Chapter – 14: Factorization

Exercise 14.1

Q1: Find the common factors of the terms

(i) $12x,\ 36$

(ii) $2y,\ 22xy$

(iii) $14pq,\ 28p^2q^2$

(iv) $2x,\ 3x^2,\ 4$

(iv) $6abc,\ 24ab^2,\ 12a^2b$

(vi) $16x^3,\ -4x^2,\ 32x$

(vii) $10pq,\ 20qr,\ 30rp$

(viii) $3x^2y^3,\ 10x^3y^2,\ 6x^2y^2z$

Difficulty level: Easy

What is known:
Terms.

What is unknown:
Common factors of given terms.

Reasoning:
First, we will find factors of each terms then find out which factors are common in each term.

Solution:

(i) $12x = 2\times 2\times 3\times x$

$36 = 2\times 2\times 3\times 3$

The common factors are 2, 2, 3.
And, $2\times 2\times 3 = 12$

(ii) $2y = 2\times y$

$22xy = 2\times 11\times x\times y$

The common factors are 2, y.
And, $2\times y = 2y$

(iii) $14pq = 2\times 7\times p\times q$

$28p^2q^2 = 2\times 2\times 7\times p\times p\times q\times q$

The common factors are 2, 7, p, q.
And, $2\times 7\times p\times q = 14pq$

(iv) $2x = 2\times x$

$3x^2 = 3\times x\times x$

$4 = 2\times 2$

The common factor is 1.

(v) $6abc = 2\times 3\times a\times b\times c$

$24ab^2 = 2\times 2\times 2\times 3\times a\times b\times b$

$12a^2b = 2\times 2\times 3\times a\times a\times b$

The common factors are 2, 3, a, b.
And, $2\times 3\times a\times b = 6ab$

(vi) $16x^3 = 2\times 2\times 2\times 2\times x\times x\times x$

$-4x^2 = -1\times 2\times 2\times x\times x$

$32x = 2\times 2\times 2\times 2\times 2\times x$

The common factors are 2, 2, x.
And, $2\times 2\times x = 4x$

(vii) $10pq = 2\times 5\times p\times q$

$20qr = 2\times 2\times 5\times q\times r$

$30rp = 2\times 3\times 5\times r\times p$

The common factors are 2, 5.
And, $2\times 5 = 10$

(viii) $3x^2y^3 = 3\times x\times x\times y\times y\times y$

$10x^3y^2 = 2\times 5\times x\times x\times x\times y\times y$

$6x^2y^2z = 2\times 3\times x\times x\times y\times y\times z$

The common factors are x, x, y, y.
And, $x\times x\times y\times y = x^2y^2$

Q2: Factorise the following expressions

(i) $7x-42$

(ii) $6p-12q$

(iii) $7a^2+14a$

(iv) $-16z+20z^3$

(v) $20l^2m+30alm$

(vi) $5x^2y-15xy^2$

(vii) $10a^2-15b^2+20c^2$

(viii) $-4a^2+4ab-4ca$

(ix) $x^2yz+xy^2z+xyz^2$

(x) ax^2y+bxy^2+cxyz

Difficulty level: Easy

What is known:

Algebraic expression.

What is unknown:

Factorisation of given algebraic expression.

Reasoning:

First, we will find factors of each terms then find out which factors are common in each term and take out that common factor from expression.

Solution:

(i) $7x = 7 \times x$

$42 = 2 \times 3 \times 7$

The common factor is 7.

$$\therefore 7x - 42 = (7 \times x) - (2 \times 3 \times 7) = 7\ (x - 6)$$

(ii) $6p = 2 \times 3 \times p$

$12q = 2 \times 2 \times 3 \times q$

The common factors are 2 and 3.

$$\begin{aligned}\therefore 6p - 12q &= (2 \times 3 \times p) - (2 \times 2 \times 3 \times q) \\ &= 2 \times 3[p - (2 \times q)] \\ &= 6(p - 2q)\end{aligned}$$

(iii) $7a^2 = 7 \times a \times a$

$14a = 2 \times 7 \times a$

The common factors are 7 and a.

$$\begin{aligned}\therefore 7a^2 + 14a &= (7 \times a \times a) + (2 \times 7 \times a) \\ &= 7 \times a[a + 2] \\ &= 7a(a + 2)\end{aligned}$$

(iv) $16z = 2 \times 2 \times 2 \times 2 \times z$

$20z^3 = 2 \times 2 \times 5 \times z \times z \times z$

The common factors are 2, 2, and z.

$$\begin{aligned}\therefore -16z + 20z^3 &= -(2 \times 2 \times 2 \times 2 \times z) + (2 \times 2 \times 5 \times z \times z \times z) \\ &= (2 \times 2 \times z)[-(2 \times 2) + (5 \times z \times z)] \\ &= 4z\left(-4 + 5z^2\right)\end{aligned}$$

(v) $20l^2m = 2 \times 2 \times 5 \times l \times l \times m$

$30\,alm = 2 \times 3 \times 5 \times a \times l \times m$

The common factors are 2, 5, l and m.

$$\begin{aligned}\therefore 20l^2m + 30alm &= (2 \times 2 \times 5 \times l \times l \times m) + (2 \times 3 \times 5 \times a \times l \times m) \\ &= (2 \times 5 \times l \times m)[(2 \times l) + (3 \times a)] \\ &= 10lm(2l + 3a)\end{aligned}$$

(vi) $5x^2y = 5\times x\times x\times y$

$15xy^2 = 3\times 5\times x\times y\times y$

The common factors are 5, x, and y.

$$\therefore 5x^2y - 15xy^2 = (5\times x\times x\times y) - (3\times 5\times x\times y\times y)$$
$$= 5\times x\times y[x - (3\times y)]$$
$$= 5xy(x - 3y)$$

(vii) $10a^2 = 2\times 5\times a\times a$

$15b^2 = 3\times 5\times b\times b$

$20c^2 = 2\times 2\times 5\times c\times c$

The common factor is 5.

$$10a^2 - 15b^2 + 20c^2 = (2\times 5\times a\times a) - (3\times 5\times b\times b) + (2\times 2\times 5\times c\times c)$$
$$= 5[(2\times a\times a) - (3\times b\times b) + (2\times 2\times c\times c)]$$
$$= 5\left(2a^2 - 3b^2 + 4c^2\right)$$

(viii) $4a^2 = 2\times 2\times a\times a$

$4ab = 2\times 2\times a\times b$

$4ca = 2\times 2\times c\times a$

The common factors are 2, 2, and a.

$$\therefore -4a^2 + 4ab - 4ca = -(2\times 2\times a\times a) + (2\times 2\times a\times b) - (2\times 2\times c\times a)$$
$$= 2\times 2\times a[-(a) + b - c]$$
$$= 4a(-a + b - c)$$

(ix) $x^2yz = x\times x\times y\times z$

$xy^2z = x\times y\times y\times z$

$xyz^2 = x\times y\times z\times z$

The common factors are x, y, and z.

$$\therefore x^2yz + xy^2z + xyz^2 = (x\times x\times y\times z) + (x\times y\times y\times z) + (x\times y\times z\times z)$$
$$= x\times y\times z[x + y + z]$$
$$= xyz(x + y + z)$$

(x) $ax^2y = a\times x\times x\times y$

$bxy^2 = b\times x\times y\times y$

$cxyz = c\times x\times y\times z$

The common factors are x and y.

$$ax^2y + bxy^2 + cxyz = (a\times x\times x\times y) + (b\times x\times y\times y) + (c\times x\times y\times z)$$
$$= (x\times y)[(a\times x) + (b\times y) + (c\times z)]$$
$$= xy(ax + by + cz)$$

Q3: Factorize:

(i) $x^2 + xy + 8x + 8y$

(ii) $15xy - 6x + 5y - 2$

(iii) $ax + bx - ay - by$

(iv) $15pq + 15 + 9q + 25p$

(v) $z - 7 + 7xy - xyz$

Difficulty level: Medium

What is known:
Algebraic expression.

What is unknown:
Factorisation of given algebraic expression.

Reasoning:
There are 4 terms in each expression. First, we will make pair of two terms from which we can take out common factors and convert the expression of 4 terms into 2 terms expression then take out common factors from remaining 2 terms.

Solution:

(i)
$$\begin{aligned} x^2 + xy + 8x + 8y &= x \times x + x \times y + 8 \times x + 8 \times y \\ &= x(x+y) + 8(x+y) \\ &= (x+y)(x+8) \end{aligned}$$

(ii)
$$\begin{aligned} 15xy - 6x + 5y - 2 &= 3 \times 5 \times x \times y - 3 \times 2 \times x + 5 \times y - 2 \\ &= 3x(5y-2) + 1(5y-2) \\ &= (5y-2)(3x+1) \end{aligned}$$

(iii)
$$\begin{aligned} ax + bx - ay - by &= a \times x + b \times x - a \times y - b \times y \\ &= x(a+b) - y(a+b) \\ &= (a+b)(x-y) \end{aligned}$$

(iv)
$$\begin{aligned} 15pq + 15 + 9q + 25p &= 15pq + 9q + 25p + 15 \\ &= 3 \times 5 \times p \times q + 3 \times 3 \times q + 5 \times 5 \times p + 3 \times 5 \\ &= 3q(5p+3) + 5(5p+3) \\ &= (5p+3)(3q+5) \end{aligned}$$

(v)
$$\begin{aligned} z - 7 + 7xy - xyz &= z - xyz - 7 + 7xy \\ &= z - x \times y \times z - 7 + 7 \times x \times y \\ &= z(1-xy) - 7(1-xy) \\ &= (1-xy)(z-7) \end{aligned}$$

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Chapter – 14: Factorization

Exercise 14.2

Q1: Factorize the following expressions.

(i) $a^2+8a+16$

(ii) $p^2-10p+25$

(iii) $25m^2+30m+9$

(iv) $49y^2+84yz+36z^2$

(v) $4x^2-8x+4$

(vi) $121b^2-88bc+16c^2$

(vii) $(l+m)^2-4lm$ (Hint: *Expand* $(l+m)^2$ *first*)

(viii) $a^4+2a^2b^2+b^4$

Difficulty level: Medium

What is known:
Algebraic expression.

What is unknown:
Factorisation of the algebraic expression.

Reasoning:
Use identity:

$$(x+y)^2=x^2+2xy+y^2$$
$$(x-y)^2=x^2-2xy+y^2$$

Solution:

(i) $a^2+8a+16=(a)^2+2\times a\times 4+(4)^2$

$=(a+4)^2$ $\left[\begin{array}{l}\text{Using identity } (x+y)^2=x^2+2xy+y^2,\\ \text{considering } x=a \text{ and } y=4\end{array}\right]$

(ii) $p^2-10p+25=(p)^2-2\times p\times 5+(5)^2$

$=(p-5)^2$ $\left[\begin{array}{l}\text{Using identity } (a-b)^2=a^2-2ab+b^2,\\ \text{considering } a=p \text{ and } y=5\end{array}\right]$

(iii) $25m^2+30m+9=(5m)^2+2\times 5m\times 3+(3)^2$

$=(5m+3)^2$ $\left[\begin{array}{l}\text{Using identity } (a+b)^2=a^2+2ab+b^2,\\ \text{considering } a=5m \text{ and } b=3\end{array}\right]$

(iv) $49y^2+84yz+36z^2=(7y)^2+2\times(7y)\times(6z)+(6z)^2$

$=(7y+6z)^2$ $\left[\begin{array}{l}\text{Using identity } (a+b)^2=a^2+2ab+b^2, \\ \text{considering } a=7y \text{ and } b=6z\end{array}\right]$

(v) $4x^2-8x+4=(2x)^2-2(2x)(2)+(2)^2$

$=(2x-2)^2$ $\left[\begin{array}{l}\text{Using identity } (a-b)^2=a^2-2ab+b^2, \\ \text{Considering } a=2x \text{ and } y=2\end{array}\right]$

$=[(2)(x-1)]^2=4(x-1)^2$

(vi) $121b^2-88bc+16c^2=(11b)^2-2(11b)(4c)+(4c)^2$

$=(11b-4c)^2$ $\left[\begin{array}{l}\text{Using identity } (a-b)^2=a^2-2ab+b^2, \\ \text{Considering } a=11b \text{ and } b=4c\end{array}\right]$

(vii) $(l+m)^2-4lm=l^2+2lm+m^2-4lm$

$=l^2-2lm+m^2$

$=(l-m)^2$ $\left[\begin{array}{l}\text{Using identity } (a-b)^2=a^2-2ab+b^2, \\ \text{considering } a=l \text{ and } b=m\end{array}\right]$

(viii) $a^4+2a^2b^2+b^4=\left(a^2\right)^2+2\left(a^2\right)\left(b^2\right)+\left(b^2\right)^2$

$=\left(a^2+b^2\right)^2$ $\left[\begin{array}{l}\text{Using identity } (x+y)^2=x^2+2xy+y^2, \\ \text{considering } x=a^2 \text{ and } y=b^2\end{array}\right]$

Q2: Factorize

(i) $4p^2-9q^2$

(ii) $63a^2-112b^2$

(iii) $49x^2-36$

(iv) $16x^5-144x^3$

(v) $(l+m)^2-(l-m)^2$

(vi) $9x^2y^2-16$

(vii) $\left(x^2-2xy+y^2\right)-z^2$

(viii) $25a^2-4b^2+28bc-49c^2$

Difficulty level: Easy

What is known:
Algebraic expression.

What is unknown:
Factorisation of the algebraic expression.

Reasoning:

Use identity:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$a^2 - b^2 = (a-b)(a+b)$$

Solution:

(i) $4p^2 - 9q^2 = (2p)^2 - (3q)^2$

$= (2p+3q)(2p-3q)$ $\left[\begin{array}{l}\text{Using identity } a^2-b^2=(a-b)(a+b),\\ \text{considering } a = 2p \text{ and } b = 3q\end{array}\right]$

(ii) $63a^2 - 112b^2 = 7\left(9a^2 - 16b^2\right)$

$= 7\left[(3a)^2 - (4b)^2\right]$

$= 7\left[(3a+4b)(3a-4b)\right]$ $\left[\begin{array}{l}\text{Using identity } x^2-y^2=(x-y)(x+y),\\ \text{considering } x = 3a \text{ and } y = 4b\end{array}\right]$

(iii) $49x^2 - 36 = (7x)^2 - (6)^2$

$= (7x-6)(7x+6)$ $\left[\begin{array}{l}\text{Using identity } a^2-b^2=(a-b)(a+b),\\ \text{considering } a = 7x \text{ and } b = 6.\end{array}\right]$

(iv) $16x^5 - 144x^3 = 16x^3\left(x^2 - 9\right)$

$= 16x^3\left[(x)^2 - (3)^2\right]$

$= 16x^3\left[(x-3)(x+3)\right]$ $\left[\begin{array}{l}\text{Using identity } a^2-b^2=(a-b)(a+b)\\ \text{Considering } a = x \text{ and } b = 3.\end{array}\right]$

(v) $(l+m)^2 - (l-m)^2 = \left[(l+m)-(l-m)\right]\left[(l+m)+(l-m)\right]$

$= (l+m-l+m)(l+m+l-m)$ $\left[\begin{array}{l}\text{Using identity } a^2-b^2=(a-b)(a+b),\\ \text{considering } a = (l+m) \text{ and } b = (l-m)\end{array}\right]$

$= 2m \times 2l$
$= 4ml$
$= 4lm$

(vi) $9x^2y^2 - 16 = (3xy)^2 - (4)^2$

$= (3xy-4)(3xy+4)$ $\left[\begin{array}{l}\text{Using the identity } a^2-b^2=(a-b)(a+b),\\ \text{considering } a = 3xy \text{ and } b = 4.\end{array}\right]$

(vii) $\left(x^2 - 2xy + y^2\right) - z^2 = (x-y)^2 - (z)^2$ $\left[\begin{array}{l}\text{Using identity } (a-b)^2 = a^2 - 2ab + b^2\\ \text{for } (x-y)^2 = x^2 - 2xy + y^2\end{array}\right]$

$= (x-y-z)(x-y+z)$ $\left[\begin{array}{l}\text{Using identity } a^2-b^2=(a-b)(a+b)\\ \text{considering } a = x-y \text{ and } b = z.\end{array}\right]$

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(viii) $25a^2 - 4b^2 + 28bc - 49c^2 = 25a^2 - \left(4b^2 - 28bc + 49c^2\right)$

$$= (5a)^2 - \left[(2b)^2 - 2 \times 2b \times 7c + (7c)^2\right]$$

$$\left[\begin{array}{l}\text{Using identity } (x-y)^2 = x^2 - 2xy + y^2 \\ \text{considering } x = 2b \text{ and } y = 7c.\end{array}\right]$$

$$= (5a)^2 - (2b - 7c)^2$$

$$\left[\begin{array}{l}\text{Using identity } x^2 - y^2 = (x-y)(x+y) \\ \text{considering } x = 5a \text{ and } y = 2b - 7c.\end{array}\right]$$

$$= [5a + (2b - 7c)][5a - (2b - 7c)]$$

$$= (5a + 2b - 7c)(5a - 2b + 7c)$$

Q3: Factorise the expressions

(i) $ax^2 + bx$

(ii) $7p^2 + 21q^2$

(iii) $2x^3 + 2xy^2 + 2xz^2$

(iv) $am^2 + bm^2 + bn^2 + an^2$

(v) $(lm + l) + m + 1$

(vi) $y(y + z) + 9(y + z)$

(vii) $5y^2 - 20y - 8z + 2yz$

(viii) $10ab + 4a + 5b + 2$

(ix) $6xy - 4y + 6 - 9x$

Difficulty level: Medium

What is known:
Algebraic expression.

What is unknown:
Factorisation of given algebraic expression.

Reasoning:
For part (i), (ii), (iii) and (vi) - First we will find factors of each terms then find out which factors are common in each term and take out that common factor from expression.

For part (iv), (v), (vii), (viii), (ix) - There are 4 terms in each expression. First, we will make pair of two terms from which we can take out common factors and convert the expression of 4 terms into 2 terms expression then take out common factors from remaining 2 terms.

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Solution:

(i) $ax^2 + bx = a \times x \times x + b \times x = x(ax + b)$

(ii) $7p^2 + 21q^2 = 7 \times p \times p + 3 \times 7 \times q \times q = 7\left(p^2 + 3q^2\right)$

(iii) $2x^3 + 2xy^2 + 2xz^2 = 2x\left(x^2 + y^2 + z^2\right)$

(iv) $$\begin{aligned} am^2 + bm^2 + bn^2 + an^2 &= am^2 + bm^2 + an^2 + bn^2 \\ &= m^2(a+b) + n^2(a+b) \\ &= (a+b)\left(m^2 + n^2\right) \end{aligned}$$

(v) $$\begin{aligned} (lm + l) + m + 1 &= lm + m + l + 1 \\ &= m(l+1) + 1(l+1) \\ &= (l+1)(m+1) \end{aligned}$$

(vi) $y(y+z) + 9(y+z) = (y+z)(y+9)$

(vii) $$\begin{aligned} 5y^2 - 20y - 8z + 2yz &= 5y^2 - 20y + 2yz - 8z \\ &= 5y(y-4) + 2z(y-4) \\ &= (y-4)(5y+2z) \end{aligned}$$

(viii) $$\begin{aligned} 10ab + 4a + 5b + 2 &= 10ab + 5b + 4a + 2 \\ &= 5b(2a+1) + 2(2a+1) \\ &= (2a+1)(5b+2) \end{aligned}$$

(ix) $$\begin{aligned} 6xy - 4y + 6 - 9x &= 6xy - 9x - 4y + 6 \\ &= 3x(2y-3) - 2(2y-3) \\ &= (2y-3)(3x-2) \end{aligned}$$

Q4: Factorise

(i) $a^4 - b^4$

(ii) $p^4 - 81$

(iii) $x^4 - (y+z)^4$

(iv) $x^4 - (x-z)^4$

(v) $a^4 - 2a^2b^2 + b^4$

Difficulty level: Easy

What is known:

Algebraic expression.

What is unknown:
Factorisation of the algebraic expression.

Reasoning:
Use identity:

$$(a-b)^2 = a^2 - 2ab + b^2$$
$$a^2 - b^2 = (a-b)(a+b)$$

Solution:
(i) $a^4 - b^4 = \left(a^2\right)^2 - \left(b^2\right)^2$

$$= \left(a^2 - b^2\right)\left(a^2 + b^2\right)$$
$$= (a-b)(a+b)\left(a^2 + b^2\right)$$

(ii) $p^4 - 81 = \left(p^2\right)^2 - (9)^2$

$$= \left(p^2 - 9\right)\left(p^2 + 9\right)$$
$$= \left[(p)^2 - (3)^2\right]\left(p^2 + 9\right)$$
$$= (p-3)(p+3)\left(p^2 + 9\right)$$

(iii) $x^4 - (y+z)^4 = \left(x^2\right)^2 - \left[(y+z)^2\right]^2$

$$= \left[x^2 - (y+z)^2\right]\left[x^2 + (y+z)^2\right]$$
$$= [x - (y+z)][x + (y+z)]\left[x^2 + (y+z)^2\right]$$
$$= (x - y - z)(x + y + z)\left[x^2 + (y+z)^2\right]$$

(iv) $x^4 - (x-z)^4 = \left(x^2\right)^2 - \left[(x-z)^2\right]^2$

$$= \left[x^2 - (x-z)^2\right]\left[x^2 + (x-z)^2\right]$$
$$= \left[x - (x-z)\right]\left[x + (x-z)\right]\left[x^2 + (x-z)^2\right]$$
$$= z(2x - z)\left[x^2 + x^2 - 2xz + z^2\right]$$
$$= z(2x - z)\left(2x^2 - 2xz + z^2\right)$$

(v) $a^4 - 2a^2b^2 + b^4 = \left(a^2\right)^2 - 2\left(a^2\right)\left(b^2\right) + \left(b^2\right)^2$

$$= \left(a^2 - b^2\right)^2$$
$$= [(a-b)(a+b)]^2$$
$$= (a-b)^2(a+b)^2$$

Q5: Factorise the following expressions

(i) p^2+6p+8

(ii) $q^2-10q+21$

(iii) $p^2+6p-16$

Difficulty level: Medium

What is known:
Algebraic expression.

What is unknown:
Factorisation of the algebraic expression.

Reasoning:
In general, for factorising an algebraic expression of the type x^2+px+q, we find two factors a and b of q (i.e., the constant term) such that $ab = q$ and $a + b = p$.

Solution:

(i) p^2+6p+8

It can be observed that, $8=4\times 2$ and $4+2=6$

$$\begin{aligned}\therefore p^2+6p+8 &= p^2+2p+4p+8\\ &= p(p+2)+4(p+2)\\ &=(p+2)(p+4)\end{aligned}$$

(ii) $q^2-10q+21$

It can be observed that, $21=(-7)\times(-3)$ and $(-7)+(-3)=-10$

$$\begin{aligned}\therefore q^2-10q+21 &= q^2-7q-3q+21\\ &= q(q-7)-3(q-7)\\ &=(q-7)(q-3)\end{aligned}$$

(iii) $p^2+6p-16$

It can be observed that, $-16 = (-2)\times 8$ *and* $8+(-2) = 6$

$$\begin{aligned}p^2+6p-16 &= p^2+8p-2p-16\\ &= p(p+8)-2(p+8)\\ &=(p+8)(p-2)\end{aligned}$$

Chapter – 14: Factorization

Exercise 14.3

Q1: Carry out the following divisions.

(i) $28x^4 \div 56x$

(ii) $-36y^3 \div 9y^2$

(iii) $66pq^2r^3 \div 11qr^2$

(iv) $34x^3y^3z^3 \div 51xy^2z^3$

(v) $12a^8b^8 \div \left(-6a^6b^4\right)$

(i) $28x^4 \div 56x$

Difficulty level: Easy

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Find out factors of $28x^4$ and $56x$ then cancel out common factors of $28x^4$ and $56x$.

Solution:

$28x^4$ can be written as $2\times2\times7\times x\times x\times x\times x$

and

$56x$ can be written as $2\times2\times2\times7\times x$

Then,

$$28x^4 \div 56x = \frac{2\times2\times7\times x\times x\times x\times x}{2\times2\times2\times7\times x}$$
$$= \frac{x^3}{2}$$
$$= \frac{1}{2}x^3$$

(ii) $-36y^3 \div 9y^2$

Difficulty level: Easy

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Find out factors of $-36y^3$ and $9y^2$ then cancel out common factors of $-36y^3$ and $9y^2$.

Solution:

$-36y^3$ can be written as $-2\times2\times3\times3\times y\times y\times y$

and

$9y^2$ can be written as $3\times3\times y\times y$

Then,

$$-36y^3 \div 9y^2 = \frac{-2\times2\times3\times3\times y\times y\times y}{3\times3\times y\times y}$$
$$= -4y$$

(iii) $66pq^2r^3 \div 11qr^2$

Difficulty level: Easy

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Find out factors of $66pq^2r^3$ and $11qr^2$ then cancel out common factors of $66pq^2r^3$ and $11qr^2$.

Solution:

$66pq^2r^3$ can be written as $2\times3\times11\times p\times q\times q\times r\times r\times r$

and

$11qr^2$ can be written as $11\times q\times r\times r$

Then,

$$66pq^2r^3 \div 11qr^2 = \frac{2\times3\times11\times p\times q\times q\times r\times r\times r}{11\times q\times r\times r}$$
$$= 6pqr$$

(iv) $34x^3y^3z^3 \div 51xy^2z^3$

Difficulty level: Easy

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Find out factors of $34x^3y^3z^3$ and $51xy^2z^3$ then cancel out common factors of $34x^3y^3z^3$ and $51xy^2z^3$.

Solution:

$34x^3y^3z^3$ can be written as $2\times17\times x\times x\times x\times y\times y\times y\times z\times z\times z$

and

$51xy^2z^3$ can be written as $3\times17\times x\times y\times y\times z\times z\times z$

Then,

$$34x^3y^3z^3 \div 51xy^2z^3 = \frac{2\times17\times x\times x\times x\times y\times y\times y\times z\times z\times z}{3\times17\times x\times y\times y\times z\times z\times z}$$
$$= \frac{2}{3}x^2y$$

(v) $12a^8b^8 \div \left(-6a^6b^4\right)$

Difficulty level: Easy

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Find out factors of $12a^8b^8$ and $-6a^6b^4$ then cancel out common factors of $12a^8b^8$ and $-6a^6b^4$.

Solution:
$12a^8b^8$ can be written as $2\times2\times3\times a^8\times b^8$

and

$-6a^6b^4$ can be written as $-2\times3\times a^6\times b^4$

Then,

$$12a^8b^8 \div \left(-6a^6b^4\right) = \frac{2\times2\times3\times a^8\times b^8}{-2\times3\times a^6\times b^4}$$
$$= -2a^2b^4$$

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Q2: Divide the given polynomial by the given monomial.

(i) $\left(5x^2-6x\right)\div 3x$

(ii) $\left(3y^8-4y^6+5y^4\right)\div y^4$

(iii) $8\left(x^3y^2z^2+x^2y^3z^2+x^2y^2z^3\right)\div 4x^2y^2z^2$

(iv) $\left(x^3+2x^2+3x\right)\div 2x$

(v) $\left(p^3q^6-p^6q^3\right)\div p^3q^3$

(i) $\left(5x^2-6x\right)\div 3x$

Difficulty level: Medium

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Find out factors of $\left(5x^2-6x\right)$ and $3x$ then cancel out common factors of $\left(5x^2-6x\right)$ and $3x$

Solution:

$5x^2-6x$ can be written as $x(5x-6)$

Then,

$$\left(5x^2-6x\right)\div 3x=\frac{x(5x-6)}{3x}$$
$$=\frac{1}{3}(5x-6)$$

(ii) $\left(3y^8-4y^6+5y^4\right)\div y^4$

Difficulty level: Medium

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Find out factors of $\left(3y^8-4y^6+5y^4\right)$ and y^4 then cancel out common factors of $\left(3y^8-4y^6+5y^4\right)$ and y^4.

Solution:

$3y^8-4y^6+5y^4$ can be written as $y^4\left(3y^4-4y^2+5\right)$

Then,

$$\left(3y^8-4y^6+5y^4\right)\div y^4=\frac{y^4\left(3y^4-4y^2+5\right)}{y^4}$$
$$=3y^4-4y^2+5$$

(iii) $8\left(x^3y^2z^2+x^2y^3z^2+x^2y^2z^3\right)\div 4x^2y^2z^2$

Difficulty level: Medium

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Find out factors of $8\left(x^3y^2z^2+x^2y^3z^2+x^2y^2z^3\right)$ and $4x^2y^2z^2$ then cancel out common factors of $8\left(x^3y^2z^2+x^2y^3z^2+x^2y^2z^3\right)$ and $4x^2y^2z^2$.

Solution:

$8\left(x^3y^2z^2+x^2y^3z^2+x^2y^2z^3\right)$ can be written as $8x^2y^2z^2(x+y+z)$

Then,

$$8\left(x^3y^2z^2+x^2y^3z^2+x^2y^2z^3\right)\div 4x^2y^2z^2=\frac{8x^2y^2z^2(x+y+z)}{4x^2y^2z^2}$$
$$=2(x+y+z)$$

(iv) $\left(x^3+2x^2+3x\right)\div 2x$

Difficulty level: Medium

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Find out factors of $\left(x^3+2x^2+3x\right)$ and $2x$ then cancel out common factors of $\left(x^3+2x^2+3x\right)$ and $2x$.

Solution:

x^3+2x^2+3x can be written as $x\left(x^2+2x+3\right)$

Then,

$$\begin{aligned}\left(x^3+2x^2+3x\right)\div 2x &= \frac{x\left(x^2+2x+3\right)}{2x}\\ &= \frac{1}{2}\left(x^2+2x+3\right)\end{aligned}$$

(v) $\left(p^3q^6-p^6q^3\right)\div p^3q^3$

Difficulty level: Medium

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Find out factors of $\left(p^3q^6-p^6q^3\right)$ and p^3q^3 then cancel out common factors of $\left(p^3q^6-p^6q^3\right)$ and p^3q^3.

Solution:
$p^3q^6-p^6q^3$ can be written as $p^3q^3\left(q^3-p^3\right)$

Then,

$$\begin{aligned}\left(p^3q^6-p^6q^3\right)\div p^3q^3 &= \frac{p^3q^3\left(q^3-p^3\right)}{p^3q^3}\\ &= q^3-p^3\end{aligned}$$

Q3: Work out the following divisions.

(i) $(10x-25)\div 5$

(ii) $(10x-25)\div(2x-5)$

(iii) $10y(6y+21)\div 5(2y+7)$

(iv) $9x^2y^2(3z-24)\div 27xy(z-8)$

(v) $96abc(3a-12)(5b-30)\div 144(a-4)(b-6)$

(i) $(10x-25)\div 5$

Difficulty level: Easy

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Find out factors of $(10x-25)$ then cancel out common factors of $(10x-25)$ and 5.

Solution:
Factorising $(10x-25)$, we get

$$\begin{aligned}(10x-25) &= 5\times 2\times x-5\times 5\\ &= 5(2x-5)\end{aligned}$$

$$\begin{aligned}(10x-25)\div 5 &= \frac{5(2x-5)}{5}\\ &= 2x-5\end{aligned}$$

(ii) $(10x-25)\div(2x-5)$

Difficulty level: Easy

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Find out factors of $(10x-25)$ then cancel out common factors of $(10x-25)$ and $(2x-5)$

Solution:
Factorising $(10x-25)$, we get

$$\begin{aligned}(10x-25) &= 5\times 2\times x-5\times 5\\ &= 5(2x-5)\end{aligned}$$

$$\begin{aligned}(10x-25)\div(2x-5) &= \frac{5(2x-5)}{2x-5}\\ &= 5\end{aligned}$$

(iii) $10y(6y+21)\div 5(2y+7)$

Difficulty level: Easy

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Find out factors of $10y(6y+21)$ then cancel out common factors of $10y(6y+21)$ and $5(2y+7)$.

Solution:
Factorising $10y(6y+21)$,
we get,

$$\begin{aligned} 10y(6y+21) &= 5\times 2\times y\times\left(2\times 3\times y+3\times 7\right) \\ &= 5\times 2\times y\times 3\left(2\times y+7\right) \\ &= 30y\left(2y+7\right) \end{aligned}$$

$$\begin{aligned} 10y(6y+21)\div 5(2y+7) &= \frac{30y(2y+7)}{5(2y+7)} \\ &= 6y \end{aligned}$$

(iv) $9x^2y^2(3z-24)\div 27xy(z-8)$

Difficulty level: Easy

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Find out factors of $9x^2y^2(3z-24)$ then cancel out common factors of $9x^2y^2(3z-24)$ and $27xy(z-8)$.

Solution:
Factorising $9x^2y^2(3z-24)$,
we get,

$$\begin{aligned} 9x^2y^2(3z-24) &= 3\times 3\times x\times x\times y\times y\times\left(3\times z-2\times 2\times 2\times 3\right) \\ &= 3\times 3\times x\times x\times y\times y\times 3\left(z-2\times 2\times 2\right) \\ &= 27x^2y^2\left(z-8\right) \end{aligned}$$

$$\begin{aligned} 9x^2y^2(3z-24)\div 27xy(z-8) &= \frac{27x^2y^2\times(z-8)}{27xy(z-8)} \\ &= xy \end{aligned}$$

(v) $96abc(3a-12)(5b-30) \div 144(a-4)(b-6)$

Difficulty level: Easy

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Find out factors of $96abc(3a-12)(5b-30)$ then cancel out common factors of $96abc(3a-12)(5b-30)$ and $144(a-4)(b-6)$.

Solution:
Factorising $96abc(3a-12)(5b-30)$,
we get,

$$\begin{aligned} 96abc(3a-12)(5b-30) &= 96abc \times (3 \times a - 2 \times 2 \times 3) \times (5 \times b - 5 \times 2 \times 3) \\ &= 96abc \times 3(a - 2 \times 2) \times 5(b - 2 \times 3) \\ &= 1440abc(a-4)(b-6) \end{aligned}$$

$$\begin{aligned} 96abc(3a-12)(5b-30) \div 144(a-4)(b-6) \\ &= \frac{1440abc(a-4)(b-6)}{144(a-4)(b-6)} \\ &= 10abc \end{aligned}$$

Q4: Divide as directed.

(i) $5(2x+1)(3x+5) \div (2x+1)$

(ii) $26xy(x+5)(y-4) \div 13x(y-4)$

(iii) $52\,\mathrm{pqr}(p+q)(q+r)(r+p) \div 104pq(q+r)(r+p)$

(iv) $20(y+4)\left(y^2+5y+3\right) \div 5(y+4)$

(v) $x(x+1)(x+2)(x+3) \div x(x+1)$

Difficulty level: Easy

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Cancel out common factors of the following.

Solution:

(i) $5(2x+1)(3x+5) \div (2x+1) = \dfrac{5(2x+1)(3x+1)}{(2x+1)}$

$= 5(3x+1)$

(ii) $26xy(x+5)(y-4) \div 13x(y-4) = \dfrac{2 \times 13 \times xy(x+5)(y-4)}{13x(y-4)}$

$= 2y(x+5)$

(iii) $52pqr(p+q)(q+r)(r+p) \div 104pq(q+r)(r+p)$

$= \dfrac{2 \times 2 \times 13 \times p \times q \times r \times (p+q) \times (q+r) \times (r+p)}{2 \times 2 \times 2 \times 13 \times p \times q \times (q+r) \times (r+p)}$

$= \dfrac{1}{2} r(p+q)$

(iv) $20(y+4)\left(y^2+5y+3\right) \div 5(y+4)$

$= \dfrac{2 \times 2 \times 5 \times (y+4) \times \left(y^2+5y+3\right)}{5 \times (y+4)}$

$= 4\left(y^2+5y+3\right)$

(v) $x(x+1)(x+2)(x+3) \div x(x+1) = \dfrac{x(x+1)(x+2)(x+3)}{x(x+1)}$

$= (x+2)(x+3)$

Q5: Factorize the expressions and divide them as directed.

(i) $\left(y^2+7y+10\right) \div (y+5)$

(ii) $\left(m^2-14m-32\right) \div (m+2)$

(iii) $\left(5p^2-25p+20\right) \div (p-1)$

(iv) $4yz\left(z^2+6z-16\right) \div 2y(z+8)$

(v) $5pq\left(p^2-q^2\right) \div 2p(p+q)$

(vi) $12xy\left(9x^2-16y^2\right) \div 4xy(3x+4y)$

(vii) $39y^3\left(50y^2-98\right) \div 26y^2(5y+7)$

(i) $\left(y^2+7y+10\right) \div (y+5)$

Difficulty level: Medium

What is known:

Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Factorise $\left(y^2+7y+10\right)$ then cancel out common factors of $\left(y^2+7y+10\right)$ and $(y+5)$.

Solution:
$\left(y^2+7y+10\right)$ can be written as,

$$\begin{aligned} y^2+2y+5y+10 &= y(y+2)+5(y+2) \\ &= (y+2)(y+5) \end{aligned}$$

Then,

$$\begin{aligned} \left(y^2+7y+10\right)\div(y+5) &= \frac{(y+2)(y+5)}{(y+5)} \\ &= y+2 \end{aligned}$$

(ii) $\left(m^2-14m-32\right)\div(m+2)$

Difficulty level: Medium

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Factorise $\left(m^2-14m-32\right)$ then cancel out common factors of $\left(m^2-14m-32\right)$ and

Solution:
$m^2-14m-32$ can be written as,

$$\begin{aligned} m^2+2m-16m-32 &= m(m+2)-16(m+2) \\ &= (m+2)(m-16) \end{aligned}$$

Then,

$$\begin{aligned} \left(m^2-14m-32\right)\div(m+2) &= \frac{(m+2)(m-16)}{(m+2)} \\ &= m-16 \end{aligned}$$

(iii) $\left(5p^2-25p+20\right)\div(p-1)$

Difficulty level: Medium

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning: Factorise $\left(5p^2-25p+20\right)$ then cancel out common factors of $\left(5p^2-25p+20\right)$ and $(p-1)$.

Solution:

$5p^2-25p+20$ can be written as,

$$\begin{aligned}5\left(p^2-5p+4\right)&=5\left(p^2-p-4p+4\right)\\&=5\left[p(p-1)-4(p-1)\right]\\&=5(p-1)(p-4)\end{aligned}$$

Then,

$$\begin{aligned}\left(5p^2-25p+20\right)\div(p-1)&=\frac{5(p-1)(p-4)}{(p-1)}\\&=5(p-4)\end{aligned}$$

(iv) $4yz\left(z^2+6z-16\right)\div 2y(z+8)$

Difficulty level: Medium

What is known:

Algebraic expression.

What is unknown:

Division of the algebraic expression.

Reasoning:

Factorise $4yz\left(z^2+6z-16\right)$ then cancel out common factors of $4yz\left(z^2+6z-16\right)$ and $2y(z+8)$.

Solution:

$4yz\left(z^2+6z-16\right)$ can be written as,

$$\begin{aligned}4yz\left(z^2-2z+8z-16\right)&=4yz\left[z(z-2)+8(z-2)\right]\\&=4yz(z-2)(z+8)\end{aligned}$$

Then,

$$\begin{aligned}4yz\left(z^2+6z-16\right)\div 2y(z+8)&=\frac{4yz(z-2)(z+8)}{2y(z+8)}\\&=2z(z-2)\end{aligned}$$

(v) $5pq\left(p^2-q^2\right)\div 2p(p+q)$

Difficulty level: Medium

What is known:

Algebraic expression.

What is unknown:

Division of the algebraic expression.

Reasoning:

Factorise $5pq\left(p^2-q^2\right)$ by using identity $a^2-b^2=(a-b)(a+b)$ then cancel out common factors of $5pq\left(p^2-q^2\right)$ and $2p(p+q)$.

Solution:

$5pq\left(p^2-q^2\right)$ can be written as $5pq(p-q)(p+q)$

Then,

$$\begin{aligned}5pq\left(p^2-q^2\right)\div 2p(p+q)&=\frac{5pq(p-q)(p+q)}{2p(p+q)}\\&=\frac{5}{2}q(p-q)\end{aligned}$$

(vi) $12xy\left(9x^2-16y^2\right)\div 4xy(3x+4y)$

Difficulty level: Medium

What is known:

Algebraic expression.

What is unknown:

Division of the algebraic expression.

Reasoning:

Factorise $12xy\left(9x^2-16y^2\right)$ by using identity $a^2-b^2=(a-b)(a+b)$ then cancel out common factors of $12xy\left(9x^2-16y^2\right)$ and $4xy(3x+4y)$.

Solution:

$12xy\left(9x^2-16y^2\right)$ can be written as,

$$\begin{aligned}12xy\left[(3x)^2-(4y)^2\right]&=12xy(3x-4y)(3x+4y)\\&=2\times 2\times 3\times x\times y\times(3x-4y)\times(3x+4y)\end{aligned}$$

Then,

$$\begin{aligned}12xy\left(9x^2-16y^2\right)\div 4xy(3x+4y)&=\frac{2\times 2\times 3\times x\times y\times(3x-4y)\times(3x+4y)}{2\times 2\times x\times y\times(3x+4y)}\\&=3(3x-4y)\end{aligned}$$

(vii) $39y^3\left(50y^2-98\right)\div 26y^2(5y+7)$

Difficulty level: Medium

What is known:
Algebraic expression.

What is unknown:
Division of the algebraic expression.

Reasoning:
Factorise $39y^3(50y^2-98)$ by using identity $a^2-b^2=(a-b)(a+b)$ then cancel out common factors of $39y^3(50y^2-98)$ and $26y^2(5y+7)$.

Solution:
$39y^3(50y^2-98)$ can be written as,

$$
\begin{aligned}
3\times13\times y\times y\times y\times\left[2\times(25y^2-49)\right] &= 3\times13\times2\times y\times y\times y\times\left[(5y)^2-(7)^2\right] \\
&= 3\times13\times2\times y\times y\times y(5y-7)(5y+7)
\end{aligned}
$$

and

$26y^2(5y+7)$ can be written as $2\times13\times y\times y\times(5y+7)$

Then,

$$
\begin{aligned}
39y^3(50y^2-98)\div26y^2(5y+7) &= \frac{3\times13\times2\times y\times y\times y(5y-7)(5y+7)}{2\times13\times y\times y\times(5y+7)} \\
&= 3y(5y-7)
\end{aligned}
$$

Chapter – 14: Factorization

Exercise 14.4

Q1: Find and correct the errors in the statement: $4(x-5)=4x-5$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:
$S = 4(x-5) \neq R.H.S.$

The correct statement is $4(x-5)=4x-20$

Q2: Find and correct the errors in the statement: $x(3x+2)=3x^2+2$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$L.H.S = x(3x+2) = 3x^2+2x$$
$$L.H.S \neq R.H.S.$$

The correct statement is $x(3x+2)=3x^2+2x$

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Q3: Find and correct the errors in the statement: $2x+3y=5xy$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$L.H.S = 2x+3y = 2x+3y$$
$$L.H.S \neq R.H.S.$$

The correct statement is $2x+3y=2x+3y$

Q4: Find and correct the errors in the statement: $x+2x+3x=5x$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$\begin{aligned} L.H.S. &= x+2x+3x = 1x+2x+3x \\ &= x(1+2+3) = 6x \end{aligned}$$
$$L.H.S. \neq R.H.S.$$

The correct statement is $x+2x+3x=6x$

Q5: Find and correct the errors in the statement: $5y+2y+y-7y=0$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$L.H.S. = 5y + 2y + y - 7y = 8y - 7y = y \neq R.H.S.$$

The correct statement is $5y + 2y + y - 7y = y$

Q6: Find and correct the errors in the statement: $3x + 2x = 5x^2$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$L.H.S. = 3x + 2x = 5x$$
$$L.H.S. \neq R.H.S.$$

The correct statement is $3x + 2x = 5x$

Q7: Find and correct the errors in the statement: $(2x)^2 + 4(2x) + 7 = 2x^2 + 8x + 7$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$L.H.S. = (2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7$$
$$L.H.S. \neq R.H.S$$

The correct statement is $(2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7$

Q8: Find and correct the errors in the statement: $(2x)^2 + 5x = 4x + 5x = 9x$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$L.H.S. = (2x)^2 + 5x = 4x^2 + 5x$$
$$L.H.S. \neq R.H.S.$$

The correct statement is $(2x)^2 + 5x = 4x^2 + 5x$

Q9: Find and correct the errors in the statement: $(3x + 2)^2 = 3x^2 + 6x + 4$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$L.H.S. = (3x+2)^2 = (3x)^2 + 2(3x)(2) + (2)^2 \qquad \left[\text{Using identity } (a+b)^2 = a^2 + 2ab + b^2\right]$$
$$= 9x^2 + 12x + 4$$
$$L.H.S. \neq R.H.S$$

The correct statement is $(3x + 2)^2 = 9x^2 + 12x + 4$

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THE MATH EXPERT

Q10: Find and correct the errors in the following mathematical statement.
Substituting $x = -3$ in

(a) $x^2 + 5x + 4$ gives $(-3)^2 + 5(-3) + 4 = 9 + 2 + 4 = 15$

(b) $x^2 - 5x + 4$ gives $(-3)^2 - 5(-3) + 4 = 9 - 15 + 4 = -2$

(c) $x^2 + 5x$ gives $(-3)^2 + 5(-3) = -9 - 15 = -24$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Put value of x in L.H.S and find correct solution.

Solution:
a) For $x = -3$

$$\begin{aligned} L.H.S &= x^2 + 5x + 4 \\ &= (-3)^2 + 5(-3) + 4 \\ &= 9 - 15 + 4 \\ &= 13 - 15 \\ &= -2 \end{aligned}$$

$L.H.S \neq R.H.S$

The correct answer is $x^2 + 5x + 4 = -2$

b) For $x = -3$

$$\begin{aligned} x^2 - 5x + 4 &= (-3)^2 - 5(-3) + 4 \\ &= 9 + 15 + 4 \\ &= 28 \end{aligned}$$

The correct answer is $x^2 - 5x + 4 = 28$

c) For $x = -3$

$$\begin{aligned} x^2 + 5x &= (-3)^2 + 5(-3) \\ &= 9 - 15 \\ &= -6 \end{aligned}$$

The correct answer is $x^2 + 5x = -6$

Q11: Find and correct the errors in the statement: $(y-3)^2 = y^2 - 9$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Use identity $(a-b)^2 = a^2 - 2ab + b^2$

Solution:

$$L.H.S. = (y-3)^2 = (y)^2 - 2(y)(3) + (3)^2 \quad \left[\text{Using identity } (a-b)^2 = a^2 - 2ab + b^2\right]$$
$$= y^2 - 6y + 9$$
$$L.H.S. \neq R.H.S.$$

The correct statement is $(y-3)^2 = y^2 - 6y + 9$

Q12: Find and correct the errors in the statement: $(z+5)^2 = z^2 + 25$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Use identity $(a+b)^2 = a^2 + 2ab + b^2$

Solution:

$$L.H.S. = (z+5)^2 = (z)^2 + 2(z)(5) + (5)^2 \quad \left[\text{Using identity } (a+b)^2 = a^2 + 2ab + b^2\right]$$
$$= z^2 + 10z + 25$$
$$L.H.S. \neq R.H.S.$$

The correct statement is $(z+5)^2 = z^2 + 10z + 25$

Q13: Find and correct the errors in the statement: $(2a+3b)(a-b) = 2a^2 - 3b^2$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$\begin{aligned} L.H.S. &= (2a+3b)(a-b) = 2a\times a - 2a\times b + 3b\times a - 3b\times b \\ &= 2a^2 - 2ab + 3ab - 3b^2 \\ &= 2a^2 + ab - 3b^2 \end{aligned}$$

$L.H.S. \neq$ R.H.S.

The correct statement is $(2a+3b)(a-b) = 2a^2 + ab - 3b^2$

Q14: Find and correct the errors in the statement : $(a+4)(a+2) = a^2 + 8$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$\begin{aligned} L.H.S. &= (a+4)(a+2) = a\times a + 2\times a + 4\times a + 4\times 2 \\ &= (a)^2 + a(4+2) + (4\times 2) \\ &= a^2 + 6a + 8 \end{aligned}$$

$L.H.S. \neq R.H.S.$

The correct statement is $(a+4)(a+2) = a^2 + 6a + 8$

Q15: Find and correct the errors in the statement: $(a-4)(a-2) = a^2 - 8$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$L.H.S. = (a-4)(a-2) = a\times a + (-2)\times a + (-4)\times a + [(-4)\times(-2)]$$
$$= a^2 - 2a - 4a + 8$$
$$= a^2 - 6a + 8$$
$$L.H.S. \neq R.H.S.$$

The correct statement is $(a-4)(a-2) = a^2 - 6a + 8$

Q16: Find and correct the errors in the statement: $\frac{3x^2}{3x^2} = 0$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$L.H.S. = \frac{3x^2}{3x^2} = \frac{3\times x\times x}{3\times x\times x} = 1$$
$$L.H.S. \neq R.H.S.$$

The correct statement is $\frac{3x^2}{3x^2} = 1$

Q17: Find and correct the errors in the statement: $\frac{3x^2+1}{3x^2} = 1 + 1 = 2$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$L.H.S. = \frac{3x^2+1}{3x^2} = \frac{3x^2}{3x^2} + \frac{1}{3x^2}$$

$$= 1 + \frac{1}{3x^2}$$

$$L.H.S. \neq R.H.S.$$

The correct statement is $\frac{3x^2+1}{3x^2} = 1 + \frac{1}{3x^2}$

Q18: Find and correct the errors in the statement: $\frac{3x}{3x+2} = \frac{1}{2}$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$L.H.S. = \frac{3x}{3x+2} \neq R.H.S.$$

The correct statement is $\frac{3x}{3x+2} = \frac{3x}{3x+2}$

Q19: Find and correct the errors in the statement: $\frac{3}{4x+3} = \frac{1}{4x}$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$L.H.S = \frac{3}{4x+3} \neq R.H.S$$

The correct statement is $\frac{3}{4x+3} = \frac{3}{4x+3}$

Q20: Find and correct the errors in the statement: $\frac{4x+5}{4x}=5$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$L.H.S.=\frac{4x+5}{4x}=\frac{4x}{4x}+\frac{5}{4x}$$
$$=1+\frac{5}{4x}$$
$$L.H.S.\neq R.H.S.$$

The correct statement is $\frac{4x+5}{4x}=1+\frac{5}{4x}$

Q21: Find and correct the errors in the statement: $\frac{7x+5}{5}=7x$

Difficulty level: Easy

What is known:
Incorrect mathematical statement.

What is unknown:
Correct mathematical statement.

Reasoning:
Solve L.H.S.

Solution:

$$L.H.S.=\frac{7x+5}{5}=\frac{7x}{5}+\frac{5}{5}$$
$$=\frac{7x}{5}+1$$
$$L.H.S.\neq R.H.S.$$

The correct statement is $\frac{7x+5}{5}=\frac{7x}{5}+1$