NCERT Solutions Class 11 Maths Chapter 12 Introduction to Three Dimensional Geometry

Question 1:
A point is on the $x-$axis. What are its $y-$coordinate and $z-$coordinates?

Solution:
If a point is on the $x-$axis, then the coordinates of y and z are 0.
So, the point is $(x,0,0)$.

Question 2:
A point is in the $XZ-$plane. What can you say about its $y-$coordinate?

Solution:
If a point is in $XZ-$plane, then its $y-$coordinate is 0.

Question 3:
Name the octants in which the following points lie:
$(1,2,3),(4,-2,3),(4,-2,-5),(4,2,-5),(-4,2,-5),(-4,2,5),(-3,-1,6),(2,-4,-7)$.

Solution:
Here is the table which represents the octants:

Octants	I	II	III	IV	V	VI	VII	VIII
x	+	−	−	+	+	−	−	+
y	+	+	−	−	+	+	−	−
z	+	+	+	+	−	−	−	−

(i) $(1,2,3)$
Here x is positive, y is positive, and z is positive.
So, it lies in I octant.

(ii) $(4,-2,3)$
Here x is positive, y is negative, and z is positive.
So, it lies in IV octant.

(iii) $(4,-2,-5)$
Here x is positive, y is negative, and z is negative.
So, it lies in VIII octant.

(iv) $(4,2,-5)$
Here x is positive, y is positive, and z is negative.
So, it lies in V octant.

(v) $(-4,2,-5)$
Here x is negative, y is positive, and z is negative.
So, it lies in VI octant.

(vi) $(-4,2,5)$
Here x is negative, y is positive, and z is positive.
So, it lies in II octant.

(vii) $(-3,-1,6)$
Here x is negative, y is negative, and z is positive.
So, it lies in III octant.

(viii) $(2,-4,-7)$
Here x is positive, y is negative, and z is negative.
So, it lies in VIII octant.

Question 4:
Fill in the blanks:

(i) The x-axis and y-axis taken together determine a plane known as **XY plane**.

(ii) The coordinates of points in the XY-plane are of the form **(*x*, *y*, 0)**.

(iii) Coordinate planes divide the space into **eight** octants.

EXERCISE 12.2

Question 1:

Find the distance between the following pairs of points:

(i) $(2,3,5)$ and $(4,3,1)$

(ii) $(-3,7,2)$ and $(2,4,-1)$

(iii) $(-1,3,-4)$ and $(1,-3,4)$

(iv) $(2,-1,3)$ and $(-2,1,3)$

Solution:

(i) $(2,3,5)$ and $(4,3,1)$

Let P be $(2,3,5)$ and Q be $(4,3,1)$

By using the formula,

Distance $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

So here,

$x_1=2, y_1=3, z_1=5$

$x_2=4, y_2=3, z_2=1$

$$PQ=\sqrt{(4-2)^2+(3-3)^2+(1-5)^2}$$
$$=\sqrt{2^2+0^2+(-4)^2}$$
$$=\sqrt{4+16}$$
$$=\sqrt{20}$$
$$=2\sqrt{5}$$

Therefore, the required distance is $2\sqrt{5}$ units.

(ii) $(-3,7,2)$ and $(2,4,-1)$

Let P be $(-3,7,2)$ and Q be $(2,4,-1)$

By using the formula,

Distance $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

So here,

$x_1=-3, y_1=7, z_1=2$

$x_2=2, y_2=4, z_2=-1$

$$PQ = \sqrt{\left(2-(-3)\right)^2 + (4-7)^2 + (-1-2)^2}$$
$$= \sqrt{5^2 + (-3)^2 + (-3)^2}$$
$$= \sqrt{25+9+9}$$
$$= \sqrt{43}$$

Therefore, the required distance is $\sqrt{43}$ units.

(iii) $(2,-1,3)$ and $(-2,1,3)$

Let P be $(-1,3,-4)$ and Q be $(1,-3,4)$

By using the formula,

Distance $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

So here,

$x_1 = -1,\ y_1 = 3,\ z_1 = -4$

$x_2 = 1,\ y_2 = -3,\ z_2 = 4$

$$PQ = \sqrt{\left(1-(-1)\right)^2 + (-3-3)^2 + \left(4-(-4)\right)^2}$$
$$= \sqrt{2^2 + (-6)^2 + 8^2}$$
$$= \sqrt{4+36+64}$$
$$= \sqrt{104}$$
$$= 2\sqrt{26}$$

Therefore, the required distance is $2\sqrt{26}$ units.

(iv) $(2,-1,3)$ and $(-2,1,3)$

Let P be $(2,-1,3)$ and Q be $(-2,1,3)$

By using the formula,

Distance $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

So here,

$x_1 = 2,\ y_1 = -1,\ z_1 = 3$

$x_2 = -2,\ y_2 = 1,\ z_2 = 3$

$$PQ=\sqrt{(-2-2)^2+(1-(-1))^2+(3-3)^2}$$
$$=\sqrt{(-4)^2+2^2+0^2}$$
$$=\sqrt{16+4}$$
$$=\sqrt{20}$$
$$=2\sqrt{5}$$

Therefore, the required distance is $2\sqrt{5}$ units.

Question 2:

Show that the points $(-2,3,5),(1,2,3)$ and $(7,0,-1)$ are collinear.

Solution:

If three points are collinear, then they lie on a line.
Firstly, let us calculate distance between the 3 points i.e. PQ, QR and PR.

Calculating PQ:

$P=(-2,3,5)$ and $Q=(1,2,3)$
By using the formula,

Distance $PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
So here,
$x_1=-2, y_1=3, z_1=5$
$x_2=1, y_2=2, z_2=3$

$$PQ=\sqrt{\left[(1-(-2))^2+(2-3)^2+(3-5)^2\right]}$$
$$=\sqrt{(3)^2+(-1)^2+(-2)^2}$$
$$=\sqrt{9+1+4}$$
$$=\sqrt{14}$$

Calculating QR:

$Q=(1,2,3)$ and $R=(7,0,-1)$
By using the formula,

Distance $QR=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
So here,

$x_1 = 1, y_1 = 2, z_1 = 3$
$x_2 = 7, y_2 = 0, z_2 = -1$

$$\begin{aligned} QR &= \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2} \\ &= \sqrt{6^2 + (-2)^2 + (-4)^2} \\ &= \sqrt{36+4+16} \\ &= \sqrt{56} \\ &= 2\sqrt{14} \end{aligned}$$

Calculating PR:

$P = (-2,3,5)$ and $R = (7,0,-1)$

By using the formula,

Distance $PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

So here,

$x_1 = -2, y_1 = 3, z_1 = 5$
$x_2 = 7, y_2 = 0, z_2 = -1$

$$\begin{aligned} PR &= \sqrt{(7-(-2))^2 + (0-3)^2 + (-1-5)^2} \\ &= \sqrt{9^2 + (-3)^2 + (-6)^2} \\ &= \sqrt{81+9+36} \\ &= \sqrt{126} \\ &= 3\sqrt{14} \end{aligned}$$

Thus, $PQ = \sqrt{14}$, $QR = 2\sqrt{14}$ and $PR = 3\sqrt{14}$

So,

$$\begin{aligned} PQ + QR &= \sqrt{14} + 2\sqrt{14} \\ &= 3\sqrt{14} \\ &= PR \end{aligned}$$

Therefore, the points P, Q and R are collinear.

Question 3:

Verify the following:

(i) $(0,7,-10),(1,6,-6)$ and $(4,9,-6)$ are the vertices of an isosceles triangle.

(ii) $(0,7,10),(-1,6,6)$ and $(-4,9,6)$ are the vertices of a right angled triangle.

(iii) $(-1,2,1),(1,-2,5),(4,-7,8)$ and $(2,-3,4)$ are the vertices of a parallelogram.

Solution:

(i) $(0,7,-10),(1,6,-6)$ and $(4,9,-6)$ are the vertices of an isosceles triangle.

Let us consider the points be $P(0,7,-10), Q(1,6,-6)$ and $R(4,9,-6)$
If any 2 sides are equal, hence it will be an isosceles triangle.
So, firstly let us calculate the length of the sides.

Calculating PQ:

$P(0,7,-10)$ and $Q(1,6,-6)$
By using the formula,

Distance $PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
So here,
$x_1=0, y_1=7, z_1=-10$
$x_2=1, y_2=6, z_2=-6$

$$\begin{aligned}PQ&=\sqrt{(1-0)^2+(6-7)^2+(-6-(-10))^2}\\&=\sqrt{1^2+(-1)^2+4^2}\\&=\sqrt{1+1+16}\\&=\sqrt{18}\end{aligned}$$

Calculating QR:

$Q(1,6,-6)$ and $R(4,9,-6)$
By using the formula,

Distance $QR=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
So here,
$x_1=1, y_1=6, z_1=-6$
$x_2=4, y_2=9, z_2=-6$

$$\begin{aligned}QR&=\sqrt{(4-1)^2+(9-6)^2+(-6-(-6))^2}\\&=\sqrt{3^2+3^2+0^2}\\&=\sqrt{9+9}\\&=\sqrt{18}\end{aligned}$$

Here, $PQ=QR=\sqrt{18}$
Since, two sides are equal, ΔPQR is an isosceles triangle.

Thus, $(0,7,-10),(1,6,-6)$ and $(4,9,-6)$ are the vertices of an isosceles triangle.

(ii) $(0,7,10),(-1,6,6)$ and $(-4,9,6)$ are the vertices of a right angled triangle.

Let the points be $P(0,7,10), Q(-1,6,6)$ and $R(-4,9,6)$
Firstly, let us calculate the length of sides PQ, QR and PR.

<u>Calculating PQ</u>:

$P(0,7,10)$ and $Q(-1,6,6)$
By using the formula,

Distance $PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
So here,
$x_1=0,\ y_1=7,\ z_1=10$
$x_2=-1,\ y_2=6,\ z_2=6$

$$\begin{aligned}PQ&=\sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}\\&=\sqrt{(-1)^2+(-1)^2+(-4)^2}\\&=\sqrt{1+1+16}\\&=\sqrt{18}\end{aligned}$$

<u>Calculating QR</u>:

$Q(-1,6,6)$ and $R(-4,9,6)$
By using the formula,

Distance $QR=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

So here,
$x_1=-1,\ y_1=6,\ z_1=6$
$x_2=-4,\ y_2=9,\ z_2=6$

$$\begin{aligned}QR&=\sqrt{(-4-(-1))^2+(9-6)^2+(6-6)^2}\\&=\sqrt{(-3)^2+3^2+0^2}\\&=\sqrt{9+9}\\&=\sqrt{18}\end{aligned}$$

<u>Calculating PR</u>:

$P(0,7,10)$ and $R(-4,9,6)$
By using the formula,

Distance $PR=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
So here,
$x_1=0, y_1=7, z_1=10$
$x_2=-4, y_2=9, z_2=6$

$$\begin{aligned}PR&=\sqrt{(-4-0)^2+(9-7)^2+(6-10)^2}\\&=\sqrt{(-4)^2+2^2+(-4)^2}\\&=\sqrt{16+4+16}\\&=\sqrt{36}\end{aligned}$$

Now,

$$\begin{aligned}PQ^2+QR^2&=18+18\\&=36\\&=PR^2\end{aligned}$$

By using Converse of Pythagoras theorem,
The given vertices P, Q and R are the vertices of a right-angled triangle at Q.

Thus, $(0,7,10),(-1,6,6)$ and $(-4,9,6)$ are the vertices of a right angled triangle.

(iii) $(-1,2,1),(1,-2,5),(4,-7,8)$ and $(2,-3,4)$ are the vertices of a parallelogram.

Let the points be $A(-1,2,1), B(1,-2,5), C(4,-7,8)$ and $D(2,-3,4)$
if pairs of opposite sides are equal then only ABCD can be a parallelogram.
i.e., $AB=CD$ and $BC=AD$.

Firstly, let us calculate the lengths of the sides

Calculating AB:
$A(-1,2,1)$ and $B(1,-2,5)$
By using the distance formula,

Distance $AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
So here,
$x_1=-1, y_1=2, z_1=1$
$x_2=1, y_2=-2, z_2=5$

$$\begin{aligned}AB &= \sqrt{(1-(-1))^2+(-2-2)^2+(5-1)^2}\\ &= \sqrt{2^2+(-4)^2+4^2}\\ &= \sqrt{4+16+16}\\ &= \sqrt{36}\\ &= 6\end{aligned}$$

<u>Calculating BC</u>:

$B(1,-2,5)$ and $C(4,-7,8)$

By using the distance formula,

Distance $BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

So here,

$x_1 = 1, y_1 = -2, z_1 = 5$

$x_2 = 4, y_2 = -7, z_2 = 8$

$$\begin{aligned}BC &= \sqrt{(4-1)^2+(-7-(-2))^2+(8-5)^2}\\ &= \sqrt{3^2+(-5)^2+3^2}\\ &= \sqrt{9+25+9}\\ &= \sqrt{43}\end{aligned}$$

<u>Calculating CD</u>:

$C(4,-7,8)$ and $D(2,-3,4)$

By using the distance formula,

Distance $CD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

So here,

$x_1 = 4, y_1 = -7, z_1 = 8$

$x_2 = 2, y_2 = -3, z_2 = 4$

$$\begin{aligned}CD &= \sqrt{\left[(2-4)^2+(-3-(-7))^2+(4-8)^2\right]}\\ &= \sqrt{(-2)^2+4^2+(-4)^2}\\ &= \sqrt{4+16+16}\\ &= \sqrt{36}\\ &= 6\end{aligned}$$

<u>Calculating DA</u>:

$D(2,-3,4)$ and $A(-1,2,1)$

By using the formula,

Distance $DA=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

So here,

$x_1=2, y_1=-3, z_1=4$

$x_2=-1, y_2=2, z_2=1$

$$DA=\sqrt{(-1-2)^2+(2-(-3))^2+(1-4)^2}$$
$$=\sqrt{(-3)^2+5^2+(-3)^2}$$
$$=\sqrt{9+25+9}$$
$$=\sqrt{43}$$

Since, in quadrilateral ABCD both the pairs of opposite sides are equal i.e., $AB=CD$ and $BC=AD$, ABCD is a parallelogram.

Thus, $(-1,2,1),(1,-2,5),(4,-7,8)$ and $(2,-3,4)$ are the vertices of a parallelogram.

Question 4:

Find the equation of the set of points which are equidistant from the points $(1,2,3)$ and $(3,2,-1)$.

Solution:

Let $A(1,2,3)$ and $B(3,2,-1)$

Let point P be (x,y,z)

Since it is given that point $P(x,y,z)$ is equidistant from the points $A(1,2,3)$ and $B(3,2,-1)$ i.e., $PA=PB$

Firstly, let us calculate distances PA and PB

<u>Calculating PA</u>:

$P(x,y,z)$ and $A(1,2,3)$

By using the distance formula,

Distance $PA=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

So here,

$x_1=x, y_1=y, z_1=z$

$x_2=1, y_2=2, z_2=3$

$$PA=\sqrt{(1-x)^2+(2-y)^2+(3-z)^2}$$

<u>Calculating PB</u>:

$P(x,y,z)$ and $B(3,2,-1)$

By using the distance formula,

Distance $PB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

So here,

$x_1=x,\ y_1=y,\ z_1=z$

$x_2=3,\ y_2=2,\ z_2=-1$

$$PB=\sqrt{(3-x)^2+(2-y)^2+(-1-z)^2}$$

Since, $PA=PB$

On squaring both the sides, we get

$$PA^2=PB^2$$

Therefore,

$$(1-x)^2+(2-y)^2+(3-z)^2=(3-x)^2+(2-y)^2+(-1-z)^2$$
$$(1+x^2-2x)+(4+y^2-4y)+(9+z^2-6z)=(9+x^2-6x)+(4+y^2-4y)+(1+z^2+2z)$$
$$-2x-4y-6z+14=-6x-4y+2z+14$$
$$4x-8z=0$$
$$x-2z=0$$

Thus, the required equation is $x-2z=0$

Question 5:

Find the equation of the set of points P, the sum of whose distances from $A(4,0,0)$ and $B(-4,0,0)$ is equal to 10.

Solution:

Let $A(4,0,0)$ and $B(-4,0,0)$

Let the coordinates of point P be (x,y,z)

<u>Calculating PA</u>:

$P(x,y,z)$ and $A(4,0,0)$

By using the distance formula,

Distance $PA=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
So here,
$x_1=x, y_1=y, z_1=z$
$x_2=4, y_2=0, z_2=0$
Distance $PA=\sqrt{(4-x)^2+(0-y)^2+(0-z)^2}$

Calculating PB:
$P(x,y,z)$ and $B(-4,0,0)$
By using the distance formula,
Distance $PB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
So here,
$x_1=x, y_1=y, z_1=z$
$x_2=-4, y_2=0, z_2=0$
Distance $PB=\sqrt{(-4-x)^2+(0-y)^2+(0-z)^2}$

Now it is given that

$$PA+PB=10$$
$$PA=10-PB$$

On squaring both the sides, we get

$$PA^2=(10-PB)^2$$
$$PA^2=100+PB^2-20PB$$

Therefore,

$$(4-x)^2+(0-y)^2+(0-z)^2=100+(-4-x)^2+(0-y)^2+(0-z)^2-20PB$$
$$(16+x^2-8x)+y^2+z^2=100+(16+x^2+8x)+y^2+z^2-20PB$$
$$20PB=16x+100$$
$$5PB=(4x+25)$$

On squaring both the sides again, we get

$$25PB^2=16x^2+200x+625$$
$$25\left[(-4-x)^2+(0-y)^2+(0-z)^2\right]=16x^2+200x+625$$
$$25\left[x^2+y^2+z^2+8x+16\right]=16x^2+200x+625$$
$$25x^2+25y^2+25z^2+200x+400=16x^2+200x+625$$
$$9x^2+25y^2+25z^2-225=0$$

Thus, the required equation is $9x^2+25y^2+25z^2-225=0$.

EXERCISE 12.3

Question 1:

Find the coordinates of the point which divides the line segment joining the points $(-2,3,5)$ and $(1,-4,6)$ in the ratio (i) $2:3$ internally, (ii) $2:3$ externally.

Solution:

(i) The coordinates of point R that divides the line segment joining the points $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ internally in the ratio $m:n$ are $\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m+n}\right)$

Let $R(x,y,z)$ be the point that divides the line segment joining points $(-2,3,5)$ and $(1,-4,6)$ internally in the ratio $2:3$

Hence,

$$x=\frac{2(1)+3(-2)}{2+3},\ y=\frac{2(-4)+3(3)}{2+3} \text{ and } z=\frac{2(6)+3(5)}{2+3}$$

i.e., $x=\frac{-4}{5},\ y=\frac{1}{5}$ and $z=\frac{27}{5}$

Thus, the coordinates of the required point are $\left(\frac{-4}{5},\frac{1}{5},\frac{27}{5}\right)$.

(ii) The coordinates of point R that divides the line segment joining the points $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ internally in the ratio $m:n$ are $\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m+n}\right)$

Let $R(x,y,z)$ be the point that divides the line segment joining points $(-2,3,5)$ and $(1,-4,6)$ externally in the ratio $2:3$

Hence,

$$x=\frac{2(1)-3(-2)}{2-3},\ y=\frac{2(-4)-3(3)}{2-3} \text{ and } z=\frac{2(6)-3(5)}{2-3}$$

i.e., $x=-8,\ y=17$ and $z=3$

Thus, the coordinates of the required point are $(-8,17,3)$.

Question 2:

Given that $P(3,2,-4), Q(5,4,-6)$ and $R(9,8,-10)$ are collinear. Find the ratio in which Q divides PR.

Solution:

Let point $Q(5,4,-6)$ divides the line segment joining points $P(3,2,-4)$ and $R(9,8,-10)$ in the ratio $k:1$.

Therefore, by section formula,

$$(5,4,-6)=\left(\frac{k(9)+3}{k+1},\frac{k(8)+2}{k+1},\frac{k(-10)-4}{k+1}\right)$$

Hence,

$$\begin{aligned}\frac{9k+3}{k+1}&=5\\ 9k+3&=5k+5\\ 4k&=2\\ k&=\frac{2}{4}\\ &=\frac{1}{2}\end{aligned}$$

Thus, point Q divides PR in the ratio $1:2$.

Question 3:

Find the ratio in which the YZ-plane divides the line segment formed by joining the points $(-2,4,7)$ and $(3,-5,8)$.

Solution:

Let the YZ plane divide the line segment joining points $(-2,4,7)$ and $(3,-5,8)$ in the ratio $k:1$.

Hence, by section formula, the coordinates of point of intersection are given by

$$\left(\frac{k(3)-2}{k+1},\frac{k(-5)+4}{k+1},\frac{k(8)+7}{k+1}\right)$$

On the YZ plane, the $x-$coordinate of any point is zero.

$$\Rightarrow \frac{3k-2}{k+1}=0$$
$$\Rightarrow 3k-2=0$$
$$\Rightarrow k=\frac{2}{3}$$

Thus, the YZ-plane divides the line segment formed by joining the given points in the ratio $2:1$.

Question 4:

Using section formula, show that the points $A(2,-3,4)$, $B(-1,2,1)$ and $C\left(0,\frac{1}{3},2\right)$ are collinear.

Solution:

Let P be a point that divides AB in the ratio $k:1$.

Hence, by section formula, the coordinates of point of intersection are given by

$$\left(\frac{k(-1)+2}{k+1},\frac{k(2)-3}{k+1},\frac{k(1)+4}{k+1}\right)$$

Now, we find the value of k at which point P coincides with point C.

By taking $\frac{-k+2}{k+1}=0$, we obtain $k=2$.

For $k=2$, the coordinates of point P are $\left(0,\frac{1}{3},2\right)$

i.e., $\left(0,\frac{1}{3},2\right)$ is a point that divides AB externally in the ratio $2:1$ and is the same as point C.

Hence, points A, B, and C are collinear.

Question 5:

Find the coordinates of the points which trisect the line segment joining the points $P(4,2,-6)$ and $Q(10,-16,6)$.

Solution:

Let A and B be the points that trisect the line segment joining the points $P(4,2,-6)$ and $Q(10,-16,6)$.

Point A divides PQ in the ratio $1:2$.

Therefore, by section formula, the coordinates of point A are given by

$$\left(\frac{1(10)+2(4)}{1+2},\frac{1(-16)+2(2)}{1+2},\frac{1(6)+2(-6)}{1+2}\right)$$
$$\Rightarrow(6,-4,-2)$$

Point B divides PQ in the ratio $2:1$.

Therefore, by section formula, the coordinates of point B are given by

$$\left(\frac{2(10)+1(4)}{1+2},\frac{2(-16)+1(2)}{1+2},\frac{2(6)+1(-6)}{1+2}\right)$$
$$\Rightarrow(8,-10,2)$$

Thus, $(6,-4,-2)$ and $(8,-10,2)$ are the points that trisect the line segment joining the points $P(4,2,-6)$ and $Q(10,-16,6)$.

MISCELLANEOUS EXERCISE

Question 1:

Three vertices of a parallelogram ABCD are $A(3,-1,2)$, $B(1,2,-4)$ and $C(-1,1,2)$. Find the coordinates of the fourth vertex.

Solution:

The three vertices of a parallelogram ABCD are given as $A(3,-1,2)$, $B(1,2,-4)$ and $C(-1,1,2)$.
Let the coordinates of the fourth vertex be $D(x,y,z)$.

We know that the diagonals of a parallelogram bisect each other.
Therefore, in parallelogram ABCD, diagonals AC and BD bisect each other.

i.e., Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{3-1}{2},\frac{-1+1}{2},\frac{2+2}{2}\right)=\left(\frac{x+1}{2},\frac{y+2}{2},\frac{z-4}{2}\right)$$

$$\Rightarrow (1,0,2)=\left(\frac{x+1}{2},\frac{y+2}{2},\frac{z-4}{2}\right)$$

Hence,

$$\frac{x+1}{2}=1,\ \frac{y+2}{2}=0 \text{ and } \frac{z-4}{2}=2$$

$$\Rightarrow x=1,\ y=-2 \text{ and } z=8$$

Thus, the coordinates of the fourth vertex D are $(1,-2,8)$.

Question 2:

Find the lengths of the medians of the triangle with vertices $A(0,0,6)$, $B(0,4,0)$ and $C(6,0,0)$.

Solution:

Let AD, BE and CF be the medians of the given triangle.

Since, AD is the median, D is the mid-point of BC

Coordinates of point $D=\left(\frac{0+6}{2},\frac{4+0}{2},\frac{0+0}{2}\right)=(3,2,0)$

$$AD = \sqrt{(0-3)^2 + (0-2)^2 + (6-0)^2}$$
$$= \sqrt{9+4+36}$$
$$= \sqrt{49}$$
$$= 7$$

Since, BE is the median, E is the mid-point of AC

Coordinates of point $E = \left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{0+6}{2}\right) = (3,0,3)$

$$BE = \sqrt{(3-0)^2 + (0-4)^2 + (3-0)^2}$$
$$= \sqrt{9+16+9}$$
$$= \sqrt{34}$$

Since CF is the median, F is the mid-point of AB

Coordinates of point $F = \left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{0+6}{2}\right) = (0,2,3)$

$$CF = \sqrt{(6-0)^2 + (0-2)^2 + (0-3)^2}$$
$$= \sqrt{36+4+9}$$
$$= \sqrt{49}$$
$$= 7$$

Thus, the lengths of the medians of triangle ABC are $7, \sqrt{34}$ and 7.

Question 3:

If the origin is the centroid of the triangle PQR with vertices $P(2a,2,6), Q(-4,3b,-10)$ and $R(8,14,2c)$, then find the values of a, b and c.

Solution:

It is known that the coordinates of the centroid of the triangle, whose vertices are $(x_1,y_1,z_1),(x_2,y_2,z_2)$ and (x_3,y_3,z_3) are $\left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{2}, \frac{z_1+z_2+z_3}{2}\right)$

Therefore, coordinates of the centroid of

$$\Delta PQR = \left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right)$$
$$= \left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$$

It is given that origin $(0,0,0)$ is the centroid of the ΔPQR
Hence,

$$\frac{2a+4}{3}=0, \frac{3b+16}{3} \text{ and } \frac{2c-4}{3}=0$$

Thus, the values of $a=-2, b=-\frac{16}{3}$ and $c=2$.

Question 4:

Find the coordinates of a point on $y-$axis which are at distance of $5\sqrt{2}$ from the point $P(3,-2,5)$.

Solution:

If a point is on the $y-$axis, then $x-$coordinate and the $z-$coordinate of the point are zero.
Let $A(0,b,0)$ be the point on the $y-$axis at a distance of $5\sqrt{2}$ from point $P(3,-2,5)$.
Accordingly, $AP=5\sqrt{2}$

On squaring both sides, we obtain $AP^2=50$

Therefore,

$$\begin{aligned}(3-0)^2+(-2-b)^2+(5-0)^2&=50\\ 9+4+b^2+4b+25&=50\\ b^2+4b-12&=0\\ b^2+6b-2b-12&=0\\ (b+6)(b-2)&=0\end{aligned}$$

$$\Rightarrow b=-6 \text{ and } b=2$$

Thus, the coordinates of the required point are $(0,2,0)$ and $(0,-6,0)$.

Question 5:

A point R with $x-$coordinate 4 lies on the line segment joining the points $P(2,-3,4)$ and $Q(8,0,10)$. Find the coordinates of the point R.
[Hint Suppose R divides PQ in the ratio $k:1$. The coordinates of the point R are given by $\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$]

Solution:

The coordinates of points P and Q are given as $P(2,-3,4)$ and $Q(8,0,10)$.
Let R divide the line segment PQ in the ratio $k:1$.

Hence, by section formula, the coordinates of point R are given by

$$\left(\frac{k(8)+2}{k+1},\frac{k(0)-3}{k+1},\frac{k(10)+4}{k+1}\right)$$
$$\Rightarrow\left(\frac{8k+2}{k+1},\frac{-3}{k+1},\frac{10k+4}{k+1}\right)$$

It is given that the $x-$coordinate of point R is 4.

Hence,

$$\frac{8k+2}{k+1}=4$$
$$8k+2=4k+4$$
$$4k=2$$
$$k=\frac{1}{2}$$

Therefore, the coordinates of point R are

$$\left(4,\frac{-3}{\frac{1}{2}+1},\frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1}\right)$$
$$\Rightarrow(4,-2,6)$$

Thus, the coordinates of point R are $(4,-2,6)$.

Question 6:

If A and B be the points $(3,4,5)$ and $(-1,3,-7)$ respectively, find the equation of the set of points P such that $PA^2+PB^2=k^2$, where k is a constant.

Solution:

The coordinates of points A and B are given as $A(3,4,5)$ and $B(-1,3,-7)$ respectively.

Let the coordinates of point P be (x,y,z).
On using distance formula, we obtain

$$\begin{aligned} PA^2 &= (x-3)^2 + (y-4)^2 + (z-5)^2 \\ &= x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z \\ &= x^2 - 6x + y^2 - 8y + z^2 - 10z + 50 \end{aligned}$$

$$\begin{aligned} PB^2 &= (x+1)^2 + (y-3)^2 + (z+7)^2 \\ &= x^2 + 1 + 2x + y^2 + 9 - 6y + z^2 + 49 + 14z \\ &= x^2 + 2x + y^2 - 6y + z^2 + 14z + 59 \end{aligned}$$

Now, if $PA^2 + PB^2 = k^2$, then,

$$\Rightarrow \left(x^2 - 6x + y^2 - 8y + z^2 - 10z + 50\right) + \left(x^2 + 2x + y^2 - 6y + z^2 + 14z + 59\right) = k^2$$

$$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$$

$$\Rightarrow 2\left(x^2 + y^2 + z^2 - 2x - 7y + 2z\right) = k^2 - 109$$

$$\Rightarrow x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$$

Thus, the required equation is $x^2 + y^2 + z^2 - 2x - 7y + 2z = \dfrac{k^2 - 109}{2}$.