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NCERT Solutions Class 11 Maths Chapter 2 Relations and Functions

Question 1:

If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$, find the values of x and y.

Solution:

It is given that $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$.
Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore, $\frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$

$$\frac{x}{3}+1=\frac{5}{3}$$

$$\Rightarrow \frac{x}{3}=\frac{5}{3}-1$$

$$\Rightarrow \frac{x}{3}=\frac{5-3}{3} \qquad y-\frac{2}{3}=\frac{1}{3}$$

$$\Rightarrow \frac{x}{3}=\frac{2}{3} \qquad \Rightarrow y=\frac{1}{3}+\frac{2}{3}$$

$$\Rightarrow x=2 \quad \text{and} \quad \Rightarrow y=1$$

Hence, $x=2$ and $y=1$

Question 2:

If the set A has 3 elements and the set $B=\{3,4,5\}$, then find the number of elements in $(A\times B)$?

Solution:

It is given that set A has 3 elements and the set $B=\{3,4,5\}$

Number of elements in set A, $n(A)=3$

Number of elements in set B, $n(B)=3$

Number of elements in $(A\times B)$ = (Number of elements in A)×(Number of elements in B)

$$\begin{aligned} n(A\times B) &= n(A)\times n(B) \\ &= 3\times 3 \\ &= 9 \end{aligned}$$

Thus, the number of elements in $(A\times B)$ is 9.

Question:3

If $G=\{7,8\}$ and $H=\{5,4,2\}$, find $G\times H$ and $H\times G$.

Solution:

It is given that $G=\{7,8\}$ and $H=\{5,4,2\}$

We know that the Cartesian product $P\times Q$ of two non-empty sets P and Q is defined as

$$P\times Q=\{(p,q):p\in P,q\in Q\}$$

Therefore,

$$G\times H=\{(7,5),(7,4),(7,2),(8,5),(8,4),(8,2)\}$$
$$H\times G=\{(5,7),(5,8),(4,7),(4,8),(2,7),(2,8)\}$$

Question 4:

State whether each of the following statements are true or false. If the statement is false, rewrite the given statement correctly.

(i) If $P=\{m,n\}$ and $Q=\{n,m\}$, then $P\times Q=\{(m,n),(n,m)\}$

(ii) If A and B are non-empty sets, then $(A\times B)$ is a non-empty set of ordered pairs such that $x\in A$ and $y\in B$.

(iii) If $A=\{1,2\}$, $B=\{3,4\}$, then $A\times(B\cap\phi)=\phi$

Solution:

(i) False, $P\times Q=\{(m,n),(m,m),(n,n),(n,m)\}$
(ii) True
(iii) True

Question 5:

If $A=\{-1,1\}$, find $A\times A\times A$.

Solution:

It is known that for any non-empty set A, $A\times A\times A$ is defined as

$$A\times A\times A=\{(a,b,c):a,b,c\in A\}$$

It is given that $A=\{-1,1\}$
Therefore,
$A\times A\times A=\{(-1,-1,-1),(-1,-1,1),(-1,1,-1),(-1,1,1),(1,-1,-1),(1,-1,1),(1,1,-1),(1,1,1)\}$

Question 6:

If $A\times B=\{(a,x),(a,y),(b,x),(b,y)\}$. Find A and B.

Solution:

It is given that $A\times B=\{(a,x),(a,y),(b,x),(b,y)\}$
We know that the Cartesian product of two non-empty sets P and Q is defined as

$$P\times Q=\{(p,q):p\in P,q\in Q\}$$

Therefore, A is the set of all first elements and B is the set of all second elements.

Thus, $A=\{a,b\}$ and $B=\{x,y\}$

Question 7:

Let $A=\{1,2\}$, $B=\{1,2,3,4\}$, $C=\{5,6\}$ and $D=\{5,6,7,8\}$. Verify that

(i) $A\times(B\cap C)=(A\times B)\cap(A\times C)$.

(ii) $A\times C$ is a subset of $B\times D$.

Solution:

(i) To verify: $A\times(B\cap C)=(A\times B)\cap(A\times C)$

We have $B\cap C=\{1,2,3,4\}\cap\{5,6\}=\phi$

$$\begin{aligned} LHS &= A\times(B\cap C)\\ &= A\times\phi\\ &=\phi \end{aligned}$$

Now,

$$A\times B=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4)\}$$
$$A\times C=\{(1,5),(1,6),(2,5),(2,6)\}$$

Therefore,

$$\begin{aligned} RHS &= (A\times B)\cap(A\times C)\\ &=\phi \end{aligned}$$

Therefore, L.H.S. = R.H.S

Hence, $A\times(B\cap C)=(A\times B)\cap(A\times C)$

(ii) To verify: $A\times C$ is a subset of $B\times D$

We have

$$A \times C = \{(1,5),(1,6),(2,5),(2,6)\}$$

$$B \times D = \begin{Bmatrix} (1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8), \\ (3,5),(3,6),(3,7),(3,8),(4,5),(4,6),(4,7),(4,8) \end{Bmatrix}$$

We can observe that all the elements of set $A \times C$ are the elements of set $B \times D$.
Therefore, $A \times C$ is a subset of $B \times D$.

Question 8:

Let $A = \{1,2\}$ and $B = \{3,4\}$. Write $A \times B$. How many subsets will $A \times B$ have? List them.

Solution:

$A = \{1,2\}$ and $B = \{3,4\}$

Therefore, $A \times B = \{(1,3),(1,4),(2,3),(2,4)\}$

Hence, $n(A \times B) = 4$

We know that if C is a set with $n(C) = m$, then $n[P(C)] = 2^m$.

Therefore, the set $A \times B$ has $2^4 = 16$ subsets. These are

$$\begin{Bmatrix} \phi, \{(1,3)\}, \{(1,4)\}, \{(2,3)\}, \{(2,4)\}, \\ \{(1,3),(1,4)\}, \{(1,3),(2,3)\}, \{(1,3),(2,4)\}, \\ \{(1,4),(2,3)\}, \{(1,4),(2,4)\}, \{(2,3),(2,4)\}, \\ \{(1,3),(1,4),(2,3)\}, \{(1,3),(1,4),(2,4)\}, \\ \{(1,3),(2,3),(2,4)\}, \{(1,4),(2,3),(2,4)\}, \{(1,3),(1,4),(2,3),(2,4)\} \end{Bmatrix}$$

Question 9:

Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x,1),(y,2),(z,1)$ are in $A \times B$, find A and B, where x, y and z are distinct elements.

Solution:

It is given that $n(A) = 3$ and $n(B) = 2$; and $(x,1),(y,2),(z,1)$ are in $A \times B$.

We know that
A = Set of first elements of the ordered pair elements of $A \times B$.
B = Set of second elements of the ordered pair elements of $A \times B$.

Therefore, x, y, and z are the elements of A; 1 and 2 are the elements of B.

Since $n(A)=3$ and $n(B)=2$,

It is clear that $A=\{x,y,z\}$ and $B=\{1,2\}$.

Question 10:

The Cartesian product $A\times A$ has 9 elements among which are found $(-1,0)$ and $(0,1)$. Find the set A and the remaining elements of $A\times A$.

Solution:

We know that if $n(A)=p$ and $n(B)=q$, then $n(A\times B)=pq$.

Therefore, $n(A\times A)=n(A)\times n(A)$

It is given that $n(A\times A)=9$

$$n(A)\times n(A)=9$$

Hence,

$$n(A)=3$$

The ordered pairs $(-1,0)$ and $(0,1)$ are two of the nine elements of $A\times A$.

We know that $A\times A=\{(a,a):a\in A\}$.

Therefore, –1, 0, and 1 are elements of A.

Since $n(A)=3$, it is clear that $A=\{-1,0,1\}$.

The remaining elements of set $A\times A$ are $(-1,-1),(-1,1),(0,-1),(0,0),(1,-1),(1,0)$, and $(1,1)$

EXERCISE 2.2

Question 1:

Let $A=\{1,2,3,\ldots,14\}$. Define a relation R from A to A by $R=\{(x,y),3x-y=0;x,y\in A\}$. Write down its domain, co-domain and range.

Solution:

The relation R from A to A is given as $R=\{(x,y),3x-y=0;x,y\in A\}$.

Thus, $R=\{(x,y),3x=y;x,y\in A\}$.

Therefore, $R=\{(1,3),(2,6),(3,9),(4,12)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation.

Hence, Domain of $R=\{1,2,3,4\}$

The whole set A is the co-domain of the relation R.

Therefore, Co-domain of $R=A=\{1,2,3,\ldots,14\}$

The range of R is the set of all second elements of the ordered pairs in the relation.

Therefore, Range of $R=\{3,6,9,12\}$

Question 2:

Define a relation R on the set N of natural numbers by

$\mathrm{R}=\{(x,\ y):\ y=x\ +\ 5, x \text{ is a natural number less than } 4; x,\ y\in \mathrm{N}\}$.

Depict this relationship using roster form. Write down the domain and the range.

Solution:

$R=\{(x,\ y):\ y=x\ +\ 5, x \text{ is a natural number less than } 4; x,\ y\in N\}$

The natural numbers less than 4 are 1, 2, and 3.

Therefore, $\mathrm{R}=\{(1,6),(2,7),(3,8)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation.

Hence, Domain of $\mathrm{R}=\{1,2,3\}$.

The range of R is the set of all second elements of the ordered pairs in the relation.

Therefore, Range of $R=\{6,7,8\}$.

Question 3:

$A=\{1,2,3,5\}$ and $B=\{4,6,9\}$. Define a relation R from A to B by $R=\{(x,y):\text{the difference between } x \text{ and } y \text{ is odd}; x\in A, y\in B\}$. Write R in roster form.

Solution:

$A=\{1,2,3,5\}$, $B=\{4,6,9\}$ and $R=\{(x,y):\text{the difference between } x \text{ and } y \text{ is odd}; x\in A, y\in B\}$

Therefore, $R=\{(1,4),(1,6),(2,9),(3,4),(3,6),(5,4),(5,6)\}$

Question 4:

The Fig2.7 shows a relationship between the sets P and Q. Write this relation
(i) in set-builder form (ii) in roster form.

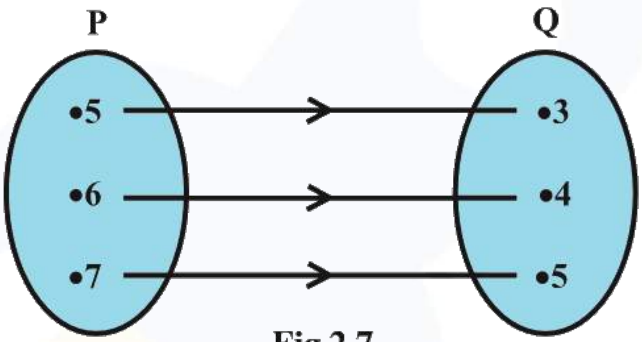

Fig 2.7

What is its domain and range?

Solution:

As per the Fig2.7, $P=\{5,6,7\}$ and $Q=\{3,4,5\}$

(i) $R=\{(x,y): y=x-2; x\in P\}$ or $R=\{(x,y): y=x-2, \text{for } x=5,6,7\}$

(ii) $R=\{(5,3),(6,4),(7,5)\}$

Domain of $R=\{5,6,7\}$

Range of $R=\{3,4,5\}$

Question 5:

Let $A=\{1,2,3,4,6\}$. Let R be the relation on A defined by $\{(a,b): a,b\in \text{A}, b \text{ is exactly divisible by } a\}$

(i) Write R in roster form
(ii) Find the domain of R
(iii) Find the range of R.

Solution:

It is given that $A=\{1,2,3,4,6\}$ and $R=\{(a,b):a,b\in \text{A}, b \text{ is exactly divisible by } a\}$

(i) $R=\{(1,1),(1,2),(1,3),(1,4),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(6,6)\}$

(ii) Domain of $R=\{1,2,3,4,6\}$

(iii) Range of $R=\{1,2,3,4,6\}$

Question 6:

Determine the domain and range of the relation R defined by
$R=\{(x,x+5):x\in\{0,1,2,3,4,5\}\}$.

Solution:

It is given that $R=\{(x,x+5):x\in\{0,1,2,3,4,5\}\}$

Therefore, $R=\{(0,5),(1,6),(2,7),(3,8),(4,9),(5,10)\}$

Hence,

Domain of $R=\{0,1,2,3,4,5\}$

Range of $R=\{5,6,7,8,9,10\}$

Question 7:

Write the relation $R=\{(x,x^3):x \text{ is a prime number less than } 10\}$ in roster form.

Solution:

It is given that $R=\{(x,x^3):x \text{ is a prime number less than } 10\}$
The prime numbers less than 10 are 2, 3, 5, and 7.

Therefore, $R=\{(2,8),(3,27),(5,125),(7,343)\}$

Question 8:

Let $A=\{x,y,z\}$ and $B=\{1,2\}$. Find the number of relations from A to B.

Solution:

It is given that $A=\{x,y,z\}$ and $B=\{1,2\}$.

Therefore, $A\times B=\{(x,1),(x,2),(y,1),(y,2),(z,1),(z,2)\}$

Since $n(A\times B)=6$, the number of subsets of $(A\times B)=2^6$.

Hence, the number of relations from A to B is 2^6.

Question 9:

Let R be the relation on **Z** defined by $R=\{(a,b):a,b\in\mathbf{Z}, a-b \text{ is an integer}\}$. Find the domain and range of R.

Solution:

It is given that $R=\{(a,b):a,b\in\mathbf{Z}, a-b \text{ is an integer}\}$

It is known that the difference between any two integers is always an integer.

Therefore,

Domain of $R=\mathbf{Z}$

Range of $R=\mathbf{Z}$

EXERCISE 2.3

Question 1:
Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$

(ii) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$

(iii) $\{(1,3),(1,5),(2,5)\}$

Solution:

(i) $\{(2,1),(5,1),(8,1),(11,1),(14,1),(17,1)\}$

Since $2,5,8,11,14,\text{and } 17$ are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain $=\{2,5,8,11,14,17\}$ and range $=\{1\}$

(ii) $\{(2,1),(4,2),(6,3),(8,4),(10,5),(12,6),(14,7)\}$

Since $2,4,6,8,10,12 \text{ and } 14$ are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain $=\{2,4,6,8,10,12,14\}$ and range $=\{1,2,3,4,5,6,7\}$

(iii) $\{(1,3),(1,5),(2,5)\}$

Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

Question 2:
Find the domain and range of the following real function:

(i) $f(x)=-|x|$ (ii) $f(x)=\sqrt{9-x^2}$

Solution:

(i) $f(x)=-|x|, x\in \mathbf{R}$

We know that, $|x|=\begin{cases} x, & \text{if } x\geq 0 \\ -x, & \text{if } x<0 \end{cases}$

Therefore, $f(x)=|-x|=\begin{cases} -x, & \text{if } x\geq 0 \\ x, & \text{if } x<0 \end{cases}$

Since $f(x)$ is defined for $x\in\mathbf{R}$, the domain of $f=\mathbf{R}$

It can be observed that the range of $f(x) = -|x|$ is all real numbers except positive real numbers.

Therefore, the range of is $f = (-\infty, 0]$

(ii) $f(x) = \sqrt{9 - x^2}$

Since $\sqrt{9 - x^2}$ is defined for all real numbers that are greater than or equal to -3 and less than or equal to 3, the domain of $f(x)$ is $\{x : -3 \le x \le 3\}$ or $[-3, 3]$.

For any value of x such that $-3 \le x \le 3$, the value of $f(x)$ will lie between 0 and 3.

Therefore, the range of $f(x)$ is $\{x : 0 \le x \le 3\}$ or $[0, 3]$

Question 3:

A function f is defined by $f(x) = 2x - 5$. Write down the values of

(i) $f(0)$ (ii) $f(7)$ (iii) $f(-3)$

Solution:

The given function is $f(x) = 2x - 5$.
Therefore,

(i) $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$

(ii) $f(7) = 2 \times 7 - 5 = 14 - 5 = 9$

(iii) $f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

Question 4:

The function 't' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$.

Find (i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) The value of C, when $t(C) = 212$.

Solution:

The given function is $t(C) = \frac{9C}{5} + 32$
Therefore,

(i) $t(0)=\dfrac{9\times 0}{5}+32=0+32=32$

(ii) $t(28)=\dfrac{9\times 28}{5}+32=\dfrac{252+160}{5}=\dfrac{412}{5}=82.4$

(iii) $t(-10)=\dfrac{9\times(-10)}{5}+32=9\times(-2)+32=-18+32=14$

(iv) It is given that

$$t(C)=212$$
$$\Rightarrow \frac{9C}{5}+32=212$$
$$\Rightarrow \frac{9C}{5}=212-32$$
$$\Rightarrow C=\frac{180\times 5}{9}=100$$

Thus, the value of 't', when $t(C)=212$ is 100.

Question 5:

Find the range of each of the following functions.

(i) $f(x)=2-3x, x\in R, x>0.$

(ii) $f(x)=x^2+2, x$ is a real number.

(iii) $f(x)=x, x$ is a real number.

Solution:

(i) $f(x)=2-3x, x\in \mathbf{R}, x>0.$

The values of $f(x)$ for various values of real numbers $x>0$ can be written in the tabular form as

x	0.01	0.1	0.9	1	2	2.5	4	5	…
$f(x)$	1.97	1.7	-0.7	-1	-4	-5.5	-10	-13	…

Thus, it can be clearly observed that the range of f is the set of all real numbers less than 2. i.e., range of $f=(-\infty,2)$

Alternative Method:

Let $x>0$

$$\Rightarrow 3x > 0$$
$$\Rightarrow 2 - 3x < 2$$
$$\Rightarrow f(x) < 2$$

Therefore, Range of $f = (-\infty, 2)$

(ii) $f(x) = x^2 + 2, x$ is a real number.

The values of f(x) for various values of real numbers x can be written in the tabular form as

x	0	± 0.3	± 0.8	± 1	± 2	± 3	...
$f(x)$	2	2.09	2.64	3	6	11	...

Thus, it can be clearly observed that the range of f is the set of all real numbers greater than 2. i.e., range of $f = [2, \infty)$

Alternative Method:

Let x be any real number i.e., $x^2 \geq 0$.
Accordingly,

$$x^2 \geq 0$$
$$\Rightarrow x^2 + 2 \geq 0 + 2$$
$$\Rightarrow x^2 + 2 \geq 2$$
$$\Rightarrow f(x) \geq 2$$

Therefore, Range of $f = [2, \infty)$

(iii) $f(x) = x, x$ is a real number
It is clear that, the range of f is the set of all real numbers.
Therefore, Range of $f = \mathbf{R}$.

MISCELLANEOUS EXERCISE

Question 1:

The relation f is defined by $f(x)=\begin{cases} x^2, 0 \le x \le 3 \\ 3x, 3 \le x \le 10 \end{cases}$

The relation g is defined by $g(x)=\begin{cases} x^2, 0 \le x \le 2 \\ 3x, 2 \le x \le 10 \end{cases}$

Show that f is a function and g is not a function.

Solution:

The relation f is defined as $f(x)=\begin{cases} x^2, 0 \le x \le 3 \\ 3x, 3 \le x \le 10 \end{cases}$

It can be observed that for

$0 \le x < 3, f(x)=x^2$ and

$3 < x \le 10, f(x)=3x$

Also, at $x=3$

$$f(x)=3^2$$
$$=9$$

and

$$f(x)=3\times 3$$
$$=9$$

i.e., at $x=3, f(x)=9$

Therefore, for $0 \le x \le 10$, the images of $f(x)$ are unique.
Thus, the given relation is a function.

The relation g is defined as $g(x)=\begin{cases} x^2, 0 \le x \le 2 \\ 3x, 2 \le x \le 10 \end{cases}$

It can be observed that for

$0 \le x \le 2, g(x)=x^2$ and

$2 \le x \le 10, g(x)=3x$

Also, at $x = 2$

$$g(x) = 2^2$$
$$= 4$$

and

$$g(x) = 3 \times 2$$
$$= 6$$

Hence, element 2 of the domain of the relation g corresponds to two different images i.e., 4 and 6.
Hence, this relation is not a function.

Thus, f is a function and g is not a function.

Question 2:

If $f(x) = x^2$, find $\dfrac{f(1.1) - f(1)}{(1.1 - 1)}$.

Solution:

It is given that $f(x) = x^2$
Therefore,

$$\frac{f(1.1) - f(1)}{(1.1 - 1)} = \frac{(1.1)^2 - (1)^2}{(1.1 - 1)}$$
$$= \frac{1.21 - 1}{0.1}$$
$$= \frac{0.21}{0.1}$$
$$= 2.1$$

Question 3:

Find the domain of the function $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - 8x + 12}$

Solution:

The given function is $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - 8x + 12}$.

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$$
$$= \frac{x^2 + 2x + 1}{(x - 6)(x - 2)}$$

It can be seen that, the function f is defined for all real numbers except at $x=6$ and $x=2$.

Hence, the domain of the function f is set of real numbers except 6 and 2. i.e., $\mathbf{R}-\{2,6\}$.

Question 4:

Find the domain and the range of the real function f defined by $f(x)=\sqrt{(x-1)}$.

Solution:

The given real function is $f(x)=\sqrt{(x-1)}$.

It can be seen that $\sqrt{(x-1)}$ is defined for $x \geq 1$.

Therefore, the domain of f is the set of all real numbers greater than or equal to 1 i.e., the domain of $f=[1,\infty)$.

As

$$\begin{aligned}&\Rightarrow x \geq 1\\&\Rightarrow (x-1) \geq 0\\&\Rightarrow \sqrt{(x-1)} \geq 0\end{aligned}$$

Therefore, the range of f is the set of all real numbers greater than or equal to 0 i.e., the range of $f=[0,\infty)$.

Question 5:

Find the domain and the range of the real function f defined by $f(x)=|x-1|$.

Solution:

The given real function is $f(x)=|x-1|$

It is clear that $|x-1|$ is defined for all real numbers.

Therefore, Domain of $f=\mathbf{R}$

Also, for $x \in \mathbf{R}, |x-1|$ assumes all real numbers.

Hence, the range of f is the set of all non-negative real numbers.

Question 6:

Let $f=\left\{\left(x,\frac{x^2}{1+x^2}\right):x\in R\right\}$ be a function from **R** into **R**. Determine the range of f.

Solution:

It is given that $f=\left\{\left(x,\frac{x^2}{1+x^2}\right):x\in R\right\}$

Therefore, $\left\{(0,0),\left(\pm0.5,\frac{1}{5}\right),\left(\pm1,\frac{1}{2}\right),\left(\pm1.5,\frac{9}{13}\right),\left(\pm2,\frac{4}{5}\right),\left(\pm3,\frac{9}{10}\right),\left(\pm4,\frac{16}{17}\right),\ldots\right\}$

The range of f is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1 (denominator is greater than numerator).

Thus, range of $f=[0,\ 1)$

Question 7:

Let $f,g:\mathbf{R}\to\mathbf{R}$ be defined, respectively by $f(x)=x+1,\ g(x)=2x-3$. Find $f+g,\ f-g$ and $\frac{f}{g}$.

Solution:

Since $f,g:\mathbf{R}\to\mathbf{R}$ is defined as $f(x)=x+1,\ g(x)=2x-3$
Hence,

$$\begin{aligned}(f+g)(x)&=f(x)+g(x)\\&=(x+1)+(2x-3)\\&=3x-2\end{aligned}$$

Therefore, $(f+g)(x)=3x-2$

Now,

$$\begin{aligned}(f-g)(x)&=f(x)-g(x)\\&=(x+1)-(2x-3)\\&=x+1-2x+3\\&=-x+4\end{aligned}$$

Therefore, $(f-g)(x)=-x+4$

Now,

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}, g(x)\neq 0, x\in R$$
$$=\frac{x+1}{2x-3}, 2x-3, x\neq\frac{3}{2}$$

Therefore, $\left(\frac{f}{g}\right)(x)=\frac{x+1}{2x-3}, x\neq\frac{3}{2}$

Question 8:

Let $f=\{(1,1),(2,3),(0,-1),(-1,-3)\}$ be a function from **Z** to **Z** defined by $f(x)=ax+b,$ for some integers *a*, *b*. Determine *a*, *b*.

Solution:

It is given that $f=\{(1,1),(2,3),(0,-1),(-1,-3)\}$ and $f(x)=ax+b$

$(1,\ 1)\in f \Rightarrow f(1)=1 \Rightarrow a\times 1+b=1$
$\Rightarrow a+b=1$
$(0,-1)\in f \Rightarrow f(0)=-1 \Rightarrow a\times 0+b=-1$
$\Rightarrow b=-1$

On substituting $b=-1$ in $a+b=1$, we obtain

$$a+(-1)=1$$
$$a=1+1$$
$$=2$$

Thus, the respective values of *a* and *b* are 2 and –1.

Question 9:

Let R be a relation from **N** to **N** defined by $R=\{(a,b): a,b\in \mathbf{N} \text{ and } a=b^2\}$. Are the following true?

(i) $(a,a)\in R,$ for all $a\in \mathbf{N}$

(ii) $(a,\ b)\in R,$ implies $(b,a)\in R$

(iii) $(a,b)\in R, (b,c)\in R$ implies $(a,c)\in R$

Justify your answer in each case.

Solution:

It is given that $R=\{(a,b): a,b\in \mathbf{N} \text{ and } a=b^2\}$

(i) It can be seen that $2\in \mathbf{N}$; however, $2\neq 2^2=4$

Therefore, the statement "$(a,a)\in R,$ for all $a\in \mathbf{N}$" is not true.

(ii) It can be seen that $(9,3)\in \mathbf{N}$ because $9,3\in \mathbf{N}$ and $9=3^2$.

Now, $3\neq 9^2=81$; therefore, $(3,9)\notin \mathbf{N}$

Therefore, the statement "$(a,\ b)\in R,$ implies $(b,a)\in R$" is not true.

(iii) It can be seen that $(9,3)\in R,(16,\ 4)\in R$ because $9,3,16,4\in \mathbf{N}$ and $9=3^2, 16=4^2$

Now, $9\neq 4^2=16$; therefore, $(9,4)\notin \mathbf{N}$.

Therefore, the statement "$(a,b)\in R,(b,c)\in R$ implies $(a,c)\in R$" is not true.

Question 10:

Let $A=\{1,2,3,4\}$, $B=\{1,5,9,11,15,16\}$ and $f=\{(1,5),(2,9),(3,1),(4,5),(2,11)\}$. Are the following are true?

(i) f is a relation from A to B (ii) f is a function from A to B.

Justify your answer in each case.

Solution:

It is given that $A=\{1,2,3,4\}$, $B=\{1,5,9,11,15,16\}$ and $f=\{(1,5),(2,9),(3,1),(4,5),(2,11)\}$.

Therefore,

$$A\times B=\begin{Bmatrix}(1,1),(1,5),(1,9),(1,11),(1,15),(1,16),(2,1),(2,5),(2,9),\\(2,11),(2,15),(2,16),(3,1),(3,5),(3,9),(3,11),(3,15),\\(3,16),(4,1),(4,\ 5),(4,9),(4,11),(4,15),(4,16)\end{Bmatrix}$$

(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product $A\times B$.

It is observed that f is a subset of $A\times B$.

Thus, f is a relation from A to B.

(ii) Since the same first element i.e., 2 corresponds to two different images i.e., 9 and 11.

Thus, f is not a function from A to B.

Question 11:

Let f be the subset of $\mathbf{Z}\times\mathbf{Z}$ defined by $f=\{(ab,\ a+b):a,b\in\mathbf{Z}\}$. Is f a function from $\mathbf{Z}$ to $\mathbf{Z}$? Justify your answer.

Solution:

The relation f is defined as $f=\{(ab,\ a+b):a,b\in\mathbf{Z}\}$.

We know that a relation f from a set A to a set B is said to be a function if every element of set A has unique images in set B.

Since, $2,6,-2,-6\in\mathbf{Z},(2\times6,2+6),(-2\ \times-6,-2\ +(-6))\in f$
i.e., $(12,8),(12,-8)\in f$

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and –8.

Thus, relation f is not a function.

Question 12:

Let $A=\{9,10,11,12,13\}$ and let $f:A\rightarrow\mathbf{N}$ be defined by $f(n)=$ the highest prime factor of n. Find the range of f.

Solution:

It is given that $A=\{9,10,11,12,13\}$ and $f:A\rightarrow\mathbf{N}$ is defined by
$f(n)=$ the highest prime factor of n
Hence,

Prime factor of $9=3$
Prime factor of $10=2,5$
Prime factor of $11=11$
Prime factor of $12=2,3$
Prime factor of $13=13$

Therefore,

$f(9)=$ the highest prime factor of $9=3$
$f(10)=$ the highest prime factor of $10=5$
$f(11)=$ the highest prime factor of $11=11$
$f(12)=$ the highest prime factor of $12=3$

$f(13) =$ the highest prime factor of $13 = 13$

The range of f is the set of all $f(n)$, where $n \in A$.

Therefore, Range of $f = \{3, 5, 11, 13\}$

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