Chapter 6: The Triangle and its Properties

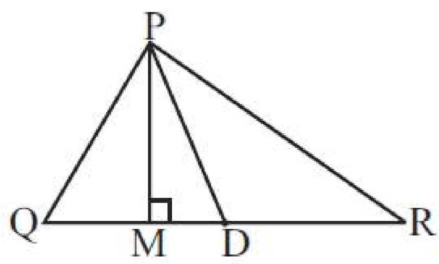
Exercise 6.1 (Page 116)

Q1. In Δ PQR, D is the mid-point of $\overline{QR}$.

$\overline{PM}$ is ________________.

$\overline{PD}$ is ________________.

Is QM $\neq$ MR?



Difficulty level: Easy

What is known:

PQR is a triangle, D is the mid-point of $\overline{QR}$.

What is unknown:

What is $\overline{PM}$ and $\overline{PD}$. Is QM $\neq$ MR?

Reasoning

Let's visually model this problem. There are three operations that done in sequence. First, check PM is perpendicular on QR or not,then check PD divides QR in equal parts or not and then find the mid-point of QR.

According to this model, the result PM is perpendicular on QR then PD divides QR in equal parts and D is the mid-point of QR.

Solution:

Given, PM is perpendicular on QR. Therefore, PM is altitude.
Also, D is the mid-point of QR.
QD = DR
PD is median
No, QM $\neq$ MR, because D is the mid-point of QR.

Q 2. Draw rough sketches for the following:

(i) In ΔABC, BE is a median.

(ii) In ΔPQR, PQ and PR are altitudes of the triangle.

(iii) In ΔXYZ, YL is an altitude in the exterior of the triangle.

Difficulty level: Easy

What is known:

Statement.

What is unknown:

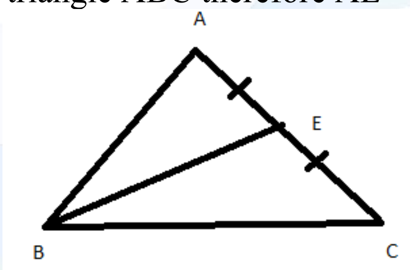
Rough sketch for given statement.

Reasoning:

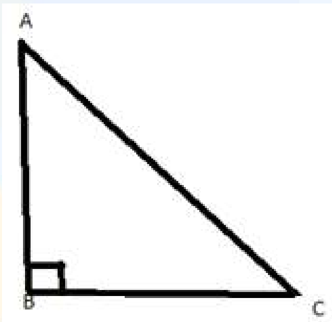
Read the statement carefully and draw a rough figure according to the statement.

Solution:

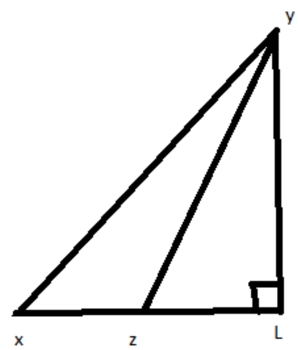
(i) Here BE is a median in triangle ABC therefore AE = EC.



(ii) Here PQ and PR are the altitudes of triangle PQR and PQ ⊥ PR.



(iii) YL is an altitude in the exterior of triangle XYZ.



Q3. Verify by drawing a diagram if the median and altitude of an isosceles triangle can be same.

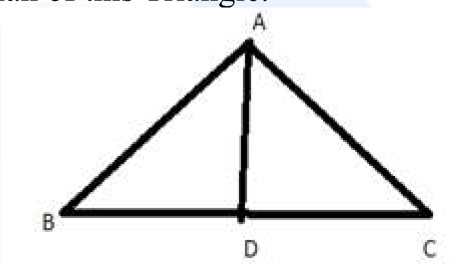
Difficulty level: Medium

What is known:
An isosceles triangle.

What is unknown:
If the median and the altitude of an isosceles triangle can be same.

Reasoning:
First, draw an isosceles triangle and draw a line which divides them into two equal parts. The two sides of this triangle are equal because it is an isosceles triangle and the line drawn is perpendicular to the triangle also it divides base into two equal parts.

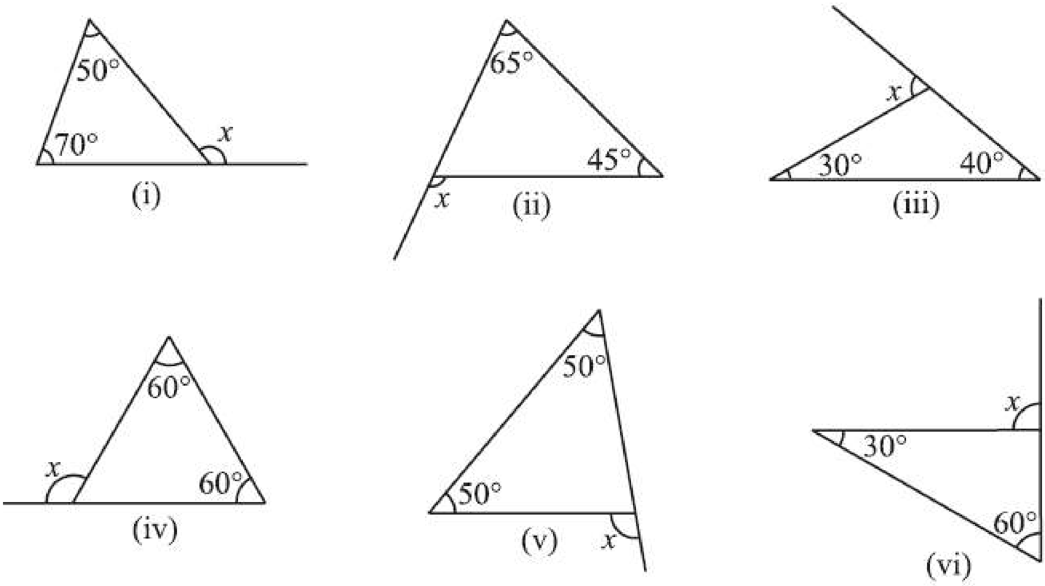
Solution:
Draw a triangle ABC and then draw a line segment AD perpendicular to BC. AD is an Altitude of the triangle. It can be observed that length of BD and DC is also same. Therefore, AD is also a median of this Triangle.



Chapter 6: The Triangle and its Properties

Exercise 6.2 (Page 118)

Q1. Find the value of the unknown exterior angle x in the following diagrams:



Difficulty level: Easy

What is known:
Measurement of interior opposite angles.

What is unknown:
Value of the unknown exterior angle x.

Reasoning:
We know that an exterior angle of a triangle is the sum of interior opposite angles. By using this fact, we can find out the unknown exterior angle x.

Solution:

(i) Interior angles are 50° and 70°
Exterior angle = sum of interior opposite angles
$x = 50° + 70°$
$x = 120°$

(ii) Interior angles are 65° and 45°
Exterior angle = sum of interior opposite angles
$x = 65° + 45°$
$x = 110°$

(iii) Interior angles are 30° and 70°
Exterior angle = sum of interior opposite angles
$x = 30° + 40°$
$x = 70°$

(iv) Interior angles are 60° and 60°

Exterior angle = sum of interior opposite angles

$x = 60° + 60°$

$x = 120°$

(v) Interior angles are 50° and 50°

Exterior angle = sum of interior opposite angles

$x = 50° + 50°$

$x = 100°$

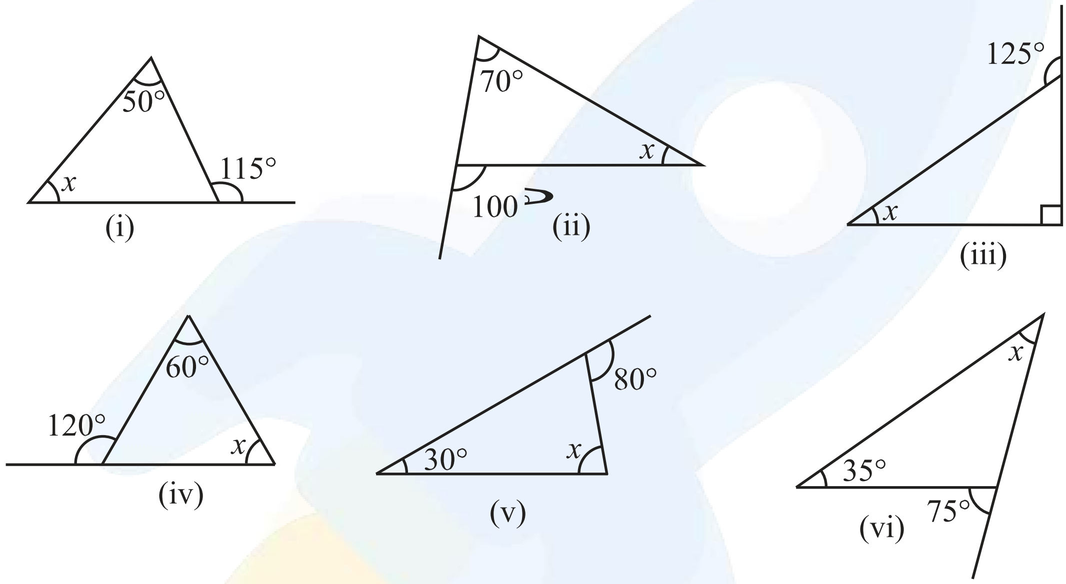
(vi) Interior angles are 30° and 60°

Exterior angle = sum of interior opposite angles

$x = 30° + 60°$

$x = 90°$

Q2. Find the value of the unknown interior angle *x* in the following figures:



Difficulty level: Medium

What is known:

One of the interior opposite angle and exterior angle.

What is unknown:

Interior opposite angle *x*.

Reasoning:

We know that the sum of interior opposite angle is equal to the exterior angle. By using this fact, we can find the unknown interior angle.

Solution:

(i) Exterior angle is 115°

Exterior angle = sum of interior opposite angles

$115° = x + 50°$

$115° - 50° = x$

$x = 65°$

(ii) Exterior angle is 100°
Exterior angle = sum of interior opposite angles
$100° = x + 70°$
$x = 100° - 70°$
$x = 30°$

(iii) Exterior angle is 125°
Exterior angle = sum of interior opposite angles
$125° = x + 90°$ (90° given)
$x = 125° - 90°$
$x = 35°$

(iv) Exterior angle is 120°
Exterior angle = sum of interior opposite angles
$120° = x + 60°$
$x = 120° - 60°$
$x = 60°$

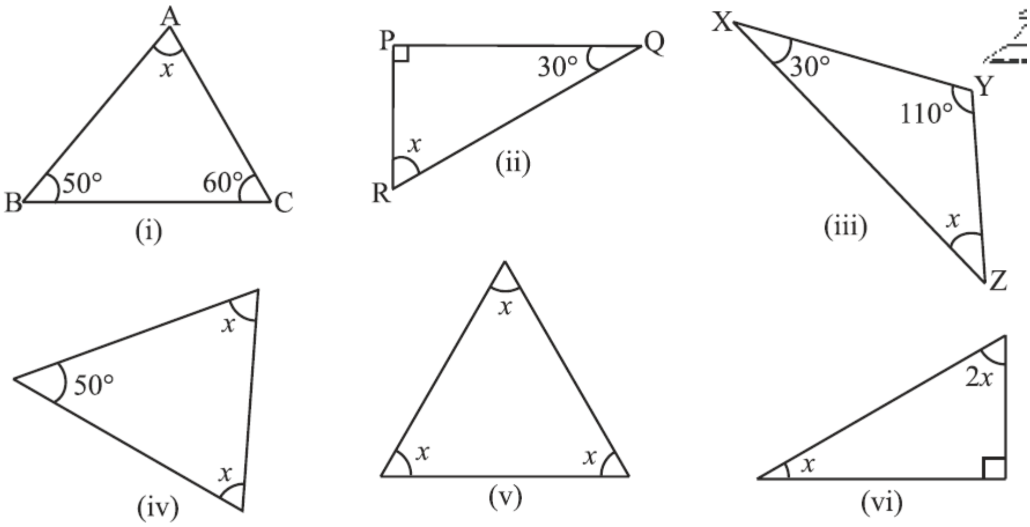
(v) Exterior angle is 80°
Exterior angle = sum of interior opposite angles
$80° = x + 30°$
$x = 80° - 30°$
$x = 50°$

(vi) Exterior angle is 75°
Exterior angle = sum of interior opposite angles
$75° = x + 35°$
$x = 75° - 35°$
$x = 40°$

Chapter 6: The Triangle and its Properties

Exercise 6.3 (Page 121)

Q1. Find the value of the unknown x in the following diagrams:



Difficulty level: Easy

What is given / known:
Measure of two interior angles of triangles.

What is the unknown:
Value of one of the angles of the given triangles.

Reasoning:
We make use of the angle sum property of a triangle according to which the sum of the interior angles of a triangle is always equal to 180º. If the given unknown interior angle of a triangle is x, then it can be obtained by subtracting sum of the other two angles from 180º.

Solution:

(i) Sum of interior angles of a triangle = 180°

$\angle A + \angle B + \angle C = 180°$

$x + 50° + 60° = 180°$

$x = 180° - 110° = 70°$

(ii) Sum of interior angles of a triangle = 180°

$\angle P + \angle Q + \angle R = 180°$

$90° + 30° + x = 180°$ (∠P=90° from figure)

$x = 180° - 120° = 60°$

(iii) Sum of interior angles of a triangle = 180°

$\angle X + \angle Y + \angle Z = 180°$

$30° + 110° + x = 180°$

$x = 180° - 140° = 40°$

(iv) Sum of interior angles of a triangle = 180° (From step1)

$x + x + 50° = 180°$

$2x + 50° = 180°$

$2x = 180° - 50°$

$x = \frac{130}{2} = 65°$

(v) Sum of interior angles of a triangle = 180° (From step1)

$x + x + x = 180°$

$3x = 180°$

$x = \frac{180}{3} = 60°$

(vi) Sum of interior angles of a triangle = 180° (From step 1)

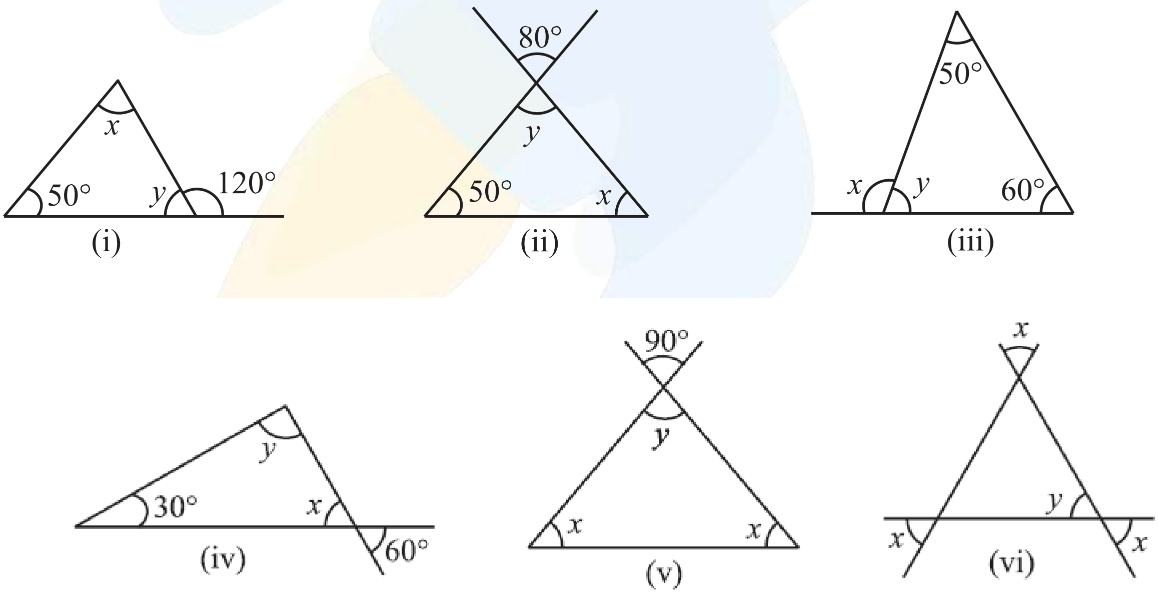
$x + 2x + 90° = 180°$

$3x + 90° = 180°$

$3x = 180° - 90°$

$x = \frac{90}{3} = 30°$

Q2. Find the values of the unknowns *x* and *y* in the following diagrams:



Difficulty level: Medium

What is given / known:

Measure of one interior angle and exterior angle.

What is the unknown:

Value of *x* and *y* in the given diagrams

Reasoning:

Let us assume the unknown interior angle of the given triangle as "x" and "y". Now, let us model this problem visually. We must know the two properties.

1. Keeping in mind that sum of the interior opposite angles of a triangle is always equal to the exterior angle, we can find out the value of unknown interior value.
2. Knowing the angle sum property i.e. sum of the interior angles of a triangle is always equal to180^0, we can then find out another unknown interior angle.

Solution:

We know that, Sum of interior opposite angles of a triangle = Exterior angle

$$50° + x = 120°$$
$$x = 120° - 50°$$
$$x = 70°$$

By angle sum property,

Sum of interior angles = 180°

$$50° + 70° + y = 180°$$
$$120° + y = 180°$$
$$y = 180° - 120°$$
$$y = 60°$$

(ii)

What is given / known?

Measure of one interior angle and exterior angle.

What is the unknown?

Value of "x" & "y".

Reasoning:

This question is simple based on two concepts. First vertically opposite angles are equal by using this we can find out the angle opposite to 80° i.e. one of the interior angles. Now, one more interior angle is unknown here so, we use the angle sum property i.e. sum of three interior angles of a triangle is 180° and find out the value of another unknown angle.

Solution:

$y = 80°$ (vertically opposite angles)

By angle sum property

Sum of interior angles = 180°

$$50° + x + y = 180°$$
$$50° + x + 80° = 180°$$
$$130° + x = 180°$$
$$x = 180° - 130°$$
$$x = 50°$$

(iii)

What is given / known:

Measure of two interior angles.

What is the unknown:

Value of "x" & "y".

Reasoning:

Visually identify the unknown interior angle and then follow two steps. First, by using angle sum property we can find out the value of unknown interior angle and then for angle x we know that sum of interior opposite angles of a triangle is equal to exterior angle.

Solution:

Solve for x and y

We know that, Sum of interior opposite angles of a triangle = Exterior angle

$$50° + 60° = x$$
$$110° = x$$
$$x = 110°$$

By angle sum property,

Sum of interior angles = 180°

$$50° + 60° + y = 180°$$
$$110° + y = 180°$$
$$y = 180° - 110°$$
$$y = 70°$$

(iv)

What is given / known?

Measure of one interior angle and exterior angle.

What is the unknown?

Value of "x" & "y"

Reasoning:

This question is simple based on two concepts. First vertically opposite angles are equal by using this we can find out the angle opposite to 60° i.e. one of the interior angles. Now, one more interior angle is unknown here so, we use the angle sum property i.e. sum of three interior angles of a triangle is 180° and find out the value of another unknown angle.

Solution:

Solve for x and y

$x = 60°$ (vertically opposite angles are equal)

We know that,

Sum of interior angles of a triangle =180°

$$30° + x + y = 180°$$
$$30° + 60° + y = 180°$$
$$90° + y = 180°$$
$$y = 180° - 90°$$
$$y = 90°$$

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(v)

What is given / known:

Measure of Exterior angle.

What is the unknown:

Value of "x" & "y".

Reasoning:

This question is simply based on two concepts. First, vertically opposite angles are equal by using this we can find out the value of one interior angle and then by applying angle sum property we can find out another interior angle. Here the measure of two interior angles is equal.

Solution:

Solve for x and y

$y = 90°$ (vertically opposite angles are equal)

We know that,

Sum of interior angles of a triangle = 180°

$$y + x + x = 180°$$
$$90° + 2x = 180°$$
$$2x = 180° - 90°$$
$$x = \frac{90°}{2}$$
$$x = 45°$$

(vi)

What is the unknown:

Two angles i.e. "x" & "y".

Reasoning:

This question is simply based on two concepts. First, vertically opposite angles are equal by using this we can find out that x and y are equal and then by applying angle sum property we can find all angles.

Solution:

Solve for x and y

$x = y$ (vertically opposite angles are equal)

We know that,

Sum of interior angles of a triangle =180°

$$x + x + y = 180°$$
$$2x + y = 180°$$
$$2x + x = 180° \quad (x = y)$$
$$3x = 180°$$
$$x = \frac{180°}{3}$$
$$x = 60°$$

Since $x = y$
Therefore,

$$y = 60°$$

Reasoning Shortcut / Tip

Whenever you encounter problems of this kind, its best to recall the angle sum property of a triangle.

Chapter 6: The Triangle and its Properties

Exercise 6.4

Q1. Is it possible to have a triangle with the following sides?

(i) 2 cm, 3 cm, 5 cm (ii) 3 cm, 6 cm, 7 cm (iii) 6 cm, 3 cm, 2 cm

Difficulty level: Medium

What is known:

Sides of triangle.

What is unknown:

Is it possible to have a triangle with these sides.

Reasoning:

This problem is based on a very simple property that if the sum of lengths of any two sides of a triangle is greater than the third side, then it is possible to draw a triangle. By using this property, you just have to check by taking the sum of any two sides of a triangle whether it is greater than the third side or not. For better understanding take any two sides one by one and check if sum of these two sides is greater than the third side or not, if it is greater than the third sides then it is possible to draw a triangle.

(i) $2 + 3 > 5$ No
$3 + 5 > 2$ Yes
$5 + 2 > 3$ Yes

No, the triangle is not possible

(ii) $3 + 6 > 7$ yes
$6 + 7 > 3$ yes
$7 + 3 > 3$ yes

Yes, the triangle is possible

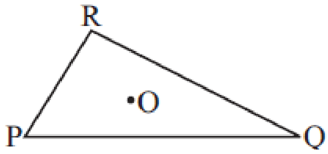
(iii) $6 + 3 > 2$ yes
$3 + 2 > 6$ No
$2 + 6 > 3$ Yes

No, the triangle is not possible

Useful Tip

Whenever you encounter problems of this kind, it is best to think of the property based on sum of lengths of any two sides of a triangle.

Q2. Take any point O in the interior of a triangle PQR. Is
(i) OP + OQ > PQ? (ii) OQ + OR > QR? (iii) OR + OP > RP?

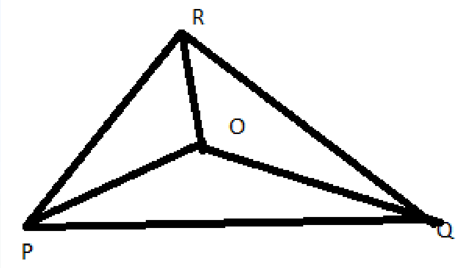


Difficulty level: Medium

What is known:
Triangle PQR and point in the interior of triangle.

What is unknown:
i) OP + OQ > PQ?
ii) OQ + OR > QR?
iii) OR + OP > RP?

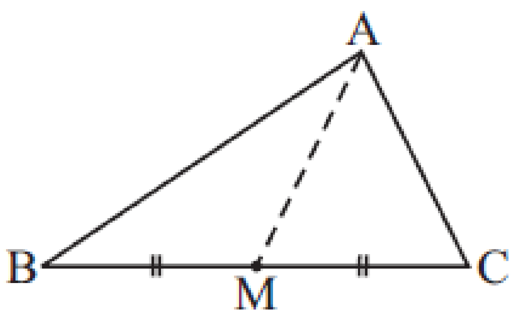
Reasoning:
This question is straight forward. If "o" is a point in the interior of a given triangle, then the three triangles can be constructed. You must join all the three vertices of a triangle with the centre "o". This question is based on a property that in a triangle the sum of lengths of any two sides is always greater than the third side.



Solution:

Yes, POQ forms a triangle
Yes, OQR forms a triangle.
Yes, ORP forms a triangle.

Q3. AM is a median of a triangle ABC. Is AB + BC + CA > 2 AM? (Consider the sides of triangles ΔABM and ΔAMC.)



Difficulty level: Difficult

What is known:
ABC is a triangle and AM is a median of triangle ABC.

What is unknown:
Is AB + BC + CA > 2 AM?

Reasoning:
In this question it is asked if AB + BC + CA > 2 AM or not. This question is also based on the property that the sum of lengths of two sides of a triangle is always greater than the third side. In such kind of problems, you just visually identify the triangle ABC and AM is the median which further divides triangle ABC into two more triangles i.e. triangle ABM and AMC. Now consider any two sides of first triangle ABM and use the above property then consider the any two sides of another triangle AMC and use the above property. Now, add L.H.S. and R.H.S. of both the triangles.

Solution:

In triangle ABM,

$$AB + BM > AM \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In triangle AMC,

$$AC + MC > AM \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

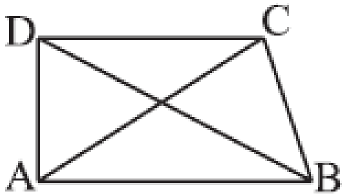
Adding equation (1) and (2) we get,

$$AB + BM + +AC + MC > AM + AM$$
$$AB + AC + BM + MC > 2AM$$
$$AB + AC + BC > 2AM$$

Hence, it is true

Useful Tip:
Whenever you encounter problems of this kind, it is best to think of the property based on sum of lengths of any two sides of a triangle is always greater than the third side.

Q4. ABCD is a quadrilateral. Is AB + BC + CD + DA > AC + BD?



Difficulty level: Difficult

What is known:
ABCD is a quadrilateral. DB and AC are diagonals.

What is unknown:
Is AB + BC + CD + DA > AC + BD?

Reasoning:
In this question, it is asked to check Is AB + BC + CD + DA > AC + BD or not. This question is based on the property that the sum of lengths of two sides of a triangle is always greater than the third side. Now visually identify that the quadrilateral ABCD is divided by diagonals AC and BD into four triangles. Now, take each triangle separately and apply the above property and then add L.H.S and R.H.S of the equation formed.

Solution:

In triangle ABC,

$AB + BC > AC$ .... (1)

In triangle ADC,

$AD + CD > AC$ .... (2)

In triangle ADB,

$AD + AB > DB$ (3)

In triangle DCB,

$DC + CB > DB$ (4)

Adding equation (1), (2), (3) and (4) we get,

$AB + BC + AD + CD + AD + AB + DC + CB > AC + AC + DB + DB$

$AB + AB + BC + BC + CD + CD + AD + AD > 2AC + 2DB$

$2AB + 2BC + 2CD + 2AD > 2AC + 2DB$

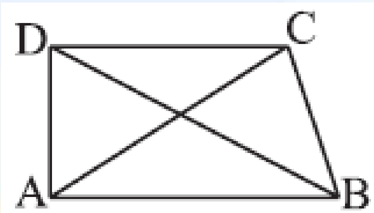
$AB + BC + CD + AD > AC + DB$

Hence, it is true

Useful Tip

Whenever you encounter problems of this kind, it is best to think of the property based on sum of lengths of any two sides of a triangle is always greater than the third side.

Q5. ABCD is quadrilateral. Is AB + BC + CD + DA < 2 (AC + BD)?



Difficulty level: Difficult

What is known:

ABCD is a quadrilateral. DB and AC are diagonals.

What is unknown:

Is AB + BC + CD + DA < 2 (AC + BD)?

Reasoning:

In this question, it is asked to check Is AB + BC + CD + DA <2(AC + BD) or not.
This question is based on the property that the sum of lengths of two sides of a triangle is always greater than the third side. Now visually identify that the quadrilateral ABCD is divided by diagonals AC and BD into four triangles. "O" is the centre of the quadrilateral. Now, take each triangle separately i.e. triangle AOB, COD, BOC and AOD and apply the above property and then add L.H.S and R.H.S of the equation formed.

Solution:

In triangle AOB,

AB < OA + OB .... (1)

In triangle COD,

CD < OC + OD .... (2)

In triangle AOD,

DA < OD + OA .... (3)

In triangle COB,

BC < OC + OB .... (4)

Adding equation (1), (2), (3) and (4) we get,

$$AB + BC + CD + DA < OA + OB + OC + OD + OD + OA + OC + OB$$
$$= AB + BC + CD + DA < 2OA + 2OB + 2OC + 2OD$$
$$= AB + BC + CD + DA < 2(OA + OB) + 2(OC + OD)$$
$$= AB + BC + CD + DA < 2(AC + BD)$$

Yes, it is true.

Useful Tip

Whenever you encounter problems of this kind, it is best to think of the property based on sum of lengths of any two sides of a triangle is always greater than the third side.

Q6. The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the lengths of third side fall.

Difficulty level: Medium

What is known:

The lengths of two sides of a triangle are 12 cm and 15 cm.

What is unknown:

Between what two measures should the lengths of third side fall.

Reasoning:

This question is straight forward. you just have to remember one property that the sum of lengths of any two sides of a triangle is always greater than the third side and also the difference between the length of any two sides of a triangle is smaller than the length of third side. In this question, two sides of a triangle are given as 12cm and 15 cm, find the sum and difference of this two sides. Remember, the third side should be lesser than their sum and also it should be greater than their difference.

Solution:

Given,

Sides of a triangle are 12cm and 15 cm

We know that, Sum of lengths of any two sides of a triangle is always greater than the third side and also, the difference between the lengths of any two sides is always smaller than the third side.

Hence the third side will be lesser than the sum of these two sides 12cm + 15cm =27 cm and also it will be greater than the difference of these two sides 15cm – 12cm =3cm. Therefore, length of third side will be smaller than 27cm and greater than 3cm.

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Chapter 6: The Triangle and its Properties

Exercise 6.5

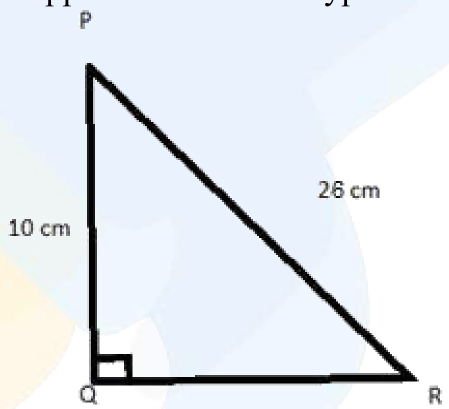
Q1. PQR is a triangle right angled at P. If PQ = 10 cm and PR = 24 cm, find QR.

Difficulty Level: Easy

What is given /known:
PQR is a triangle right angled at P and the length of two sides PQ = 10 cm and PR = 24 cm.

What is the unknown:
Length of one side of the triangle.

Reasoning:
This question is straight forward, as it is given in the question that PQR is a right- angled triangle and it is right angled at P. So, we can apply Pythagoras theorem here, if it is right angled at P then the side opposite to P will be the hypotenuse of the triangle i.e. QR and the other side is given PQ = 10cm and PR = 24. Now, by applying Pythagoras theorem i.e. in a right-angled triangle, the square of hypotenuse is equal to the sum of square of other two sides, we can find QR. For better visual understanding draw a right-angled triangle which is right angled at Q and consider the side opposite to it PR as hypotenuse



Solution:

Given, PQ = 10 cm, PR = 24 cm and QR =?

By applying Pythagoras theorem in triangle PQR, we get

$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

$= (QR)^2 = (PQ)^2 + (PR)^2$

$= (QR)^2 = (10)^2 + (24)^2$

$=(QR)^2 = 100 + 576$

$= (QR)^2 = 676$

$QR = 26$ cm

Thus, QR is equal to 26 cm

Useful Tip:

Whenever you encounter problems of this kind, it is best to think of the Pythagoras theorem for right angled triangle.

Q2. ABC is a triangle right angled at C. If AB = 25 cm and AC = 7 cm, find BC.

Difficulty Level: Easy

What is given / known:
ABC is a triangle right angled at C and the length of two sides AB = 25 cm and AC = 7 cm.

What is the unknown:
Length of BC.

Reasoning:
This question is straight forward, as it is given in the question that ABC is a triangle, right-angled at C. So, we can apply Pythagoras theorem here, if it is right angled at C then the side opposite to C will be the hypotenuse of the triangle i.e. AB = 25 cm and the other two side is AC = 7 cm and BC. Now, by applying Pythagoras theorem i.e. in a right-angled triangle, the square of hypotenuse is equal to the sum of square of other two sides, we can find BC.

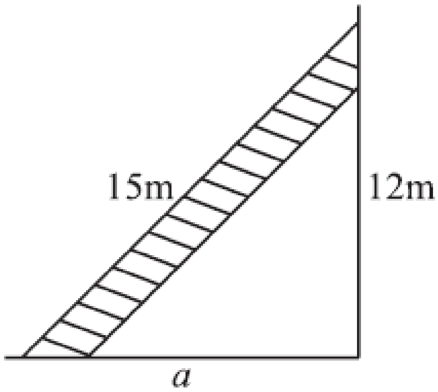
For better visual understanding draw a right-angled triangle which is right angled at C and consider the side opposite to it AB as hypotenuse.

Solution:
Given, ABC is a triangle which is right-angled at C.
AB = 25cm, AC = 7 cm and BC =?

In triangle ACB, By Pythagoras theorem,
$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$
$= (AB)^2 = (AC)^2 + (BC)^2$
$= (25)^2 = (7)^2 + (BC)^2$
$625 = 49 + (BC)^2$
$(BC)^2 = 625 - 49$
$(BC)^2 = 576$
$BC = 24$ cm
Thus, BC is equal to 24cm

Useful Tip
Whenever you encounter problems of this kind, it is best to think of the Pythagoras theorem for right – angled triangle.

Q3. A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance a. Find the distance of the foot of the ladder from the wall.



Difficulty Level: Moderate

What is given / known:
Length of ladder is 15 m and the height of the window from the ground is 12m.

What is the unknown:
The distance of the foot of the ladder from the wall.

Reasoning:
This question is based on the concept of right – angled triangle. As it is clear from the figure that the ladder is kept slanted on the wall so, consider length of the ladder as hypotenuse i.e.: AB =15m and as it is kept slanted on the wall so, we consider wall as perpendicular i.e.: AC = 12 m. Now, you must find out the distance of the foot of the ladder from the wall i.e.: BC = a. Now by applying Pythagoras theorem in triangle ABC, we can find out BC.

For better visual understanding draw a right-angled triangle consider ladder as hypotenuse, wall as perpendicular and distance between the foot of the ladder and wall as base.

Solution:

Given, Length of ladder AB = 15 m
Length of wall AC = 12 m

To find (BC) = distance of the foot of the ladder from the wall.
According to Pythagoras theorem,

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$
$$(AB)^2 = (AC)^2 + (BC)^2$$
$$(15)^2 = (12)^2 + (a)^2$$
$$225 = 144 + (a)^2$$
$$225 - 144 = (a)^2$$
$$81 = a^2 \qquad a = 9 \text{ m}$$

Therefore, the distance of the foot of the ladder from the wall is 9 m.

Useful Tip:
Whenever you encounter problem of this kind, it is better to understand its visually.

Q4. Which of the following can be the sides of a right triangle?

(i) 2.5 cm, 6.5 cm, 6 cm

(ii) 2 cm, 2 cm, 5 cm.

(iii) 1.5 cm, 2cm, 2.5 cm.

In the case of right-angled triangles, identify the right angles.

Difficulty Level: Easy

What is given / known:

Sides of the triangles.

What is the unknown:

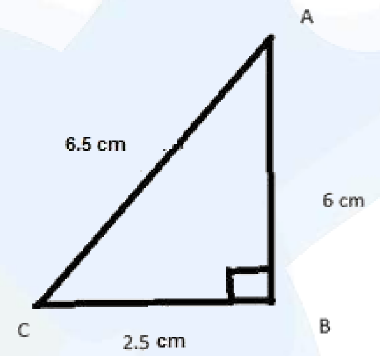
Given sides are the sides of right-angled triangle or not.

Reasoning:

This question is very simple. Consider the greater side as the hypotenuse and if the square of hypotenuse is equal to the sum of square of other two sides then these are the sides of a right - angled triangle.

Solution:

(i)



$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$

$(6.5)^2 = (6)^2 + (2.5)^2$

$42.25 = 36 + 6.25$

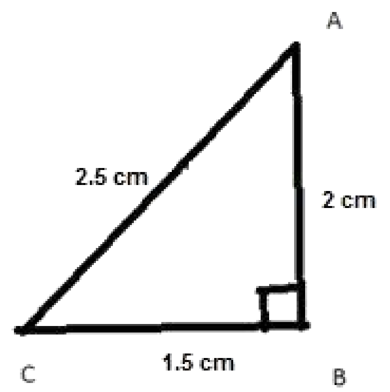
$42.25 = 42.25$

L.H.S = R.H.S

Therefore, given sides are of the right-angled triangle.

Right angle lies opposite to the greater side.

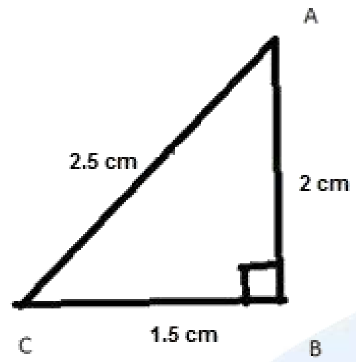
(ii)



$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$
$$(5)^2 = (2)^2 + (2)^2$$
$$25 = 4 + 4$$
$$25 \neq 8$$

Therefore, the given sides are not of a right- angled triangle.

(iii)



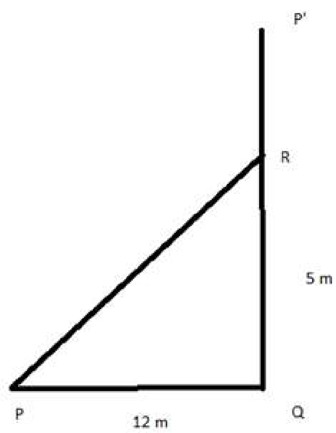
$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$
$$(2.5)^2 = (2)^2 + (1.5)^2$$
$$6.25 = 4 + 2.25$$
$$6.25 = 6.25$$
$$\text{L.H.S} = \text{R.H.S}$$

Therefore, given sides are of a right-angled triangle. Right angle lies on the opposite to the greater side.

Useful Tip:

Whenever you encounter problem of this kind, it is best to think of the Pythagoras property that if the square of hypotenuse or greater side is equal to the sum of square of other two sides then it is a right- angled triangle.

Q5. A tree is broken at a height of 5 m from the ground and its top touches the ground at 12 m from the base of the tree. Find the original height of the tree.



Difficulty Level: Moderate

What is given / known:

Height of the point where the tree broke from the ground is 5 m and distance between the base of the tree and the top of the tree when it touches the ground is 12 m.

What is the unknown:
Original height of the tree.

Reasoning:
This question is also based on the concept of right-angled triangle and Pythagoras theorem. Suppose P'Q is the height of the tree, as it is mentioned in the question that the tree is broken at a height of 5m from the ground. suppose tree is broken from R So, consider RQ as perpendicular and PR as broken part of the tree and, as hypotenuse. Remember, length of PR i.e. broken part of the tree will remain same as P'R. Now, a right – angled triangle PQR is formed, apply Pythagoras theorem and find the length of broken part i.e. PR. As we must find out the original height of the tree for this, add the length of the broken part and the length where it broke i.e. QR.

For better visual understanding draw a right-angled triangle and visualise all the parts.

Solution:
Let P'Q represents the original height of the tree before it broken at R and RP represents the broken part of the tree.
Triangle PQR is right angled at Q. So, in this triangle, according to Pythagoras theorem,

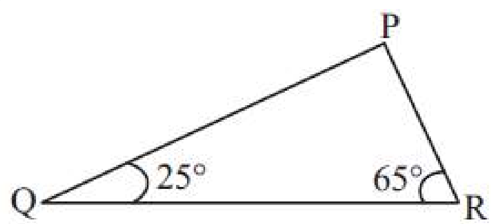
$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$
$$(PR)^2 = (RQ)^2 + (PQ)^2$$
$$(PR)^2 = (5)^2 + (12)^2$$
$$(PR)^2 = 25+144$$
$$(PR)^2 = 169$$
$$PR = 13 \text{ m}$$

Thus, the original height of the tree = PR + RQ
= 13 m + 5 m
= 18 m
Thus, the original height of the tree is 18 m.

Useful Tip:
Whenever you encounter problem of this kind, it is best to think of the concept of a right-angled triangle.

Q6. Angles Q and R of a ΔPQR are 25º and 65º.
Write which of the following is true:

(i) $PQ^2 + QR^2 = RP^2$
(ii) $PQ^2 + RP^2 = QR^2$
(iii) $RP^2 + QR^2 = PQ^2$



Difficulty Level: Moderate

What is given / known:
Measure of two angles of a triangle i.e. 25°and 65°.

What is the unknown:
Which option is correct.

Reasoning:

This question is straight forward, in this question it is clear from the figure two angles of a triangle are given and we must find out the third angle by using angle sum property i.e. the sum of three interior angles of a triangle is 180°. By using this property, we got the value of third angle i.e. P = 90° that means side opposite to P is hypotenuse i.e. QR. As one of the angles is 90° that means it is a right-angled triangle and the square of hypotenuse is equal to the sum of square of other two sides.

Solution:

We know that, sum of interior angles of a triangle is 180°.

$$\angle P + \angle Q + \angle R = 180^\circ$$
$$\angle P + 25^\circ + 65^\circ = 180^\circ$$
$$\angle P + 90^\circ = 180^\circ$$
$$\angle P = 180^\circ - 90^\circ$$
$$\angle P = 90^\circ$$

Thus, triangle PQR is a right angled at P

Therefore, by Pythagoras theorem

$$(P)^2 + (B)^2 = (H)^2$$
$$(QP)^2 + (PR)^2 = (QR)^2$$

Hence, option (ii) is correct.

Useful Tip:

Whenever you encounter problem of this kind, it is best to think of the Pythagoras property if one of the three angles is 90°then the square of hypotenuse or greater side is equal to the sum of square of other two sides.

Q7. Find the perimeter of the rectangle whose length is 40 cm and diagonal is 41 cm.

Difficulty Level: Moderate

What is given / known:

Length of rectangle is given 40 cm and diagonal is 41 cm.

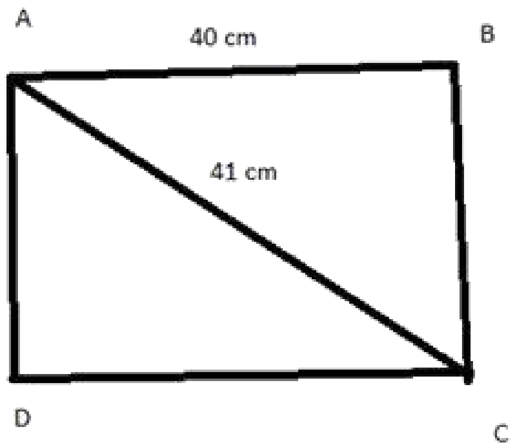
What is the unknown:

Breadth and perimeter of rectangle.

Reasoning:

This question is based on the two concepts i.e. the concept of rectangle and Pythagoras theorem. For better understanding of this question understand it with the help of a figure.

As it is mentioned in the question suppose there is a rectangle ABCD and whose length is given 40 cm. As, we know the opposite sides of a rectangle are equal if AB = 40 cm that means side opposite to AB i.e. CD will also be 40 cm. Now, one of the diagonals of a rectangle is given AC = 41cm it divides the rectangle into two right-angled triangles. Now, you can apply Pythagoras theorem and find the third side i.e. breadth of the rectangle. Now, you have the measure of both the sides of a rectangle i.e. length and breadth, you can easily find out the perimeter of the rectangle.



Solution:

Given, in rectangle ABCD, AB and CD are the lengths of the rectangle.
AB = 40 cm, CD = 40 cm and AC is the diagonal
Therefore, AC = 41 cm
Let breadth of rectangle be x.

Now, in triangle ADC, by Pythagoras theorem

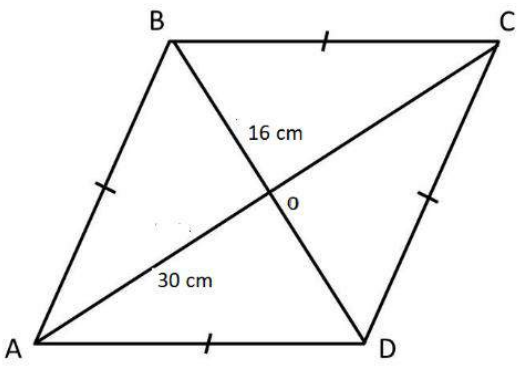
$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$
$$(AC)^2 = (DC)^2 + (AD)^2$$
$$(41)^2 = (40)^2 + (x)^2$$
$$1681 = 1600 + (x)^2$$
$$1681 - 1600 = (x)^2$$
$$x^2 = 81$$
$$\text{or } x = 9 \text{ cm}$$

Therefore, breadth of rectangle = 9 cm

Now, we know that
Perimeter of rectangle = 2(l + b)
= 2 (40 + 9)
= 2(49)
= 98 cm
Hence, the perimeter of rectangle is 98 cm

Useful Tip: whenever you encounter problems of this kind, remember the Pythagoras property of right -angled triangle and the formulas related to rectangle.

Q8. The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.



Difficulty Level: Tough

What is given / known:
Diagonals of rhombus are given as 16cm and 30cm.

What is the unknown:
Perimeter of rhombus.

Reasoning:
This question is also based on the two concepts i.e. the concept of rhombus and Pythagoras theorem. For better understanding of this question understand it with the help of a figure.

As it is mentioned in the question suppose there is a rhombus ABCD and length of whose diagonals are given as 16 cm and 30 cm. As, we know the diagonals of a rhombus bisect each other at 90° i.e. at o. Now, o divides DB and AC into two equal parts i.e. OD is half of DB and OC is half AC. Now, you can apply Pythagoras theorem in triangle DOC and get the measure of side DC. Now, the side of rhombus is known, you can easily find the perimeter of rhombus.

Solution:
Given, AC and BD are the two diagonals of the rhombus.

AC = 30 cm, BD = 16 cm

$$OD = \frac{DB}{2} = \frac{16}{2} = 8\,cm$$

Since, the diagonal of a rhombus bisect each other at 90°.

Therefore, $OC = \frac{AC}{2} = \frac{30}{2} = 15\,cm$

$\Delta DOC,$

Now, in right angled triangle

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$
$$(DC)^2 = (OD)^2 + (OC)^2$$
$$(DC)^2 = (8)^2 + (15)^2$$
$$(DC)^2 = 64 + 225$$
$$(DC)^2 = 289$$
$$DC = 17\text{ cm}$$

Perimeter of rhombus $= 4 \times$ side
$= 4 \times 17$
$= 68$ cm

Thus, the perimeter of rhombus is 68 cm.

Useful Tip:
Whenever you encounter problem of this kind, it is best to think of the properties related to rhombus.