NCERT Solutions Class 11 Maths Chapter 3 Trigonometric Functions

Question 1:

Find the radian measures corresponding to the following degree measures:
(i) 25° (ii) – 47°30′ (iii) 240° (iv) 520°

Solution:
We know that $180° = \pi$ radian
Therefore,

(i) $$25° = \frac{\pi}{180} \times 25 \text{ radian}$$
$$= \frac{5\pi}{36} \text{ radian}$$

(ii) $$-47°30' = -\left(47\frac{1}{2}\right)° = -\left(\frac{95}{2}\right)°$$
$$-\left(\frac{95}{2}\right)° = \frac{\pi}{180} \times -\left(\frac{95}{2}\right) \text{radian}$$
$$= -\frac{19\pi}{72} \text{ radian}$$

(iii) $$245° = \frac{\pi}{180} \times 240 \text{ radian}$$
$$= \frac{3\pi}{4} \text{ radian}$$

(iv) $$520° = \frac{\pi}{180} \times 520 \text{ radian}$$
$$= \frac{26\pi}{9} \text{ radian}$$

Question 2:

Find the degree measures corresponding to the following radian measures $\left(\text{Use } \pi = \frac{22}{7}\right)$.
(i) $\frac{11}{16}$ (ii) – 4 (iii) $\frac{5\pi}{3}$ (iv) $\frac{7\pi}{6}$

Solution:

We know that $180° = \pi$ radian

Therefore,

(i) $\frac{11}{16}$ radian $= \frac{180°}{\pi} \times \frac{11}{16}$

$$= 180 \times \frac{7}{22} \times \frac{11}{16} \deg$$

$$= \frac{315}{8} \deg$$

$$= 39\frac{3}{8} \deg$$

$$= 39 \deg + \frac{3}{8} \times 60 \min \qquad [\because 1° = 60']$$

$$= 39 \deg + 22\frac{1}{2} \min$$

$$= 39 \deg + 22 \min + \frac{60}{2} \sec \qquad [\because 1' = 60'']$$

$$= 39°22'30''$$

(ii) -4 radian $= \frac{180°}{\pi} \times (-4)$

$$= 180 \times \frac{7}{22} \times (-4) \deg$$

$$= -\frac{2520}{11} \deg$$

$$= -229\frac{1}{11} \deg$$

$$= -229 \deg + \frac{60}{11} \min \qquad [\because 1° = 60']$$

$$= -22 \deg + 5\frac{5}{11} \min$$

$$= -22 \deg + 5 \min + \frac{5}{11} \times 60 \sec \qquad [\because 1' = 60'']$$

$$= -22°5'27''$$

(iii) $\frac{5\pi}{3}$ radian $= \frac{180°}{\pi} \times \frac{5\pi}{3}$

$$= 300°$$

(iv) $\frac{7\pi}{6}$ radian $= \frac{180°}{\pi} \times \frac{7\pi}{6}$

$= 210°$

Question 3:
A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Solution:
A wheel makes 360 revolutions in 1 minute (60 seconds)

Therefore,

Number of revolutions made by the wheel in 1 second $= \frac{360}{60} = 6$
In one complete revolution, the wheel turns an angle of 2π radians
Hence, in 6 complete revolutions, it will turn an angle of $6 \times 2\pi = 12\pi$ radians

Thus, in one second, the wheel turns an angle of 12π radians.

Question 4:
Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Solution:
As we know that if in a circle of radius *r*, an arc of length *l* subtends an angle of θ radians,
Then $l = r\theta$

Therefore, $\theta = \frac{l}{r}$ radian

$$\theta = \frac{22cm}{100cm} \text{ radian}$$

$$\theta = \frac{11}{50} \times \frac{180}{\pi} \text{deg}$$

$$= \frac{11}{50} \times 180 \times \frac{7}{22} \text{deg}$$

$$= \frac{63}{5} \text{deg}$$

$$= 12\frac{3}{5} \text{deg}$$

$$= 12 \text{deg} + \frac{3}{5} \times 60 \text{min} \qquad [\because 1° = 60']$$

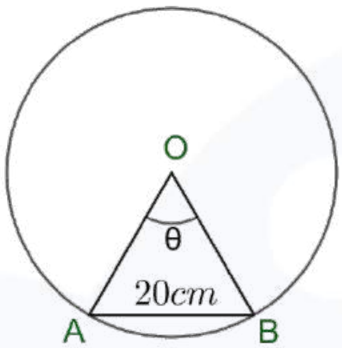
$$= 12°36'$$

Thus, the required angle is $12°36'$

Question 5:
In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Solution:
Diameter of the circle = 40 cm

Therefore, Radius of the circle $r = \frac{40cm}{2} = 20cm$
Let AB be a chord of length 20 cm of the circle.



In ΔAOB
$AB = 20cm$
$OA = OB = r = 20cm$

Hence, ΔAOB is an equilateral triangle

Thus, $\theta = 60°$ or $\theta = \frac{\pi}{3}$ radian

As we know that if in a circle of radius *r*, an arc of length *l* subtends an angle of θ radians,
Then $l = r\theta$
Therefore,

$$l = r\theta$$
$$\overset{\frown}{AB} = 20cm \times \frac{\pi}{3}$$
$$= \frac{20\pi}{3} cm$$

Hence, the length of the minor arc of the chord is $\frac{20\pi}{3} cm$.

Question 6:
If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

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Solution:

Let the radii of the two circles be r and R.

Let an arc of length l subtend an angle of 60° at the centre of the circle of radius r, and 75° at the centre of the circle of radius R.

Now,

$$60° = \frac{\pi}{3} \text{ radian and } 75° = \frac{5\pi}{12} \text{ radian}$$

As we know that if in a circle of radius r, an arc of length l subtends an angle of θ radians,

Then $l = r\theta$

Therefore,

$$l = r \times \frac{\pi}{3} = \frac{\pi r}{3} \quad \text{and} \quad l = R \times \frac{5\pi}{12} = \frac{5\pi R}{12}$$

Thus,

$$\frac{\pi r}{3} = \frac{5\pi R}{12}$$

$$\frac{r}{R} = \frac{5}{4}$$

$$r : R = 5 : 4$$

Hence, the ratio of their radii is $5:4$.

Question 7:

Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc of length

(i) 10 cm (ii) 15 cm (iii) 21 cm

Solution:

As we know that if in a circle of radius r, an arc of length l subtends an angle of θ radians,

Then $l = r\theta$

(i) Radius, $r = 75cm$ and length of the arc, $l = 10cm$

$$\theta = \frac{l}{r} = \frac{10cm}{75cm} = \frac{2}{15}$$

Thus, $\theta = \frac{2}{15}$ radian

(ii) Radius, $r = 75cm$ and length of the arc, $l = 15cm$

$$\theta = \frac{l}{r}$$
$$= \frac{15cm}{75cm}$$
$$= \frac{1}{5}$$

Thus, $\theta = \frac{1}{5}$ radian

(iii) Radius, $r = 75cm$ and length of the arc, $l = 21cm$

$$\theta = \frac{l}{r}$$
$$= \frac{21cm}{75cm}$$
$$= \frac{7}{25}$$

Thus, $\theta = \frac{7}{25}$ radian

EXERCISE 3.2

Find the values of other five trigonometric functions in Exercises 1 to 5.

Question 1:

$\cos x = -\frac{1}{2}$, x lies in third quadrant.

Solution:

As we know that

$$\begin{aligned}\sec x &= \frac{1}{\cos x}\\ &= \frac{1}{\left(-\frac{1}{2}\right)}\\ &= -2\end{aligned}$$

Now, $\sin^2 x + \cos^2 x = 1$

$$\begin{aligned}\sin^2 x &= 1 - \cos^2 x\\ \sin x &= \pm\sqrt{1-\cos^2 x}\\ &= \pm\sqrt{1-\left(-\frac{1}{2}\right)^2}\\ &= \pm\sqrt{1-\frac{1}{4}}\\ &= \pm\sqrt{\frac{3}{4}}\\ &= \pm\frac{\sqrt{3}}{2}\end{aligned}$$

Since, x lies in third quadrant, the value of $\sin x$ will be negative.
Therefore,

$$\sin x = -\frac{\sqrt{3}}{2}$$

Now,

$$\begin{aligned}\operatorname{cosec} x &= \frac{1}{\sin x}\\ &= \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)}\\ &= -\frac{2}{\sqrt{3}}\end{aligned}$$

Now,

$$\tan x = \frac{\sin x}{\cos x}$$
$$= \frac{\left(-\frac{1}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)}$$
$$= \sqrt{3}$$

Now,

$$\cot x = \frac{1}{\tan x}$$
$$= \frac{1}{\sqrt{3}}$$

Hence, $\sin x = -\frac{\sqrt{3}}{2}$, $\operatorname{cosec} x = -\frac{2}{\sqrt{3}}$, $\sec x = -2$, $\tan x = \sqrt{3}$, and $\cot x = \frac{1}{\sqrt{3}}$

Question 2:

$\sin x = \frac{3}{5}$, x lies in second quadrant.

Solution:

As we know that

$$\operatorname{cosec} x = \frac{1}{\sin x}$$
$$= \frac{1}{\left(\frac{3}{5}\right)}$$
$$= \frac{5}{3}$$

Now, $\sin^2 x + \cos^2 x = 1$

$$\cos^2 x = 1 - \sin^2 x$$
$$\cos x = \pm\sqrt{1-\sin^2 x}$$
$$= \pm\sqrt{1-\left(\frac{3}{5}\right)^2}$$
$$= \pm\sqrt{1-\frac{9}{25}}$$
$$= \pm\sqrt{\frac{16}{25}}$$
$$= \pm\frac{4}{5}$$

Since, x lies in second quadrant, the value of $\cos x$ will be negative.
Therefore,

$$\cos x = -\frac{4}{5}$$

Now,

$$\sec x = \frac{1}{\cos x}$$
$$= \frac{1}{\left(-\frac{4}{5}\right)}$$
$$= -\frac{5}{4}$$

Now,

$$\tan x = \frac{\sin x}{\cos x}$$
$$= \frac{\left(\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)}$$
$$= -\frac{3}{4}$$

Now,

$$\cot x = \frac{1}{\tan x}$$
$$= \frac{1}{\left(-\frac{3}{4}\right)}$$
$$= -\frac{4}{3}$$

Hence, $\operatorname{cosec} x = \frac{5}{3}$, $\cos x = -\frac{4}{5}$, $\sec x = -\frac{5}{4}$, $\tan x = -\frac{3}{4}$, and $\cot x = -\frac{4}{3}$

Question 3:

$\cot x = \frac{3}{4}$, x lies in third quadrant.

Solution:

As we know that

$$\begin{aligned}\tan x &= \frac{1}{\cot x}\\ &= \frac{1}{\left(\frac{3}{4}\right)}\\ &= \frac{4}{3}\end{aligned}$$

Now, $1+\tan^2 x = \sec^2 x$

$$\begin{aligned}\sec x &= \pm\sqrt{1+\tan^2 x}\\ &= \pm\sqrt{1+\left(\frac{4}{3}\right)^2}\\ &= \pm\sqrt{1+\frac{16}{9}}\\ &= \pm\sqrt{\frac{25}{9}}\\ &= \pm\frac{5}{3}\end{aligned}$$

Since, x lies in third quadrant, the value of $\sec x$ will be negative.

Therefore,

$$\sec x = -\frac{5}{3}$$

Now,

$$\begin{aligned}\cos x &= \frac{1}{\sec x}\\ &= \frac{1}{\left(-\frac{5}{3}\right)}\\ &= -\frac{3}{5}\end{aligned}$$

Now,

$$\tan x = \frac{\sin x}{\cos x}$$
$$\sin x = \tan x \cos x$$
$$= \left(\frac{4}{3}\right) \times \left(-\frac{3}{5}\right)$$
$$= -\frac{4}{5}$$

Now,

$$\text{cosec } x = \frac{1}{\sin x}$$
$$= \frac{1}{\left(-\frac{4}{5}\right)}$$
$$= -\frac{5}{4}$$

Hence, $\sin x = -\frac{4}{5}$, $\text{cosec } x = -\frac{5}{4}$, $\cos x = -\frac{3}{5}$, $\sec x = -\frac{5}{3}$, and $\tan x = \frac{4}{3}$

Question 4:

$\sec x = \frac{13}{5}$, x lies in fourth quadrant.

Solution:

As we know that

$$\cos x = \frac{1}{\sec x}$$
$$= \frac{1}{\left(\frac{13}{5}\right)}$$
$$= \frac{5}{13}$$

Now, $\sin^2 x + \cos^2 x = 1$

$$\sin^2 x = 1 - \cos^2 x$$

$$\sin x = \pm\sqrt{1-\cos^2 x}$$

$$= \pm\sqrt{1-\left(\frac{5}{13}\right)^2}$$

$$= \pm\sqrt{1-\frac{25}{169}}$$

$$= \pm\sqrt{\frac{144}{169}}$$

$$= \pm\frac{12}{13}$$

Since, x lies in fourth quadrant, the value of $\sin x$ will be negative.

Therefore,

$$\sin x = -\frac{12}{13}$$

Now,

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

$$= \frac{1}{\left(-\frac{12}{13}\right)}$$

$$= -\frac{13}{12}$$

Now,

$$\tan x = \frac{\sin x}{\cos x}$$

$$= \frac{\left(-\frac{12}{13}\right)}{\left(\frac{5}{13}\right)}$$

$$= -\frac{12}{5}$$

Now,

$$\cot x = \frac{1}{\tan x}$$
$$= \frac{1}{\left(-\frac{12}{5}\right)}$$
$$= -\frac{5}{12}$$

Hence, $\sin x = -\frac{12}{13}$, $\operatorname{cosec} x = -\frac{13}{12}$, $\cos x = \frac{5}{13}$, $\tan x = -\frac{12}{5}$, and $\cot x = -\frac{5}{12}$

Question 5:

$\tan x = -\frac{5}{12}$, x lies in second quadrant.

Solution:

As we know that

$$\cot x = \frac{1}{\tan x}$$
$$= \frac{1}{\left(-\frac{5}{12}\right)}$$
$$= -\frac{12}{5}$$

Now, $1 + \tan^2 x = \sec^2 x$

$$\sec x = \pm\sqrt{1 + \tan^2 x}$$
$$= \pm\sqrt{1 + \left(-\frac{5}{12}\right)^2}$$
$$= \pm\sqrt{1 + \frac{25}{144}}$$
$$= \pm\sqrt{\frac{169}{144}}$$
$$= \pm\frac{13}{12}$$

Since, x lies in second quadrant, the value of $\sec x$ will be negative.
Therefore,

$$\sec x = -\frac{13}{12}$$

Now,

$$\cos x = \frac{1}{\sec x}$$
$$= \frac{1}{\left(-\frac{13}{12}\right)}$$
$$= -\frac{12}{13}$$

Now,

$$\tan x = \frac{\sin x}{\cos x}$$
$$\sin x = \tan x \cos x$$
$$= \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right)$$
$$= \frac{5}{13}$$

Now,

$$\operatorname{cosec} x = \frac{1}{\sin x}$$
$$= \frac{1}{\left(\frac{5}{13}\right)}$$
$$= \frac{13}{5}$$

Hence, $\sin x = \frac{5}{13}$, $\operatorname{cosec} x = \frac{13}{5}$, $\cos x = -\frac{12}{13}$, $\sec x = -\frac{13}{12}$, and $\cot x = -\frac{12}{5}$

Find the values of the trigonometric functions in Exercises 6 to 10.

Question 6:

$\sin 765°$

Solution:

It is known that the value of $\sin x$ repeat after an interval of 2n or 360°.

Therefore,

$$\begin{aligned}\sin 765° &= \sin\left(2\times 360° + 45°\right)\\ &= \sin 45°\\ &= \frac{1}{\sqrt{2}}\end{aligned}$$

Question 7:

$\operatorname{cosec}\left(-1410°\right)$

Solution:

It is known that the value of $\sin x$ repeat after an interval of 2n or 360°.
Therefore,

$$\begin{aligned}\operatorname{cosec}\left(-1410°\right) &= \operatorname{cosec}\left(4\times 360° - 1410°\right)\\ &= \operatorname{cosec}\left(1440° - 1410°\right)\\ &= \operatorname{cosec}30°\\ &= 2\end{aligned}$$

Question 8:

$\tan\frac{19\pi}{3}$

Solution:

It is known that the value of $\tan x$ repeat after an interval of n or 180°.
Therefore,

$$\begin{aligned}\tan\frac{19\pi}{3} &= \tan 6\frac{1}{3}\pi\\ &= \tan\left(6\pi + \frac{1}{3}\pi\right)\\ &= \tan\frac{\pi}{3}\\ &= \tan 60°\\ &= \sqrt{3}\end{aligned}$$

Question 9:

$\sin\left(-\frac{11\pi}{3}\right)$

Solution:

It is known that the value of $\sin x$ repeat after an interval of 2π or 360°.

Therefore,

$$\sin\left(-\frac{11\pi}{3}\right)=\sin\left(2\times 2\pi-\frac{11\pi}{3}\right)$$
$$=\sin\frac{\pi}{3}$$
$$=\sin 60°$$
$$=\frac{\sqrt{3}}{2}$$

Question 10:

$\cot\left(-\frac{15\pi}{4}\right)$

Solution:

It is known that the value of $\cos x$ repeat after an interval of π or 180°.
Therefore,

$$\cot\left(-\frac{15\pi}{4}\right)=\cot\left(4\pi-\frac{15\pi}{4}\right)$$
$$=\cot\frac{\pi}{4}$$
$$=\cot 45°$$
$$=1$$

EXERCISE 3.3

Prove that:

Question 1:

$$\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$$

Solution:

$$\begin{aligned}LHS &= \sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}\\ &=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2\\ &=\frac{1}{4}+\frac{1}{4}-1\\ &=\frac{1+1-4}{4}\\ &=\frac{-2}{4}\\ &=-\frac{1}{2}=RHS\end{aligned}$$

Question 2:

$$2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$$

Solution:

$$\begin{aligned}LHS &= 2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}\\ &=2\left(\frac{1}{2}\right)^2+\operatorname{cosec}^2\left(\pi+\frac{\pi}{6}\right)\times\left(\frac{1}{2}\right)^2\\ &=2\times\frac{1}{4}+\left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\times\frac{1}{4}\\ &=\frac{1}{2}+(-2)^2\times\frac{1}{4}\\ &=\frac{1}{2}+1\\ &=\frac{1+2}{2}\\ &=\frac{3}{2}=RHS\end{aligned}$$

Question 3:

$$\cot^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=6$$

Solution:

$$\begin{aligned}
LHS &= \cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}\\
&=\left(\sqrt{3}\right)^2+\operatorname{cosec}\left(\pi-\frac{\pi}{6}\right)+3\left(\frac{1}{\sqrt{3}}\right)^2\\
&=3+\operatorname{cosec}\frac{\pi}{6}+3\times\frac{1}{3}\\
&=3+2+1\\
&=6=RHS
\end{aligned}$$

Question 4:

$$2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=10$$

Solution:

$$\begin{aligned}
LHS &= 2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}\\
&=2\sin^2\left(\pi-\frac{\pi}{4}\right)+2\times\left(\frac{1}{\sqrt{2}}\right)^2+2\times(2)^2\\
&=2\sin^2\frac{\pi}{4}+2\times\frac{1}{2}+2\times 4\\
&=2\times\left(\frac{1}{\sqrt{2}}\right)^2+1+8\\
&=1+9\\
&=10=RHS
\end{aligned}$$

Question 5:

Find the value of:

(i) $\sin 75°$

(ii) $\tan 15°$

Solution:

(i) $\sin 75°$

$$\sin 75° = \sin(45° + 30°)$$
$$= \sin 45° \cos 30° + \cos 45° \sin 30° \qquad [\because \sin(x+y) = \sin x \cos y + \cos x \sin y]$$
$$= \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii) $\tan 15°$

$$\tan 15° = \tan(45° - 30°)$$
$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} \qquad \left[\because \tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}\right]$$
$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}}$$
$$= \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$
$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$
$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} \qquad [\text{By rationalizing}]$$
$$= \frac{\left(\sqrt{3}-1\right)^2}{3-1}$$
$$= \frac{3+1-2\sqrt{3}}{2}$$
$$= \frac{2\left(2-\sqrt{3}\right)}{2}$$
$$= 2 - \sqrt{3}$$

Prove the following:

Question 6:

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$$

Solution:

$$LHS=\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$$

$$=\frac{1}{2}\left[2\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)\right]+\frac{1}{2}\left[-2\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)\right]$$

$$=\left(\begin{array}{l}\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}+\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]\\ \qquad+\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}-\cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]\end{array}\right)$$

$$\left[\begin{array}{l}\because 2\cos A\cos B=\cos(A+B)+\cos(A-B)\\ -2\sin A\sin B=\cos(A+B)-\cos(A-B)\end{array}\right]$$

$$=\sin(x+y)=RHS$$

$$=\left(\begin{array}{l}\frac{1}{2}\left[\cos\left\{\frac{\pi}{4}-x+\frac{\pi}{4}-y\right\}+\cos\left\{\frac{\pi}{4}-x-\frac{\pi}{4}+y\right\}\right]\\ \qquad+\frac{1}{2}\left[\cos\left\{\frac{\pi}{4}-x+\frac{\pi}{4}-y\right\}-\cos\left\{\frac{\pi}{4}-x-\frac{\pi}{4}+y\right\}\right]\end{array}\right)$$

$$=\frac{1}{2}\left[\cos\left\{\frac{\pi}{2}-(x+y)\right\}+\cos\{-(x-y)\}+\cos\left\{\frac{\pi}{2}-(x+y)\right\}-\cos\{-(x-y)\}\right]$$

$$=\frac{1}{2}\left[2\cos\left\{\frac{\pi}{2}-(x+y)\right\}\right]$$

$$=\sin(x+y) \qquad \left[\because \cos\left(\frac{\pi}{2}-A\right)=\sin A\right]$$

$$=RHS$$

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Question 7:

$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2$$

Solution:

$$LHS=\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$$

$$=\frac{\left(\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\left(\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)} \qquad \left[\because \tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B} \quad \& \tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}\right]$$

$$=\frac{\left(\dfrac{1+\tan x}{1-\tan x}\right)}{\left(\dfrac{1-\tan x}{1+\tan x}\right)}$$

$$=\left(\frac{1+\tan x}{1-\tan x}\right)\times\left(\frac{1+\tan x}{1-\tan x}\right)$$

$$=\left(\frac{1+\tan x}{1-\tan x}\right)^2$$

$$=RHS$$

Question 8:

$$\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$$

Solution:

$$LHS=\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$$

$$=\frac{(-\cos x)\times(\cos x)}{(\sin x)\times(-\sin x)} \qquad \left[\begin{array}{l}\because \cos(\pi+x)=-\cos x\\ \Rightarrow \cos(-x)=\cos x\\ \Rightarrow \cos\left(\frac{\pi}{2}+x\right)=-\sin x\\ \Rightarrow \sin(\pi-x)=\sin x\end{array}\right]$$

$$=\frac{-\cos^2 x}{-\sin^2 x}$$

$$=\left(\frac{\cos x}{\sin x}\right)^2$$

$$=\cot^2 x \qquad \left[\because \cot x=\frac{\cos x}{\sin x}\right]$$

$$=RHS$$

Question 9:

$$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$$

Solution:

$$LHS = \cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$$

$$= \cos\left\{\pi+\left(\frac{\pi}{2}+x\right)\right\}\cos x\left[\cot\left\{\pi+\left(\frac{\pi}{2}-x\right)\right\}+\cot x\right] \qquad \left[\begin{array}{l}\because \cos(2n\pi+\theta)=\cos\theta \\ \Rightarrow \cot(2n\pi+\theta)=\cot\theta\end{array}\right]$$

$$= -\cos\left(\frac{\pi}{2}+x\right)\cos x\left[\cot\left(\frac{\pi}{2}-x\right)+\cot x\right] \qquad \left[\begin{array}{l}\because \cos(\pi+\theta)=-\cos\theta \\ \Rightarrow \cot(\pi+\theta)=\cot\theta\end{array}\right]$$

$$= -(-\sin x)\cos x\left[\tan x+\cot x\right] \qquad \left[\begin{array}{l}\because \cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta \\ \Rightarrow \cot(2n\pi+\theta)=\cos\theta\end{array}\right]$$

$$= \sin x\cos x\left[\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right]$$

$$= \sin^2 x+\cos^2 x$$

$$= 1$$

$$= RHS$$

Question 10:

$$\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x=\cos x$$

Solution:

$$LHS = \sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x$$

$$= \cos(n+2)x.\cos(n+1)x+\sin(n+2)x.\sin(n+1)x$$

$$= \cos\{(n+2)x-(n+1)x\} \qquad [\because \cos(A-B)=\cos A\cos B+\sin A\sin B]$$

$$= \cos\{n+2-n-1\}x$$

$$= \cos x$$

$$= RHS$$

Question 11:

$$\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$$

Solution:

$$LHS = \cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$$

$$= -2\sin\left(\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right)\sin\left(\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right)$$

$$\left[\because \cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= -2\sin\left(\frac{\frac{3\pi}{4}+x+\frac{3\pi}{4}-x}{2}\right)\sin\left(\frac{\frac{3\pi}{4}+x-\frac{3\pi}{4}+x}{2}\right)$$

$$= -2\sin\left(\frac{3\pi}{4}\right)\sin\left(\frac{2x}{2}\right)$$

$$= -2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$$

$$= -2\sin\frac{\pi}{4}\sin x \qquad \left[\because \sin(\pi-\theta)=\sin\theta\right]$$

$$= -2\times\frac{1}{\sqrt{2}}\times\sin x$$

$$= -\sqrt{2}\sin x$$

$$= RHS$$

Question 12:

$\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Solution:

$$\begin{aligned}
LHS &= \sin^2 6x - \sin^2 4x \\
&= (\sin 6x + \sin 4x)(\sin 6x - \sin 4x) \\
&= \left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right] \times \left[2\cos\left(\frac{6x+4x}{2}\right)\sin\left(\frac{6x-4x}{2}\right)\right]
\end{aligned}$$

$$\begin{bmatrix} \because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\ \& \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \end{bmatrix}$$

$$\begin{aligned}
&= [2\sin 5x \cos x] \times [2\cos 5x \sin x] \\
&= [2\sin 5x \cos 5x] \times [2\sin x \cos x] \\
&= [\sin(5x+5x) + \sin(5x-5x)] \times [\sin(x+x) + \sin(x-x)]
\end{aligned}$$

$$[\because \ 2\sin A \cos B = \sin(A+B) + \sin(A-B)]$$

$$\begin{aligned}
&= [\sin 10x + \sin 0] \times [\sin 2x + \sin 0] \\
&= [\sin 10x + 0] \times [\sin 2x + 0] \\
&= \sin 2x \sin 10x \\
&= RHS
\end{aligned}$$

Question 13:

$$\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$$

Solution:

$$\begin{aligned}
LHS &= \cos^2 2x - \cos^2 6x \\
&= (\cos 2x + \cos 6x)(\cos 2x - \cos 6x) && [\because a^2 - b^2 = (a+b)(a-b)] \\
&= \left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right] \times \left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\left(\frac{2x-6x}{2}\right)\right]
\end{aligned}$$

$$\left[\begin{aligned}&\because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\ &\& \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\end{aligned}\right]$$

$$\begin{aligned}
&= [2\cos 4x \cos(-2x)] \times [-2\sin 4x \sin(-2x)] \\
&= [2\cos 4x \cos 2x] \times [-2\sin 4x(-\sin 2x)] && \left[\begin{aligned}&\because \cos(-\theta) = \cos\theta \\ &\& \sin(-\theta) = -\sin\theta\end{aligned}\right] \\
&= [2\cos 4x \cos 2x] \times [2\sin 4x \sin 2x] \\
&= [2\cos 4x \sin 4x] \times [2\cos 2x \sin 2x] \\
&= [\sin(4x+4x) - \sin(4x-4x)] \times [\sin(2x+2x) - \sin(2x-2x)] \\
& && [\because 2\cos A \sin B = \sin(A+B) - \sin(A-B)] \\
&= [\sin 8x - \sin 0] \times [\sin 4x - \sin 0] \\
&= [\sin 8x - 0] \times [\sin 4x - 0] \\
&= \sin 4x \sin 8x \\
&= RHS
\end{aligned}$$

Question 14:

$$\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$$

Solution:

$$\begin{aligned}
LHS &= \sin 2x + 2\sin 4x + \sin 6x \\
&= [\sin 2x + \sin 6x] + 2\sin 4x \\
&= \left[2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right] + 2\sin 4x && \left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right] \\
&= [2\sin 4x\cos(-2x)] + 2\sin 4x \\
&= 2\sin 4x\cos 2x + 2\sin 4x \\
&= 2\sin 4x(\cos 2x + 1) \\
&= 2\sin 4x(2\cos^2 x - 1 + 1) && [\because \cos 2x = 2\cos^2 x - 1] \\
&= 2\sin 4x(2\cos^2 x) \\
&= 4\cos^2 x\sin 4x \\
&= RHS
\end{aligned}$$

Question 15:

$\cot 4x(\sin 5x + \sin 3x) = \cot x(\sin 5x - \sin 3x)$

Solution:

$$\begin{aligned}
LHS &= \cot 4x(\sin 5x + \sin 3x) \\
&= \cot 4x\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right] && \left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right] \\
&= \frac{\cos 4x}{\sin 4x}[2\sin 4x\cos x] && [\because \cos 2x = 2\cos^2 x - 1] \\
&= 2\cos 4x\cos x \\
&= 2\cos 4x\cos x \times \frac{\sin x}{\sin x} \\
&= \frac{\cos x}{\sin x} \times [2\cos 4x\sin x] \\
&= \cot x[\sin(4x+x) - \sin(4x-x)] && [\because 2\cos A\sin B = \sin(A+B) - \sin(A-B)] \\
&= \cot x(\sin 5x - \sin 3x) \\
&= RHS
\end{aligned}$$

Question 16:

$$\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$$

Solution:

$$\begin{aligned} LHS &= \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} \\ &= \frac{\left[-2\sin\left(\frac{9x+5x}{2}\right)\sin\left(\frac{9x-5x}{2}\right)\right]}{\left[2\cos\left(\frac{17x+3x}{2}\right)\sin\left(\frac{17x-3x}{2}\right)\right]} \\ &= \frac{[-2\sin 7x \sin 2x]}{[2\cos 10x \sin 7x]} \\ &= -\frac{\sin 2x}{\cos 10x} \\ &= RHS \end{aligned}$$

$$\left[\begin{aligned} &\because \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \\ &\& \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \end{aligned}\right]$$

Question 17:

$$\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$$

Solution:

$$\begin{aligned} LHS &= \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} \\ &= \frac{\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]} \\ &= \frac{\sin 4x}{\cos 4x} \\ &= \tan 4x \\ &= RHS \end{aligned}$$

$$\left[\begin{aligned} &\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\ &\& \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \end{aligned}\right]$$

Question 18:

$$\frac{\sin x - \sin y}{\cos x + \cos y} = \tan \frac{x-y}{2}$$

Solution:

$$LHS = \frac{\sin x - \sin y}{\cos x + \cos y}$$

$$= \frac{\left[2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right]}{\left[2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]} \qquad \left[\begin{array}{l}\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \\ \& \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\end{array}\right]$$

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$= \tan\frac{x-y}{2}$$

$$= RHS$$

Question 19:

$$\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$$

Solution:

$$LHS = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$$

$$= \frac{\left[2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)\right]} \qquad \left[\begin{array}{l}\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\ \& \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\end{array}\right]$$

$$= \frac{\sin 2x}{\cos 2x}$$

$$= \tan 2x$$

$$= RHS$$

Question 20:

$$\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$$

Solution:

$$LHS = \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$$

$$= \frac{\left[2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)\right]}{-\left[\cos^2 x - \sin^2 x\right]} \qquad \left[\begin{array}{l}\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \\ \& \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\end{array}\right]$$

$$= \frac{2\cos 2x \sin(-x)}{-\cos 2x}$$

$$= 2\sin x$$

$$= RHS$$

Question 21:

$$\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$$

Solution:

$$LHS = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

$$= \frac{[\cos 4x + \cos 2x] + \cos 3x}{[\sin 4x + \sin 2x] + \sin 3x}$$

$$= \frac{\left[2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)\right] + \cos 3x}{\left[2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)\right] + \sin 3x} \qquad \left[\begin{array}{l}\because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\ \& \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\end{array}\right]$$

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

$$= \frac{\cos 3x(2\cos x + 1)}{\sin 3x(2\cos x + 1)}$$

$$= \frac{\cos 3x}{\sin 3x}$$

$$= \cot 3x$$

$$= RHS$$

Question 22:

$\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

Solution:

$$
\begin{aligned}
LHS &= \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x \\
&= \cot x \cot 2x - \cot 3x(\cot 2x + \cot x) \\
&= \cot x \cot 2x - \cot(2x + x)(\cot 2x + \cot x) \\
&= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot 2x + \cot x}\right](\cot 2x + \cot x) \qquad \left[\because \cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}\right] \\
&= \cot x \cot 2x - [\cot 2x \cot x - 1] \\
&= \cot x \cot 2x - \cot x \cot 2x + 1 \\
&= 1 \\
&= RHS
\end{aligned}
$$

Question 23:

$$\tan 4x = \frac{4 \tan x(1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$$

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Solution:

$$LHS = \tan 4x$$
$$= \tan 2(2x)$$
$$= \frac{2\tan 2x}{1-\tan^2 2x} \qquad \left[\because \tan 2A = \frac{2\tan A}{1-\tan^2 A}\right]$$
$$= \frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2} \qquad \left[\because \tan 2A = \frac{2\tan A}{1-\tan^2 A}\right]$$
$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{4\tan^2 x}{1+\tan^4 x-2\tan^2 x}\right)} \qquad \left[\because (a-b)^2 = a^2+b^2-2ab\right]$$
$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left(\frac{1+\tan^4 x-2\tan^2 x-4\tan^2 x}{1+\tan^4 x-2\tan^2 x}\right)}$$
$$= \left(\frac{4\tan x}{1-\tan^2 x}\right)\times\left(\frac{1+\tan^4 x-2\tan^2 x}{1+\tan^4 x-6\tan^2 x}\right)$$
$$= \frac{4\tan x\left(1-\tan^2 x\right)^2}{\left(1-\tan^2 x\right)\left(1+\tan^4 x-6\tan^2 x\right)} \qquad \left[\because a^2+b^2-2ab = (a-b)^2\right]$$
$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x}$$
$$= RHS$$

Question 24:

$\cos 4x = 1-8\sin^2 x\cos^2 x$

Solution:

$$LHS = \cos 4x$$
$$= \cos 2(2x)$$
$$= 1-2\sin^2 2x \qquad \left[\because \cos 2A = 1-2\sin^2 x\right]$$
$$= 1-2(2\sin x\cos x)^2 \qquad \left[\because \sin 2A = 2\sin x\cos x\right]$$
$$= 1-2\left(4\sin^2 x\cos^2 x\right)$$
$$= 1-8\sin^2 x\cos^2 x$$
$$= RHS$$

Question 25:

$\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Solution:

$$\begin{aligned}
LHS &= \cos 6x \\
&= \cos 3(2x) \\
&= 4\cos^3 2x - 3\cos 2x && [\because \cos 3A = 4\cos^3 x - 3\cos x] \\
&= 4(2\cos^2 x - 1)^3 - 3(2\cos^2 x - 1) && [\because \cos 2A = 2\cos^2 x - 1] \\
&= 4(8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x) - 6\cos^2 x + 3 && [\because (a-b)^3 = a^3 - b^3 - 3a^2 b + 3ab^2] \\
&= 32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x - 6\cos^2 x + 3 \\
&= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1 \\
&= RHS
\end{aligned}$$

EXERCISE 3.4

Find the principal and general solutions of the following equations:

Question 1:

$\tan x=\sqrt{3}$

Solution:

It is given that $\tan x=\sqrt{3}$

We know that $\tan\frac{\pi}{3}=\sqrt{3}$ and

$$\begin{aligned}\tan\frac{4\pi}{3}&=\tan\left(\pi+\frac{\pi}{3}\right)\\&=\tan\frac{\pi}{3}\\&=\sqrt{3}\end{aligned}$$

Hence the principal solutions are $x=\frac{\pi}{3}$ and $\frac{4\pi}{3}$.

Now,

$\tan x=\tan\frac{\pi}{3}$

Therefore, $x=n\pi+\frac{\pi}{3}$, where $n\in Z$.

Hence the general solution is $x=n\pi+\frac{\pi}{3}$, where $n\in Z$.

Question 2:

$\sec x=2$

Solution:

It is given that $\sec x=2$

We know that $\sec\frac{\pi}{3}=2$ and

$$\begin{aligned}\sec\frac{5\pi}{3}&=\sec\left(2\pi-\frac{\pi}{3}\right)\\&=\sec\frac{\pi}{3}\\&=\sqrt{3}\end{aligned}$$

Hence the principal solutions are $x = \frac{\pi}{3}$ and $\frac{5\pi}{3}$.

Now,

$$\sec x = \sec\frac{\pi}{3}$$

$$\cos x = \cos\frac{\pi}{3}$$

Therefore, $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$.

Hence the general solution is $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$.

Question 3:

$\cot x = -\sqrt{3}$

Solution:

It is given that $\cot x = -\sqrt{3}$

We know that $\cot\frac{\pi}{6} = \sqrt{3}$

Therefore,

$$\cot\left(\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6}$$

$$\cot\left(\frac{5\pi}{6}\right) = -\sqrt{3}$$

and

$$\cot\left(2\pi - \frac{\pi}{6}\right) = -\cot\frac{\pi}{6}$$

$$\cot\left(\frac{11\pi}{6}\right) = -\sqrt{3}$$

Hence the principal solutions are $x = \frac{5\pi}{6}$ and $\frac{11\pi}{6}$.

Now,

$$\cot x = \cot\left(\frac{5\pi}{6}\right)$$

$$\tan x = \tan\left(\frac{5\pi}{6}\right)$$

Therefore, $x = n\pi + \frac{5\pi}{6}$, where $n \in Z$.

Hence the general solution is $x = n\pi + \frac{5\pi}{6}$, where $n \in Z$.

Question 4:

$\operatorname{cosec} x = -2$

Solution:

It is given that $\operatorname{cosec} x = -2$

We know that $\operatorname{cosec}\frac{\pi}{6} = 2$ and

Therefore,

$$\operatorname{cosec}\left(\pi + \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6}$$

$$\operatorname{cosec}\left(\frac{7\pi}{6}\right) = -2$$

and

$$\operatorname{cosec}\left(2\pi - \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6}$$

$$\operatorname{cosec}\left(\frac{11\pi}{6}\right) = -2$$

Hence the principal solutions are $x = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$.

Now,

$$\operatorname{cosec} x = \operatorname{cosec}\frac{7\pi}{6}$$

$$\sin x = \sin\frac{7\pi}{6}$$

Therefore, $x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in Z$.

Hence the general solution is $x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in Z$.

Find the general solution for each of the following equations:

Question 5:

$\cos 4x = \cos 2x$

Solution:

$\cos 4x - \cos 2x = 0$

$-2\sin\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = 0$ $\quad \left[\because \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$\sin 3x \sin x = 0$

$\sin 3x = 0$ or $\sin x = 0$

$3x = n\pi$ or $x = n\pi$ where $n \in Z$

$x = \frac{n\pi}{3}$ or $x = n\pi$

Therefore, $x = \frac{n\pi}{3}$ or $n\pi$, where $n \in Z$

Question 6:

$\cos 3x + \cos x - \cos 2x = 0$

Solution:

$\cos 3x + \cos x - \cos 2x = 0$

$2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0$ $\quad \left[\because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$2\cos 2x \cos x - \cos 2x = 0$

$\cos 2x(2\cos x - 1) = 0$

$\cos 2x = 0$ or $2\cos x - 1 = 0$

$2x = (2n+1)\frac{\pi}{2}$ or $\cos x = \frac{1}{2}$

$x = (2n+1)\frac{\pi}{4}$ or $x = 2n\pi \pm \frac{\pi}{3}$

Therefore, $x = (2n+1)\frac{\pi}{4}$ or $\left(2n\pi \pm \frac{\pi}{3}\right)$, where $n \in Z$

Question 7:

$\sin 2x + \cos x = 0$

Solution:

$$\sin 2x+\cos x=0$$
$$2\sin x\cos x+\cos x=0 \qquad [\because \sin 2x=2\sin x\cos x]$$
$$\cos x(2\sin x+1)=0$$

Now,

$\cos x=0$ or $2\sin x+1=0$

Therefore,

$$\cos x=0$$
$$x=(2n+1)\frac{\pi}{2} \qquad n\in Z$$

Or,

$$2\sin x+1=0$$
$$\sin x=-\frac{1}{2}$$
$$=-\sin\frac{\pi}{6}$$
$$=\sin\left(\pi+\frac{\pi}{6}\right)$$
$$\sin x=\sin\frac{7\pi}{6}$$
$$x=n\pi+(-1)^n\frac{7\pi}{6} \qquad n\in Z$$

Therefore, $x=(2n+1)\frac{\pi}{2}$ or $n\pi+(-1)^n\frac{7\pi}{6}$, where $n\in Z$

Question 8:

$$\sec^2 2x=1-\tan 2x$$

Solution:

$$\sec^2 2x=1-\tan 2x$$
$$1-\tan^2 2x=1-\tan 2x$$
$$\tan^2 2x-\tan 2x=0$$
$$\tan 2x(\tan 2x-1)=0$$

Now,

$\tan 2x=0$ or $\tan 2x-1=0$

Therefore,

$$\tan 2x = 0$$

$$2x = n\pi \qquad n \in Z$$

$$x = \frac{n\pi}{2}$$

Or,

$$\tan 2x - 1 = 0$$

$$\tan 2x = 1$$

$$= -\tan\frac{\pi}{4}$$

$$= \tan\left(\pi - \frac{\pi}{4}\right)$$

$$= \tan\frac{3\pi}{4}$$

$$2x = n\pi + \frac{3\pi}{4} \qquad n \in Z$$

$$x = \frac{n\pi}{2} + \frac{3\pi}{8}$$

Therefore, $x = \frac{n\pi}{2}$ or $\left(\frac{n\pi}{2} + \frac{3\pi}{8}\right)$, where $n \in Z$

Question 9:

$$\sin x + \sin 3x + \sin 5x = 0$$

Solution:

$$\sin x + \sin 3x + \sin 5x = 0$$

$$2\sin\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right) + \sin 3x = 0 \qquad \left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$2\sin 3x \cos 2x + \sin 3x = 0$$

$$\sin 3x(2\cos 2x + 1) = 0$$

Now,

$\sin 3x = 0$ or $2\cos 2x + 1 = 0$

Therefore,

$\sin 3x = 0$

$$3x = n\pi \qquad n \in Z$$

$$x = \frac{n\pi}{3}$$

Or,

$2\cos 2x + 1 = 0$

$$\cos 2x = -\frac{1}{2}$$

$$= -\cos\frac{\pi}{3}$$

$$= \cos\left(\pi - \frac{\pi}{3}\right)$$

$$= \cos\frac{2\pi}{3}$$

$$2x = 2n\pi \pm \frac{2\pi}{3} \qquad n \in Z$$

$$x = n\pi \pm \frac{\pi}{3}$$

Therefore, $x = \frac{n\pi}{3}$ or $\left(n\pi \pm \frac{\pi}{3}\right)$, where $n \in Z$

MISCELLANEOUS EXERCISE

Prove that:

Question 1:

$$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$$

Solution:

$$\begin{aligned}
LHS &= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}\\
&= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\left(\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}\right) && \left[\because \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]\\
&= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\left(-\frac{\pi}{13}\right)\\
&= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}\\
&= 2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13}+\cos\frac{4\pi}{13}\right]\\
&= 2\cos\frac{\pi}{13}\left[2\cos\left(\frac{\frac{9\pi}{13}+\frac{4\pi}{13}}{2}\right)\cos\left(\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right)\right] && \left[\because \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]\\
&= 2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}\cos\frac{5\pi}{26}\right]\\
&= 2\cos\frac{\pi}{13}\times 2\times 0\times\cos\frac{5\pi}{26} && \left[\because \cos\frac{\pi}{2}=0\right]\\
&= 0\\
&= RHS
\end{aligned}$$

Question 2:

$$(\sin 3x+\sin x)\sin x+(\cos 3x-\cos x)\cos x=0$$

Solution:

$$
\begin{aligned}
LHS &= (\sin 3x + \sin x)\sin x + (\cos 3x - \cos x)\cos x \\
&= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x \\
&= \cos 3x \cos x + \sin 3x \sin x - (\cos^2 x - \sin^2 x) \\
&= \cos(3x - x) - \cos 2x \qquad \left[\begin{array}{l} \because \cos(A-B) = \cos A \cos B + \sin A \sin B \\ \& \cos 2A = \cos^2 A - \sin^2 A \end{array}\right] \\
&= \cos 2x - \cos 2x \\
&= 0 \\
&= RHS
\end{aligned}
$$

Question 3:

$$(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2 \frac{x+y}{2}$$

Solution:

$$
\begin{aligned}
LHS &= (\cos x + \cos y)^2 + (\sin x - \sin y)^2 \\
&= \cos^2 x + \cos^2 y + 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y \qquad \left[\begin{array}{l} \because (a+b)^2 = a^2 + b^2 + 2ab \\ \& (a-b)^2 = a^2 + b^2 - 2ab \end{array}\right] \\
&= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) + 2(\cos x \cos y - \sin x \sin y) \\
&= 1 + 1 + 2\cos(x+y) \qquad \left[\begin{array}{l} \because (\cos^2 A + \sin^2 A) = 1 \\ \& \cos(A+B) = \cos A \cos B - \sin A \sin B \end{array}\right] \\
&= 2 + 2\cos(x+y) \\
&= 2\left[1 + \cos 2\left(\frac{x+y}{2}\right)\right] \\
&= 2\left[1 + 2\cos^2\left(\frac{x+y}{2}\right) - 1\right] \qquad [\because \cos 2A = 2\cos^2 A - 1] \\
&= 4\cos^2 \frac{x+y}{2} \\
&= RHS
\end{aligned}
$$

Question 4:

$$(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2 \frac{x-y}{2}$$

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Solution:

$$LHS = (\cos x - \cos y)^2 + (\sin x - \sin y)^2$$

$$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y \quad \left[\begin{array}{l}\because (a+b)^2 = a^2 + b^2 + 2ab \\ \& (a-b)^2 = a^2 + b^2 - 2ab\end{array}\right]$$

$$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2(\cos x \cos y + \sin x \sin y)$$

$$= 1 + 1 - 2\cos(x-y) \quad \left[\begin{array}{l}\because (\cos^2 A + \sin^2 A) = 1 \\ \& \cos(A-B) = \cos A \cos B + \sin A \sin B\end{array}\right]$$

$$= 2 - 2\cos(x-y)$$

$$= 2\left[1 - \cos 2\left(\frac{x-y}{2}\right)\right]$$

$$= 2\left[1 - \left\{1 - 2\sin^2\left(\frac{x-y}{2}\right)\right\}\right] \quad [\because \cos 2A = 2\cos^2 A - 1]$$

$$= 2\left[1 - 1 + 2\sin^2\left(\frac{x-y}{2}\right)\right]$$

$$= 4\sin^2 \frac{x-y}{2}$$

$$= RHS$$

Question 5:

$\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x \cos 2x \sin 4x$

Solution:

$$LHS = \sin x + \sin 3x + \sin 5x + \sin 7x$$

$$= (\sin 5x + \sin x) + (\sin 7x + \sin 3x)$$

$$= \left[2\sin\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right)\right] + \left[2\sin\left(\frac{7x+3x}{2}\right)\cos\left(\frac{7x-3x}{2}\right)\right]$$

$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= 2\sin 3x \cos 2x + 2\sin 5x \cos 2x$$

$$= 2\cos 2x(\sin 5x + \sin 3x)$$

$$= 2\cos 2x\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right] \quad \left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= 2\cos 2x[2\sin 4x \cos x]$$

$$= 4\cos x \cos 2x \sin 4x$$

$$= RHS$$

Question 6:

$$\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}=\tan 6x$$

Solution:

$$LHS=\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$$

$$=\frac{2\sin\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)+2\sin\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)}{2\cos\left(\frac{7x+5x}{2}\right)\cos\left(\frac{7x-5x}{2}\right)+2\cos\left(\frac{9x+3x}{2}\right)\cos\left(\frac{9x-3x}{2}\right)}$$

$$\left[\begin{array}{l}\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\\ \& \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\end{array}\right]$$

$$=\frac{2\sin 6x\cos x+2\sin 6x\cos 3x}{2\cos 6x\cos x+2\cos 6x\cos 3x}$$

$$=\frac{2\sin 6x(\cos x+\cos 3x)}{2\cos 6x(\cos x+\cos 3x)}$$

$$=\frac{\sin 6x}{\cos 6x}$$

$$=\tan 6x$$

$$=RHS$$

Question 7:

$$\sin 3x+\sin 2x-\sin x=4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$$

Solution:

$$LHS = \sin 3x + \sin 2x - \sin x$$

$$= \sin 3x + (\sin 2x - \sin x)$$

$$= \sin 2\left(\frac{3x}{2}\right) + \left[2\cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right)\right] \qquad \left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= 2\sin\frac{3x}{2}\cos\frac{3x}{2} + 2\cos\frac{3x}{2}\sin\frac{x}{2} \qquad [\because \sin 2A = 2\sin A\cos A]$$

$$= 2\cos\frac{3x}{2}\left(\sin\frac{3x}{2} + \sin\frac{x}{2}\right)$$

$$= 2\cos\frac{3x}{2}\left[2\sin\left(\frac{\frac{3x}{2}+\frac{x}{2}}{2}\right)\cos\left(\frac{\frac{3x}{2}-\frac{x}{2}}{2}\right)\right] \qquad \left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= 2\cos\frac{3x}{2}\left[2\sin x\cos\frac{x}{2}\right]$$

$$= 4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$$

$$= RHS$$

Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ in each of the following:

Question 8:

$\tan x = -\frac{4}{3}$, x in quadrant II.

Solution:

Since x lies in quadrant II

$$\frac{\pi}{2} < x < \pi$$

Therefore,

$$\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Hence, $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ lie in quadrant I and all are positive.

It is given that $\tan x = -\frac{4}{3}$

$$\sec^2 x = 1 + \tan^2 x$$
$$= 1 + \left(-\frac{4}{3}\right)^2$$
$$= 1 + \frac{16}{9}$$
$$= \frac{25}{9}$$
$$\sec x = \pm\sqrt{\frac{25}{9}}$$
$$\frac{1}{\cos x} = \pm\frac{5}{3}$$
$$\cos x = \pm\frac{3}{5}$$

As x is in quadrant II, $\cos x$ is negative.

$$\cos x = -\frac{3}{5}$$
$$\cos 2\left(\frac{x}{2}\right) = -\frac{3}{5}$$
$$2\cos^2\frac{x}{2} - 1 = -\frac{3}{5}$$
$$2\cos^2\frac{x}{2} = 1 - \frac{3}{5}$$
$$\cos^2\frac{x}{2} = \frac{2}{5} \times \frac{1}{2}$$
$$\cos^2\frac{x}{2} = \frac{1}{5}$$
$$\cos\frac{x}{2} = \pm\sqrt{\frac{1}{5}}$$

Since, $\cos\frac{x}{2}$ lies in quadrant I and positive, $\cos\frac{x}{2} = \frac{1}{\sqrt{5}}$

Now,

$$\sin^2\frac{x}{2} = 1-\cos^2\frac{x}{2} \qquad \left[\because \sin^2 A+\cos^2 A = 1\right]$$

$$= 1-\left(\frac{1}{\sqrt{5}}\right)^2$$

$$= 1-\frac{1}{5}$$

$$= \frac{4}{5}$$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{4}{5}}$$

$$\sin\frac{x}{2} = \pm\frac{2}{\sqrt{5}}$$

Since, $\sin\frac{x}{2}$ lies in quadrant I and positive, $\sin\frac{x}{2} = \frac{2}{\sqrt{5}}$

Now,

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$$

$$= \frac{\left(\frac{2}{\sqrt{5}}\right)}{\left(\frac{1}{\sqrt{5}}\right)}$$

$$= \frac{2}{\sqrt{5}}\times\frac{\sqrt{5}}{1}$$

$$= 2$$

Therefore, $\sin\frac{x}{2} = \frac{2}{\sqrt{5}}$, $\cos\frac{x}{2} = \frac{1}{\sqrt{5}}$ and $\tan\frac{x}{2} = 2$.

Question 9:

$\cos x = -\frac{1}{3}, x$ in quadrant III.

Solution:

Since x lies in quadrant III

$$\pi < x < \frac{3\pi}{2}$$

Therefore,

$$\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

Hence, , $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are negative while $\sin\frac{x}{2}$ is positive as all lie in quadrant II.

It is given that $\cos x = -\frac{1}{3}$

$$\cos 2\left(\frac{x}{2}\right) = -\frac{1}{3}$$

$$2\cos^2\frac{x}{2} - 1 = -\frac{1}{3}$$

$$2\cos^2\frac{x}{2} = 1 - \frac{1}{3}$$

$$\cos^2\frac{x}{2} = \frac{2}{3}\times\frac{1}{2}$$

$$\cos^2\frac{x}{2} = \frac{1}{3}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1}{3}}$$

Since, $\cos\frac{x}{2}$ is negative

So,

$$\cos\frac{x}{2} = -\frac{1}{\sqrt{3}}$$

$$= -\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$= -\frac{\sqrt{3}}{3}$$

Now,

$$\sin^2\frac{x}{2} = 1 - \cos^2\frac{x}{2} \qquad \left[\because \sin^2 A + \cos^2 A = 1\right]$$

$$= 1 - \left(-\frac{\sqrt{3}}{3}\right)^2$$

$$= 1 - \frac{1}{3}$$

$$= \frac{2}{3}$$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{2}{3}}$$

Since, $\sin\frac{x}{2}$ positive,

So,

$$\sin\frac{x}{2}=\sqrt{\frac{2}{3}}$$
$$=\frac{\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$
$$=\frac{\sqrt{6}}{3}$$

Now,

$$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$$
$$=\frac{\frac{\sqrt{6}}{3}}{\left(-\frac{\sqrt{3}}{3}\right)}$$
$$=\frac{\sqrt{6}}{3}\times\left(-\frac{3}{\sqrt{3}}\right)$$
$$=-\sqrt{2}$$

Therefore, $\sin\frac{x}{2}=\frac{\sqrt{6}}{3}$, $\cos\frac{x}{2}=-\frac{\sqrt{3}}{3}$ and $\tan\frac{x}{2}=-\sqrt{2}$.

Question 10:

$\sin x=\frac{1}{4}, x$ in quadrant II.

Solution:

Since x lies in quadrant II

$$\frac{\pi}{2}<x<\pi$$

Therefore,

$$\frac{\pi}{4}<\frac{x}{2}<\frac{\pi}{2}$$

Hence, $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ lie in quadrant I and all are positive.

It is given that $\sin x=\frac{1}{4}$

Therefore,

$$\cos^2 x = 1 - \sin^2 x \qquad \left[\because \sin^2 A + \cos^2 A = 1\right]$$

$$= 1 - \left(\frac{1}{4}\right)^2$$

$$= 1 - \frac{1}{16}$$

$$= \frac{15}{16}$$

$$\cos x = \pm\sqrt{\frac{15}{16}}$$

Since, $\cos x$ lies in quadrant II and negative
So,

$$\cos x = -\sqrt{\frac{15}{16}}$$

$$\cos 2\left(\frac{x}{2}\right) = -\frac{\sqrt{15}}{4}$$

$$2\cos^2\frac{x}{2} - 1 = -\frac{\sqrt{15}}{4}$$

$$2\cos^2\frac{x}{2} = 1 - \frac{\sqrt{15}}{4}$$

$$2\cos^2\frac{x}{2} = \frac{4-\sqrt{15}}{4}$$

$$\cos^2\frac{x}{2} = \frac{4-\sqrt{15}}{8}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{4-\sqrt{15}}{8}}$$

Since, $\cos\frac{x}{2}$ lies in quadrant I and is positive.

So,

$$\cos\frac{x}{2} = \sqrt{\frac{4-\sqrt{15}}{8}}$$

$$= \sqrt{\frac{4-\sqrt{15}}{8} \times \frac{2}{2}}$$

$$= \frac{\sqrt{8-2\sqrt{15}}}{4}$$

Now,

$$\sin^2\frac{x}{2} = 1-\cos^2\frac{x}{2} \qquad \left[\because \sin^2 A+\cos^2 A=1\right]$$

$$= 1-\left(\frac{\sqrt{8-2\sqrt{15}}}{4}\right)^2$$

$$= 1-\frac{8-2\sqrt{15}}{16}$$

$$= \frac{16-8+2\sqrt{15}}{16}$$

$$= \frac{8+2\sqrt{15}}{16}$$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{8+2\sqrt{15}}{16}}$$

Since, $\sin\frac{x}{2}$ positive,

So,

$$\sin\frac{x}{2} = \frac{\sqrt{8+2\sqrt{15}}}{4}$$

Now,

$$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$$

$$=\frac{\left(\frac{\sqrt{8+2\sqrt{15}}}{4}\right)}{\left(\frac{\sqrt{8-2\sqrt{15}}}{4}\right)}$$

$$=\left(\frac{\sqrt{8+2\sqrt{15}}}{4}\right)\times\left(\frac{4}{\sqrt{8-2\sqrt{15}}}\right)$$

$$=\sqrt{\frac{2\left(4+\sqrt{15}\right)}{2\left(4-\sqrt{15}\right)}}$$

$$=\sqrt{\frac{4+\sqrt{15}}{4-\sqrt{15}}\times\frac{4+\sqrt{15}}{4+\sqrt{15}}}$$

$$=\sqrt{\frac{\left(4+\sqrt{15}\right)^2}{16-15}}$$

$$=4+\sqrt{15}$$

Therefore, $\sin\frac{x}{2}=\frac{\sqrt{8+2\sqrt{15}}}{4}$, $\cos\frac{x}{2}=\frac{\sqrt{8-2\sqrt{15}}}{4}$ and $\tan\frac{x}{2}=4+\sqrt{15}$.