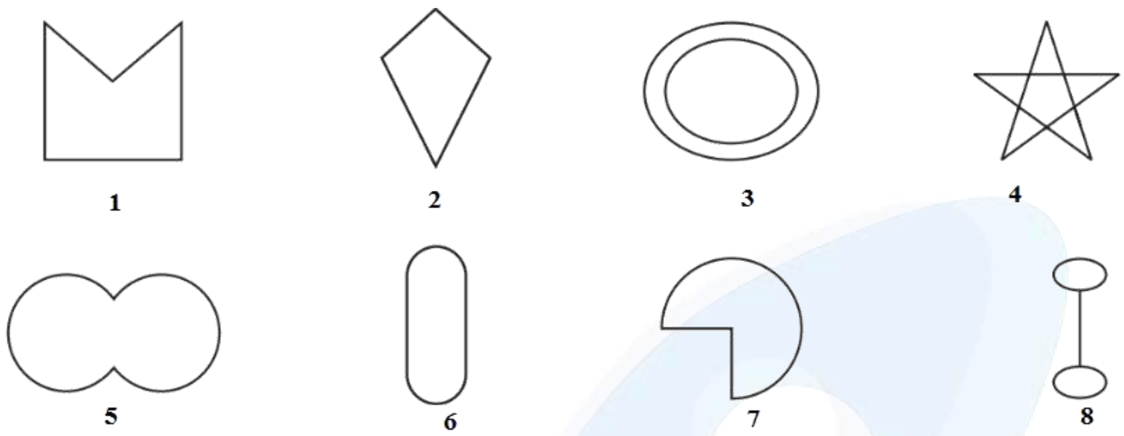
Chapter-3: Understanding Quadrilaterals

Exercise 3.1 (Page 41 of Grade 8 NCERT)

Q1. Given here are some figures.



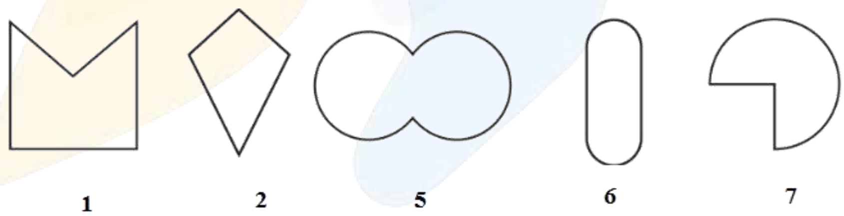
Classify each of them on the basis of the following.

(a) Simple curve
(b) Simple closed curve
(c) Polygon
(d) Convex polygon
(e) Concave polygon

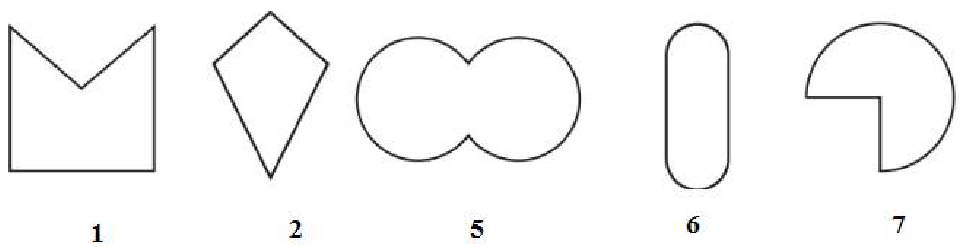
Difficulty Level: Easy

Solution:

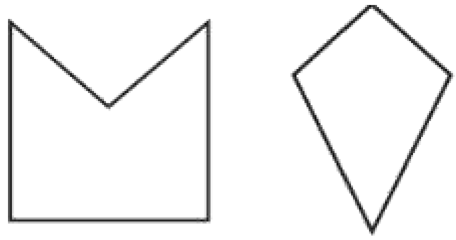
(a) Simple curve - A simple curve is a curve that does not cross itself.



(b) Simple closed curve -In simple closed curves the shapes are closed by line-segments or by a curved line.



(c) Polygon-A simple closed curve made up of only line segments is called a polygon.



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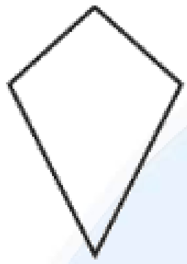
(d) Convex polygon-A Convex polygon is defined as a polygon with no portions of their diagonals in their exteriors.



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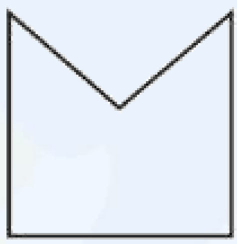
(e) Concave polygon- A concave polygon is defined as a polygon with one or more interior angles greater than 180°.



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Q2. How many diagonals does each of the following have?

(a) A convex quadrilateral
(b) A regular hexagon
(c) A triangle

Difficulty Level: Easy

Known:
Polygons with known number of vertices.

Unknown:
Number of diagonals

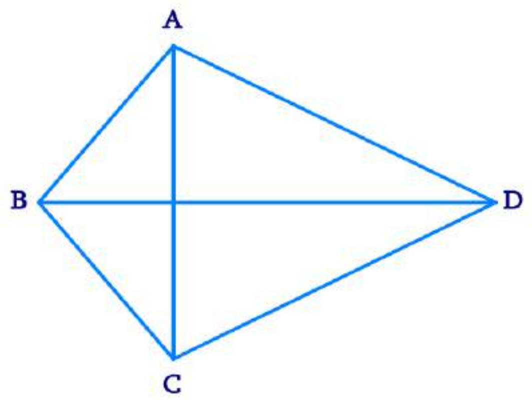
Reasoning:

A diagonal is a line segment connecting two non-consecutive vertices of a polygon. Draw the above given polygon and mark vertices and then, draw lines joining the two non-consecutive vertices. From this, we can calculate the number of diagonals.

Solution:

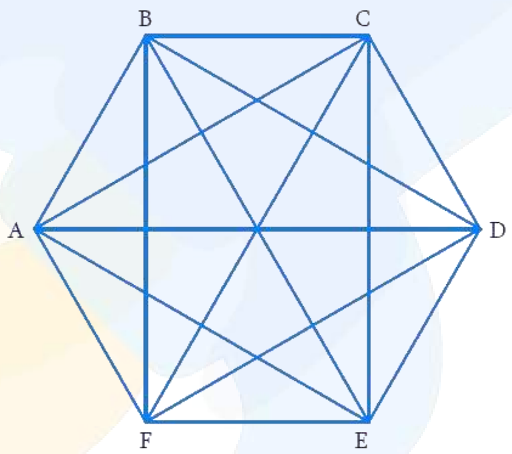
(a) Convex quadrilateral

A convex quadrilateral has two diagonals.



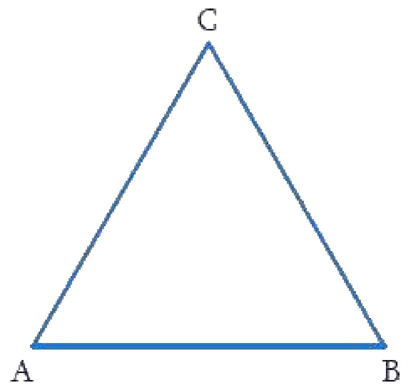
Here, AC and BD are two diagonals.

(b)A regular hexagon



Here, the diagonals are AD, AE, BD, BE, FC, FB, AC, EC and FD. Totally there are 9 diagonals.

(c) A triangle



A triangle has no diagonal because there no two non-consecutive vertices.

Q3. What is the sum of the measures of the angles of a convex quadrilateral? Will this property hold if the quadrilateral is not convex? (Make a non-convex quadrilateral and try!)

Difficulty Level: Medium

Known:

Quadrilateral ABCD

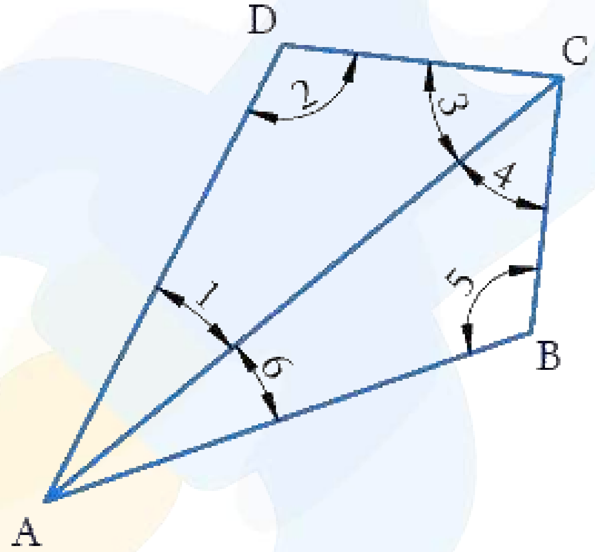
Unknown:

Sum of the measures of the angles of a convex quadrilateral.

Reasoning:

Let ABCD be a convex quadrilateral. Then, we draw a diagonal AC which divides the Quadrilateral into two triangles. We know that the sum of the angles of a triangle is 180 degree, so by calculating the sum of the angles of a ΔABC and ΔADC, we can measure the sum of angles of convex quadrilateral.

Solution:



ABCD is a convex quadrilateral made of two triangles ΔABC and ΔADC. We know that the sum of the angles of a triangle is 180 degree. So:

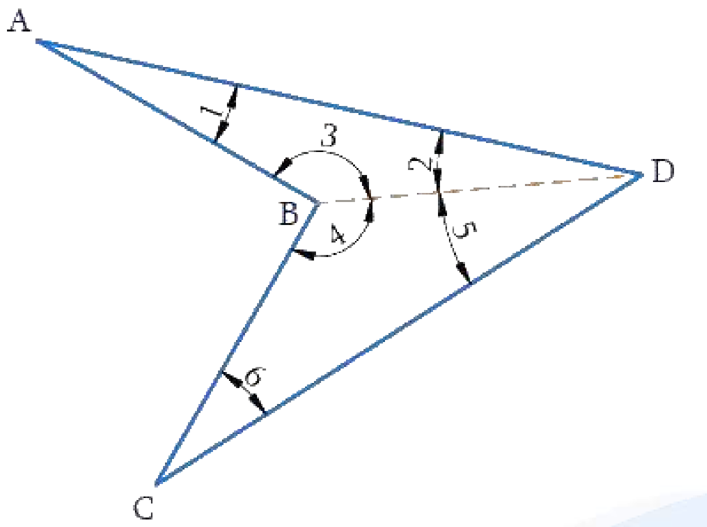
$$\angle 6+\angle 5+\angle 4=180° \ [\text{sum of the angles of } \Delta ABC=180°]$$

$$\angle 1+\angle 2+\angle 3=180° \ [\text{sum of the angles of } \Delta ADC=180°]$$

Adding we get

$$\angle 6+\angle 5+\angle 4+\angle 1+\angle 2+\angle 3$$
$$=180^o+180^o$$
$$=360^o$$

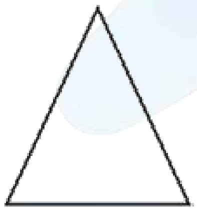
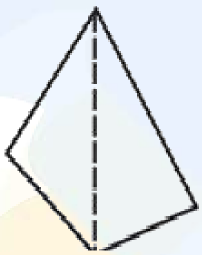
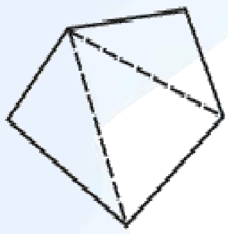
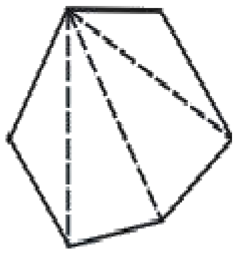
Hence, the sum of measures of the triangles of a convex quadrilateral is 360°. Yes, even if quadrilateral is not convex then, this property applies. Let ABCD be a non-convex quadrilateral; join BD, which also divides the quadrilateral in two triangles.



Using the angle sum property of triangle, again ABCD is a concave quadrilateral, made of two triangles ΔABD and ΔBCD. Therefore, the sum of all the interior angles of this quadrilateral will also be,

$$180^\circ + 180^\circ = 360^\circ$$

Q4. Examine the table. (Each figure is divided into triangles and the sum of the angles deduced from that.)

Figures				
Side	3	4	5	6
Angle Sum	180°	$2\times180^\circ = (4-2)\times180^\circ$ $= 360^\circ$	$3\times180^\circ = (5-2)\times180^\circ$ $= 540$	$4\times180^\circ = (6-2)\times180^\circ$ $= 720^\circ$

What can you say about the angle sum of a convex polygon with number of sides?

(a) 7
(b) 8
(c) 10
(d) n

Difficulty Level: Easy

Reasoning:

From the table, it can be observed that the angle sum of a convex polygon of n sides is $(n-2) \times 180^\circ$.

Solution:

(a) When n = 7

Then Angle sum of a polygon $=(n-2)\times 180^{o}=(7-2)\times 180^{o}=5\times 180^{o}=900^{o}$

(b) When n = 8

Then Angle sum of a polygon $=(n-2)\times 180^{o}=(8-2)\times 180^{o}=6x180^{o}=1080^{o}$

(c) When n = 10

Then Angle sum of a polygon $=(n-2)\times 180^{o}=(10-2)\times 180^{o}=8\times 180^{o}=1440^{o}$

(d) When n = n

Then Angle sum of a polygon $=(n-\ 2)\times 180°$

Q5. What is a regular polygon? State the name of a regular polygon of

(i) 3 sides
(ii) 4 sides
(iii) 6 sides

Difficulty Level: Easy

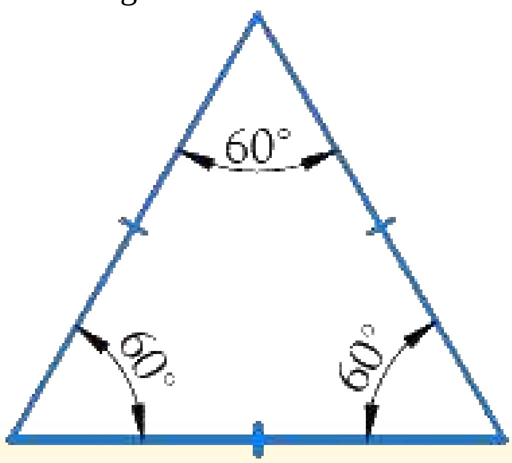
Known:
Number of sides of polygon

Unknown:
Name of a regular polygon of

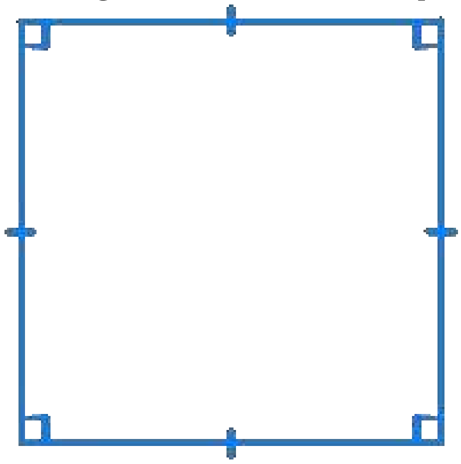
(i) 3 sides
(ii) 4 sides
(iii) 6 sides

Solution:
Regular polygon - A polygon having all sides of equal length and the interior angles of equal measure is known as regular polygon i.e. a regular polygon is both ‘equiangular’ and ‘equilateral’.

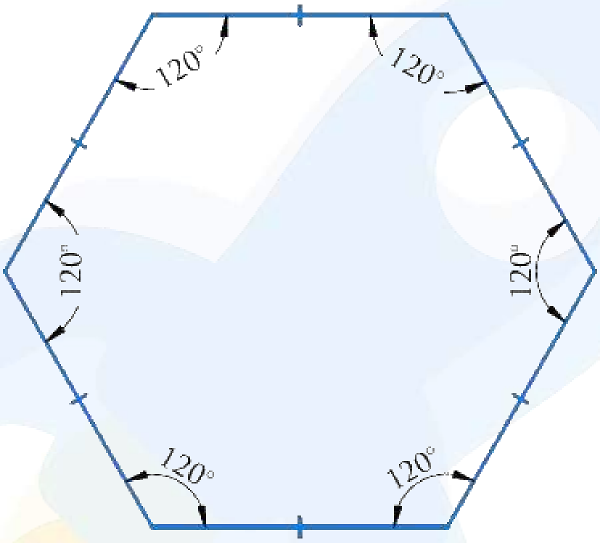
(i) 3 sides = polygons having three sides is called a Triangle.



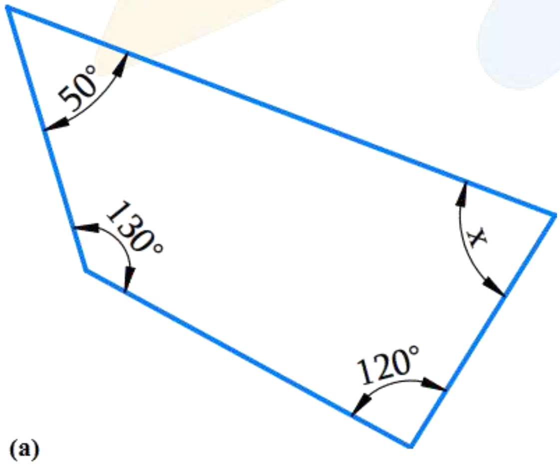
(ii) 4 sides = polygons having four sides is called Square/quadrilateral.

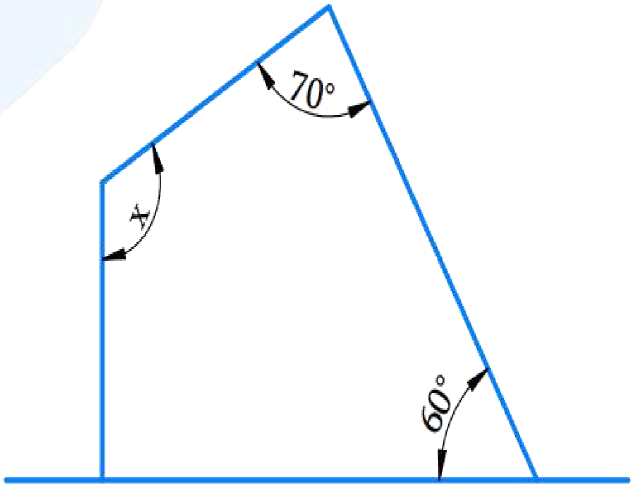


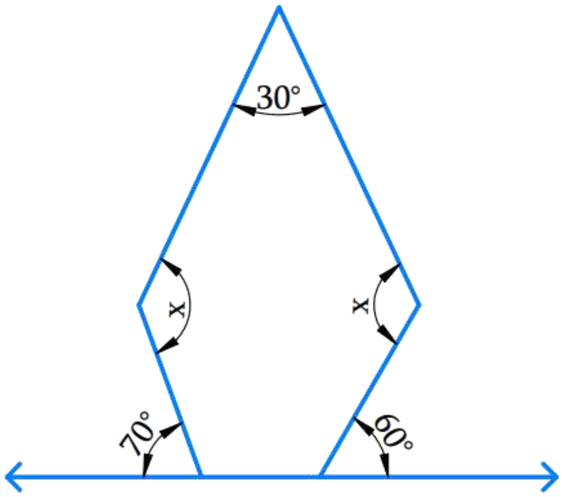
(iii) 6 sides= polygons having six sides is called a Hexagon.



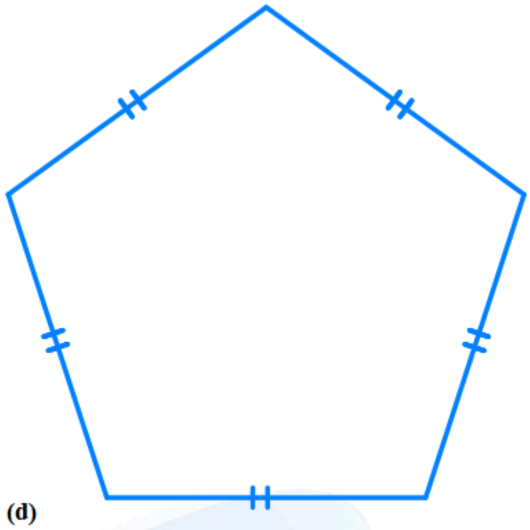
Q6. Find the angle measure x in the following figures:







(c)

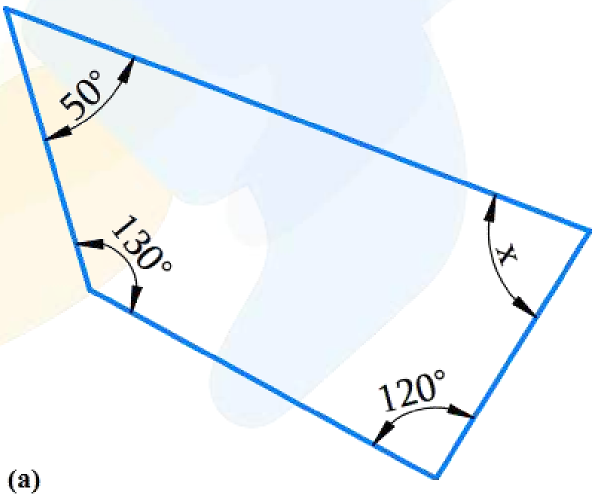

(d)

Difficulty Level: Medium

Known:
Sum of the measures of all interior angles of a quadrilateral is 360° and that of a pentagon is 540°.

Unknown:
Angle x in the above figures a, b, c and d

Solution:



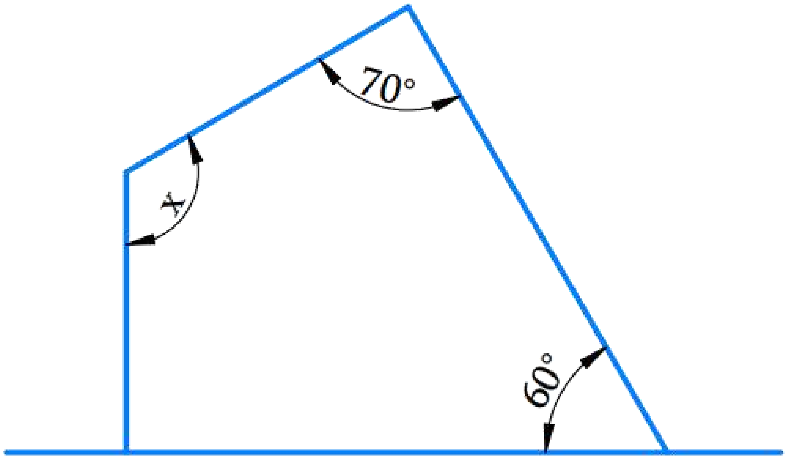
(a)

a) The above figure has 4 sides and hence it is a quadrilateral.

Using the angle sum property of a quadrilateral,

$$50^\circ + 130^\circ + 120^\circ + x = 360^\circ$$
$$300^\circ + x = 360^\circ$$
$$x = 360^\circ - 300^\circ$$
$$x = 60^\circ$$

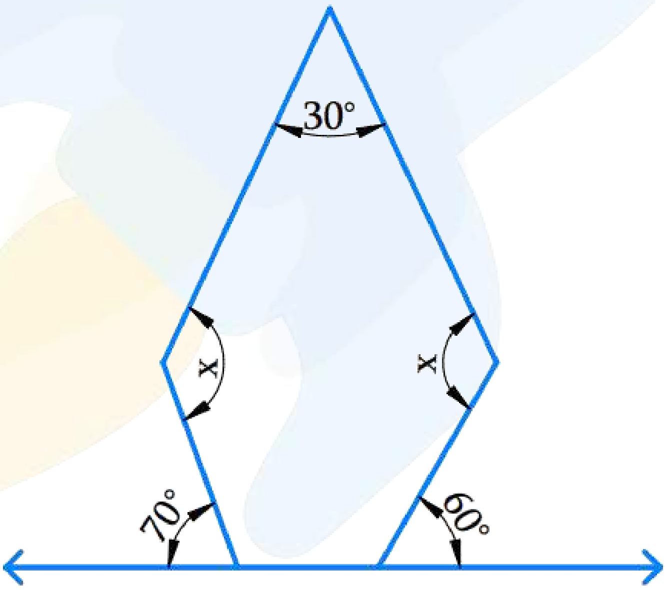
b) Using the angle sum property of a quadrilateral.



(b)

$$90^\circ + 60^\circ + 70^\circ + x = 360^\circ$$
$$220^\circ + x = 360^\circ$$
$$220^\circ + x = 360^\circ$$
$$x = 360^\circ - 220^\circ$$
$$x = 140^\circ$$

c) The given figure is a pentagon (n=5)



(c)

Angle sum of a polygon $= (n-2)\times 180^\circ$

$= (5-2)\times 180^\circ$

$= 3\times 180^\circ$

$= 540^\circ$

Sum of the interior angle of pentagon is 540°.
Angles at the bottom are linear pair.

$\therefore$ First base interior angle i.e. a $= 180^o - 70^o$ (angle of straight line is 180^o)
$= 110^o$

$\therefore$ Second base interior angle i.e. b $= 180^o - 60^o$
$= 120^o$

Angle sum of a polygon $= (n-2)\times 180^o$

i.e. $30^o + x + 110^o + 120^o + x = 540^o$

i.e. $2x + 260^o = 540^o$

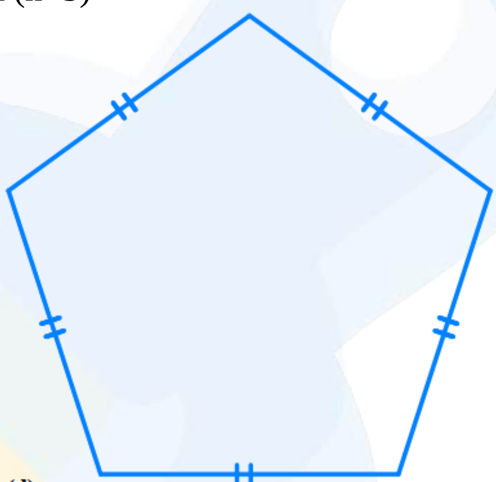
i.e. $2x = 540^o - 260^o$

i.e. $2x = 280^o$

i.e. $x = \frac{280^o}{2}$

i.e. $x = 140^o$

d) The given figure is pentagon (n=5)



(d)

Sum of the interior angle of pentagon is $540°$.

Angle sum of a polygon $= (n-2)\times 180^o$
$= (5-2)\times 180^o$
$= 3\times 180^o$
$= 540^o$

Angle sum of a polygon $=$ $x + x + x + x + x = 540^o$

i.e. $5x = 540^o$

$x = \frac{540^o}{5}$

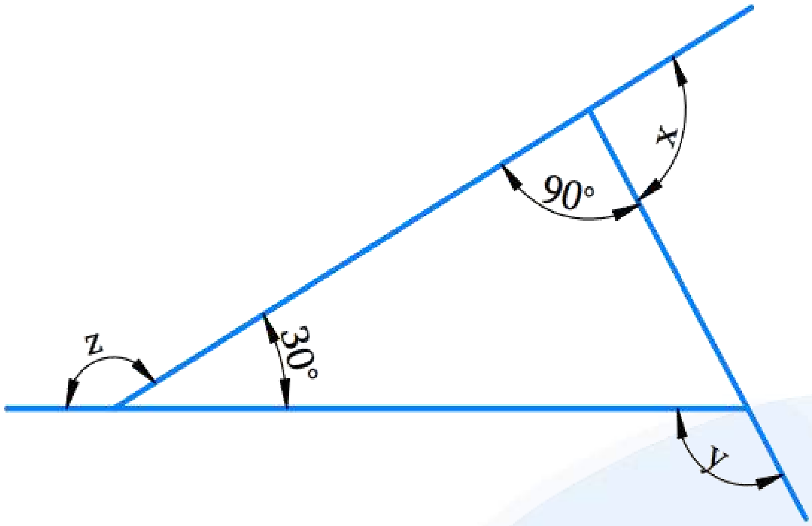
$x = 108^o$

A pentagon is a regular polygon i.e. it is both ‘equilateral’ and ‘equiangular’. Thus, the measure of each interior angle of the pentagon are equal.

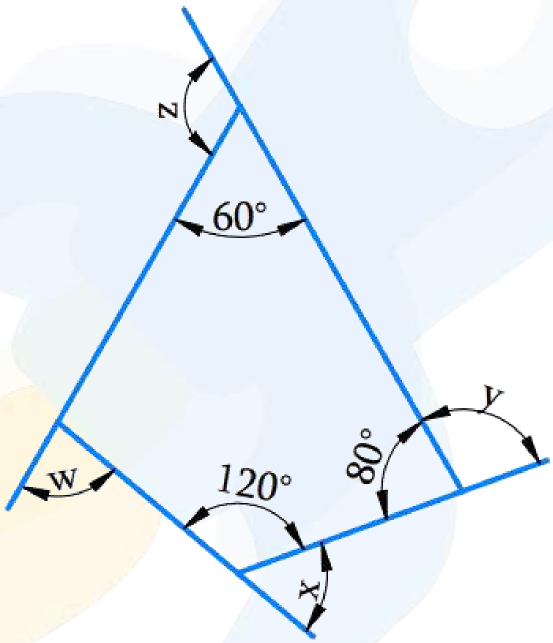
Hence each interior angle is $108°$.

Q7.

a) Find x +y +z



b) Find x +y +z +w



Difficulty Level: Medium

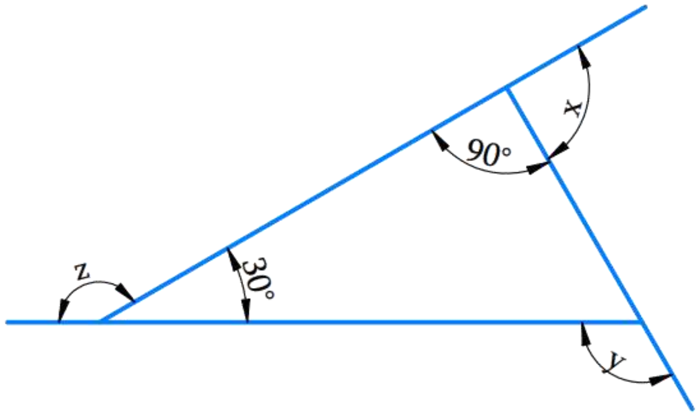
Known:
The sum of the measures of all the interior angles of a quadrilateral is 360º and that of a triangle is 180°.

Unknown:
Angle x, y, z and w in the above figures a and b.

Reasoning:
The unknown angles can be estimated by using the angle sum property of a quadrilateral and triangle accordingly.

Solution:

(a) Find $x + y + z$



Sum of linear pair of angles is $= 180^o$

$$x + 90^o = 180^o \text{ (Linear pair)}$$
$$x = 180^o - 90^o$$
$$x = 90^o$$

And

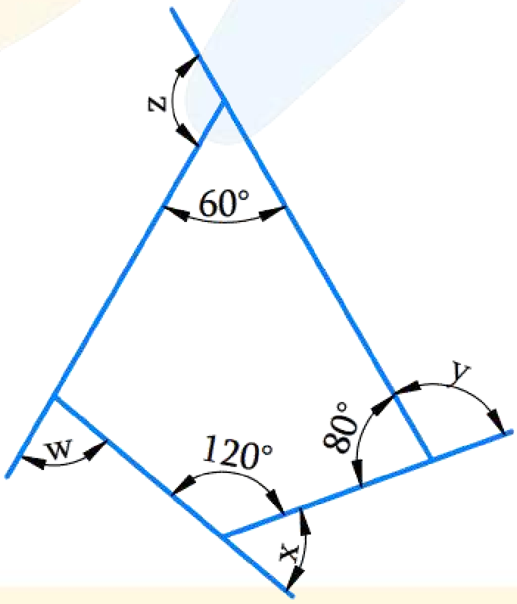
$$z + 30^o = 180^o \text{ (Linear pair)}$$
$$z = 180^o - 30^o$$
$$z = 150^o$$

And

$$y = 90^o + 30^o \text{ (Exterior angle theorem)}$$
$$y = 120^o$$

$$x + y + z = 90^o + 120^o + 150^o$$
$$= 360^o$$

(b) Find $x + y + z + w$



The sum of the measures of all the interior angles of a quadrilateral is 360º.

Using the angle sum property of a quadrilateral,

Let n is the fourth interior angle of the quadrilateral.

$$\begin{aligned} 60^\circ + 80^\circ + 120^\circ + n &= 360^\circ \\ 260^\circ + n &= 360^\circ \\ n &= 360^\circ - 260^\circ \\ n &= 100^\circ \end{aligned}$$

sum of linear pair of angles is 180° .

$$w + 100^\circ = 180^\circ \quad \ldots(1)$$
$$x + 120^\circ = 180^\circ \quad \ldots(2)$$
$$y + 80^\circ = 180^\circ \quad \ldots(3)$$
$$z + 60^\circ = 180^\circ \quad \ldots(4)$$

Adding equation (1), (2), (3) and (4),

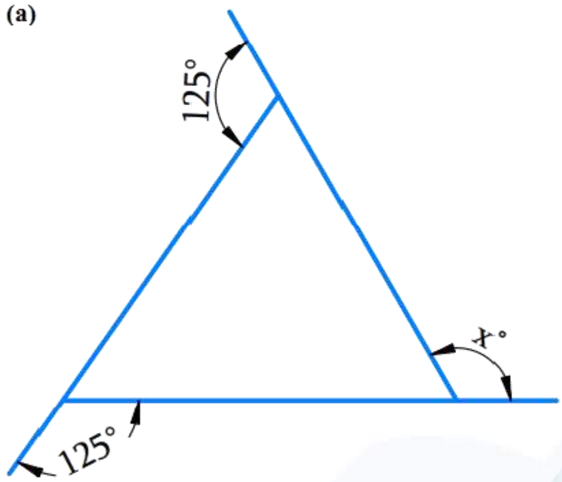
$$\begin{aligned} w + 100^\circ + x + 120^\circ + y + 80^\circ + z + 60^\circ &= 180^\circ + 180^\circ + 180^\circ + 180^\circ \\ w + x + y + z + 360^\circ &= 720^\circ \\ w + x + y + z &= 720^\circ - 360^\circ \\ w + x + y + z &= 360^\circ \end{aligned}$$

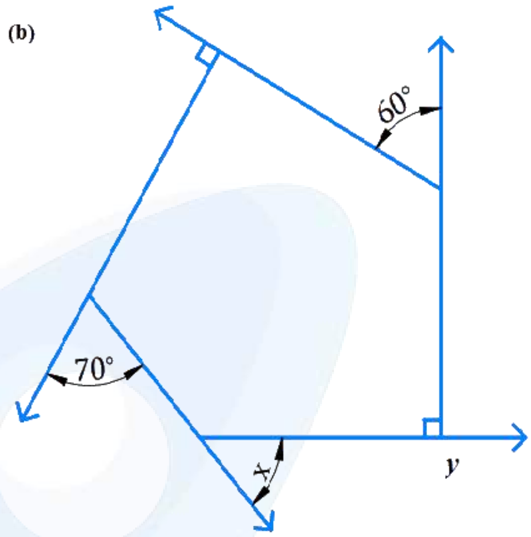
The sum of the measures of the external angles of any polygon is 360° .

Chapter-3: Understanding Quadrilaterals

Exercise 3.2 (Page 44 of Grade 8 NCERT)

Q1. Find x in the following figures.





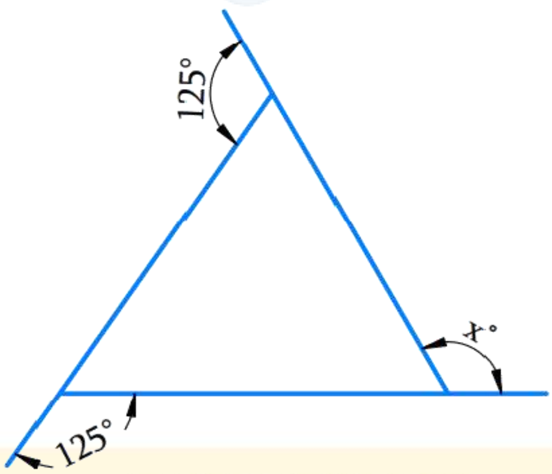
Difficulty Level: Easy

Known:
Other angles of given polygon.

Unknown:
Angle x in the above figures a and b.

Reasoning:
We know that the sum of the measures of the exterior angles of any polygon is 360°. So equate all the angle sum to 360° and find out the unknown angle.

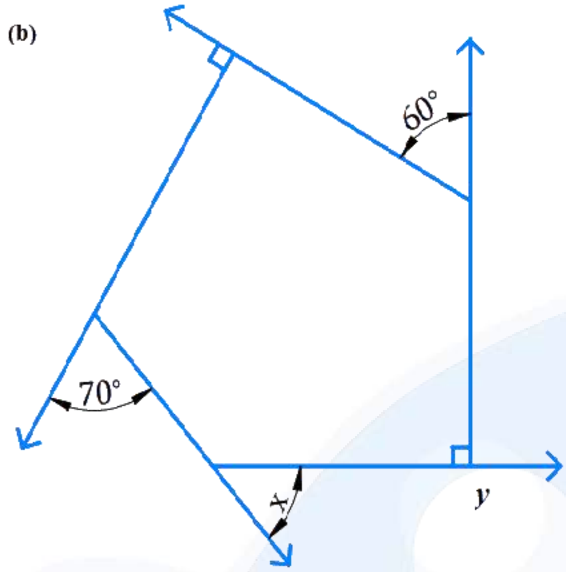
Solution:
(a)



Sum of the measures of the external angles,

$$125° + 125° + x° = 360°$$
$$250° + x° = 360°$$
$$x° = 110°$$

(b)



$y = 180^o - 90^o$ [linear pair angles]

$y = 90^o$

Sum of the measures of the external angles is $360°$,

$$60° + 90° + 70° + x + y = 360^o$$
$$60° + 90° + 70° + x + 90° = 360°$$
$$310° + x = 360°$$
$$x = 50°$$

Q2. Find the measure of each exterior angle of a regular polygon of

(i) 9 sides (ii) 15 sides

Difficulty Level:
Medium

Known:
The number of sides of the polygon.

Unknown:
Exterior angle of a regular polygon of 9 sides.

Exterior angle of a regular polygon of 15 sides.

Reasoning:
Irrespective of the number of sides of the polygon, the measure of the exterior angles is equal and the sum of the measure of all the exterior angles of the regular polygon is equal to 360°.

Solution:

(i) 9 sides

Total measure of all exterior angles $= 360^\circ$

$$\text{Each exterior angle} = \frac{\text{sum of exterior angle}}{\text{number of sides}}$$
$$= \frac{360^\circ}{9}$$
$$= 40^\circ$$
$$\therefore \text{ Each exterior angle} = 40^\circ$$

(ii) 15 sides

Total measure of all exterior angles = 360°

$$\text{Each exterior angle} = \frac{\text{sum of exterior angle}}{\text{number of sides}}$$
$$= \frac{360^\circ}{15}$$
$$= 24^\circ$$
$$\therefore \text{ Each exterior angle} = 24^\circ$$

Q3. How many sides does a regular polygon have if the measure of an exterior angle is 24°?

Difficulty Level: Medium

Known:
Measure of an exterior angle is 24°.

Unknown:
The number of sides of the regular polygon.

Solution:
Total measure of all the exterior angles of the regular polygon $= 360^\circ$

Let number of sides be = n.

Measure of each exterior angle $= 24^\circ$

$$\therefore \text{ number of sides} = \frac{\text{Sum of exterior angles}}{\text{Each exterior angle}}$$

$$= \frac{360°}{24°}$$

$$= 15$$

∴ Regular polygon has 15 sides.

Q4. How many sides does a regular polygon have if each of its interior angles is 165°?

Difficulty Level: Medium

Known:
Measure of an interior angle is 165°.

Unknown:
The number of sides of the regular polygon.

Reasoning:
We know that:

a) Irrespective of the number of sides of the polygon, the measure of the exterior angles is equal and the sum of the measures of all the exterior angles of the regular polygon is equal to 360°.

b) The measure of interior angle of regular polygon is (n-2) *180/n, where ‘n’ is the number of sides of the polygon.

Solution:
Let number of sides be n.

Measure of each interior angle = 165°

Measure of each exterior angle = 180° − 165° = 15° [linear pair angles]

$$\text{Number of sides} = \frac{\text{Sum of exterior angles}}{\text{Each exterior angle}}$$

$$= \frac{360°}{15°}$$

$$= 24$$

Hence, the regular polygon has 24 sides.

Q5.

(a) Is it possible to have a regular polygon with measure of each exterior angle as 22°?
(b)Can it be an interior angle of a regular polygon? Why?

Difficulty Level: Medium

Known:

Measure of an exterior angle is 22°.

Unknown:

To find whether a regular polygon with exterior angle = 22° is possible or not.

Reasoning:

$$\text{Number of sides of any polygon} = \frac{360^\circ}{\text{Exterior angle}}$$

Hence,if the number of side will be a whole number , then the given polygon is possible.

Solution:

(a) Is it possible to have a regular polygon with measure of each exterior angle as 22°?

Total measure of all exterior angles = $360°$
Let number of sides be = n.
Measure of each exterior angle = $22°$

$$\text{Therefore, the number of sides} = \frac{\text{Sum of exterior angles}}{\text{Each exterior angle}}$$
$$= \frac{360^\circ}{22^\circ}$$
$$= 16.36$$

We cannot have regular polygon with each exterior angle = $22°$ as the number of sides is not a whole number.[22 is not a perfect divisor of $360°$]

(b) Can it be an interior angle of a regular polygon? Why?

$$\text{Measure of each interior angle} = 22^\circ$$
$$\text{Measure of each exterior angle} = 180^\circ - 22^\circ$$
$$= 158^\circ$$

$$\text{Number of sides} = \frac{\text{Sum of exterior angles}}{\text{Each exterior angle}}$$
$$= \frac{360^\circ}{158^\circ}$$
$$= 2.27$$

We cannot have regular polygon with each interior angle as $22°$ because the number of sides is not a whole number. [22 is not a perfect divisor of $360°$]

Q6.

(a) What is the minimum interior angle possible for a regular polygon? Why?
(b) What is the maximum exterior angle possible for a regular polygon?

Difficulty Level: Medium

Known:
We know polygons according to the number of sides (or vertices) they have.

Unknown:
Minimum interior angle possible for a regular polygon.
Maximum exterior angle possible for a regular polygon.

Reasoning:
We know that the sum of measure of interior angle of triangle is 180°.

Equilateral triangle is a regular polygon having maximum exterior angle because it consists of least number of sides.

Solution:
(a) What is the minimum interior angle possible for a regular polygon? Why?
Consider a regular polygon having the least number of sides (i.e., an equilateral triangle).

We know Sum of all the angles of a triangle = 180°

$$x + x + x = 180^\circ$$
$$3x = 180^\circ$$
$$x = \frac{180^\circ}{3}$$
$$x = 60^\circ$$

Thus, minimum interior angle possible for a regular polygon $= 60^\circ$

(b) What is the maximum exterior angle possible for a regular polygon?

We know that the exterior angle and an interior angle will always form a linear pair.
Thus, exterior angle will be maximum when interior angle is minimum.

$$\text{Exterior angle} = 180^\circ - 60^\circ$$
$$= 120^\circ$$

Therefore, maximum exterior angle possible for a regular polygon is 120°.

Equilateral triangle is a regular polygon having maximum exterior angle because it consists of least number of sides.

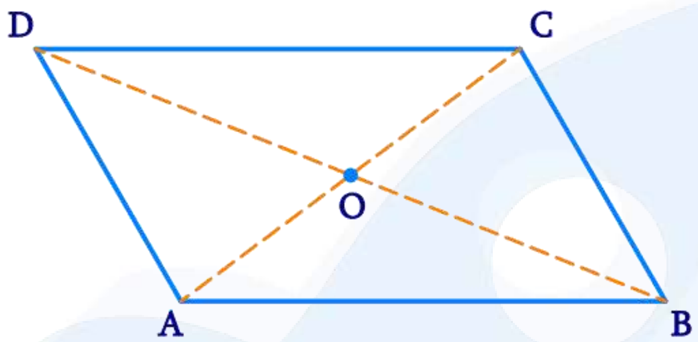
Chapter-3: Understanding Quadrilaterals

Exercise 3.3 (Page 50 of Grade 8 NCERT)

Q1. Given a parallelogram ABCD. Complete each statement along with the definition or property used.

(i) $AD =$ __________ (ii) $\angle DCB =$ _______

(iii) $OC =$ _________ (iv) $m\angle DAB + m\angle CDA =$ ________



Difficulty Level: Medium

Known:
ABCD is a parallelogram.

Unknown:
AD, $\angle DCB$, OC, m$\angle DAB$ + m$\angle CDA$

Reasoning:
We can use the properties of parallelogram to determine the solution.

Solution:
i) The opposite sides of a parallelogram are of equal length.

$$AD = BC$$

(ii) In a parallelogram, opposite angles are equal in measure.

$$\angle DCB = \angle DAB$$

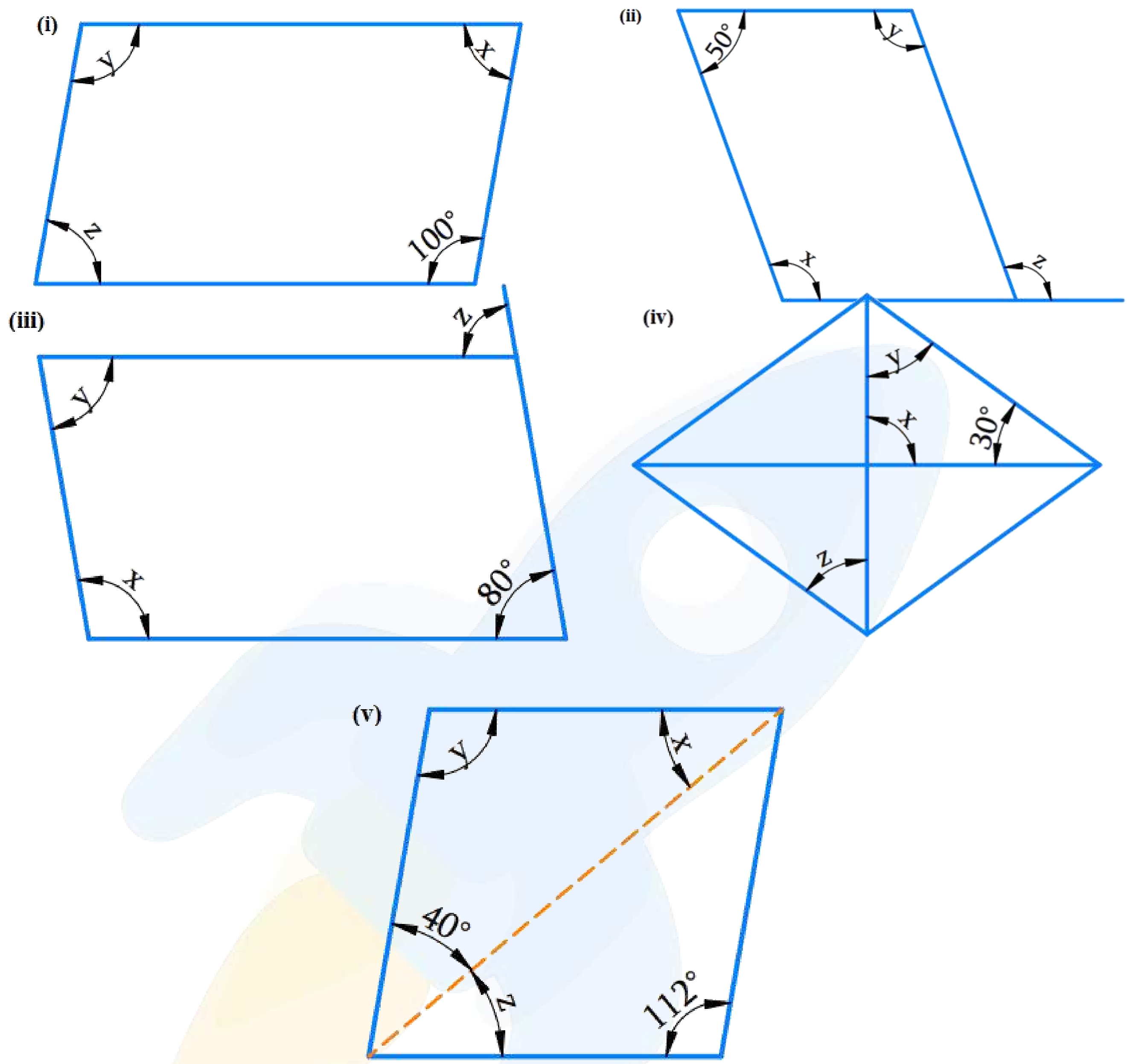
(iii) In a parallelogram, diagonals bisect each other. Hence,

$$OC = OA$$

(iv)In a parallelogram, adjacent angles are supplementary to each other. Hence,

$$m\angle DAB + m\angle CDA = 180°$$

Q2. Consider the following parallelograms. Find the values of the unknowns x, y, z.



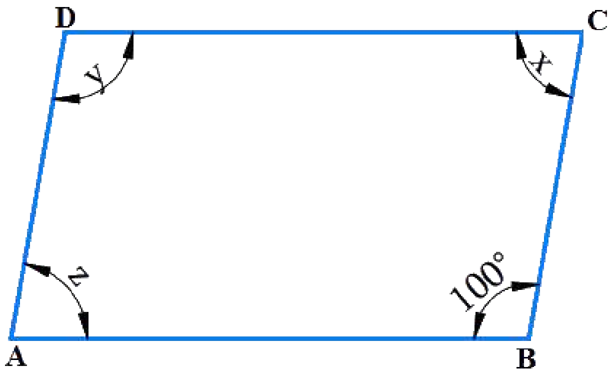
Difficulty Level: Medium

Known:
ABCD is a parallelogram.

Unknown:
Values of x, y, z.

Reasoning:
In a parallelogram, opposite angles are equal and adjacent angles are supplementary.
Using this property, we can calculate the measure of unknown angles.

Solution:



Since D is opposite to B.

So, $y = 100°$ [Since opposite angles of a parallelogram are equal]

$\angle C + \angle B = 180°$ (The adjacent angles in a parallelogram are supplementary)

$x + 100° = 180°$ (The adjacent angles in a parallelogram are supplementary)

Therefore $x = 180^{\circ} - 100^{\circ}$
$= 80^{\circ}$

$x = z = 80°$ [Since opposite angles of a parallelogram are equal]

ii)

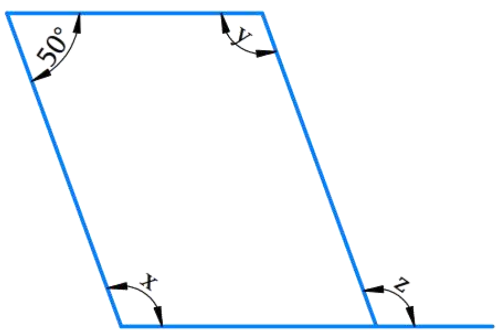
Difficulty Level: Medium

Known:
Given figure is a parallelogram.

Unknown:
values of *x*, *y*, *z*.

Reasoning:
In a parallelogram, opposite angles are equal and adjacent angles are supplementary. Using this property, we can calculate the measure of the unknown angles.

Solution:



$x + 50° = 180°$ (The adjacent angles in a parallelogram are supplementary)

$x = 180° - 50°$

$= 130°$

$x = y = 130°$ (Since opposite angles of a parallelogram are equal)

$x = z = 130°$ (Corresponding angles)

iii)

Difficulty Level: Medium

Known:

Given figure is a parallelogram.

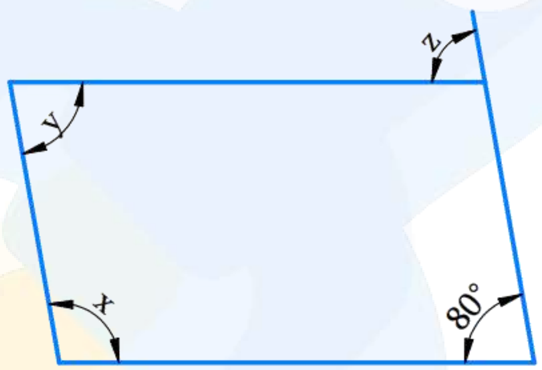
Unknown:

values of x, y, z.

Reasoning:

In a parallelogram, opposite angles are equal and adjacent angles are supplementary. Using this property, we can calculate the measure of the unknown angles.

Solution:



$z = 80^{o}$ (Corresponding angles)

$y = 80^{o}$ (since opposite angles of a parallelogram are equal)

$x + y = 180^{o}$ (Adjacent angles are supplementary)

$x + 80^{o} = 180^{o}$

$x = 180^{o} - 80^{o}$

$x = 100^{\circ}$

Therefore, $x = 100^{\circ}$, $y = 80^{\circ}$, $z = 80^{\circ}$

iv)

Difficulty Level: Medium

What is the known/given?

Given figure is a parallelogram.

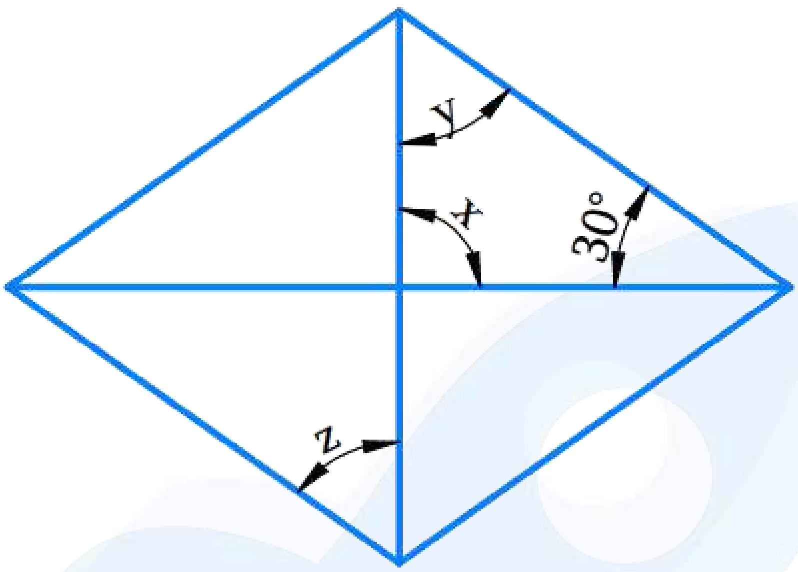
What is the unknown?

Values of *x*, *y*, *z*.

Reasoning:

In a parallelogram, opposite angles are equal and adjacent angles are supplementary. Using this property, we can calculate the measure of the unknown angles.

Solution:



$$x + y + 30^o = 180^o \text{ (Angle sum property of triangles)}$$

$$x = 90^o \text{ (Vertically opposite angles)}$$

$$90^{\circ} + y + 30^o = 180^o$$

$$y + 120 = 180^o$$

$$y = 180^o$$

$$\therefore\ z = y = 60^o \text{ (Alternate interior angles are equal)}$$

v)

Difficulty Level: Medium

Known:

Given figure is a parallelogram.

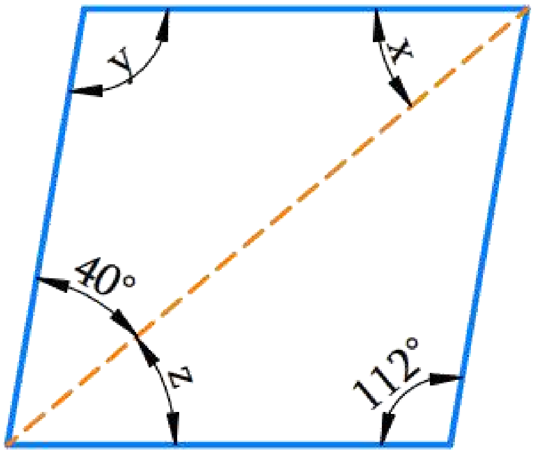
Unknown:

Values of x, y, z.

Reasoning:

In parallelogram opposite angles are equal and Adjacent angles are supplementary. Using this property, we can calculate the unknown angles.

Solution:



$y = 112^\circ$ (Since opposite angles of a parallelogram are equal)

$x + y + 40^\circ = 180^\circ$ (Angle sum property of triangles)

$x + 112^\circ + 40^\circ = 180^\circ$

$x + 152^\circ = 180^\circ$

$x = 180^\circ - 152^\circ$

$x = 28^\circ$

$z = x = 28^\circ$ (Alternate interior angles)

$\therefore x = 28^\circ,\ y = 112^\circ,\ z = 28^\circ$

Q3. Can a quadrilateral ABCD be a parallelogram if

(i) $\angle D + \angle B = 180°$?

(ii) AB = DC = 8 cm, AD = 4 cm and BC = 4.4 cm?

(iii) $\angle A = 70°$ and $\angle C = 65°$?

i)

Difficulty Level: Medium

Known:

Given figure is a quadrilateral

Unknown:

If ABCD is a parallelogram when $\angle D + \angle B = 180°$?

Reasoning:

A parallelogram is a quadrilateral whose opposite sides are parallel.

In parallelogram opposite angles are equal and adjacent angles are supplementary.

Using this property, we can calculate the unknown angles.

Solution:

Using the angle sum property of a quadrilateral,

$$\angle A + \angle B + \angle D + \angle C = 360^{o}$$
$$\angle A + \angle C + 180^{o} = 360^{o}$$
$$\angle A + \angle C = 360^{o} - 180^{o}$$
$$\angle A + \angle C = 180^{o}$$ (Opposite angles should also be of same measures.)

For $\angle D + \angle B = 180°$, is a parallelogram.

If the following conditions is fulfilled, then ABCD is a parallelogram.
The sum of the measures of the adjacent angles should be 180º.
Opposite angles should also be of same measure.

ii)

Difficulty Level: Easy

Known:

Given figure is a quadrilateral.

Unknown:

ABCD be a parallelogram if AB = DC = 8 cm, AD = 4 cm and BC = 4.4 cm

Reasoning:

A parallelogram is a quadrilateral whose opposite sides are parallel.

Solution:

Property of parallelogram: The opposite sides of a parallelogram are of equal length.
Opposite sides AD and BC are of different lengths. So, it's not parallelogram.

iii)

Difficulty Level: Medium

Known:

Given figure is a quadrilateral.

Unknown:

ABCD be a parallelogram if $\angle A = 70°$ and $\angle C = 65°$?

Reasoning:
A parallelogram is a quadrilateral whose opposite sides and angles are equal.

Solution:
Property: In a parallelogram opposite angles are equal.
So, $\angle A = 70°$ and $\angle C = 65°$ are not equal.
So ABCD is not parallelogram.

Q4. Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

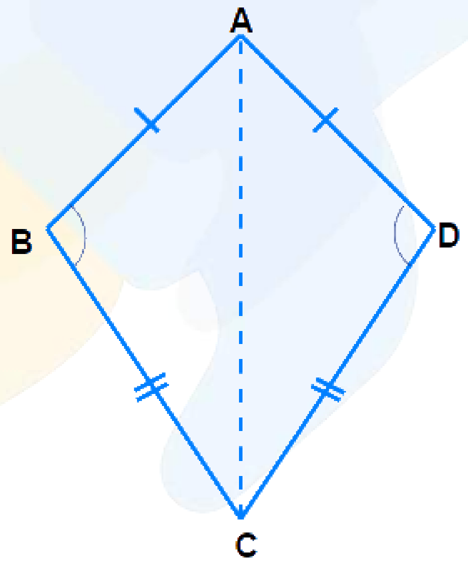
Difficulty Level: Medium

Known:
Draw a figure of quadrilateral having two opposite angles of equal measure.

Unknown:
ABCD is quadrilateral whose opposite angles are equal.

Reasoning:
The opposite angles of a parallelogram are equal.

Solution:



In a kite, the angle between unequal sides are equal.

Draw line from A to C and we will get two triangles with common base AC.

In ΔABC and ΔADC we have,

$AB = AD, \quad BC = CD$; AC is common to both

$\Delta ABC = \Delta ADC$ [congruent triangles]

Hence corresponding parts of congruent triangles are equal.

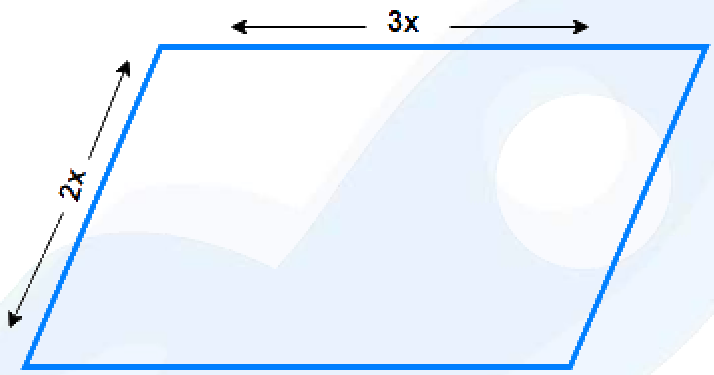
Therefore $\angle B = \angle D$

However, the quadrilateral ABCD is not a parallelogram as the measures of the remaining pair of opposite angles, $\angle A$ and $\angle C$, are not equal. Since they form angle between equal sides.

Q5. The measures of two adjacent angles of a parallelogram are in the ratio $3:2$. Find the measure of each of the angles of the parallelogram.

Difficulty Level: Medium

Known:



Given figure is a parallelogram and two adjacent angles are having ratio of $3:2$ quadrilateral.

Unknown:
Measure of Each angles of parallelogram.

Reasoning:
A parallelogram is a quadrilateral whose opposite angles are equal.

Solution:
We know that the sum of the measures of adjacent angles is 180° for a parallelogram.

$$\angle A + \angle B = 180^\circ$$
$$3x + 2x = 180^\circ$$
$$5x = 180^\circ$$
$$x = \frac{180^\circ}{5}$$
$$x = 36^\circ$$

$$\angle A = \angle C = 3x$$
$$= 108^{\circ} \ (\text{ Opposite angles })$$
$$\angle B = \angle D = 2x$$
$$= 72^{\circ} \ (\text{Opposite angles})$$

Thus, the measures of the angles of the parallelogram are 108º, 72º, 108º, and 72º.

Q6. Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.

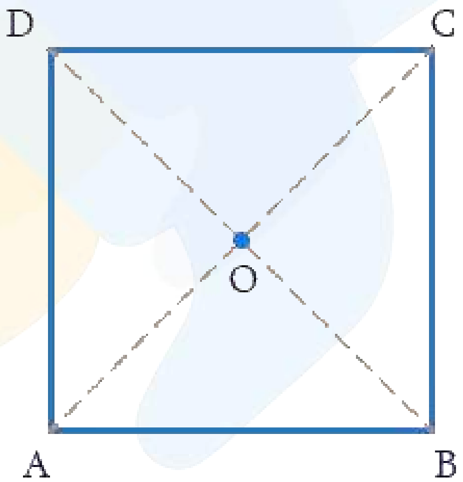
Difficulty Level: Medium

Known:
Two adjacent angles of a parallelogram have equal measure.

Unknown:
Measure of each of the angles of the parallelogram.

Reasoning:
In parallelogram opposite angles are equal and adjacent angles are supplementary. Using this property, we can calculate the unknown angles.

Solution:



In parallelogram ABCD,
$\angle A$ and $\angle D$ are supplementary since DC is parallel to AB and with transversal DA, making $\angle A$ and $\angle D$ interior opposite.

$\angle A$ and $\angle B$ are also supplementary since AD is parallel to BC and with transversal BA, making $\angle A$ and $\angle B$ interior opposite.

Sum of adjacent angles = 180°

Let each adjacent angle be x

Since the adjacent angles in a parallelogram are supplementary.

$$x + x = 180°$$
$$2x = 180°$$
$$x = \frac{180°}{2}$$

Hence, each adjacent angle is 90.

$$\angle A = \angle B = 90^o \quad \text{(adjacent angles)}$$
$$\angle C = \angle A = 90^o \quad \text{(Opposite angles)}$$
$$\angle D = \angle B = 90^o \quad \text{(Opposite angles)}$$

Thus, each angle of the parallelogram measures 90º.

Q7. The adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the properties you use to find them.

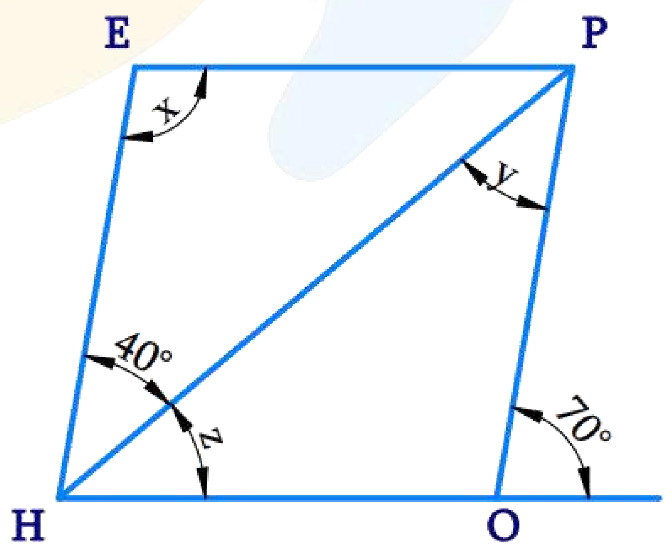
Difficulty Level: Medium

Known:
Given figure is a parallelogram.

Unknown:
Values of x, y, z.

Reasoning:
In a parallelogram, opposite angles are equal and adjacent angles are supplementary. Using this property, we can calculate the unknown angles.

Solution:



Here,

$$\angle HOP + 70^\circ = 180^\circ \qquad [\text{Angles of linear pair}]$$

$$\angle HOP = 180^\circ - 70^\circ$$

$$\angle HOP = 110^\circ$$

$$\angle O = \angle E \qquad (\text{opposite angles are equal})$$

$$\therefore x = 110^\circ$$

$$y = 40^\circ \qquad (\text{Alternate interior angles are equal})$$

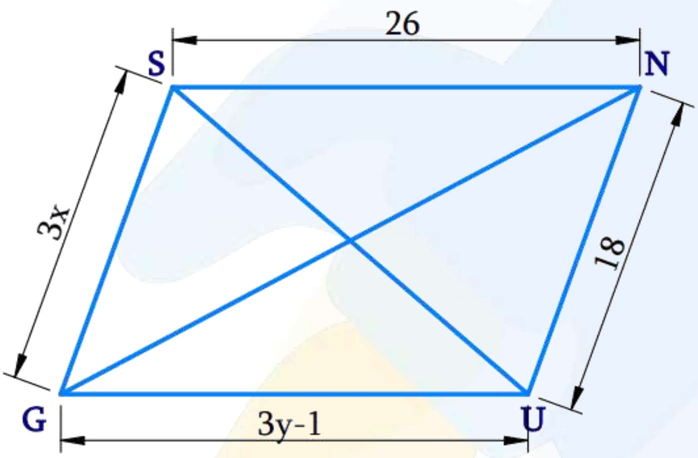
$$z + 40^\circ = 70^\circ \quad (\text{Corresponding angles})$$

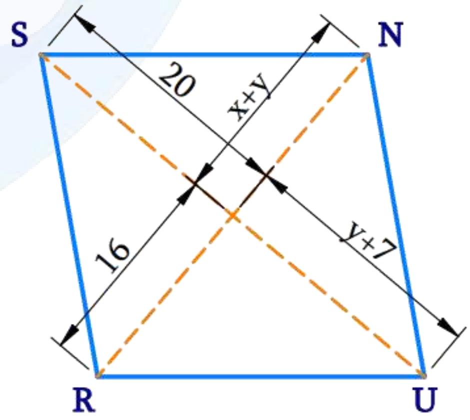
$$z = 70^\circ - 40^\circ$$

$$z = 30^\circ$$

$$\therefore\ x = 110^\circ, \quad y = 40^\circ, \quad z = 30^\circ$$

Q8. The following figures GUNS and RUNS are parallelograms. Find *x* and *y*. (Lengths are in cm)





i)

Difficulty Level: Medium

Known:

Given figure is a parallelogram.

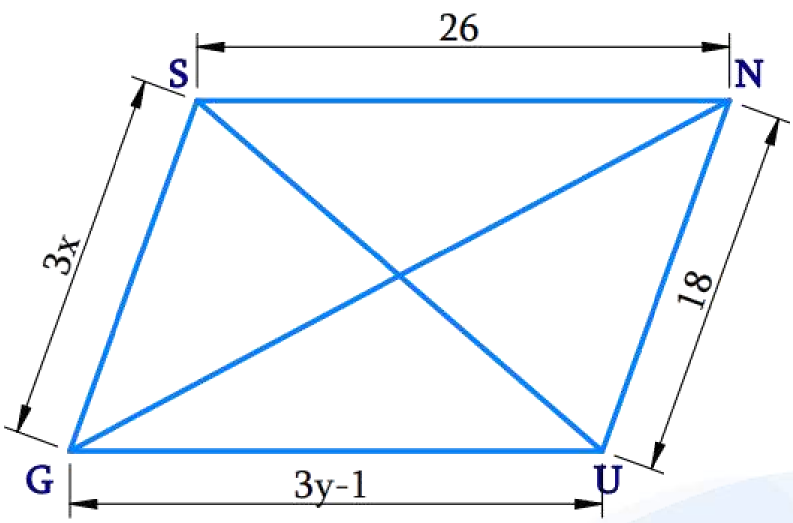
Unknown:

Values of x, y

Reasoning:

The diagonals of a parallelogram bisect each other, in a parallelogram, the opposite sides have same length.

Solution:



In a parallelogram, the opposite sides have same length.

$$SG = NU$$
$$3x = 18$$
$$x = \frac{18}{3}$$
$$x = 6$$

And,

$$SN = GU$$
$$26 = 3y - 1$$
$$3y = 26 + 1$$
$$y = \frac{27}{3}$$
$$y = 9$$

Hence, the measures of x and y are 6 cm and 9 cm respectively.

(ii)

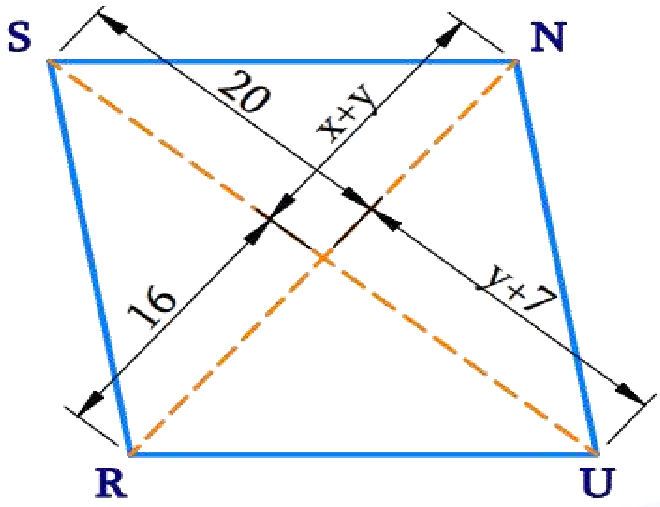
Difficulty Level: Medium

Known:
Given figure is a parallelogram.

Unknown:
Values of x, y

Reasoning:
The diagonals of a parallelogram bisect each other. In a parallelogram, the opposite sides have same length.

Solution:

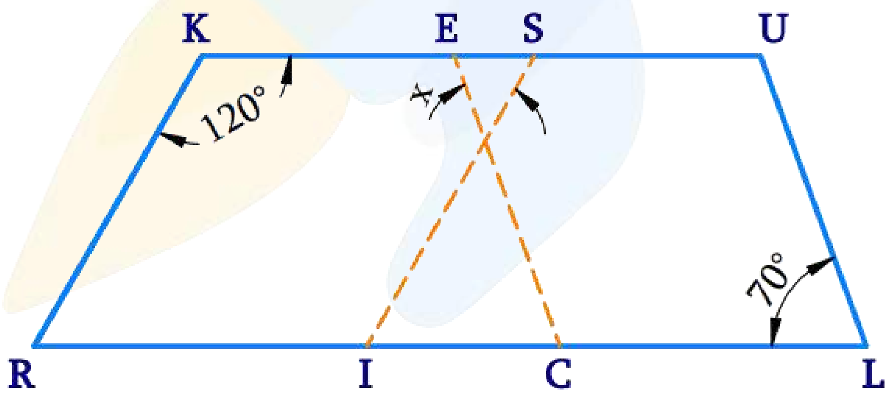


Property: The diagonals of a parallelogram bisect each other.

$$y + 7 = 20$$
$$y = 20 - 7$$
$$y = 13$$
$$x + y = 16$$
$$x + 13 = 16$$
$$x = 3$$

Hence, the measures of x and y are 3 cm and 13 cm respectively.

Q9.



In the above figure both RISK and CLUE are parallelograms. Find the value of x.

Difficulty Level: Medium

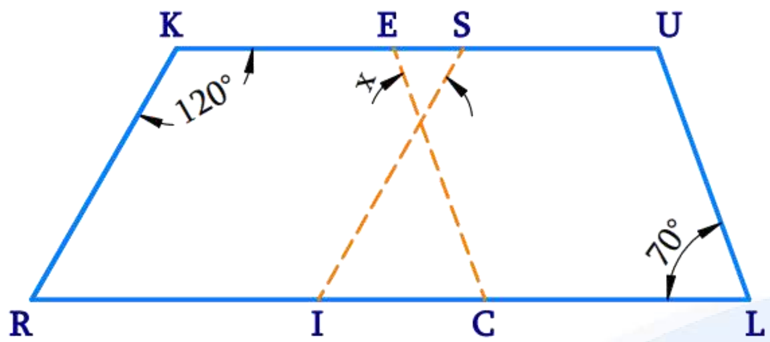
Known:
In the given figure RISK and CLUE are parallelograms.

Unknown:
Values of x

Reasoning:

The diagonals of a parallelogram bisect each other. Also, in a parallelogram, opposite angles are equal and adjacent angles are supplementary. Using this property, we can calculate the unknown angles.

Solution:



In parallelogram RISK

$$\begin{aligned} RKS + \angle ISK &= 180^\circ \\ 120^\circ + \angle ISK &= 180^\circ \\ \angle ISK &= 180^\circ - 120^\circ \\ \angle ISK &= 60^\circ \end{aligned}$$

$$\begin{aligned} \angle I &= \angle K \quad \text{(In parallelogram opposite angles are equal)} \\ &= 120^\circ \end{aligned}$$

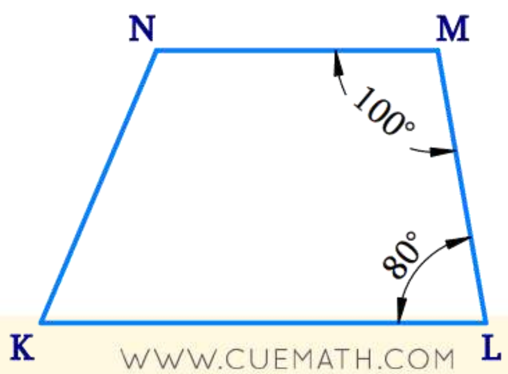
In parallelogram CLUE

$$\begin{aligned} \angle L &= \angle E \quad \text{(In parallelogram opposite angles are equal)} \\ &= 70^\circ \end{aligned}$$

The sum of the measures of all the interior angles of a triangle is 180º.

$$\begin{aligned} x + 60^\circ + 70^\circ &= 180^\circ \\ x + 130^\circ &= 180^\circ \\ x &= 180^\circ - 130^\circ \\ x &= 50^\circ \end{aligned}$$

Q10. Explain how this figure is a trapezium. Which of its two sides are parallel? (Fig 3.32)



Difficulty Level: Medium

Known:
Given figure is a Quadrilateral.

Unknown:
To identify the two parallel sides of the figure and to prove that it is a trapezium.

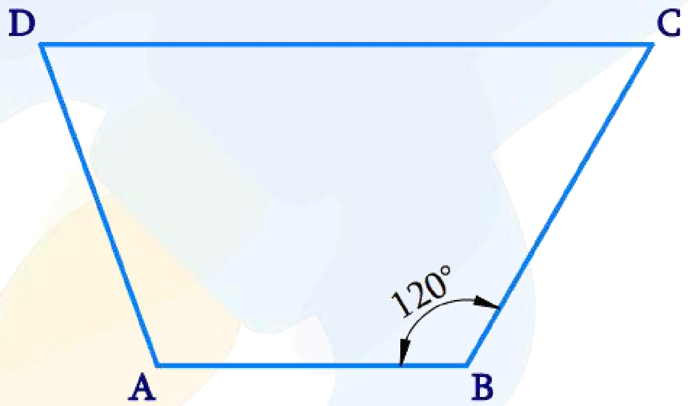
Reasoning:
Trapezium is a quadrilateral having one pair of parallel sides.

Solution:
In the given figure KLMN,
$\angle L + \angle M = 180°$ [two pair of adjacent angles (which form pairs of consecutive interior angles) are supplementary]
$$= 80° + 100° = 180°$$

Therefore, KN is parallel to ML

Hence, KLMN is a trapezium as it has a pair of parallel sides: KN and ML.

Q11. Find $m\angle C$ in Fig 3.33 if $\overline{AB}$ is parallel to $\overline{DC}$.



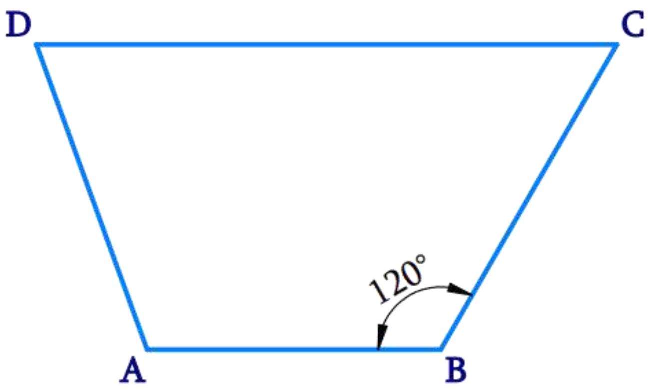
Difficulty Level: Medium

Known:
Given figure is a Quadrilateral with two sides running parallel and one angle is given.

Unknown:
Find $m\angle C$

Reasoning:
Trapezium is a quadrilateral with one pair of parallel sides.

Solution:

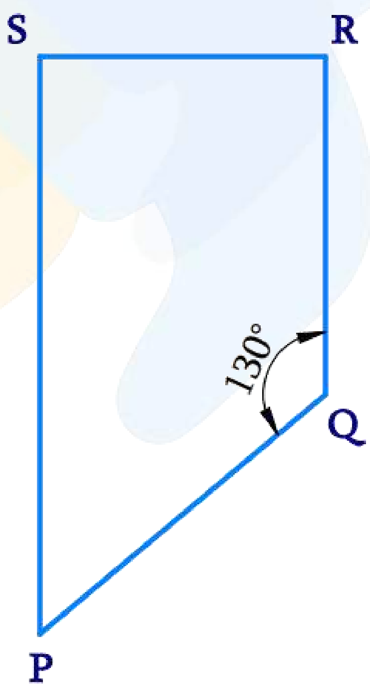


Given figure ABCD is a Trapezium, in which $\overline{AB}$ is parallel to $\overline{DC}$.

Here,

$$\angle B + \angle C = 180° \quad [\text{pair of adjacent angles are supplementary}]$$
$$120° + \angle C = 180°$$
$$\angle C = 180° - 120°$$
$$\angle C = 60°$$
$$\text{Therefore,} \quad m\angle C = 60°$$

Q12. Find the measure of $\angle P$ and $\angle S$ if $\overline{SP}$ is parallel to $\overline{RQ}$ in Fig 3.34. (If you find $m\angle R$, is there more than one method to find $m\angle P$?)



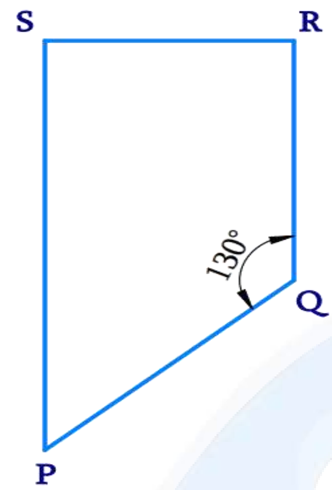
Difficulty Level: Medium

Known:
Given figure is a Quadrilateral.

Unknown:
Find $m\angle P$ and $m\angle S$

Reasoning:
Sum of the measures of all the interior angles of a quadrilateral is 360º.

Solution:



Given $\overline{SP}$ is parallel to $\overline{RQ}$ and SR is the traversal drawn to these lines. Hence,

$$\angle S + \angle R = 180^o$$
$$\angle S + 90^o = 180^o$$
$$\angle S = 180^o - 90^o$$
$$\angle S = 90^o$$

Using the angle sum property of a quadrilateral,

$$\angle S + \angle P + \angle Q + \angle R = 360^o$$
$$90^o + \angle P + 130^o + 90^o = 360^o$$
$$\angle P + 310^o = 360^o$$
$$\angle P = 360^o - 310^o$$
$$\angle P = 50^o$$

Alternate Method:

$$\angle P + \angle Q = 180^o \text{(adjacent angles)}$$
$$\angle P + 130^o = 180^o$$
$$\angle P = 180^o - 130^o$$
$$\angle P = 50^o$$

And,

$$\angle S + \angle R = 180^o \text{(adjacent angles)}$$
$$\angle S + 90^o = 180^o$$
$$\angle S = 180^o - 90^o$$
$$\angle S = 90^o$$

Chapter-3: Understanding Quadrilaterals

Exercise 3.4 (Page 55 of Grade 8 NCERT)

Q1. State whether True or False.

a) All rectangles are squares.
b) All rhombuses are parallelograms.
c) All squares are rhombuses and are also, rectangles.
d) All squares are not parallelograms.
e) All kites are rhombuses.
f) All rhombuses are kites.
g) All parallelograms are trapeziums.
h) All squares are trapeziums.

Solution:

	Shapes	**True or False**	**Reason**
A	All rectangles are Squares.	False	A rectangle need not have all sides equal hence it is not square.
B	All rhombuses are parallelograms	True	Since the opposite sides of a rhombus are equal and parallel to each other, it is also a parallelogram
C	All squares are rhombuses and are also rectangles.	True	All squares are rhombuses as all sides of a square are of equal lengths. All squares are also rectangles as each internal angle is 90 degrees.
D	All squares are not parallelograms.	False	The opposite sides of a parallelogram are of equal length hence squares with all sides equal are parallelograms.
E	All kites are Rhombuses.	False	Since rhombus have all sides of equal length. A kite does not have all sides of the same length.
F	All rhombuses are kites.	True	Since, all rhombuses have equal sides and diagonals bisect each other.
G	All parallelograms are trapeziums.	True	Since, all trapeziums have a pair of parallel sides.
H	All squares are Trapeziums.	True	All trapeziums have a pair of parallel sides; hence squares can be trapezium.

Q2. Identify all the quadrilaterals that have.

a) four sides of equal length
b) four right angles

Solution:

a) Four sides of equal length - Rhombus and Square are the quadrilaterals with 4 sides of equal length.

b) Four right angles - Square and Rectangle are the quadrilaterals with 4 right angles.

Q3. Explain how a square is.

(i) a quadrilateral (ii) a parallelogram (iii) a rhombus (iv) a rectangle

Solution:

(i)	Quadrilateral-	A square is a quadrilateral since it has four sides.
(ii)	Parallelogram- properties (i) Opposite sides are equal. (ii) Opposite angles are equal. (iii) Diagonals bisect one another.	A square is parallelogram, since it contains both pairs of opposite sides equal.
(iii)	Rhombus - properties i) A parallelogram with sides of equal length. ii)The diagonals of a rhombus are perpendicular bisectors of one another.	A square is a rhombus since i) its four sides are of same length. ii)the diagonals of a square are perpendicular bisectors of each other.
(iv)	Rectangle-properties i) Being a parallelogram, the rectangle has opposite sides of equal length and its diagonals bisect each other.	A square is rectangle since each interior angle measures 90 degree.

Q4. Name the quadrilaterals whose diagonals.

(i) bisect each other

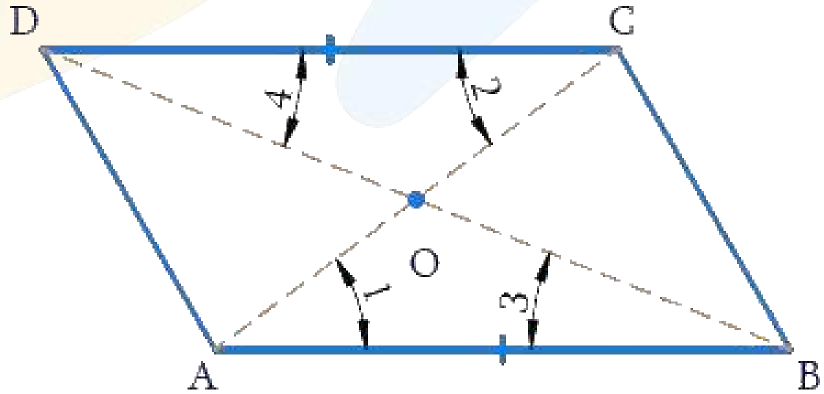
(ii) are perpendicular bisectors of each other

(iii) are equal

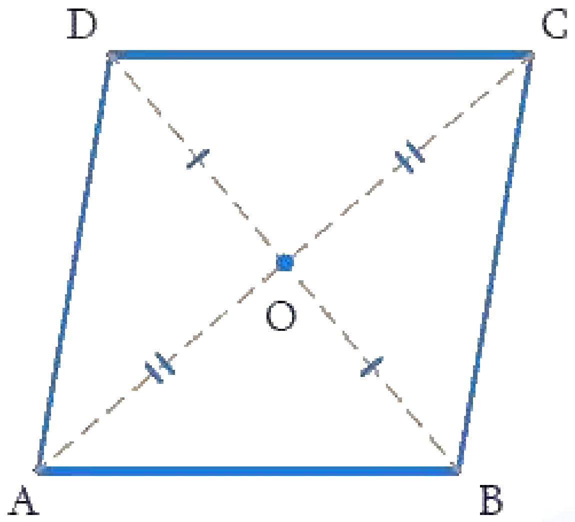
Solution:

(i) bisect each other

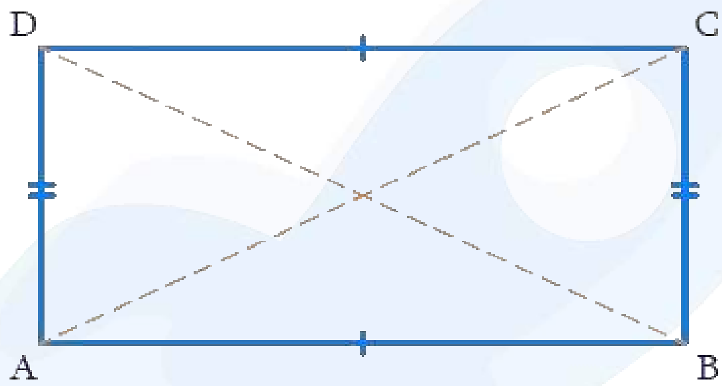
Parallelogram:



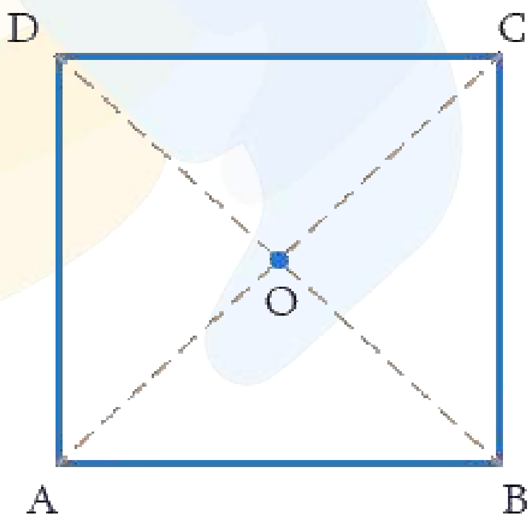
Rhombus:



Rectangle:



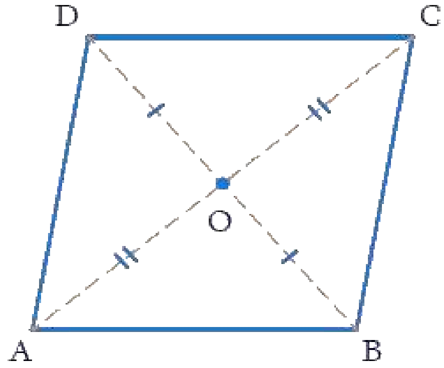
Square:

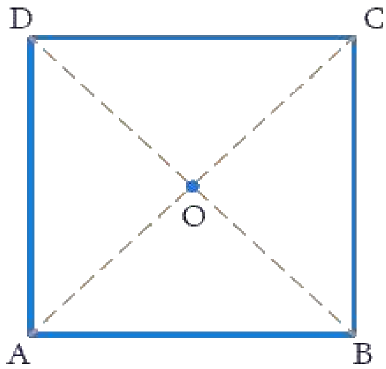


a) Parallelogram
b) b) Rhombus
c) c) Rectangle
d) d) Square

The diagonals of a parallelogram, rhombus, rectangle and square are perpendicular bisectors of each other.

(ii) Are perpendicular bisectors of each other

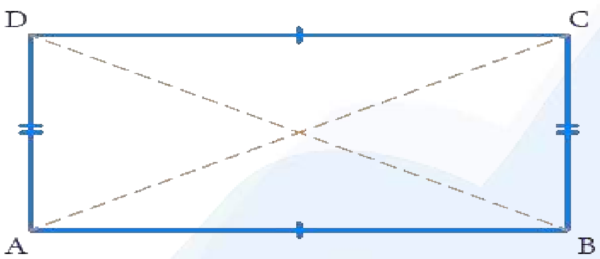


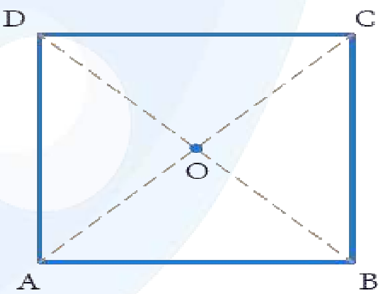


a) Rhombus
b) Square

The diagonals of a square and rhombus are perpendicular bisectors of each other.

(iii) are equal





a) Rectangle
b) Square

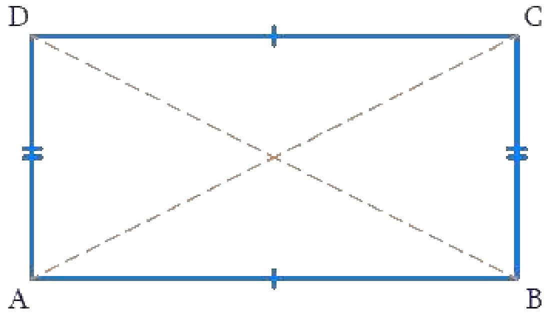
The diagonals of a rectangle and square are equal.

Q5. Explain why a rectangle is a convex quadrilateral.

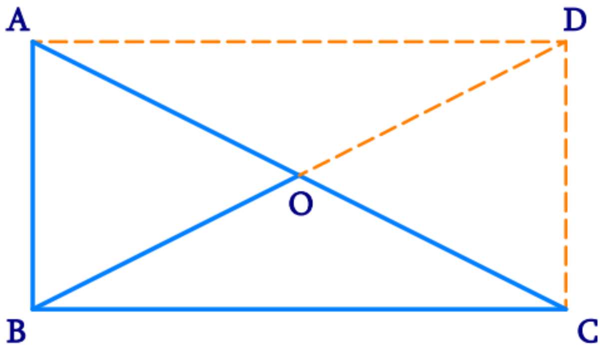
Solution:
Polygons that are convex have no portions of their diagonals in their exteriors. A rectangle is a convex quadrilateral since its vertex are raised and both of its diagonals lie in its interior.

Or

None of the angles being a reflex angle, So, rectangle is convex quadrilateral.



Q6. ABC is a right-angled triangle and O is the midpoint of the side opposite to the right angle. Explain why O is equidistant from A, B and C. (The dotted lines are drawn additionally to help you).



Difficulty Level: Medium

Known:
ABC is a right-angled triangle and O is the midpoint of the side opposite to the right angle.

Unknown:
Why O is equidistant from A, B and C

Reasoning:
Since, two right triangles make a rectangle and, in any rectangle, diagonals bisect each other.

Solution:
ABCD is a rectangle as opposite sides are equal and parallel to each other and all the interior angles are of 90^o.

$$AD \parallel BC, \quad AB \parallel DC$$

$$AD = BC, \quad AB = DC$$

In a rectangle, diagonals are of equal length and also these bisect each other.

Hence, $AO = OC = BO = OD$

Since, two right triangles make a rectangle where O is equidistant point from A, B, C and D because O is the mid-point of the two diagonals of a rectangle. So, O is equidistant from A, B, C and D.