Chapter 5: Lines and Angles
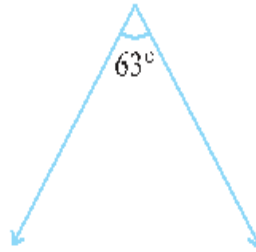
Exercise 5.1 (Page 101)

Q1. Find the complement of each of the following
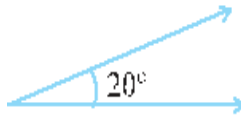
(i) (ii) (iii)

Difficulty Level: Low

What is given/known
Measure of angles.

What is the unknown?
Measure of complementary angles.

Reasoning
The sum of complementary angles is always 90°. If the given angle is x, then we can find complementary angle by subtracting x from 90°. Let us now find complement angles.

Solution:
(i) Given angle = 20°
    Complement angle of 20° = 90° − $given\ angle$
                            = 90° − 20° = 70°
(ii) Given angle = 63°
    Complement angle of 63° = 90° − $given\ angle$
                            = 90° − 63° = 27°
(iii) Given angle = 57°
    Complement angle of 57° = 90° − $given\ angle$
                            = 90° − 57° = 33°

Q2. Find the supplement of each of the following angles:
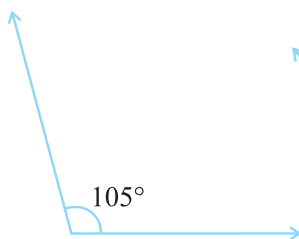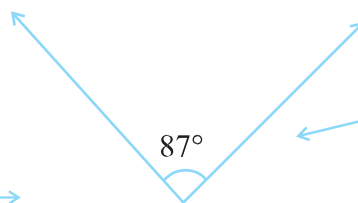
(i) (ii) (iii)

What is given /known
Measure of angles.

What is the unknown
Measure of supplement of each angle.

Reasoning
In a pair of angles, if the sum of the measures of the angles comes out to be 180°, angles are called supplementary angles. When two angles are supplementary, each angle is said to be the supplement of the other.

If the given angle is x, then we can find the supplement by subtracting x from 180°. Let us now find supplement angles.

Solution:
(i) Given angle $= 105°$
$\qquad$ Supplement angle of $105° = 180° - given\ angle$
$\qquad\qquad\qquad\qquad = 180° - 105° = 75°$
(ii) Given angle $= 87°$
$\qquad$ Supplement angle of $87° = 180° - given\ angle$
$\qquad\qquad\qquad\qquad = 180° - 87° = 93$
(iii) Given angle $= 154°$
$\qquad$ Supplement angle of $154° = 180° - given\ angle$
$\qquad\qquad\qquad\qquad = 180° - 154° = 26°$

Q3. Identify which of the following pairs of angles are complementary and which are supplementary:
$\qquad$ (i) 65°, 115° $\qquad\qquad$ (ii) 63°, 27° $\qquad\qquad$ (iii) 112°, 68 °
$\qquad$ (iv) 130°, 50° $\qquad\qquad$ (v) 45°, 45° $\qquad\qquad$ (vi) 80°, 10°

<u>NOTE</u>: The sum of the measure of complementary angle is 90° and that of supplementary angle is 180°.

Reasoning
Find out the sum of two given angles, and then check whether it is 180° or 90°. If the sum of two angles is equal to 90°, the angles are complementary and if the sum of the two angles is 180°, the angles are complementary.

Solution:
Solve for supplementary angle or complementary angle:
(i) 65°, 115°
$\qquad$ Sum of measure of these two angles $= 65° + 115° = 180°$
$\qquad$ Therefore, these two angles are supplementary.
(ii) 63°, 27°
$\qquad$ Sum of measure of these two angles $= 63° + 27° = 90°$
$\qquad$ Therefore, these two angles are complementary.

(iii) 112°, 68°

       Sum of measure of these two angles = 112° + 68° = 180°

       Therefore, these two angles are supplementary.

(iv) 130°, 50°

       Sum of measure of these two angles = 130° + 50° = 180°

       Therefore, these two angles are supplementary.

(v) 45°, 45°

       Sum of measure of these two angles = 45° + 45° = 90°

       Therefore, these two angles are complementary.

(vi) 80°, 10°

       Sum of measure of these two angles = 80° + 10° = 90°

       Therefore, these two angles are complementary.

Q4. Find the angle which is equal to its complement.

Difficulty Level: Low

What is given /known

An angle which is equal to its complement.

What is the unknown

Find measure of the angle.

Reasoning

Let us say given angle is x and according to the question, its complement angle will also be equal to x. The sum of angle and its complement angle is always equal to 90°.

Solution:

Solve for angle which is equal to its complement.

Let the angle be x.

Therefore, complement of this angle will also be x.

We know that, the sum of measure of pair of complementary angles is 90°.

      $x + x = 90°.$

      $=> 2x = 90°.$

      $=> x = \frac{90°}{2}.$

      $=> x = 45°.$

Thus, the angle which is equal to its complement is 45°.

Q5. Find the angle which is equal to its supplement.

Difficulty Level: Low

What is given /known

An angle which is equal to its supplement.

What is the unknown

Find measure of the angle.

Reasoning

Let us say given angle is x and according to the question, its supplementary angle will also be equal to x. The sum of angle and its supplement angle is always equal to 180°.

Solution:

Let the angle be x.

Therefore, supplement of this angle will also be x.

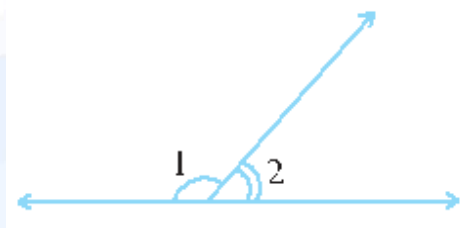We know that, the sum of measure of pair of supplementary angles is 180°.

$$x + x = 180°.$$
$$=> 2x = 180°.$$
$$=> x = \frac{180°}{2}.$$
$$=> x = 90°.$$

Thus, the angle which is equal to its supplement is 90°.

Q6. In the given figure, $\angle 1$ and $\angle 2$ are supplementary angles. If $\angle 1$ is decreased, what changes should take place in $\angle 2$ so that both the angles remain supplementary?

Difficulty Level: Low

What is given /known

Two angles which are equal to its supplementary.

What is the unknown

Change in one of the angles if other is decreased provided both angles still remain supplementary.

Reasoning

Let's solve this problem visually. There are two operations done in sequence. First, if one angle is decreasing then another angle will increase but their sum will remain same i.e. 180°

According to this model, the resultant sum of angle $\angle 1$ and $\angle 2$ will remain 180°.

Solution:

If $\angle 1$ is decreased by some degrees, then $\angle 2$ will also be increased with the same degree, so both the angles will remain supplementary.

Q7. Can two angles be supplementary if both are:
  (i) acute   (ii) obtuse   (iii) right

Solution:
 (i) No, sum of acute angles is less than 180°.
 (ii) No, sum of obtuse angles is greater than 180°.
(iii) Yes, sum of two right angles is 180°.

Q8. An angle is greater than 45°. Is its complementary angle greater than 45° or equal to 45° or less than 45° ?

Reasoning
Let us represent the angle by $\angle 1$ and its complement angle by $\angle 2$. Let's visually model this problem. There are two operations done in sequence. First take $\angle 1 > 45°$ *as it is given,* and then add $\angle 2$ to both sides of the equation.
According to this model, the result is equal to $\angle 1 + \angle 2 > 45° + \angle 2$. Now, it's a matter of finding, Is its complementary angle greater than 45° or equal to 45° or less than 45°.

Solution:
Let there be two angles $\angle 1$ and $\angle 2$.
Therefore $\angle 1 > 45°$ (given)
Adding $\angle 2$ to both sides ,we get
=> $\angle 1 + \angle 2 > 45° + \angle 2$
=> $90° > 45° + \angle 2$
=> $90° - 45° > \angle 2$
=> $45° > \angle 2$
Therefore, its complementary angle will be less than 45°.

Q9. In the adjoining figure:
  (i) Is $\angle 1$ adjacent to $\angle 2$?
  (ii) Is $\angle AOC$ adjacent to $\angle AOE$?
  (iii) Do $\angle COE$ and $\angle EOD$ form a linear pair?
  (iv) Are $\angle BOD$ and $\angle DOA$ supplementary?
  (v) Is $\angle 1$ vertically opposite to $\angle 4$?
  (vi) What is the vertically opposite angle of $\angle 5$?
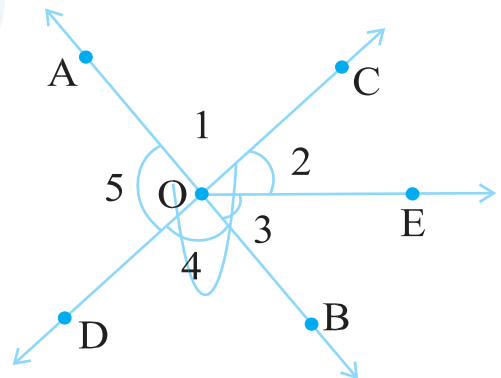
Solution:
  (i)   Yes, because they have common vertex *o* and common arm OC.
  (ii)   No, they have non-common arms on either side of common arms.
  (iii) Yes, they form linear pair.
  (iv)   Yes, they are supplementary.
  (v)   Yes, they are vertical angles because they are formed due to intersection of straight lines.
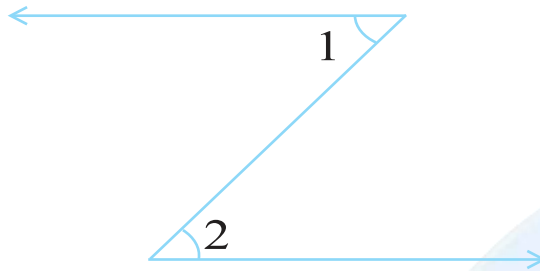 (vi)   Vertically opposite angle of $\angle 5$ is $\angle 2 + \angle 3$ i.e. $\angle COB$.

Q10. Indicate which pairs of angles are:
  (i)   Vertically opposite angles?
  (ii) Linear pairs?

Solution:
  (i) $\angle 1$ and $\angle 4$, $\angle 5$ and ($\angle 2 + \angle 3$) are vertically opposite angles as they formed due to intersection of two straight lines.
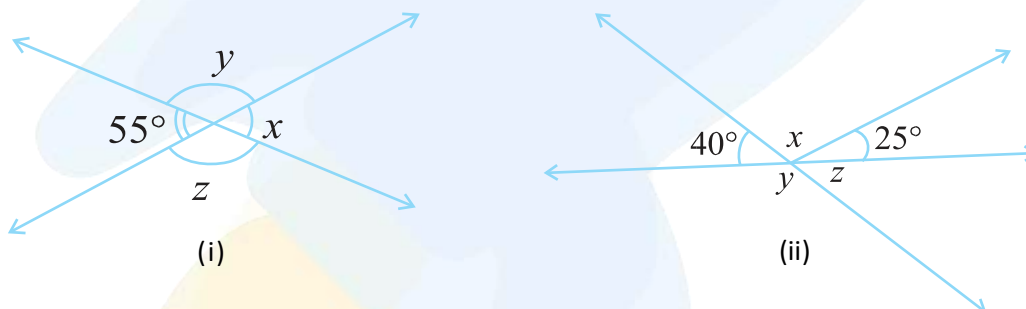 (ii) $\angle 1$ and $\angle 5$, $\angle 5$ and $\angle 4$ forms linear pair.

Q11. In the following figure, is $\angle 1$ adjacent to $\angle 2$? Give reasons.

Solution:
$\angle 1$ is not adjacent to $\angle 2$ because their vertex is not common.

Q12. Find the values of the angles x, y and z in each of the following:

(i) Reasoning
There are two operations done in sequence. First, if one angle is 55° then the angle opposite to it will also be 55° as vertically opposite angles are equal.
Also sum of $\angle x + \angle y = 180°$ and $\angle z + 55° = 180°$. Now, $\angle x, \angle y \ and \ \angle z$ can be easily calculated.

Solution:
Solve for $\angle x, \angle y \ and \ \angle z$:
(i) $\angle x = 55°$ (Vertically opposite angle)
   $\angle x + \angle y = 180°$ (Linear pair)
   $55° + \angle y = 180°$
   $\angle y = 180° - 55°$
   $\angle y = 125°$
Therefore, $\angle y = \angle z = 125°$ (Vertically opposite angle)
Hence, $\angle x = 55°$, $\angle y = 125°$, $\angle z = 125°$

By using angle sum property, find the value of x, and then find the value of y and z.
Since the sum of $y + z = 180°$. Now, it's a matter of finding y and z.

By using angle sum property,
$40° + x + 25° = 180°$ (Angles on straight line)
$x + 65° = 180°$
$x = 180° - 65° = 115°$

Also, $40° + y = 180°$ (Linear pair)
$\quad\quad y = 180° - 40°$
$\quad\quad y = 140°$
$y + z = 180°$ (Linear pair)
$140° + z = 180°$ $(y = 140°)$
$z = 180° - 140°$
$z = 40°$
Thus, $x = 115°, y = 140°$ and $z = 40°$

Q13. Fill in the blanks:

(i) If two angles are complementary, then the sum of their measures is _____.
(ii) If two angles are supplementary, then the sum of their measures is_____.
(iii) Two angles forming a linear pair are _____.
(iv) If two adjacent angles are supplementary, they form a _____.
(v) If two lines intersect a point, then the vertically opposite angles are always _____.
(vi) (vi) If two lines intersect at a point and if one pair of vertically opposite angles are acute angles, then the other pair of vertically opposite angles are_____

Solution:
(i)   90°
(ii)   180°
(iii) Supplementary
(iv)   Linear pair
(v)   Equal
(vi)   Obtuse angle

Q14. In the adjoining figure, name the following pairs of angles:

(i)   Obtuse vertically opposite angles.
(ii)   Adjacent complementary angles
(iii) Equal supplementary angles.
(iv)   Unequal supplementary angles.
(v)   Adjacent angles that do not form a linear pair.
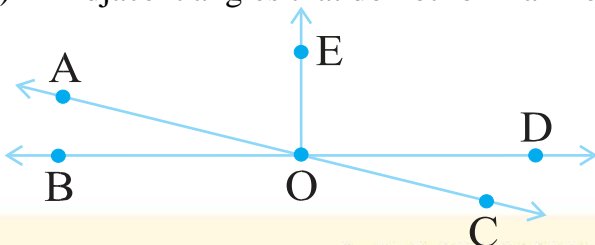
Solution:
(i) Obtuse vertically opposite angles mean angles greater than 90°and are equal

∠AOD = ∠ BOC

(ii) Adjacent complementary angles have common vertex and common arm, non - common arms are on either sides of common arm and their sum is 90°.

∠ EOA and ∠ AOB are adjacent complementary angles.

(iii) Equal supplementary angles have sum of angles 180° and supplementary angles are equal.

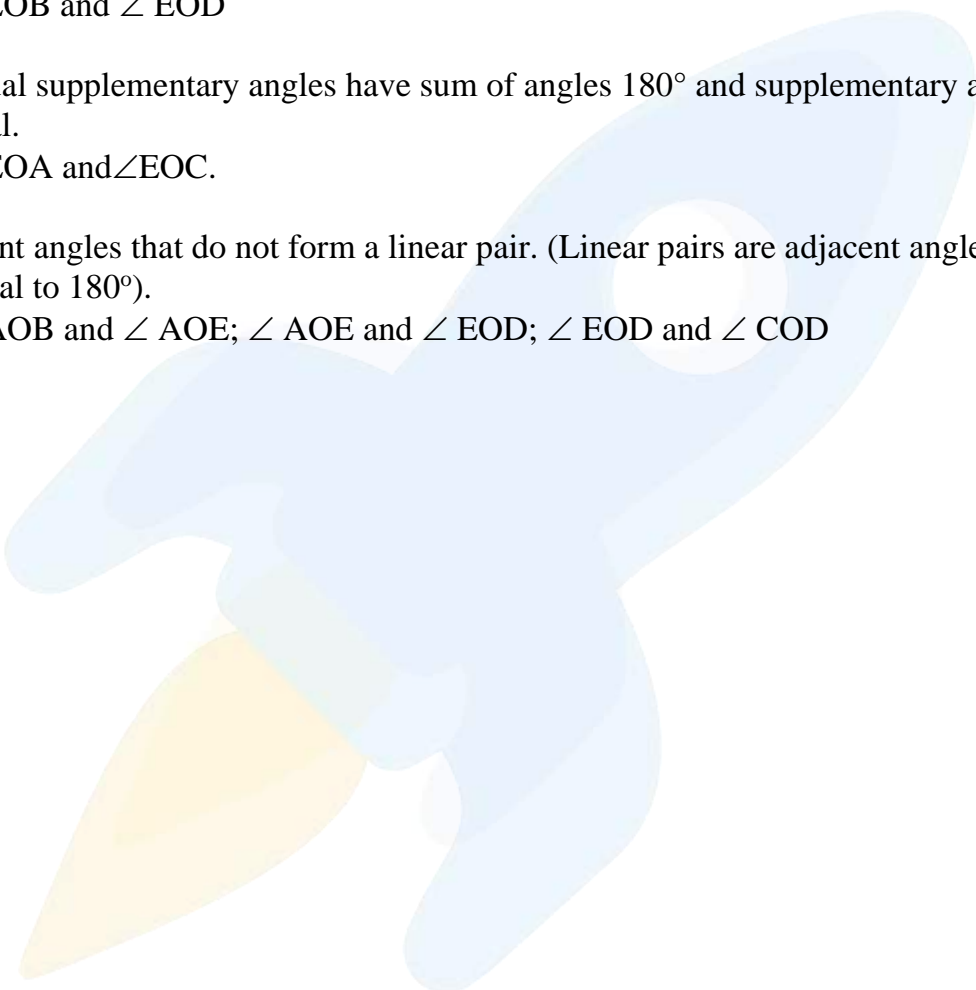∠ EOB and ∠ EOD

(iv) Unequal supplementary angles have sum of angles 180° and supplementary angles are unequal.

∠ EOA and∠EOC.

(v) Adjacent angles that do not form a linear pair. (Linear pairs are adjacent angles whose sum is equal to 180º).

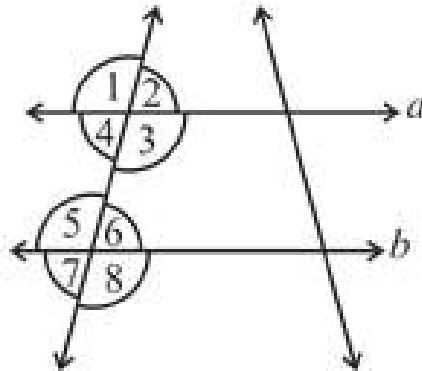∠ AOB and ∠ AOE; ∠ AOE and ∠ EOD; ∠ EOD and ∠ COD

Chapter 5: Lines and Angles

Exercise 5.2 (Page 110)

Q1. State the property that is used in each of the following statements?

      (i)   If a || b, then $\angle 1 = \angle 5$.

      (ii)   If $\angle 4 = \angle 6$, then a || b.

   (iii)   If $\angle 4 + \angle 5 = 180°$, then a || b.

Solution:

  (i)   If two parallel lines are intersected by a transversal, then each pair of corresponding angles are equal   .

  (ii) When a transversal intersects two parallel lines such that if pair of alternate interior angles are equal then the lines are parallel.

 (iii) When a transversal intersects two parallel line such that pair of interior angles on the same side of transversal are supplementary, then the lines are parallel.

Q2. In the adjoining figure, identify:
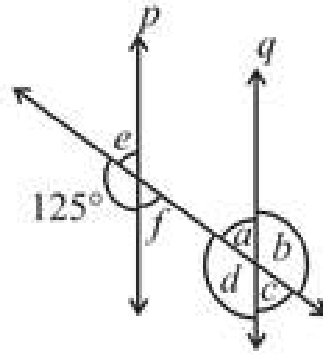
  (i)   The pairs of corresponding angles.

  (ii)   The pairs of alternate interior angles.

 (iii)   Pairs of interior angles on the same side of the transversal.

 (iv)   The vertically opposite angles.
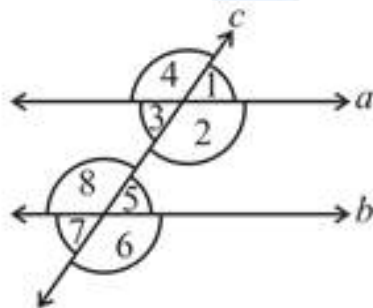
Solution:

  (i) $\angle 1$ and $\angle 5$; $\angle 2$ and $\angle 6$; $\angle 4$ and $\angle 8$; $\angle 3$ and $\angle 7$

 (ii) $\angle 3$ and $\angle 5$ *and* $\angle 2$ and $\angle 8$

 (iii) $\angle 3$ and $\angle 8$ *and* $\angle 2$ and $\angle 5$

 (iv) $\angle 1$ and $\angle 3$; $\angle 2$ and $\angle 4$; $\angle 6$ and $\angle 8$; $\angle 5$ and $\angle 7$

Q3. In the adjoining figure, $p \parallel q$. Find the unknown angles

Reasoning

First, by using linear pair find the measure of ∠e and then find the corresponding and vertically opposite angle to ∠e and again by using linear pair find the value of ∠b and its vertically opposite angle.

Solution:

Given $p \parallel q$ and it is intersected by a transversal.

$\angle d = 125°$ (Corresponding angle)

Since, $125° + \angle e = 180°$ (Linear pair)

$\angle e = 180° - 125°$

$\angle e = 55°$

$\angle e = \angle f = 55°$ (Vertically opposite angles)

$\angle e = \angle a = 55°$ (Corresponding angles)

$\angle a + \angle b = 180°$ (Linear pair)

$55° + \angle b = 180°$

$\angle b = 180° - 55°$

$\angle b = 125°$
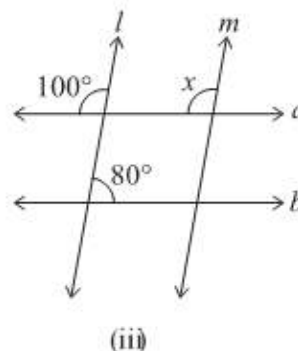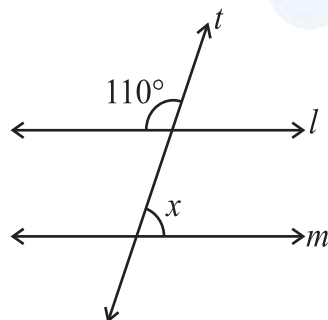
Also, $\angle b = \angle d = 125°$ (Vertically opposite angles)

$\angle a = \angle c = 55°$ (Vertically opposite angles)

Thus, $\angle a = 55°; \angle b = 125°; \angle c = 55°; \angle d = 125°;$

$\angle e = 55° \ \& \ \angle f = 55°$

Q4. Find the value of x in each of the following figures if $l \parallel m$.

(iii)

(i) **Reasoning**

There are two operations done in sequence. First, find the corresponding angle to $110°$ $i.e \angle y$, then by using Linear pair find the *value of ∠x.*

According to this model, the result $\angle x + \angle y = 180°$.
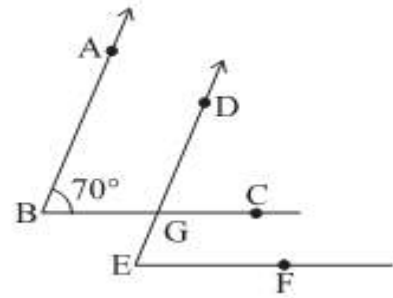Now, it's a matter of finding *value of* $\angle x$.

Solution:

Solve for x

Given l || m and t is transversal,

$\angle y = 110°$ (Corresponding angle)

$\angle x + \angle y = 180°$ (Linear pair)

$\angle x = 180° - 110°$

$\angle x = 70°$

(ii) **Reasoning**

Let's visually model this problem. There is one operation that can be done. Find the corresponding angle to x. According to this model, the resultant value of *corresponding angle will be equal to* x. Now, it's a matter of finding *measure of* x.

Solution:

Solve for x

Given l || m and a || b,

$\angle x = 100°$ (corresponding angle)

Q5. In the given figure, the arms of two angles are parallel. If $\angle ABC = 70°$, then find:

    (i) $\angle DGC$ (ii) $\angle DEF$

Reasoning

Let's visually model this problem. There is one operation that can be done. According to this model, the resultant value of *corresponding angle will be equal to* $\angle DGC$.
Now, it's a matter of finding *measure of* $\angle DGC$.

Solution:

(i) Solve for $\angle DGC$

    Given AB || DG and BC is transversal

    Also, $\angle ABC = 70°$ (Given)

    Since, $\angle ABC = \angle DGC$ (Corresponding angles)

    Therefore, $\angle DGC = 70°$→(1)

Reasoning

Let's visually model this problem. There is one operation that can be done. According to this model, the resultant value of *corresponding angle will be equal to* $\angle DGC$. Now, it's a matter of finding *measure of* $\angle DEF$.
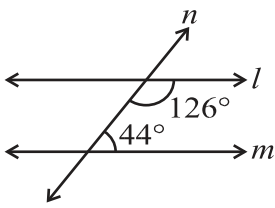
Solution:

(ii) Solve for $< DEF$

    Given BC || EF and DE is transversal

    Also, $< DGC = 70°$ (from 1)

    Since, $< DGC = < DEF$ (Corresponding angles)

    Therefore, $< DEF = 70°$
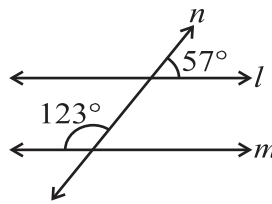
Q6. In the given figures below, decide whether l is parallel to m.
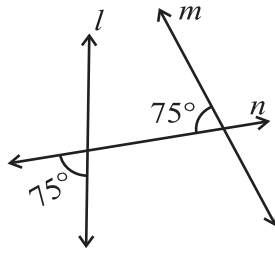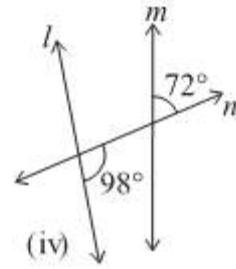

(i)   (ii)   (iii)   (iv)

(i) Reasoning

Let's visually model this problem. There is one operation that can be done, check whether interior angles are supplementary or not. According to this model, the result sum of $126° + 44°$ is $170°$. Now, it's a matter of finding l is parallel to m or not.

Solution:

(i) $126° + 44° = 170°$

As the sum of interior angles on the same side of transversal n is not $180°$.
Therefore, l is not parallel to m.

(ii) Reasoning

Let's visually model this problem. There are two operations that can be done in a sequence. First find the value of x and then check it is equal to its corresponding angle or not. According to this model, the resultant value of x is not equal to its corresponding angle. Now, it's a matter of finding l is parallel to m or not

Solution:

$\angle x + 75° = 180°$ (Linear pair)
$\angle x = 180° - 75°$
$\angle x = 105°$

For l and m to be parallel measure of their corresponding angles should be equal but here the measure of $\angle x$ is $105°$ and its corresponding angle is $75°$.
Therefore, the lines l and m are not parallel.

(iii) Reasoning

Let's visually model this problem. There are two operations that can be done in a sequence. First find the value of x and then check it is equal to its corresponding angle or not. According to this model, the resultant value of x is not equal to its corresponding angle. Now, it's a matter of finding l is parallel to m or not.

Solution:

(iii). $\angle y = 57°$ (Vertically opposite angles)
$\angle x + 123° = 180°$ (Linear pair)
$\angle x = 180° - 123°$
$\angle x = 57°$
Here, the measure of corresponding angles are *equal i.e* $57°$.
Therefore, lines l and m are parallel to each other.

(iv) **Reasoning**

Let's visually model this problem. There are two operations that can be done in a sequence. First find the value of *x by using linear pair* and then check it is equal to its corresponding angle or not. According to this model, the resultant value of *x* is not equal to its corresponding angle. Now, it's a matter of finding *l* is parallel to *m* or not
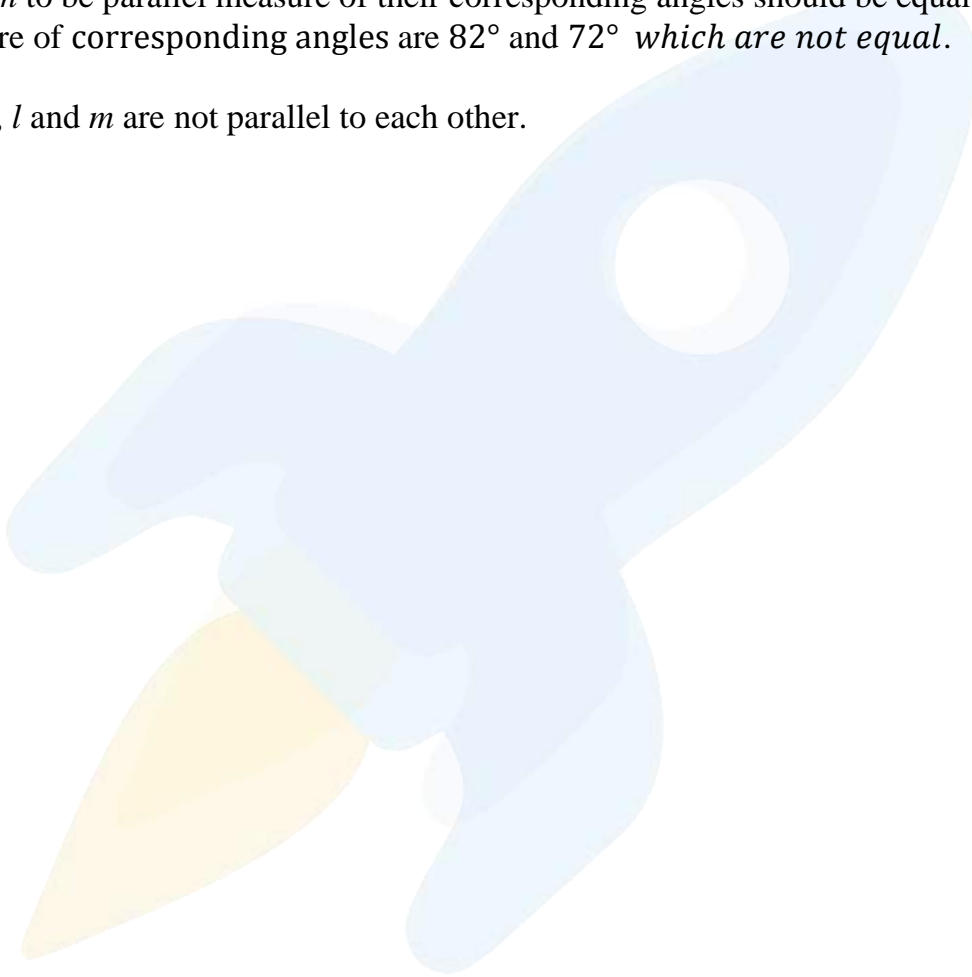
Solution:

(iv) $\angle x + 98° = 180°$ (Linear pair)

$\angle x = 180° - 98°$

$\angle x = 82°$

For *l* and *m* to be parallel measure of their corresponding angles should be equal but here the measure of corresponding angles are 82° and 72° *which are not equal*.

Therefore, *l* and *m* are not parallel to each other.

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