \frac{-2}{4-h}$$

The slope of the line $7x - 9y - 19 = 0$ or $y = \frac{7}{9}x - \frac{19}{9}$ is $m_2 = \frac{7}{9}$
It is given that the two lines are perpendicular
Therefore,

$$\Rightarrow m_1 \times m_2 = -1$$

$$\Rightarrow \frac{-14}{36 - 9h} = -1$$

$$\Rightarrow 14 = 36 - 9h$$

$$\Rightarrow 9h = 36 - 14$$

$$\Rightarrow h = \frac{22}{9}$$

Thus, the value of $h = \frac{22}{9}$.

Question 11:

Prove that the line through the point (x_1, y_1) and parallel to the line $Ax + By + C = 0$ is $A(x - x_1) + B(y - y_1) = 0$

Solution:

The slope of line $Ax + By + C = 0$ or $y = \left(-\frac{A}{B}\right)x + \left(-\frac{C}{B}\right)$ is $m = -\frac{A}{B}$
It is known that parallel lines have the same slope.

Therefore, slope of the other line $= m = -\frac{A}{B}$

The equation of the line passing through point (x_1, y_1) and having slope $m = -\frac{A}{B}$ is

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = -\frac{A}{B}(x - x_1)$$

$$B(y - y_1) = -A(x - x_1)$$

$$A(x - x_1) + B(y - y_1) = 0$$

Hence, the line through point (x_1, y_1) and parallel to line $Ax + By + C = 0$ is $A(x - x_1) + B(y - y_1) = 0$

Question 12:

Two lines passing through the points $(2,3)$ intersects each other at an angle of $60°$. If slope of one line is 2, find equation of the other line.

Solution:

It is given that the slope of the first line, $m_1 = 2$.

Let the slope of the other line be m_2.
The angle between the two lines is $60°$.

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan 60° = \left|\frac{2 - m_2}{1 + 2m_2}\right|$$

$$\sqrt{3} = \pm\left(\frac{2 - m_2}{1 + 2m_2}\right)$$

$$\sqrt{3}=\left(\frac{2-m_2}{1+2m_2}\right) \qquad \sqrt{3}=-\left(\frac{2-m_2}{1+2m_2}\right)$$

$$\sqrt{3}\left(1+2m_2\right)=2-m_2 \qquad \sqrt{3}\left(1+2m_2\right)=-\left(2-m_2\right)$$

$$\sqrt{3}+2\sqrt{3}m_2+m_2=2 \qquad \sqrt{3}+2\sqrt{3}m_2-m_2=-2$$

$$\sqrt{3}+\left(2\sqrt{3}+1\right)m_2=2 \qquad \sqrt{3}+\left(2\sqrt{3}-1\right)m_2=-2$$

$$m_2=\frac{2-\sqrt{3}}{\left(2\sqrt{3}+1\right)} \quad \text{or} \quad m_2=\frac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}-1\right)}$$

Case 1:

$$m_2=\frac{2-\sqrt{3}}{\left(2\sqrt{3}+1\right)}$$

The equation of the line passing through the point $(2,3)$ and having a slope of $\frac{2-\sqrt{3}}{\left(2\sqrt{3}+1\right)}$ is

$$\left(y-3\right)=\frac{2-\sqrt{3}}{(2\sqrt{3}+1)}\left(x-2\right)$$

$$\left(2\sqrt{3}+1\right)y-3\left(2\sqrt{3}+1\right)=\left(2-\sqrt{3}\right)x-2\left(2-\sqrt{3}\right)$$

$$\left(\sqrt{3}-2\right)x+\left(2\sqrt{3}+1\right)y=-1+8\sqrt{3}$$

In this case, the equation of the other line is $\left(\sqrt{3}-2\right)x+\left(2\sqrt{3}+1\right)y=-1+8\sqrt{3}$

Case 2:

$$m_2=\frac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}-1\right)}$$

The equation of the line passing through the point $(2,3)$ and having a slope of $\frac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}-1\right)}$ is

$$\left(y-3\right)=\frac{-\left(2+\sqrt{3}\right)}{\left(2\sqrt{3}-1\right)}\left(x-2\right)$$

$$\left(2\sqrt{3}-1\right)y-3\left(2\sqrt{3}-1\right)=-\left(2-\sqrt{3}\right)x+2\left(2-\sqrt{3}\right)$$

$$\left(2-\sqrt{3}\right)x+\left(2\sqrt{3}-1\right)y=4-2\sqrt{3}+6\sqrt{3}-3$$

$$\left(2-\sqrt{3}\right)x+\left(2\sqrt{3}-1\right)y=1+8\sqrt{3}$$

If the case of the equation of the other line is $\left(2-\sqrt{3}\right)x+\left(2\sqrt{3}-1\right)y=1+8\sqrt{3}$

Thus, the required equation of the other line is $\left(\sqrt{3}-2\right)x+\left(2\sqrt{3}+1\right)y=-1+8\sqrt{3}$ or $\left(2-\sqrt{3}\right)x+\left(2\sqrt{3}-1\right)y=1+8\sqrt{3}$.

Question 13:

Find the equation of the right bisector of the line segment joining the points $(3,4)$ and $(-1,2)$.

Solution:

The right bisector of a line segment bisects the line segment at $90°$.

The end points of the line segment are given as $A(3,4)$ and $B(-1,2)$.

Accordingly, mid-point of $AB=\left(\frac{3-1}{2},\frac{4+2}{0}\right)=(1,3)$

Slope of $AB=\frac{2-4}{-1-3}=\frac{-2}{-4}=\frac{1}{2}$

Slope of the line perpendicular to $AB=-\frac{1}{\left(\frac{1}{2}\right)}=-2$

The equation of the line passing through $(1,3)$ and having a slope of -2 is

$$\begin{aligned}&(y-3)=-2(x-1)\\&y-3=-2x+2\\&2x+y=5\end{aligned}$$

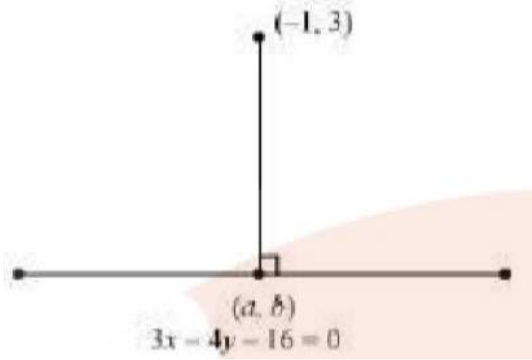
Thus, the required equation of the line is $2x+y=5$.

Question 14:

Find the coordinates of the foot of perpendicular from the points $(-1,3)$ to the line $3x-4y-16=0$.

Solution:

Let (a,b) be the coordinates of the foot of the perpendicular from the points $(-1,3)$ to the line $3x-4y-16=0$.



Slope of the line joining $(-1,3)$ and (a,b), $m_1 = \dfrac{b-3}{a+1}$

Slope of the line $3x-4y-16=0$ or $y=\dfrac{3}{4}x-4,\ m_2=\dfrac{3}{4}$

Since these two lines are perpendicular, $m_1 \times m_2 = -1$
Therefore,

$$\Rightarrow \left(\frac{b-3}{a+1}\right)\times\left(\frac{3}{4}\right)=-1$$
$$\Rightarrow \frac{3b-9}{4a+4}=-1$$
$$\Rightarrow 3b-9=-4a-4$$
$$\Rightarrow 4a+3b=5 \qquad \ldots(1)$$

Point (a,b) lies on the line $3x-4y-16=0$
Therefore,

$$\Rightarrow 3a-4b=16 \qquad \ldots(2)$$

On solving equations (1) and (2), we obtain

$$a=\frac{68}{25} \text{ and } b=-\frac{49}{25}$$

Thus, the required coordinates of the foot of the perpendicular are $\left(\dfrac{68}{25},\dfrac{49}{25}\right)$

Question 15:

The perpendicular from the origin to the line $y=mx+c$ meets it at the a point $(-1,2)$. Find the values of *m* and *c*.

Solution:

The given equation of line is $y=mx+c$

It is given that the perpendicular from the origin meets the given line at $(-1,2)$.

Therefore, the line joining the points $(0,0)$ and $(-1,2)$ is perpendicular to the given line

Slope of the line joining $(0,0)$ and $(-1,2)$ is $\frac{2}{-1}=-2$

The slope of the given line is m
Therefore,

$$m\times(-2)=-1 \quad \text{[The two lines are perpendicular]}$$
$$\Rightarrow m=\frac{1}{2}$$

Since points $(-1,2)$ lies on the given line, it satisfies the equation $y=mx+c$
Therefore,

$$\Rightarrow 2=m(-1)+c$$
$$\Rightarrow 2=2+\frac{1}{2}(-1)+c$$
$$\Rightarrow c=2+\frac{1}{2}=\frac{5}{2}$$

Thus, the respective values of m and c are $\frac{1}{2}$ and $\frac{5}{2}$

Question 16:

If p and q are the lengths of perpendicular from the origin to the lines $x\cos\theta-y\sin\theta=k\cos 2\theta$ and $x\sec\theta+y\,\mathrm{cosec}\theta=k$, respectively, prove that $p^2+4q^2=k^2$

Solution:

The equations of given lines are

$$x\cos\theta-y\sin\theta=k\cos 2\theta \qquad \ldots(1)$$
$$x\sec\theta+y\cos ec\theta=k \qquad \ldots(2)$$

The perpendicular distance(d) of a line $Ax+By+C=0$ from a point (x_1,x_2) is given by

$$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$$

On comparing equation(1) to the general equation of a line i.e., $Ax+By+C=0$, we obtain $A=\cos\theta, B=-\sin\theta$ and $C=-k\cos 2\theta$

It is given that p is the length of the perpendicular from $(0,0)$ to line (1).

Therefore, $$p=\frac{|A(0)+B(0)+C|}{\sqrt{A^2+B^2}}=\frac{|C|}{\sqrt{A^2+B^2}}=\frac{|-k\cos 2\theta|}{\sqrt{\cos^2\theta+\sin^2\theta}}=|-k\cos 2\theta| \qquad \ldots(3)$$

On comparing equation (2) to the general equation of line i.e., $Ax+By+C=0$, we obtain $A=\sec\theta, B=\text{cosec}\,\theta$ and $C=-k$

It is given that q is the length of the perpendicular from $(0,0)$ to line (2)

Therefore, $p=\dfrac{|A(0)+B(0)+C|}{\sqrt{A^2+B^2}}=\dfrac{|C|}{\sqrt{A^2+B^2}}=\dfrac{|-k|}{\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}}$ ...(4)

From (3) and (4), we have

$$
\begin{aligned}
p^2+4q^2 &= (|-k\cos 2\theta|)^2+4\left(\frac{|-k|}{\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}}\right)^2 \\
&= k^2\cos^2 2\theta+\frac{4k^2}{(\sec^2\theta+\operatorname{cosec}^2\theta)} \\
&= k^2\cos^2 2\theta+\frac{4k^2}{\left(\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}\right)} \\
&= k^2\cos^2 2\theta+\left(\frac{4k^2}{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta\cos^2\theta}}\right) \\
&= k^2\cos^2 2\theta+\left(\frac{4k^2}{\frac{1}{\sin^2\theta\cos^2\theta}}\right) \\
&= k^2\cos^2 2\theta+4k^2\sin^2\theta\cos^2\theta \\
&= k^2\cos^2 2\theta+k^2(2\sin\theta\cos\theta)^2 \\
&= k^2\cos^2 2\theta+k^2\sin^2 2\theta \\
&= k^2\left(\cos^2 2\theta+\sin^2 2\theta\right) \\
&= k^2
\end{aligned}
$$

Hence, we proved that $p^2+4q^2=k^2$

Question 17:

In the triangle ABC with vertices $A(2,3)$, $B(4,-1)$ and $C(1,2)$, find the equation and length of altitude from the vertex A.

Solution:

Let AD be the altitude of triangle ABC from vertex A.

Accordingly, $AD\perp BC$

The equation of the line passing through point $(2,3)$ and having a slope of 1 is

$$\Rightarrow (y-3) = 1(x-2)$$
$$\Rightarrow x - y + 1 = 0$$
$$\Rightarrow y - x = 1$$

Therefore, equation of the altitude from vertex $A = y - x = 1$

Length of AD = Length of the perpendicular from $A(2,3)$ to BC
The equation of BC is

$$\Rightarrow (y+1) = \frac{2+1}{1-4}(x-4)$$
$$\Rightarrow (y+1) = -1(x-4)$$
$$\Rightarrow y + 1 = -x + 4$$
$$\Rightarrow x + y - 3 = 0 \qquad \ldots(1)$$

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by
$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

On comparing equation (1) to the general equation of line $Ax + By + C = 0$, we obtain $A = 1, B = 1$ and $C = 3$.

Length of $AD = \frac{|1\times 2 + 1\times 3 - 3|}{\sqrt{1^2 + 1^2}} = \frac{2}{\sqrt{2}} = \sqrt{2} units$

Thus, the equation and length of the altitude from vertex A are $y - x = 1$ and $\sqrt{2}$ units.

Question 18:

If p is the length of perpendicular from the origin to the line whose intercepts on the *x*-axis are a and b, then show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Solution:

It is known that the equation of a line whose intercepts on the axis *a* and *b* is

$$\frac{x}{a} + \frac{y}{b} = 1$$
$$bx + ay = ab$$
$$bx + ay - ab = 0 \qquad \ldots(1)$$

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

On comparing equation (1) to the general equation of line $Ax + By + C = 0$, we obtain $A = b,\ B = a$ and $C = -ab$.

Therefore, if p is the length of the perpendicular from point $(x_1, y_1) = (0,0)$ to line (1),
We obtain

$$p = \frac{|A(0) + B(0) - ab|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|-ab|}{\sqrt{a^2 + b^2}}$$

On squaring both sides, we obtain

$$\Rightarrow p^2 = \frac{(-ab)^2}{a^2 + b^2}$$

$$\Rightarrow p^2\left(a^2 + b^2\right) = a^2b^2$$

$$\Rightarrow \frac{a^2 + b^2}{a^2b^2} = \frac{1}{p^2}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Hence, we showed $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

MISCELLANEOUS EXERCISE

Question 1:

Find the value of k for which the line $(k-3)x-(4-k^2)y+k^2-7k+6=0$ is
(a) Parallel to x-axis.
(b) Parallel to y-axis.
(c) Passing through the origin.

Solution:
The given equation of the line is
$(k-3)x-(4-k^2)y+k^2-7k+6=0$ ...(1)

(a) If the given line is parallel to the x-axis then,
Slope of the given line = Slope of the x-axis
Then given line can be written as

$$(k-3)x+k^2-7k+6=(4-k^2)y$$

$$y=\frac{(k-3)}{(4-k^2)}x+\frac{k^2-7k+6}{(4-k^2)}$$

Which is of the form $y=mx+c$

Slope of the given line $=\frac{(k-3)}{(4-k^2)}$
Slope of the x-axis = 0

$$\Rightarrow \frac{k-4}{(4-k^2)}=0$$

$$\Rightarrow k-3=0$$

$$\Rightarrow k=3$$

Thus, the given line is parallel to x-axis, then the value of $k=3$.

(b) If the given line is parallel to the y-axis, it is vertical.
Hence, its slope will be undefined.

The slope of the given line is $=\frac{(k-3)}{(4-k^2)}$

Now, $\frac{(k-3)}{(4-k^2)}$ is defined at $k^2=4$

$$\Rightarrow k^2=4$$

$$\Rightarrow k=\pm 2$$

Thus, if the given line is parallel to the y-axis, then the value of $k=\pm 2$.

(c) if the given line is passing through the origin, then point $(0,0)$ satisfies the given equation of the line.

$$(k-3)(0)-(4-k^2)(0)+k^2-7k+6=0$$
$$k^2-7k+6=0$$
$$k^2-6k-k+6=0$$
$$(k-6)(k-1)=0$$
$$\Rightarrow k=6 \text{ or } k=1$$

Thus, if the given line is passing through the origin, then the values of k is either 1 or 6.

Question 2:

Find the values of θ and p, if the equation $x\cos\theta+y\sin\theta=p$ is the normal form of the line $\sqrt{3}x+y+2=0$

Solution:

The equation of the given line is $\sqrt{3}x+y+2=0$
This equation can be reduced as

$$\Rightarrow \sqrt{3}x+y+2=0$$
$$\Rightarrow -\sqrt{3}x-y=2$$

On dividing both sides by $\sqrt{(-\sqrt{3})^2+(-1)^2}=2$, we obtain

$$\Rightarrow -\frac{\sqrt{3}}{2}x-\frac{1}{2}y=\frac{2}{2}$$
$$\Rightarrow \left(-\frac{\sqrt{3}}{2}\right)x-\left(\frac{1}{2}\right)y=1 \qquad \ldots(1)$$

On comparing equation (1) to $x\cos\theta+y\sin\theta=p$, we obtain

$\cos\theta=-\frac{\sqrt{3}}{2}, \sin\theta=-\frac{1}{2}$ and $p=1$

Since the value of $\sin\theta$ and $\cos\theta$ are negative $\theta=\pi+\frac{\pi}{6}=\frac{7\pi}{6}$

Thus, the respective values of θ and p are $\frac{7\pi}{6}$ and 1.

Question 3:

Find the equation of the line, which cut-off intercepts on the axis whose sum and product are 1 and −6 respectively.

Solution:
Let the intercepts cut by the given lines on the axis be a and b.
It is given that

$$a+b=1 \qquad \ldots(1)$$
$$ab=-6 \qquad \ldots(2)$$

On solving equation (1) and (2), we obtain
$a=3$ and $b=-2$ or $a=-3$ and $b=3$

It is known that the equation of the line whose intercepts on the axis are a and b is
$\frac{x}{a}+\frac{y}{b}=1$ or $bx+ay-ab=0$

Case I: $a=3$ and $b=-2$
In this case, the equation of the line is $-2x+3y+6=0 \Rightarrow 2x-3y=6$

Case II: $a=-3$ and $b=3$
In this case, the equation of the line is $3x-2y+6=0 \Rightarrow -3x+2y=6$

Thus, the required equations of the lines are $2x-3y=6$ and $-3x+2y=6$

Question 4:

What are the points on the y-axis whose distance from the line $\frac{x}{3}+\frac{y}{4}=1$ is 4 units.
Solution:

Let $(0,b)$ be the point on y-axis whose distance from line $\frac{x}{3}+\frac{y}{4}=1$ is 4 units.
The given line can be written as

$$4x+3y-12=0 \qquad \ldots(1)$$

On comparing equation (1) to the general equation of line $Ax+By+C=0$, we obtain
$A=4, B=-3$ and $C=-12$.

It is known that the perpendicular distance (d) of a line $Ax+By+C=0$ from a point (x_1, y_1) is
given by $d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$

Therefore, if $(0,b)$ is the point on the y-axis whose distance from line $\frac{x}{3}+\frac{y}{4}=1$ is 4 units,
Then,

$$\Rightarrow 4 = \frac{|4(0)+3(b)-12|}{\sqrt{4^2+3^2}}$$

$$\Rightarrow 4 = \frac{|3b-12|}{5}$$

$$\Rightarrow 20 = |3b-12|$$

$$\Rightarrow 20 = \pm(3b-12)$$

$$\Rightarrow (3b-12) = 20 \text{ or } (3b-12) = -20$$

$$\Rightarrow 3b = 20+12 \text{ or } 3b = -20+12$$

$$\Rightarrow b = \frac{32}{3} \text{ or } b = \frac{-8}{3}$$

Thus, the required points are $\left(0, \frac{32}{3}\right)$ and $\left(0, -\frac{8}{3}\right)$

Question 5:

Find the perpendicular distance from the origin to the line joining the points $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$.

Solution:

The equation of the line joining the points $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$ is given by

$$\frac{y-\sin\theta}{x-\cos\theta} = \frac{\sin\phi-\sin\theta}{\cos\phi-\cos\theta}$$

$$y-\sin\theta = \frac{\sin\phi-\sin\theta}{\cos\phi-\cos\theta}(x-\cos\theta)$$

$$y(\cos\phi-\cos\theta)-\sin\theta(\cos\phi-\cos\theta) = x(\sin\phi-\sin\theta)-\cos\theta(\sin\phi-\sin\theta)$$

$$x(\sin\phi-\sin\theta)+y(\cos\phi-\cos\theta)+\cos\theta\sin\phi-\cos\theta\sin\theta-\sin\theta\cos\phi+\sin\theta\cos\theta = 0$$

$$x(\sin\phi-\sin\theta)+y(\cos\phi-\cos\theta)+\sin(\phi-\theta) = 0$$

$Ax+By+C=0$, where $A=\sin\theta-\sin\phi, B=\cos\phi-\cos\theta$ and $C=\sin(\phi-\theta)$

It is known that the perpendicular distance (d) of a line $Ax+By+C=0$ from a point (x_1, y_1) is given by $d = \frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$

Therefore, the perpendicular distance (d) of the given line from point $(x_1, y_1) = (0,0)$ is

$$d=\frac{\left|(\sin\phi-\sin\theta)(0)+(\cos\phi-\cos\theta)(0)+\sin(\phi-\theta)\right|}{\sqrt{(\sin\phi-\sin\theta)^2+(\cos\phi-\cos\theta)^2}}$$

$$=\frac{|\sin(\phi-\theta)|}{\sqrt{\sin^2\phi+\sin^2\theta-2\sin\theta\sin\phi+\cos^2\phi+\cos^2\theta-2\cos\phi\cos\theta}}$$

$$=\frac{|\sin(\phi-\theta)|}{\sqrt{(\sin^2\phi+\cos^2\phi)+(\sin^2\theta+\cos^2\theta)-2(\sin\theta\sin\phi+\cos\phi\cos\theta)}}$$

$$=\frac{|\sin(\phi-\theta)|}{\sqrt{1+1-2(\cos(\phi-\theta))}}$$

$$=\frac{|\sin(\phi-\theta)|}{\sqrt{2(1-\cos(\phi-\theta))}}$$

$$=\frac{|\sin(\phi-\theta)|}{\sqrt{2\left(2\sin^2\left(\frac{\phi-\theta}{2}\right)\right)}}$$

$$=\frac{|\sin(\phi-\theta)|}{\left|2\sin\left(\frac{\phi-\theta}{2}\right)\right|}$$

Where we know that $\sin\theta = 2\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)$

Then,

$$\frac{\left|2\cos\left(\frac{\phi-\theta}{2}\right)\sin\left(\frac{\phi-\theta}{2}\right)\right|}{\left|2\sin\left(\frac{\phi-\theta}{2}\right)\right|}=\left|\cos\left(\frac{\phi-\theta}{2}\right)\right|$$

Question 6:

Find the equation of the line parallel to y-axis and draw through the point of intersection of the lines $x-7y+5=0$ and $3x+y=0$

Solution:

The equation of any line parallel to the y-axis is of the form

$$x=a \qquad \ldots(1)$$

The two given lines are

$$x-7y+5=0 \qquad \ldots(2)$$

$$3x+y=0 \qquad \ldots(3)$$

On solving equation (2) and (3), we obtain $x=-\frac{5}{22}$ and $x=-\frac{15}{22}$

Therefore, $\left(-\frac{5}{22},-\frac{15}{22}\right)$ is the point of intersection of lines (2) and (3).

Since, line $x=a$ passes through point $\left(-\frac{5}{22},-\frac{15}{22}\right)$, $a=-\frac{5}{22}$

Thus, the required equation of the line is $x=-\frac{5}{22}$.

Question 7:

Find the equation of a line drawn perpendicular to the line $\frac{x}{4}+\frac{y}{6}=1$ through the point, where it meets the *y*-axis.

Solution:

The equation of the given line is $\frac{x}{4}+\frac{y}{6}=1$
This equation can also be written as $3x+2y-12=0$

$y=-\frac{3}{2}x+6$, which is of the form $y=mx+c$

Hence, the slope of the given line is $-\frac{3}{2}$

Slope of the line perpendicular to the given line is $-\frac{1}{\left(-\frac{3}{2}\right)}=\frac{2}{3}$

Let the given line intersect the *y*-axis at $(0,y)$.

On substituting $x=0$ in the equation of the given line, we obtain $\frac{y}{6}=1\Rightarrow y=6$

The given line intersects the *y*-axis at $(0,6)$.

The equation of the line that has a slope of $\frac{2}{3}$ and passes through point $(0,6)$ is

$$(y-6)=\frac{2}{3}(x-0)$$
$$3y-18=2x$$
$$2x-3y+18=0$$

Thus, the required equation of the line is $2x-3y+18=0$.

Question 8:

Find the area of the triangle formed by the line $y-x=0, x+y=0$ and $x-k=0$.

Solution:

The equations of the given lines are:

$$y-x=0 \qquad \ldots(1)$$
$$x+y=0 \qquad \ldots(2)$$
$$x-k=0 \qquad \ldots(3)$$

The point of interaction of lines (1) and (2) is given by

$x=0$ and $y=0$.

The point of interaction of lines (2) and (3) is given by

$x=k$ and $y=-k$.

The point of interaction of lines (3) and (1) is given by

$x=k$ and $y=k$.

Thus, the vertices of the triangle formed by the three given lines are $(0,0),(k,-k)$ and $(k,-k)$.

We know that the area of a triangle whose vertices are $(x_1,y_1),(x_2,y_2)$ and (x_3,y_3) is

$$\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

Therefore, area of the triangle formed by three given lines

$$=\frac{1}{2}\left|0(-k-k)+k(k-0)+k(0+k)\right|$$
$$=\frac{1}{2}\left|k^2+k^2\right|$$
$$=\frac{1}{2}2k^2$$
$$=k^2$$

Hence, the area of the triangle is k^2 square units.

Question 9:

Find the value of p so that the three lines $3x+y-2=0, px+2y-3=0$ and $2x-y-3=0$ may intersect at one point.

Solution:

The equation of the given line are

$$3x+y-2=0 \qquad \ldots(1)$$
$$px+2y-3=0 \qquad \ldots(2)$$
$$2x-y-3=0 \qquad \ldots(3)$$

On solving equations (1) and (3), we obtain

$x=1$ and $y=-1$

Since these three lines may intersect at one point, the point of intersection of lines (1) and (3) will also satisfy line (2)

$$p(1)+2(-1)-3=0$$
$$p-2-3=0$$
$$p=5$$

Thus, the required value of $p=5$.

Question 10:

If three lines whose equations are $y_1=m_1x+c_1$, $y=m_2x+c_2$ and $y=m_3x+c_3$ are concurrent, then show that $m_1(c_2-c_3)+m_2(c_3-c_1)+m_3(c_1-c_2)=0$.

Solution:

The equations of the given lines are:

$$y=m_1x+c_1 \qquad \ldots(1)$$
$$y=m_2x+c_2 \qquad \ldots(2)$$
$$y=m_3x+c_3 \qquad \ldots(3)$$

On subtracting equation (1) from (2) we obtain

$$(m_2-m_1)x+(c_2-c_1)=0$$
$$(m_1-m_2)x=(c_2-c_1)$$
$$x=\frac{(c_2-c_1)}{(m_1-m_2)}$$

On substituting this value of x in (1), we obtain

$$y = m_1\left(\frac{c_2 - c_1}{m_1 - m_2}\right) + c_1$$
$$= \frac{m_1c_2 - m_1c_1}{m_1 - m_2} + c_1$$
$$= \frac{m_1c_2 - m_1c_1 + c_1(m_1 - m_2)}{m_1 - m_2}$$
$$= \frac{m_1c_2 - m_1c_1 + m_1c_1 - m_2c_1}{m_1 - m_2}$$
$$y = \frac{m_1c_2 - m_2c_1}{m_1 - m_2}$$

Therefore,

$\left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1c_2 - m_2c_1}{m_1 - m_2}\right)$ is the point of intersection of line (1) and (2)

It is given that lines (1), (2) and (3) are concurrent.

Hence the point of intersection of lines (1) and (2) will also satisfy equation (3).

On substituting this value of x and y in (3), we obtain

$$\frac{m_1c_2 - m_2c_1}{m_1 - m_2} = m_3\left(\frac{c_2 - c_1}{m_1 - m_2}\right) + c_3$$
$$\frac{m_1c_2 - m_2c_1}{m_1 - m_2} = \frac{m_3c_2 - m_3c_1 + c_3m_1 - c_3m_2}{m_1 - m_2}$$
$$m_1c_2 - m_2c_1 - m_3c_2 + m_3c_1 - m_1c_3 + m_2c_3 = 0$$
$$m_1c_2 - m_1c_3 - m_2c_1 + m_2c_3 - m_3c_2 + m_3c_1 = 0$$
$$m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$$

Hence, $m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$ proved.

Question 11:

Find the equation of the line through the points $(3,2)$ which make an angle of $45°$ with the line $x - 2y = 3$.

Solution:

Let the slope of the required line be m_1

The given line can be represented as $y = \frac{1}{2}x - \frac{3}{2}$, which is of the form $y = mx + c$

Slope of the given line $= m_2 = \frac{1}{2}$

It is given that the angle between the required line and line $x-2y=3$ is $45°$.

We know that if θ is the acute angle between lines l_1 and l_2 with the slopes m_1 and m_2 respectively,

Then,

$$\tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right|$$

$$\tan 45° = \left|\frac{m_2-m_1}{1+m_1m_2}\right|$$

$$1 = \left|\frac{\frac{1}{2}-m_1}{1+\frac{m_1}{2}}\right|$$

$$1 = \left|\frac{\frac{1-2m_1}{2}}{\frac{2+m_1}{2}}\right|$$

$$1 = \left|\frac{1-2m_1}{2+m_1}\right|$$

$$1 = \pm\left(\frac{1-2m_1}{2+m_1}\right)$$

$$\Rightarrow 1 = \left(\frac{1-2m_1}{2+m_1}\right)$$

$$\Rightarrow 2+m_1 = 1-2m_1$$

$$\Rightarrow m_1 = -\frac{1}{3}$$

or

$$\Rightarrow 1 = -\left(\frac{1-2m_1}{2+m_1}\right)$$

$$\Rightarrow 2+m_1 = -1+2m_1$$

$$\Rightarrow m_1 = 3$$

Case I: $m_1 = 3$

The equation of the line passing through $(3,2)$ and having a slope of 3 is:

$$y-2 = 3(x-3)$$

$$y-2 = 3x-9$$

$$3x-y = 7$$

Case I: $m_1 = -\frac{1}{3}$

The equation of the line passing through (3,2) and having a slope of $-\frac{1}{3}$ is

$$y-2=-\frac{1}{3}(x-2)$$
$$3y-6=-x+3$$
$$x+3y=9$$

Thus, the equations of the line are $3x-y=7$ and $x+3y=9$.

Question 12:

Find the equation of the line passing through the point of intersection of the line $4x+7y-3=0$ and $2x-3y+1=0$ that has equal intercepts on the axes.

Solution:

Let the equation of the line having equal intercepts on the axes be

$$\frac{x}{a}+\frac{y}{b}=1$$
$$\Rightarrow x+y=ab \qquad \ldots(1)$$

On solving equations $4x+7y-3=0$ and $2x-3y+1=0$, we obtain $x=\frac{1}{13}$ and $y=\frac{5}{13}$

Therefore,

$\left(\frac{1}{13},\frac{5}{13}\right)$ is the point of the intersection of the two given lines.

Since equation (1) passes through point $\left(\frac{1}{13},\frac{5}{13}\right)$

$$\frac{1}{13}+\frac{5}{13}=a$$
$$\Rightarrow a=\frac{6}{13}$$

Equation (1) becomes

$$\Rightarrow x+y=\frac{6}{13}$$
$$\Rightarrow 13x+13y=6$$

Thus, the required equation of the line $13x+13y=6$

Question 13:

Show that the equation of the line passing through the origin and making an angle θ with the line $y=mx+c$, is $\frac{y}{x}=\frac{m\pm\tan\theta}{1\mp m\tan\theta}$

Solution:

Let the equation of the line passing through the origin be $y = m_1 x$

If this line makes an angle of θ with line $y = mx + c$, then angle θ is given by

$$\tan\theta = \left|\frac{m_1 - m}{1 + m_1 m}\right|$$

$$= \left|\frac{\frac{y}{x} - m}{1 + \frac{y}{x}m}\right|$$

$$\tan\theta = \pm\left(\frac{\frac{y}{x} - m}{1 + \frac{y}{x}m}\right)$$

$$\Rightarrow \tan\theta = \frac{\frac{y}{x} - m}{1 + \frac{y}{x}m} \quad \text{or} \quad \tan\theta = -\left(\frac{\frac{y}{x} - m}{1 + \frac{y}{x}m}\right)$$

Case I: $\tan\theta = \dfrac{\frac{y}{x} - m}{1 + \frac{y}{x}m}$

$$\tan\theta = \frac{\frac{y}{x} - m}{1 + \frac{y}{x}m}$$

$$\tan\theta + \frac{y}{x}m\tan\theta = \frac{y}{x} - m$$

$$m + \tan\theta = \frac{y}{x}(1 - m\tan\theta)$$

$$\frac{y}{x} = \frac{m + \tan\theta}{1 - m\tan\theta}$$

Case II: $\tan\theta = -\left(\dfrac{\frac{y}{x} - m}{1 + \frac{y}{x}m}\right)$

$$\tan\theta = -\left(\frac{\frac{y}{x}-m}{1+\frac{y}{x}m}\right)$$

$$\tan\theta + \frac{y}{x}m\tan\theta = -\frac{y}{x}+m$$

$$m-\tan\theta = \frac{y}{x}(1+m\tan\theta)$$

$$\frac{y}{x} = \frac{m-\tan\theta}{1+m\tan\theta}$$

Thus, the required line is given by $\frac{y}{x} = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}$

Question 14:

In what ratio, the joining $(-1,1)$ and $(5,7)$ is divisible by the line $x+y=4$?

Solution:

The solution of the line joining the points $(-1,1)$ and $(5,7)$ is given by

$$y-1 = \frac{7-1}{5+1}(x+1)$$

$$y-1 = \frac{6}{6}(x+1)$$

$$x-y+2=0 \qquad \ldots(1)$$

The equation of the line is

$$x+y=4 \qquad \ldots(2)$$

The points of intersection of line (1) and (2) is given by

$$x=1 \text{ and } y=3$$

Let point $(1,3)$ divides the line segment joining $(-1,1)$ and $(5,7)$ in the ratio $1:k$. Accordingly, by section formula

$$(1,3) = \left(\frac{k(-1)+1(5)}{1+k}, \frac{k(1)+1(7)}{1+k}\right)$$

$$\Rightarrow (1,3) = \left(\frac{-k+5}{1+k}, \frac{k+7}{1+k}\right)$$

$$\Rightarrow \frac{-k+5}{1+k} = 1, \ \frac{k+7}{1+k} = 3$$

Therefore,

$$\Rightarrow \frac{-k+5}{1+k}=1$$
$$\Rightarrow -k+5=1+k$$
$$\Rightarrow 2k=4$$
$$\Rightarrow k=2$$

Thus, the line joining the points $(-1,1)$ and $(5,7)$ is divided by line $x+y=4$ in the ratio $1:2$.

Question 15:

Find the distance of the line $4x+7y+5=0$ from the point $(1,2)$ along the line $2x-y=0$.

Solution:

The given lines are

$$2x-y=0 \qquad \ldots(1)$$
$$4x+7y+5=0 \qquad \ldots(2)$$

Let $A(1,2)$ is a point on the line (1) and B be the point intersection of line (1) and (2).

On solving equations (1) and (2), we obtain $x=-\frac{5}{18}$ and $y=-\frac{5}{9}$

Coordinates of point B are $\left(-\frac{5}{18},-\frac{5}{9}\right)$

By using distance formula, the distance between points A and B can be obtained as

$$AB = \sqrt{\left(1+\frac{5}{18}\right)^2 + \left(2+\frac{5}{9}\right)^2}$$
$$= \sqrt{\left(\frac{23}{18}\right)^2 + \left(\frac{23}{9}\right)^2}$$
$$= \sqrt{\left(\frac{23}{9\times 2}\right)^2 + \left(\frac{23}{9}\right)^2}$$
$$= \sqrt{\left(\frac{23}{9}\right)^2\left(\frac{1}{2}\right)^2 + \left(\frac{23}{9}\right)^2}$$
$$= \sqrt{\left(\frac{23}{9}\right)^2\left(\frac{1}{4}+1\right)}$$
$$= \frac{23}{9}\sqrt{\left(\frac{5}{4}\right)}$$
$$= \frac{23}{9}\times\frac{\sqrt{5}}{2}$$
$$= \frac{23\sqrt{5}}{18}$$

Thus, the required distance is $\frac{23\sqrt{5}}{18}$ units.

Question 16:

Find the direction in which a straight line must be drawn through the points $(-1,2)$ so that its point of intersection with line $x-y=4$ may be at a distance of 3 units from this point.

Solution:

Let $y=mx+c$ be the line through point $(-1,2)$
Accordingly,

$$\Rightarrow 2 = m(-1)+c$$
$$\Rightarrow 2 = -m+c$$
$$\Rightarrow c = m+2$$
$$\Rightarrow y = mx+m+2 \qquad \ldots(1)$$

The given line is

$$x-y=4 \qquad \ldots(2)$$

On solving equation (1) and (2), we obtain

$$x=\frac{2-m}{m+1} \text{ and } y=\frac{5m+2}{m+1}$$

Therefore,

$\left(\frac{2-m}{m+1},\frac{5m+2}{m+1}\right)$ is the point of intersection of line (1) and (2)

Since this point is at a distance of 3 units from point $(-1,2)$, accordingly to distance formula,

$$\Rightarrow \sqrt{\left(\frac{2-m}{m+1}+1\right)^2+\left(\frac{5m+2}{m+1}-2\right)^2}=3$$

$$\Rightarrow \left(\frac{2-m+m+1}{m+1}\right)^2+\left(\frac{5m+2-2m-2}{m+1}\right)^2=3^2$$

$$\Rightarrow \frac{9}{(m+1)^2}+\frac{9m^2}{(m+1)^2}=9$$

$$\Rightarrow \frac{1+m^2}{(m+1)^2}=1$$

$$\Rightarrow 1+m^2=m^2+1+2m$$

$$\Rightarrow 2m=0$$

$$\Rightarrow m=0$$

Thus, the slope of the required line must be zero i.e., the line must be parallel to the *x*-axis.

Question 17:

The hypotenuse of a right-angled triangle has its end at the points $(1,3)$ and $(-4,1)$. Find the equation of the legs (perpendicular sides) of the triangle.

Solution:

Let ABC be the right-angled triangle, where $\angle C=90°$

Let the slope of $AC=m$

Hence, the slope of $BC=-\frac{1}{m}$

Equation of AC:

$$\Rightarrow y-3=m(x-1)$$

$$\Rightarrow (x-1)=\frac{y-3}{m}$$

Equation of BC:

$$x+4=-m(y-1)$$

For a given value of m, we can get these equations

For, $m = 0,\ y - 3 = 0;\ x + 4 = 0$

For $m \to \infty,\ x - 1 = 0;\ y - 1 = 0$

Question 18:

Find the image of the point $(3,8)$ with respect to the line $x + 3y = 7$ assuming the line to be a plane mirror.

Solution:

The equation of the given line is

$$x + 3y = 7 \qquad \ldots(1)$$

Let point $B(a,b)$ be the image of point $A(3,8)$

Accordingly, line (1) is the perpendicular bisector of AB

Slope of $AB = \frac{b-8}{a-3}$, while the slope of the line (1) is $-\frac{1}{3}$

Since line (1) is perpendicular to AB

$$\Rightarrow \left(\frac{b-8}{a-3}\right)\left(-\frac{1}{3}\right) = 1$$

$$\Rightarrow \frac{b-8}{3a-9} = 1$$

$$\Rightarrow b - 8 = 3a - 9$$

$$\Rightarrow 3a - b - 1$$

Mid-point of $AB = \left(\frac{a+3}{2}, \frac{b+8}{2}\right)$

The mid-point of the line segments AB will also satisfy line (1).

Hence, from equation (1), we have

$$\Rightarrow \left(\frac{a+3}{2}\right) + 3\left(\frac{b+8}{2}\right) = 7$$

$$\Rightarrow a + 3 + 3b + 24 = 14$$

$$\Rightarrow a + 3b = -13 \ldots\ldots(3)$$

On solving equations (2) and (3), we obtain

$$a = -1 \text{ and } b = -4.$$

Thus, the image of the given point with respect to the given line is $(-1,-4)$.

Question 19:

If the lines $y = 3x+1$ and $2y = x+3$ are equally indicated to the line $y = mx+4$, find the value of m.

Solution:

The equations of the given lines are:

$$y = 3x+1 \qquad \ldots(1)$$
$$2y = x+3 \qquad \ldots(2)$$
$$y = mx+4 \qquad \ldots(3)$$

Slope of line (1), $m_1 = 3$

Slope of line (2), $m_2 = \frac{1}{2}$

Slope of line (3), $m_3 = m$

It is given that lines (1) and (2) are equally inclined to line (3). This means that the given angle between lines (1) and (3) equals the angle between lines (2) and (3).

Therefore,

$$\Rightarrow \left|\frac{m_1 - m_3}{1+m_1m_3}\right| = \left|\frac{m_2 - m_3}{1+m_2m_3}\right|$$

$$\Rightarrow \left|\frac{3-m}{1+3m}\right| = \left|\frac{\frac{1}{2}-m}{1+\frac{1}{2}m}\right|$$

$$\Rightarrow \left|\frac{3-m}{1+3m}\right| = \left|\frac{1-2m}{m+2}\right|$$

$$\Rightarrow \left|\frac{3-m}{1+3m}\right| = \pm\left(\frac{1-2m}{m+2}\right)$$

$$\Rightarrow \frac{3-m}{1+3m} = \frac{1-2m}{m+2} \, or \, \frac{3-m}{1+3m} = -\left(\frac{1-2m}{m+2}\right)$$

Case I: If $\frac{3-m}{1+3m} = \frac{1-2m}{m+2}$

Then,

$$\Rightarrow (3-m)(m+2)=(1-2m)(1+3m)$$
$$\Rightarrow -m^2+m-6=1+m-6m^2$$
$$\Rightarrow 5m^2+5=0$$
$$\Rightarrow m^2+1=0$$
$$\Rightarrow m^2=-1$$
$$\Rightarrow m=\sqrt{-1}$$

Here, $m=\sqrt{-1}$, which is not real
Hence, this case is not possible

<u>Case II</u>: If $\frac{3-m}{1+3m}=-\left(\frac{1-2m}{m+2}\right)$
Then,

$$\Rightarrow (3-m)(m+2)=-(1-2m)(1+3m)$$
$$\Rightarrow -m^2+m-6=-(1+m-6m^2)$$
$$\Rightarrow 7m^2-2m-7=0$$
$$\Rightarrow m=\frac{-2\pm\sqrt{4-4(7)(-7)}}{2(7)}$$
$$\Rightarrow m=\frac{2\pm 2\sqrt{1+49}}{14}$$
$$\Rightarrow m=\frac{1\pm 5\sqrt{2}}{7}$$

Thus, the required value of $m=\frac{1\pm 5\sqrt{2}}{7}$.

Question 20:

If sum of the perpendicular distance of a variable point $P(x,y)$ from the lines $x+y-5=0$ and $3x-2y+7=0$ is always 10. Show that P must move on a line.

Solution:
The equations of the lines are

$$x+y-5=0 \qquad \ldots(1)$$
$$3x-2y+7=0 \qquad \ldots(2)$$

The perpendicular distance of $P(x,y)$ from lines (1) and (2) are respectively given by

$$d_1 = \frac{|x+y-5|}{\sqrt{(1)^2+(1)^2}} \text{ and } d_2 = \frac{|3x-2y+7|}{\sqrt{(3)^2+(-2)^2}}$$

i.e., $$d_1 = \frac{|x+y-5|}{\sqrt{2}} \text{ and } d_2 = \frac{|3x-2y+7|}{\sqrt{13}}$$

It is given that $d_1 + d_2 = 10$
Therefore,

$$\Rightarrow \frac{|x+y-5|}{\sqrt{2}} + \frac{|3x-2y+7|}{\sqrt{13}} = 10$$

$$\Rightarrow \sqrt{13}\,|x+y-5| + \sqrt{2}\,|3x-2y+7| - 10\sqrt{26} = 0$$

$$\Rightarrow \sqrt{13}(x+y-5) + \sqrt{2}(3x-2y+7) - 10\sqrt{26} = 0$$

Assuming $x+y-5=0$ and $3x-2y+7=0$ are positive.

$$\Rightarrow \sqrt{13}x + \sqrt{13}y - 5\sqrt{13} + 3\sqrt{2}x - 2\sqrt{2}y + 7\sqrt{2} - 10\sqrt{26} = 0$$

$$\Rightarrow \left(\sqrt{13}+3\sqrt{2}\right)x + \left(\sqrt{13}-2\sqrt{2}\right)y + \left(7\sqrt{2}-5\sqrt{13}-10\sqrt{26}\right) = 0$$

Since, $\left(\sqrt{13}+3\sqrt{2}\right)x + \left(\sqrt{13}-2\sqrt{2}\right)y + \left(7\sqrt{2}-5\sqrt{13}-10\sqrt{26}\right) = 0$ is the equation of a line.

Similarly, we can obtain the equation of line for any signs of $x+y-5=0$ and $3x-2y+7=0$.

Thus, point P must move on a line.

Question 21:

Find equation of the line which is equidistant from parallel lines $9x+6y-7=0$ and $3x+2y+6=0$.

Solution:
The equations of the given lines are

$$9x+6y-7=0 \qquad \ldots(1)$$

$$3x+2y+6=0 \qquad \ldots(2)$$

Let $P(h,k)$ be the arbitrary point, is equidistant from lines (1) and (2).

Then the perpendicular distance of $P(h,k)$ from the line (1) is given by

$$d_1 = \frac{|9h+6k-7|}{\sqrt{(9)^2+(6)^2}}$$
$$= \frac{|9h+6k-7|}{\sqrt{117}}$$
$$= \frac{|9h+6k-7|}{3\sqrt{13}}$$

And the perpendicular distance of $P(h,k)$ from line (2) is given by

$$d_2 = \frac{|3h+2k+6|}{\sqrt{(3)^2+(2)^2}}$$
$$= \frac{|3h+2k+6|}{\sqrt{13}}$$

Since $P(h,k)$ is equidistant from lines (1) and (2), $d_1 = d_2$

Therefore,

$$\Rightarrow \frac{|9h+6k-7|}{3\sqrt{13}} = \frac{|3h+2k+6|}{\sqrt{13}}$$
$$\Rightarrow |9h+6k-7| = 3|3h+2k+6|$$
$$\Rightarrow 9h+6k-7 = \pm 3(3h+2k+6)$$

Case I: $9h+6k-7 = 3(3h+2k+6)$

$$\Rightarrow 9h+6k-7 = 3(3h+2k+6)$$
$$\Rightarrow 9h+6k-7 = 9h+6k+18$$
$$\Rightarrow 9h+6k-7-9h-6k-18 = 0$$
$$\Rightarrow 25 = 0$$

Which is an absurd, hence this case is not possible.

Case II: $9h+6k-7 = -3(3h+2k+6)$

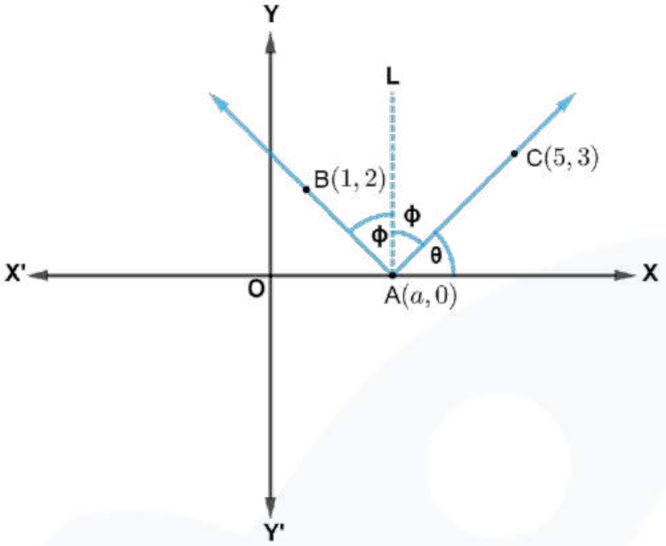
$$\Rightarrow 9h+6k-7 = -3(3h+2k+6)$$
$$\Rightarrow 9h+6k-7 = -9h-6k-18$$
$$\Rightarrow 18h+12k+11 = 0$$

Thus, the required equation of the line is $18x+12y+11=0$.

Question 22:

A ray of light passing through the point $(1,2)$ reflects on the x-axis at point A and the reflected ray passes through the point $(5,3)$. Find the coordinates of A.

Solution:



Let the coordinates of point A be $(a,0)$.
Draw a line, AL perpendicular to the x-axis

We know that angle of incidence is equal to angle of reflection.

Hence, let $\angle BAL = \angle CAL = \phi$ and $\angle CAX = \theta$
Now,

$$\begin{aligned}\angle OAB &= 180° - (\theta + 2\phi)\\ &= 180° - [\theta + 2(90° - \theta)]\\ &= 180° - [\theta + 180° - 2\theta]\\ &= 180° - 180° + \theta\\ &= \theta\end{aligned}$$

Therefore,

$$\angle BAX = 180° - \theta$$

Now,

Slope of line $AC = \frac{3-0}{5-a}$

$$\Rightarrow \tan\theta = \frac{3-0}{5-a} \qquad \ldots(1)$$

Slope of line $AC = \frac{2-0}{1-a}$

$$\Rightarrow \tan\left(180^\circ - \theta\right) = \frac{2}{1-a}$$

$$\Rightarrow -\tan\theta = \frac{2}{1-a}$$

$$\Rightarrow \tan\theta = \frac{2}{a-1} \qquad \ldots(2)$$

From equations (1) and (2), we obtain

$$\Rightarrow \frac{3}{5-a} = \frac{2}{a-1}$$

$$\Rightarrow 3a - 3 = 10 - 2a$$

$$\Rightarrow a = \frac{13}{5}$$

Thus, the coordinates of point A are $\left(\frac{13}{5}, 0\right)$.

Question 23:

Prove that the product of the lengths of the perpendiculars drawn from the points $\left(\sqrt{a^2 - b^2}, 0\right)$ and $\left(-\sqrt{a^2 + b^2}, 0\right)$ to the line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ is b^2.

Solution:

The equation of the given line is

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

$$bx\cos\theta + ay\sin\theta - ab = 0 \qquad \ldots(1)$$

Length of the perpendicular from point $\left(\sqrt{a^2 + b^2}, 0\right)$ to the line (1) is

$$p_1 = \frac{\left|b\cos\theta\left(\sqrt{a^2 - b^2}\right) + a\sin\theta(0) - ab\right|}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}}$$

$$= \frac{\left|b\cos\theta\left(\sqrt{a^2 - b^2}\right) - ab\right|}{\sqrt{b^2\cos^2\theta + a^2\sin^2\theta}} \qquad \ldots(2)$$

Length of the perpendicular from point $\left(-\sqrt{a^2 + b^2}, 0\right)$ to the line (2) is

$$p_2 = \frac{\left|b\cos\theta\left(-\sqrt{a^2-b^2}\right)+a\sin\theta(0)-ab\right|}{\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}}$$

$$= \frac{\left|b\cos\theta\left(\sqrt{a^2-b^2}\right)-ab\right|}{\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}} \qquad \ldots(3)$$

On multiplying equations (2) and (3), we obtain

$$p_1p_2 = \frac{\left|b\cos\theta\left(\sqrt{a^2-b^2}\right)-ab\right|\left|b\cos\theta\left(\sqrt{a^2-b^2}\right)-ab\right|}{\left(\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}\right)^2}$$

$$= \frac{\left|\left(b\cos\theta\left(\sqrt{a^2-b^2}\right)-ab\right)\left(b\cos\theta\left(\sqrt{a^2-b^2}\right)-ab\right)\right|}{\left(b^2\cos^2\theta+a^2\sin^2\theta\right)}$$

$$= \frac{\left|\left(b\cos\theta\sqrt{a^2-b^2}\right)^2-(ab)^2\right|}{\left(b^2\cos^2\theta+a^2\sin^2\theta\right)}$$

$$= \frac{\left|b^2\cos^2\theta\left(a^2-b^2\right)-a^2b^2\right|}{\left(b^2\cos^2\theta+a^2\sin^2\theta\right)}$$

$$= \frac{\left|a^2b^2\cos^2\theta-b^4\cos^2\theta-a^2b^2\right|}{\left(b^2\cos^2\theta+a^2\sin^2\theta\right)}$$

$$= \frac{b^2\left|a^2\cos^2\theta-b^2\cos^2\theta-a^2\right|}{\left(b^2\cos^2\theta+a^2\sin^2\theta\right)}$$

$$= \frac{b^2\left|a^2\cos^2\theta-b^2\cos^2\theta-a^2\sin^2\theta-a^2\cos^2\theta\right|}{\left(b^2\cos^2\theta+a^2\sin^2\theta\right)} \qquad \left[\because \sin^2\theta+\cos^2\theta=1\right]$$

$$= \frac{b^2\left|-\left(b^2\cos^2\theta+a^2\sin^2\theta\right)\right|}{\left(b^2\cos^2\theta+a^2\sin^2\theta\right)}$$

$$= \frac{b^2\left(b^2\cos^2\theta+a^2\sin^2\theta\right)}{\left(b^2\cos^2\theta+a^2\sin^2\theta\right)}$$

$$= b^2$$

Hence, proved

Question 24:

A person standing at the junction (crossing) of two straight paths represented by the equation $2x-3y+4=0$ and $3x+4y-5=0$ wants to reach the path whose equation is $6x-7y+8=0$ in the least time. Find equation of the path that he should follow.

Solution:

The equations of the given lines are

$$2x-3y+4=0 \qquad \ldots(1)$$
$$3x+4y-5=0 \qquad \ldots(2)$$
$$6x-7y+8=0 \qquad \ldots(3)$$

The person is standing at the junction of the paths represented by lines (1) and (2).

On solving equations (1) and (2), we obtain $x=-\frac{1}{17}$ and $y=\frac{22}{17}$

Thus, the person is standing at point $\left(-\frac{1}{17},\frac{22}{17}\right)$

The person can reach path (3) in the least time if he walks along the perpendicular line to (3) from point $\left(-\frac{1}{17},\frac{22}{17}\right)$

Now,

Slope of the line (3) $=\frac{6}{7}$

Slope of the line perpendicular to line (3) $=-\frac{1}{\left(\frac{6}{7}\right)}=-\frac{7}{6}$

The equation of the line passing through $\left(-\frac{1}{17},\frac{22}{17}\right)$ and having a slope of $-\frac{7}{6}$ is given by

$$\Rightarrow\left(y-\frac{22}{17}\right)=-\frac{7}{6}\left(x+\frac{1}{17}\right)$$
$$\Rightarrow 6(17y-22)=-7(17x+1)$$
$$\Rightarrow 102y-132=-119x-7$$
$$\Rightarrow 119x+102y=125$$

Hence, the path that the person should follow is $119x+102y=125$.