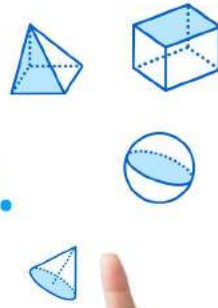


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## NCERT Solutions Class 11 Maths Chapter 3 Exercise 3.3

Prove that:

**Question 1:**

$$\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = -\frac{1}{2}$$

**Solution:**

$$\begin{aligned} LHS &= \sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} \\ &= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2 \\ &= \frac{1}{4} + \frac{1}{4} - 1 \\ &= \frac{1+1-4}{4} \\ &= \frac{-2}{4} \\ &= -\frac{1}{2} = RHS \end{aligned}$$

**Question 2:**

$$2\sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} = \frac{3}{2}$$

**Solution:**

$$\begin{aligned} LHS &= 2\sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} \\ &= 2\left(\frac{1}{2}\right)^2 + \operatorname{cosec}^2 \left(\pi + \frac{\pi}{6}\right) \times \left(\frac{1}{2}\right)^2 \\ &= 2 \times \frac{1}{4} + \left(-\operatorname{cosec} \frac{\pi}{6}\right)^2 \times \frac{1}{4} \\ &= \frac{1}{2} + (-2)^2 \times \frac{1}{4} \\ &= \frac{1}{2} + 1 \\ &= \frac{1+2}{2} \\ &= \frac{3}{2} = RHS \end{aligned}$$

**Question 3:**

$$\cot^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{5\pi}{6} + 3 \tan^2 \frac{\pi}{6} = 6$$

**Solution:**

$$\begin{aligned} LHS &= \cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3 \tan^2 \frac{\pi}{6} \\ &= (\sqrt{3})^2 + \operatorname{cosec} \left( \pi - \frac{\pi}{6} \right) + 3 \left( \frac{1}{\sqrt{3}} \right)^2 \\ &= 3 + \operatorname{cosec} \frac{\pi}{6} + 3 \times \frac{1}{3} \\ &= 3 + 2 + 1 \\ &= 6 = RHS \end{aligned}$$

**Question 4:**

$$2 \sin^2 \frac{3\pi}{4} + 2 \cos^2 \frac{\pi}{4} + 2 \sec^2 \frac{\pi}{3} = 10$$

**Solution:**

$$\begin{aligned} LHS &= 2 \sin^2 \frac{3\pi}{4} + 2 \cos^2 \frac{\pi}{4} + 2 \sec^2 \frac{\pi}{3} \\ &= 2 \sin^2 \left( \pi - \frac{\pi}{4} \right) + 2 \times \left( \frac{1}{\sqrt{2}} \right)^2 + 2 \times (2)^2 \\ &= 2 \sin^2 \frac{\pi}{4} + 2 \times \frac{1}{2} + 2 \times 4 \\ &= 2 \times \left( \frac{1}{\sqrt{2}} \right)^2 + 1 + 8 \\ &= 1 + 9 \\ &= 10 = RHS \end{aligned}$$

**Question 5:**

Find the value of:

(i)  $\sin 75^\circ$

(ii)  $\tan 15^\circ$

**Solution:**

(i)  $\sin 75^\circ$

$$\begin{aligned} \sin 75^\circ &= \sin(45^\circ + 30^\circ) \\ &= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ && [\because \sin(x+y) = \sin x \cos y + \cos x \sin y] \\ &= \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{1}{2}\right) \\ &= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \\ &= \frac{\sqrt{3}+1}{2\sqrt{2}} \end{aligned}$$

(ii)  $\tan 15^\circ$

$$\begin{aligned} \tan 15^\circ &= \tan(45^\circ - 30^\circ) \\ &= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} && [\because \tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}] \\ &= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}} \\ &= \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} \\ &= \frac{\sqrt{3}-1}{\sqrt{3}+1} \\ &= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} && [\text{By rationalizing}] \\ &= \frac{(\sqrt{3}-1)^2}{3-1} \\ &= \frac{3+1-2\sqrt{3}}{2} \\ &= \frac{2(2-\sqrt{3})}{2} \\ &= 2-\sqrt{3} \end{aligned}$$

Prove the following:

**Question 6:**

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$$

**Solution:**

$$\begin{aligned} LHS &= \cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right) \\ &= \frac{1}{2}\left[2\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)\right]+\frac{1}{2}\left[-2\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)\right] \\ &= \left(\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}+\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]\right. \\ &\quad \left.+\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}-\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]\right) \\ &\quad \left[\begin{array}{l} \because 2\cos A\cos B=\cos(A+B)+\cos(A-B) \\ -2\sin A\sin B=\cos(A+B)-\cos(A-B) \end{array}\right] \end{aligned}$$

$$= \sin(x+y) = RHS$$

$$\begin{aligned} &= \left(\frac{1}{2}\left[\cos\left\{\frac{\pi}{4}-x+\frac{\pi}{4}-y\right\}+\cos\left\{\frac{\pi}{4}-x-\frac{\pi}{4}+y\right\}\right]\right. \\ &\quad \left.+\frac{1}{2}\left[\cos\left\{\frac{\pi}{4}-x+\frac{\pi}{4}-y\right\}-\cos\left\{\frac{\pi}{4}-x-\frac{\pi}{4}+y\right\}\right]\right) \\ &= \frac{1}{2}\left[\cos\left\{\frac{\pi}{2}-(x+y)\right\}+\cos\left\{-(x-y)\right\}+\cos\left\{\frac{\pi}{2}-(x+y)\right\}-\cos\left\{-(x-y)\right\}\right] \\ &= \frac{1}{2}\left[2\cos\left\{\frac{\pi}{2}-(x+y)\right\}\right] \end{aligned}$$

$$= \sin(x+y) \quad \left[\because \cos\left(\frac{\pi}{2}-A\right)=\sin A\right]$$

$$= RHS$$

**Question 7:**

$$\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$$

**Solution:**

$$LHS = \frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)}$$

$$= \frac{\left(\frac{\tan \frac{\pi}{4} + \tan x}{1 - \tan \frac{\pi}{4} \tan x}\right)}{\left(\frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \tan x}\right)}$$

$$= \frac{\left(\frac{1 + \tan x}{1 - \tan x}\right)}{\left(\frac{1 - \tan x}{1 + \tan x}\right)}$$

$$= \left(\frac{1 + \tan x}{1 - \tan x}\right) \times \left(\frac{1 + \tan x}{1 - \tan x}\right)$$

$$= \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$$

$$= RHS$$

$$\left[ \begin{array}{l} \because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ \& \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \end{array} \right]$$

**Question 8:**

$$\frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)} = \cot^2 x$$

**Solution:**

$$LHS = \frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)}$$

$$= \frac{(-\cos x) \times (\cos x)}{(\sin x) \times (-\sin x)}$$

$$= \frac{-\cos^2 x}{-\sin^2 x}$$

$$= \left(\frac{\cos x}{\sin x}\right)^2$$

$$= \cot^2 x$$

$$= RHS$$

$$\left[ \begin{array}{l} \because \cos(\pi + x) = -\cos x \\ \Rightarrow \cos(-x) = \cos x \\ \Rightarrow \cos\left(\frac{\pi}{2} + x\right) = -\sin x \\ \Rightarrow \sin(\pi - x) = \sin x \end{array} \right]$$

$$\left[ \because \cot x = \frac{\cos x}{\sin x} \right]$$

**Question 9:**

$$\cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right] = 1$$

**Solution:**

$$LHS = \cos\left(\frac{3\pi}{2} + x\right) \cos(2\pi + x) \left[ \cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x) \right]$$

$$= \cos\left\{\pi + \left(\frac{\pi}{2} + x\right)\right\} \cos x \left[ \cot\left\{\pi + \left(\frac{\pi}{2} - x\right)\right\} + \cot x \right]$$

$$= -\cos\left(\frac{\pi}{2} + x\right) \cos x \left[ \cot\left(\frac{\pi}{2} - x\right) + \cot x \right]$$

$$= -(-\sin x) \cos x [\tan x + \cot x]$$

$$= \sin x \cos x \left[ \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right]$$

$$= \sin^2 x + \cos^2 x$$

$$= 1$$

$$= RHS$$

$$\left[ \begin{array}{l} \because \cos(2n\pi + \theta) = \cos \theta \\ \Rightarrow \cot(2n\pi + \theta) = \cot \theta \end{array} \right]$$

$$\left[ \begin{array}{l} \because \cos(\pi + \theta) = -\cos \theta \\ \Rightarrow \cot(\pi + \theta) = \cot \theta \end{array} \right]$$

$$\left[ \begin{array}{l} \because \cos\left(\frac{\pi}{2} + \theta\right) = -\sin \theta \\ \Rightarrow \cot(2n\pi + \theta) = \cos \theta \end{array} \right]$$

**Question 10:**

$$\sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x = \cos x$$

**Solution:**

$$LHS = \sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x$$

$$= \cos(n+2)x \cdot \cos(n+1)x + \sin(n+2)x \cdot \sin(n+1)x$$

$$= \cos\{(n+2)x - (n+1)x\}$$

$$[\because \cos(A-B) = \cos A \cos B + \sin A \sin B]$$

$$= \cos\{n+2-n-1\}x$$

$$= \cos x$$

$$= RHS$$

**Question 11:**

$$\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x$$

**Solution:**

$$\begin{aligned}LHS &= \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) \\&= -2 \sin\left(\frac{\left(\frac{3\pi}{4} + x\right) + \left(\frac{3\pi}{4} - x\right)}{2}\right) \sin\left(\frac{\left(\frac{3\pi}{4} + x\right) - \left(\frac{3\pi}{4} - x\right)}{2}\right) \\&\quad \left[\because \cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)\right] \\&= -2 \sin\left(\frac{\frac{3\pi}{4} + x + \frac{3\pi}{4} - x}{2}\right) \sin\left(\frac{\frac{3\pi}{4} + x - \frac{3\pi}{4} + x}{2}\right) \\&= -2 \sin\left(\frac{3\pi}{4}\right) \sin\left(\frac{2x}{2}\right) \\&= -2 \sin\left(\pi - \frac{\pi}{4}\right) \sin x \\&= -2 \sin \frac{\pi}{4} \sin x \quad \left[\because \sin(\pi - \theta) = \sin \theta\right] \\&= -2 \times \frac{1}{\sqrt{2}} \times \sin x \\&= -\sqrt{2} \sin x \\&= RHS\end{aligned}$$

**Question 12:**

$$\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$$

**Solution:**

$$LHS = \sin^2 6x - \sin^2 4x$$

$$= (\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$$

$$= \left[ 2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right) \right] \times \left[ 2\cos\left(\frac{6x+4x}{2}\right)\sin\left(\frac{6x-4x}{2}\right) \right]$$

$$\left[ \begin{array}{l} \because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\ \& \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \end{array} \right]$$

$$= [2\sin 5x \cos x] \times [2\cos 5x \sin x]$$

$$= [2\sin 5x \cos 5x] \times [2\sin x \cos x]$$

$$= [\sin(5x+5x) + \sin(5x-5x)] \times [\sin(x+x) + \sin(x-x)]$$

$$\left[ \because 2\sin A \cos B = \sin(A+B) + \sin(A-B) \right]$$

$$= [\sin 10x + \sin 0] \times [\sin 2x + \sin 0]$$

$$= [\sin 10x + 0] \times [\sin 2x + 0]$$

$$= \sin 2x \sin 10x$$

$$= RHS$$

### Question 13:

$$\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$$

#### Solution:

$$LHS = \cos^2 2x - \cos^2 6x$$

$$= (\cos 2x + \cos 6x)(\cos 2x - \cos 6x) \quad \left[ \because a^2 - b^2 = (a+b)(a-b) \right]$$

$$= \left[ 2 \cos \left( \frac{2x+6x}{2} \right) \cos \left( \frac{2x-6x}{2} \right) \right] \times \left[ -2 \sin \left( \frac{2x+6x}{2} \right) \sin \left( \frac{2x-6x}{2} \right) \right]$$

$$\left[ \begin{array}{l} \because \cos A + \cos B = 2 \cos \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \\ \& \cos A - \cos B = -2 \sin \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right) \end{array} \right]$$

$$= [2 \cos 4x \cos (-2x)] \times [-2 \sin 4x \sin (-2x)]$$

$$= [2 \cos 4x \cos 2x] \times [-2 \sin 4x (-\sin 2x)] \quad \left[ \begin{array}{l} \because \cos(-\theta) = \cos \theta \\ \& \sin(-\theta) = -\sin \theta \end{array} \right]$$

$$= [2 \cos 4x \cos 2x] \times [2 \sin 4x \sin 2x]$$

$$= [2 \cos 4x \sin 4x] \times [2 \cos 2x \sin 2x]$$

$$= [\sin(4x+4x) - \sin(4x-4x)] \times [\sin(2x+2x) - \sin(2x-2x)]$$

$$\left[ \because 2 \cos A \sin B = \sin(A+B) - \sin(A-B) \right]$$

$$= [\sin 8x - \sin 0] \times [\sin 4x - \sin 0]$$

$$= [\sin 8x - 0] \times [\sin 4x - 0]$$

$$= \sin 4x \sin 8x$$

$$= RHS$$

### Question 14:

$$\sin 2x + 2 \sin 4x + \sin 6x = 4 \cos^2 x \sin 4x$$

**Solution:**

$$\begin{aligned}
 LHS &= \sin 2x + 2 \sin 4x + \sin 6x \\
 &= [\sin 2x + \sin 6x] + 2 \sin 4x \\
 &= \left[ 2 \sin \left( \frac{2x+6x}{2} \right) \cos \left( \frac{2x-6x}{2} \right) \right] + 2 \sin 4x && \left[ \because \sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \right] \\
 &= [2 \sin 4x \cos(-2x)] + 2 \sin 4x \\
 &= 2 \sin 4x \cos 2x + 2 \sin 4x \\
 &= 2 \sin 4x (\cos 2x + 1) \\
 &= 2 \sin 4x (2 \cos^2 x - 1 + 1) && \left[ \because \cos 2x = 2 \cos^2 x - 1 \right] \\
 &= 2 \sin 4x (2 \cos^2 x) \\
 &= 4 \cos^2 x \sin 4x \\
 &= RHS
 \end{aligned}$$

**Question 15:**

$$\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$$

**Solution:**

$$\begin{aligned}
 LHS &= \cot 4x (\sin 5x + \sin 3x) \\
 &= \cot 4x \left[ 2 \sin \left( \frac{5x+3x}{2} \right) \cos \left( \frac{5x-3x}{2} \right) \right] && \left[ \because \sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \right] \\
 &= \frac{\cos 4x}{\sin 4x} [2 \sin 4x \cos x] && \left[ \because \cos 2x = 2 \cos^2 x - 1 \right] \\
 &= 2 \cos 4x \cos x \\
 &= 2 \cos 4x \cos x \times \frac{\sin x}{\sin x} \\
 &= \frac{\cos x}{\sin x} \times [2 \cos 4x \sin x] \\
 &= \cot x [\sin(4x+x) - \sin(4x-x)] && \left[ \because 2 \cos A \sin B = \sin(A+B) - \sin(A-B) \right] \\
 &= \cot x (\sin 5x - \sin 3x) \\
 &= RHS
 \end{aligned}$$

**Question 16:**

$$\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$$

**Solution:**

$$\begin{aligned}
 LHS &= \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} \\
 &= \frac{\left[ -2 \sin \left( \frac{9x+5x}{2} \right) \sin \left( \frac{9x-5x}{2} \right) \right]}{\left[ 2 \cos \left( \frac{17x+3x}{2} \right) \sin \left( \frac{17x-3x}{2} \right) \right]} \\
 &= \frac{[-2 \sin 7x \sin 2x]}{[2 \cos 10x \sin 7x]} \\
 &= -\frac{\sin 2x}{\cos 10x} \\
 &= RHS
 \end{aligned}$$

$$\begin{aligned}
 &\left[ \because \cos A - \cos B = -2 \sin \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right) \right] \\
 &\left[ \& \sin A - \sin B = 2 \cos \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right) \right]
 \end{aligned}$$

**Question 17:**

$$\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$$

**Solution:**

$$\begin{aligned}
 LHS &= \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} \\
 &= \frac{\left[ 2 \sin \left( \frac{5x+3x}{2} \right) \cos \left( \frac{5x-3x}{2} \right) \right]}{\left[ 2 \cos \left( \frac{5x+3x}{2} \right) \cos \left( \frac{5x-3x}{2} \right) \right]} \\
 &= \frac{\sin 4x}{\cos 4x} \\
 &= \tan 4x \\
 &= RHS
 \end{aligned}$$

$$\begin{aligned}
 &\left[ \because \sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \right] \\
 &\left[ \& \cos A + \cos B = 2 \cos \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \right]
 \end{aligned}$$

**Question 18:**

$$\frac{\sin x - \sin y}{\cos x + \cos y} = \tan \frac{x-y}{2}$$

**Solution:**

$$LHS = \frac{\sin x - \sin y}{\cos x + \cos y}$$

$$= \frac{\left[ 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right) \right]}{\left[ 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) \right]}$$

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$= \tan \frac{x-y}{2}$$

$$= RHS$$

$$\left[ \begin{array}{l} \because \sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right) \\ \& \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \end{array} \right]$$

**Question 19:**

$$\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$$

**Solution:**

$$LHS = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$$

$$= \frac{\left[ 2 \sin\left(\frac{x+3x}{2}\right) \cos\left(\frac{x-3x}{2}\right) \right]}{\left[ 2 \cos\left(\frac{x+3x}{2}\right) \cos\left(\frac{x-3x}{2}\right) \right]}$$

$$= \frac{\sin 2x}{\cos 2x}$$

$$= \tan 2x$$

$$= RHS$$

$$\left[ \begin{array}{l} \because \sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \\ \& \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \end{array} \right]$$

**Question 20:**

$$\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2 \sin x$$

**Solution:**

$$\begin{aligned}
 LHS &= \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} \\
 &= \frac{\left[ 2 \cos \left( \frac{x+3x}{2} \right) \sin \left( \frac{x-3x}{2} \right) \right]}{-[\cos^2 x - \sin^2 x]} \\
 &= \frac{2 \cos 2x \sin(-x)}{-\cos 2x} \\
 &= 2 \sin x \\
 &= RHS
 \end{aligned}$$

$$\left[ \begin{aligned}
 \because \sin A - \sin B &= 2 \cos \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right) \\
 \&\ \cos A + \cos B &= 2 \cos \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right)
 \end{aligned} \right]$$

**Question 21:**

$$\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$$

**Solution:**

$$\begin{aligned}
 LHS &= \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} \\
 &= \frac{[\cos 4x + \cos 2x] + \cos 3x}{[\sin 4x + \sin 2x] + \sin 3x} \\
 &= \frac{\left[ 2 \cos \left( \frac{4x+2x}{2} \right) \cos \left( \frac{4x-2x}{2} \right) \right] + \cos 3x}{\left[ 2 \sin \left( \frac{4x+2x}{2} \right) \cos \left( \frac{4x-2x}{2} \right) \right] + \sin 3x} \\
 &= \frac{2 \cos 3x \cos x + \cos 3x}{2 \sin 3x \cos x + \sin 3x} \\
 &= \frac{\cos 3x (2 \cos x + 1)}{\sin 3x (2 \cos x + 1)} \\
 &= \frac{\cos 3x}{\sin 3x} \\
 &= \cot 3x \\
 &= RHS
 \end{aligned}$$

$$\left[ \begin{aligned}
 \because \cos A + \cos B &= 2 \cos \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \\
 \&\ \sin A + \sin B &= 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right)
 \end{aligned} \right]$$

**Question 22:**

$$\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$$

**Solution:**

$$LHS = \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x$$

$$= \cot x \cot 2x - \cot 3x (\cot 2x + \cot x)$$

$$= \cot x \cot 2x - \cot (2x + x) (\cot 2x + \cot x)$$

$$= \cot x \cot 2x - \left[ \frac{\cot 2x \cot x - 1}{\cot 2x + \cot x} \right] (\cot 2x + \cot x)$$

$$\left[ \because \cot(A + B) = \frac{\cot A \cot B - 1}{\cot A + \cot B} \right]$$

$$= \cot x \cot 2x - [\cot 2x \cot x - 1]$$

$$= \cot x \cot 2x - \cot x \cot 2x + 1$$

$$= 1$$

$$= RHS$$

**Question 23:**

$$\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$$

**Solution:**

$$LHS = \tan 4x$$

$$= \tan 2(2x)$$

$$= \frac{2 \tan 2x}{1 - \tan^2 2x}$$

$$= \frac{2 \left( \frac{2 \tan x}{1 - \tan^2 x} \right)}{1 - \left( \frac{2 \tan x}{1 - \tan^2 x} \right)^2}$$

$$= \frac{\left( \frac{4 \tan x}{1 - \tan^2 x} \right)}{1 - \left( \frac{4 \tan^2 x}{1 + \tan^4 x - 2 \tan^2 x} \right)}$$

$$= \frac{\left( \frac{4 \tan x}{1 - \tan^2 x} \right)}{\left( \frac{1 + \tan^4 x - 2 \tan^2 x - 4 \tan^2 x}{1 + \tan^4 x - 2 \tan^2 x} \right)}$$

$$= \left( \frac{4 \tan x}{1 - \tan^2 x} \right) \times \left( \frac{1 + \tan^4 x - 2 \tan^2 x}{1 + \tan^4 x - 6 \tan^2 x} \right)$$

$$= \frac{4 \tan x (1 - \tan^2 x)^2}{(1 - \tan^2 x)(1 + \tan^4 x - 6 \tan^2 x)}$$

$$= \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$$

$$= RHS$$

$$\left[ \because \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \right]$$

$$\left[ \because \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \right]$$

$$\left[ \because (a-b)^2 = a^2 + b^2 - 2ab \right]$$

$$\left[ \because a^2 + b^2 - 2ab = (a-b)^2 \right]$$

**Question 24:**

$$\cos 4x = 1 - 8 \sin^2 x \cos^2 x$$

**Solution:**

$$LHS = \cos 4x$$

$$= \cos 2(2x)$$

$$= 1 - 2 \sin^2 2x$$

$$= 1 - 2(2 \sin x \cos x)^2$$

$$= 1 - 2(4 \sin^2 x \cos^2 x)$$

$$= 1 - 8 \sin^2 x \cos^2 x$$

$$= RHS$$

$$\left[ \because \cos 2A = 1 - 2 \sin^2 x \right]$$

$$\left[ \because \sin 2A = 2 \sin x \cos x \right]$$

**Question 25:**

$$\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$$

**Solution:**

$$LHS = \cos 6x$$

$$= \cos 3(2x)$$

$$= 4 \cos^3 2x - 3 \cos 2x$$

$$= 4(2 \cos^2 x - 1)^3 - 3(2 \cos^2 x - 1)$$

$$= 4(8 \cos^6 x - 12 \cos^4 x + 6 \cos^2 x) - 6 \cos^2 x + 3$$

$$= 32 \cos^6 x - 48 \cos^4 x + 24 \cos^2 x - 6 \cos^2 x + 3$$

$$= 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$$

$$= RHS$$

$$[\because \cos 3A = 4 \cos^3 x - 3 \cos x]$$

$$[\because \cos 2A = 2 \cos^2 x - 1]$$

$$[\because (a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2]$$



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