Chapter 1 Number System

Exercise 1.1(Page 5 of Grade 9 NCERT Textbook)

Q1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$?

Difficulty Level: Easy

Solution:
Yes, zero is a rational number.

Zero can be written as: $\frac{0}{\text{Any non-zero integer}}$

$\text{Example}: \frac{0}{1}, \frac{0}{-2}$

Which is in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

Q2. Find six rational numbers between 3 and 4.

Difficulty Level: Easy

Solution:
We can find any number of rational numbers between two rational numbers. First of all, we make the denominators same by multiplying or dividing the given rational numbers by a suitable number. If denominator is already same then depending on number of rational no. we need to find in question, we add one and multiply the result by numerator and denominator.

$$3 = \frac{3 \times 7}{7} \quad \text{and} \quad 4 = \frac{4 \times 7}{7}$$

$$3 = \frac{21}{7} \quad \text{and} \quad 4 = \frac{28}{7}$$

We can choose 6 rational numbers as: $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}$ and $\frac{27}{7}$

Q3. Find five rational numbers between $\frac{3}{4}$ and $\frac{4}{5}$

Difficulty Level: Easy

Solution:

Since we make the denominator same first, then

$$\frac{3}{4}=\frac{3\times5}{4\times5}=\frac{15}{20}$$

$$\frac{4}{5}=\frac{4\times4}{5\times4}=\frac{16}{20}$$

Now we need to find 5 rational no.

$$\frac{15}{20}=\frac{15\times6}{20\times6}=\frac{90}{120}$$

$$\therefore\ \frac{16}{20}=\frac{16\times6}{20\times6}=\frac{96}{120}$$

$\therefore$ Five rational numbers between $\frac{3}{4}$ and $\frac{4}{5}$ are $\frac{91}{120},\frac{92}{120},\frac{93}{120},\frac{94}{120}$ and $\frac{95}{120}$

Q4. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

Solution:

True, because the set of natural numbers is represented as N= {1, 2, 3…….} and the set of whole numbers is W = {0, 1, 2, 3 ………}. We see that every natural number is present in the set of whole numbers.

(ii) Every integer is a whole number.

Solution:

False. Negative integers are not present in the set of whole numbers.

(iii) Every rational number is a whole number.

Solution:

False. For example, $\frac{1}{2}$ is a rational number, which is not a whole number.

Chapter 1 Number System

Exercise 1.2(Page 8 of Grade 9 NCERT Textbook)

Q1. State whether the following statements are true or false. Justify your Answers.

(i) Every irrational number is a real number.

Solution:
True, because the set of real numbers consists of rational numbers and irrational numbers.

(ii) Every point on the number line in of the form $\sqrt{m}$, where 'm' is a natural number.

Solution:

False, for example $\sqrt{\frac{2}{3}}$ is a real number on the number line but $\frac{2}{3}$ is not a natural number.

(i) Every real number is an irrational number.

Solution:
False, for example $\frac{1}{2}$ is a rational number and hence it is real. But it is not an irrational number.

Q2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.
Solution:
The square roots of all positive integers are not irrationals.

Example: $\sqrt{4} = 2$ and 2 in a rational number $(\because 2 = \frac{2}{1})$

Q3. Show how $\sqrt{5}$ can be represented on the number line.

Difficulty Level: Medium

What is known/given?
Integer 5.

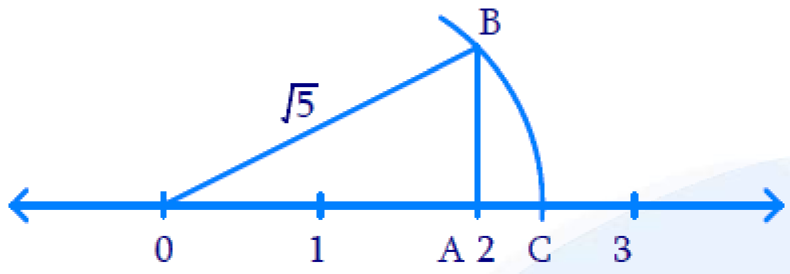
What is unknown?

Point representing $\sqrt{5}$ on the number line.

Solution:

We shall write 5 on the sum of two squares in the form $5 = 2^2 + 1^2$. This shows we need to construct a right triangle with sides 2 and 1 units. So, the hypotenuse becomes $\sqrt{5}$ units on the number line. We shall proceed as follows.

Diagram



On the number line take 2 units from O and represent the point as A. At A draw the perpendicular and mark B such that AB =1 unit with O as center and OB as radius. Draw an arc to cut the number line at C. Now C represents $\sqrt{5}$.

In ΔOAB,

$$\begin{aligned} OB^2 &= OA^2 + AB^2 \\ &= 2^2 + 1^2 \\ &= 5 \\ \therefore OB &= \sqrt{5} = OC \end{aligned}$$

Chapter 1 Number System

Exercise 1.3 (Page 14 of Grade 9 NCERT Textbook)

Q1. Write the following in decimal form and say what kind of decimal expansion each has:

i) $\frac{36}{100}$ ii) $\frac{1}{11}$ iii) $4\frac{1}{8}$

iv) $\frac{3}{13}$ v) $\frac{2}{11}$ vi) $\frac{329}{400}$

Difficulty Level: Easy

Solution:

(i) $\frac{36}{100} = 0.36$

Terminating decimal.

(ii)

$\frac{1}{11}$

$$\begin{array}{r|l} & 0.0909 \\ \hline 11 & 1.00 \\ & 99 \\ \hline & 100 \\ & 99 \\ \hline & 1 \end{array}$$

The remainder 1 keeps repeating. $\frac{1}{11} = 0.0909$ and can be written as

$\frac{1}{11} = 0.\overline{09}$

Non-terminating recurring decimal.

iii) $4\frac{1}{8} = \frac{33}{8}$

$$\begin{array}{r|l} & 4.125 \\ \hline 8 & 33.0 \\ & 32 \\ \hline & 10 \\ & 8 \\ \hline & 20 \\ & 16 \\ \hline & 40 \\ & 40 \\ \hline & 0 \end{array}$$

$$4\frac{1}{8} = 4.125$$

Terminating decimal ($\because$ The remainder is zero)

iv) $\frac{3}{13} = 0.23076923$

$$\begin{array}{r} 0.23076923 \\ 13\overline{)\ 30} \\ \underline{26} \\ 40 \\ \underline{39} \\ 100 \\ \underline{91} \\ 90 \\ \underline{78} \\ 120 \\ \underline{117} \\ 30 \\ \underline{26} \\ 40 \\ \underline{39} \\ 1 \end{array}$$

$\because$ We find the block of numbers 230769.keep repeating.

This is non-terminating recurring decimal and is written as:

$$\frac{3}{13} = 0.\overline{230769}$$

v) $\frac{2}{11} = 0.1818$

$$\begin{array}{r} 0.1818 \\ 11\overline{)\ 20} \\ \underline{11} \\ 90 \\ \underline{88} \\ 20 \\ \underline{11} \\ 90 \\ \underline{88} \\ 2 \end{array}$$

Here we find the block of numbers 18 keep repeating. Hence this is a non-terminating recurring decimal and is written as:

$$\frac{2}{11} = 0.\overline{18}$$

vi) $\dfrac{329}{400}=\dfrac{329}{4\times 100}$

$$
\begin{array}{r}
82.25 \\
4\overline{)\;329} \\
32 \\
\hline
09 \\
08 \\
\hline
10 \\
8 \\
\hline
20 \\
20 \\
\hline
0
\end{array}
$$

$\dfrac{82.25}{100}=0.8225$

Terminating decimal ($\because$ The remainder is zero)

Q2. You know that $\dfrac{1}{7}=0.\overline{142857}$. Can you predict what the decimal expansions of $\dfrac{2}{7},\dfrac{3}{7},\dfrac{4}{7},\dfrac{3}{7},\dfrac{4}{7},\dfrac{5}{7},\dfrac{6}{7}$ are, without actually doing the long division? If so, how?

Difficulty Level: Easy

Known:

The decimal expansion of $\dfrac{1}{7}$.

Unknown:

The decimal expansions of $\dfrac{2}{7},\dfrac{3}{7},\dfrac{4}{7},\dfrac{5}{7},\dfrac{6}{7}$

Solution:

$\dfrac{1}{7}=0.\overline{142587}$

$$
\begin{array}{r}
0.142857 \\
7\overline{)\;10} \\
7 \\
\hline
30 \\
28 \\
\hline
20 \\
14 \\
\hline
60 \\
56 \\
\hline
40 \\
35 \\
\hline
50 \\
49 \\
\hline
1
\end{array}
$$

This is a non-terminating recurring decimal.

We can use this to find the decimal expansion of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$.

To write the decimal expansion for

i) $\frac{2}{7}$: We observe that we get 2 as remainder after the second step in the above division. Hence, we start writing the quotient after the second decimal place and we get $\frac{2}{7} = 0.\overline{285714}$

ii) $\frac{3}{7}$: 3 is the remainder after the first step.

Hence $\frac{3}{7} = 0.\overline{428571}$

iii) $\frac{4}{7}$: 4 is the remainder at the 4^{th} step.

Hence $\frac{4}{7} = 0.\overline{571428}$

iv) $\frac{5}{7}$: 5 is the remainder at the 5^{th} step.

Hence $\frac{5}{7} = 0.\overline{714285}$

v) $\frac{6}{7}$: 6 is the remainder after the 3^{rd} step.

Hence $\frac{6}{7} = 0.\overline{857142}$

Q3. Express the following in the form of $\frac{p}{q}$, where p and q are integers and q$\neq$ 0.

i) $0.\overline{6}$ ii) $0.4\overline{7}$ iii) $0.\overline{001}$

Difficulty Level: Medium

Solution:

(i) $0.\overline{6}$

$Let\ x = 0.\overline{6}$

$x = 0.666....$ (1)

Since one digit is repeating. We should multiply both sides of (1) by 10. We get,

$$10x = 6.666...$$
$$10x = 6 + 0.666...$$
$$10x = 6 + x$$
$$10x - x = 6$$
$$9x = 6$$
$$x = \frac{6}{9} = \frac{2}{3}$$

Hence $0.\bar{6} = \frac{2}{3}$

(ii) $0.4\bar{7}$

Let $x = 0.4777\ldots\ldots$ (1)

Here the repetition starts after the first decimal place and one digit is repeated.

$10x = 4.777\ldots...$ (2)

(2) – (1) gives

$$10x - x = 4.777\ldots - 0.4777\ldots$$
$$9x = 4.3$$
$$9x = \frac{43}{10}$$
$$x = \frac{43}{90}$$

Here $0.4\bar{7} = \frac{43}{90}$

(iii) $0.\overline{001}$

Let $x = 0.001001\ldots$ …(1)
Since 3 digits are repeated multiply both the sides of (1) by 1000

$$1000x = 1.001001\ldots$$
$$1000x = 1 + 0.001001\ldots$$
$$1000x = 1 + x$$
$$1000x - x = 1$$
$$999x = 1$$
$$x = \frac{1}{999}$$

$$0.\overline{001} = \frac{1}{999}$$

cuemath
THE MATH EXPERT

Q4. Express 0.99999… in the form of $\frac{p}{q}$. Are you surprised with your Answer? With your teacher and classmates discuss why the answer makes sense?

Difficulty Level: Medium

Solution:

Let $x = 0.99999\ldots$ (1)

Since one digit is repeated.
We should multiply both the sides of (1) by 10

$$10x = 9.9999$$
$$10x = 9 + 0.9999$$
$$10x = 9 + x$$
$$10x - x = 9$$
$$9x = 9$$
$$x = 1$$

Hence 0.99999 = 1

Q5. What can be the maximum number of digits be in the repeating block of digits in decimal expansion of $\frac{1}{17}$? Perform the division to check your Answer.

Difficulty Level: Medium

Solution:

Let us perform the division $1 \div 17$

$$
\begin{array}{r}
0.0588235294117647 \\
17\overline{)\;100} \\
85 \\
\hline
150 \\
136 \\
\hline
140 \\
136 \\
\hline
40 \\
34 \\
\hline
60 \\
51 \\
\hline
90 \\
85 \\
\hline
50 \\
34 \\
\hline
160 \\
153 \\
\hline
70 \\
68 \\
\hline
20 \\
17 \\
\hline
30 \\
17 \\
\hline
130 \\
119 \\
\hline
110 \\
102 \\
\hline
80 \\
68 \\
\hline
120 \\
119 \\
\hline
1
\end{array}
$$

$$\therefore \frac{1}{17} = 0.\overline{0588235294117647}$$

There are 16 digits in the repeating block of the decimal expansion of $\frac{1}{17}$.

Q6. Look at the several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$ where p and q are integers with no common factors other than 1 and having terminating decimal representation (expansions). Can you guess what property q must satisfy?

Difficulty Level: Medium

Solution:

We shall look at some examples of rational numbers in the form of $\frac{p}{q}$ $(q \neq 0)$ where decimal representations are terminating.

$$\frac{2}{5} = 0.4 \qquad \frac{3}{100} = 0.03$$

$$\frac{27}{16} = 1.6875 \qquad \frac{33}{50} = 0.66$$

We observed that the denominators of above rational numbers are in the form of

$2^a \times 5^b$ Where, a and b are whole numbers.

Hence if q is in the form $2^a \times 5^b$ then $\frac{p}{q}$ is a terminating decimal.

Q7. Write three numbers whose decimal expansions are non terminating and non-recurring.

Difficulty Level: Medium

Solution:

(i) 0.212212221…

(ii) 0.03003000300003…

(iii) 0.825882588825…

Q8. Find three irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Difficulty Level: Medium

Solution:

Let us find the decimal expansion of $\frac{5}{7}$ and $\frac{9}{11}$.

$$
\begin{array}{r}
0.714285 \\
7\overline{)\ 50} \\
49 \\
\hline
10 \\
7 \\
\hline
30 \\
28 \\
\hline
20 \\
14 \\
\hline
60 \\
56 \\
\hline
40 \\
35 \\
\hline
5
\end{array}
$$

$$
\begin{array}{r}
0.81 \\
11\overline{)\ 90} \\
88 \\
\hline
20 \\
11 \\
\hline
9
\end{array}
$$

We can write 3 irrational numbers between them as follows:

(i) 0.721722172221…

(ii) 0.750975009750009…

(iii) 0.808008000…

Q9. Classify the following numbers as rational or irrational:

i) $\sqrt{23}$ ii) $\sqrt{225}$ iii) 0.3796

iv)7.478478…. v) 1.101001000100001….

Difficulty Level:

Solution:

i) $\sqrt{23} = \frac{\sqrt{23}}{1} = \frac{p}{q}$, but p is not an integer.

Hence $\sqrt{23}$ is an irrational number.

ii) $\sqrt{225} = \frac{15}{1} = \frac{p}{q}$, where p and q are integers. $q \neq 0$.

Hence $\sqrt{225}$ is a rational number.

iii) 0.3796

0.3796 is a rational number. Because, it is a terminating decimal number.

iv) $7.478478\ldots = 7.\overline{478}$

It is a rational number. As It is a non-terminating recurring decimal.

v) 1.101001000100001…….

It is an irrational number. Because, it is a non-terminating and non-recurring decimal.

Chapter 1 Number System

Exercise 1.4 (Page 18 of Grade 9 NCERT Textbook)

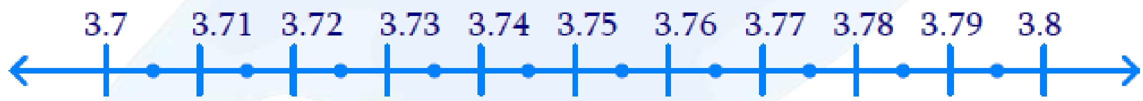
Q1. Visualize 3.765 on the number line, using successive magnification.

Difficulty Level: Medium

Solution:

i) 3.7 lies between 3 & 4. So, we divide the portion between 3 & 4 on the number line into 10 equal parts

ii) 3.76 lies between 3.7 and 3.8. So, we divide the portion between 3.7 and 3.8 on the number line into 10 equal parts.



iii) 3.765 lies between 3.76 and 3.77. Dividing the line before we got

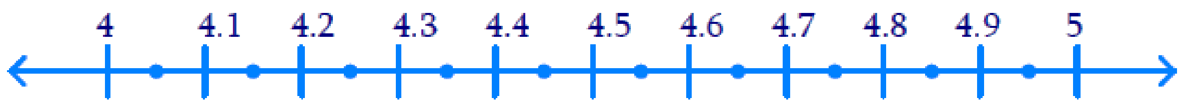
Q2. Visualize $4.\overline{26}$ on the number line, up to 4 decimal places.

Difficulty Level: Medium

Solution:

$4.\overline{26} = 4.2626\ldots$

i) 4.2 lies between 4 & 5. Observe the number line given below



ii) 4.26 lies between 4.2 & 4.3, shown below on the number line.

iii) 4.262 lies between 4.26 & 4.27 as shown below.

iv) 4.2626 lies between 4.262 & 4.263 as shown below

Hence by the successive magnification method required number is obtained in the number line.

Chapter 1 Number System

Exercise 1.5 (Page 24 of Grade 9 NCERT Textbook)

Q1. Classify the following numbers as rational or irrational:

(i) $2-\sqrt{5}$ (ii) $(3+\sqrt{23})-\sqrt{23}$ (iii) $\dfrac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\dfrac{1}{\sqrt{2}}$ (v) 2π

Difficulty Level: Easy

(i) $2-\sqrt{5}$

Solution:

The sum or difference of a rational number and an irrational number is always irrational.

Here 2 is a rational number and $\sqrt{5}$ is an irrational number. Hence $2-\sqrt{5}$ is an irrational number.

(ii) $(3+\sqrt{23})-\sqrt{23}$

Solution:

By simplifying we get only 3.

$3=\dfrac{3}{1}$, which is in the form of $\dfrac{p}{q}$,

Hence is $(3+\sqrt{23})-\sqrt{23}$ a rational number.

(iii) $\dfrac{2\sqrt{7}}{7\sqrt{7}}$

Solution:

$\dfrac{2\sqrt{7}}{7\sqrt{7}}=\dfrac{2}{7}$, which is in the form of $\dfrac{p}{q}$,

Hence $\dfrac{2\sqrt{7}}{7\sqrt{7}}$ is a rational number.

(iv) $\frac{1}{\sqrt{2}}$

Solution:

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{2}}{2} = \frac{1.414}{2}$$

$= 0.702$ is a non - terminating, non-recurring decimal which is irrational hence $\frac{1}{\sqrt{2}}$ is an irrational number.

(v) 2π

Solution:
$2\pi = 2 \times 3.1415...$

π is an irrational number whose value is non-terminating and non-recurring.

2 is a rational number.

Product of a non-zero rational number and irrational number is an irrational number.

Hence 2π is irrational.

Q2. Simplify each of the following expressions

(i) $(3+\sqrt{3})(2+\sqrt{2})$ (ii) $(3+\sqrt{3})(3-\sqrt{3})$

(iii) $(\sqrt{5}+\sqrt{2})^2$ (iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Difficulty Level: Medium

Solution:

(i) $(3+\sqrt{3})(2+\sqrt{2})$

By Distributive property

$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

Using the identity $(a + b)(a - b) = a^2 - b^2$

$= 9 - 3$

$= 6$

(iii) $(\sqrt{5}+\sqrt{2})^2$

Using (a + b) ^{2} = $a^2 + 2ab + b^2$

$$(\sqrt{5}+\sqrt{2})^2 = (\sqrt{5})^2 + 2\times\sqrt{5}\times\sqrt{2} + (\sqrt{2})^2$$
$$= (5+2\sqrt{10}+2)$$
$$= 7+2\sqrt{10}$$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Using (a + b) (a - b) = $a^2 - b^2$

$$(\sqrt{5})^2 - (\sqrt{2})^2$$
$$= 5-2$$
$$= 3$$

Q3. Recall, π is defined as the ratio of circumference (say c) to its diameter (say d). That is $\pi = \dfrac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Difficulty Level: Medium

Solution:

Writing π as $\dfrac{22}{7}$ is only an approximate value and so we can't conclude that it is in the form of a rational. In fact, the value of π is calculating as non-terminating, non-recurring decimal as $\pi = 3.14159$ Whereas

If we calculate the value of $\dfrac{22}{7}$ it gives 3.142857 and hence $\pi \neq \dfrac{22}{7}$

In conclusion π is an irrational number.

Q4. Represent $\sqrt{9.3}$ on the number line.

Difficulty Level: Medium

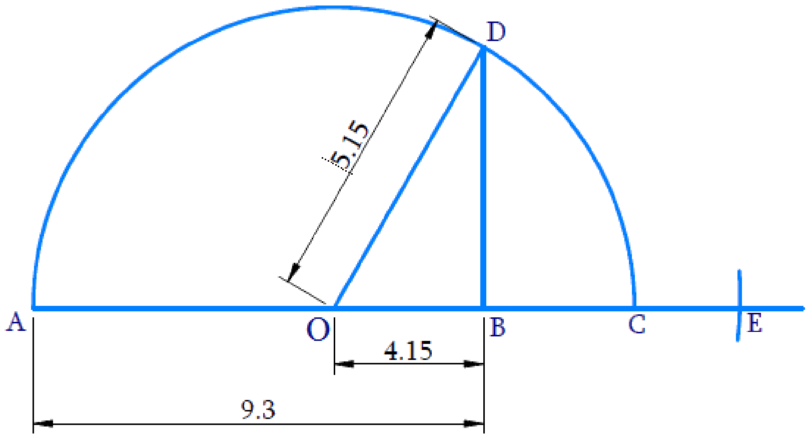
Solution:

Draw a line and take AB = 9.3 units on it.

From B measure a distance of 1 unit and mark C on the number line. Make the midpoint of AC as O.

With 'O' as center and OC as radius, draw a semicircle.

At B, draw a perpendicular to cut the semicircle at D, with B as center and BD as radius draw an arc to cut the number line at E. Taking B as the origin the distance $\text{BE} = \sqrt{9.3}$, and hence E represents $\sqrt{9.3}$.



Proof:

AB = 9.3, BC = 1

AC = AB + BC = 10.3

$$\text{OC} = \frac{\text{AC}}{2} = \frac{10.3}{2} = 5.15$$

OC = OD = 5.15

OB = OC – BC = 5.15 -1 = 4.15

In right angled Δ OBD,

$$\begin{aligned} BD^2 &= OD^2 - OB^2 \\ &= (5.15)^2 - (4.15)^2 \\ &= (5.15+4.15)(5.15-4.15) \qquad \textit{Using } a^2 - b^2 = (a+b)(a-b) \\ &= 9.3\times 1 \\ &= 9.3 \end{aligned}$$

$$BD = \sqrt{9.3} = BE$$

Q5. Rationalize the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$ (iv) $\frac{1}{\sqrt{7}-2}$

Difficulty Level:

Reasoning:

Solution:

(i) $\frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

(Dividing and multiplying by $\sqrt{7}$)

$= \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

Dividing and multiplying by $\sqrt{7}+\sqrt{6}$, we get

$$\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}-\sqrt{6}}$$

$$= \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2} \qquad \left[\text{Using } (a+b)(a-b) = a^2-b^2\right]$$

$$= \frac{\sqrt{7}+\sqrt{6}}{7-6}$$

$$= \sqrt{7}+\sqrt{6}$$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

Dividing and multiplying by $\sqrt{5}-\sqrt{2}$, we get

$$\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2} \qquad \left[\text{Using } (a+b)(a-b) = a^2-b^2\right]$$

$$= \frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{3}$$

(iv) $\frac{1}{\sqrt{7}-2}$

Dividing and multiplying by $\sqrt{7}+2$, we get

$$\frac{1}{\sqrt{7}-2} = \frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$$

$$= \frac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2} \qquad \left[\text{Using } (a+b)(a-b) = a^2-b^2\right]$$

$$= \frac{\sqrt{7}+2}{7-4}$$

$$= \frac{\sqrt{7}+2}{3}$$

Chapter 1 Number System

Exercise 1.6 (Page 26 of Grade 9 NCERT Textbook)

Q1. Find: (i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$

Difficulty Level: Easy

Solution:

(i) $64^{\frac{1}{2}}$

$$64^{\frac{1}{2}} = (8^2)^{\frac{1}{2}}$$

$$= 8^{2\times\frac{1}{2}} \qquad \left[\text{Using } (a^p)^q = a^{pq}\right]$$

$$= 8^1 \qquad \left[\text{Where } a > 0,\ p \text{ and } q \text{ are rational numbers.}\right]$$

$$= 8$$

(ii) $32^{\frac{1}{5}}$

$$32^{\frac{1}{5}} = \left(2^5\right)^{\frac{1}{5}} \qquad \left[\text{using } (a^p)^q = a^{pq}\right]$$

$$= (2)^{5\times\frac{1}{5}}$$

$$= 2^1 = 2$$

(iii) $125^{\frac{1}{3}}$

$$125^{\frac{1}{3}} = \left(5^3\right)^{\frac{1}{3}} \qquad \left[\text{using } (a^p)^q = a^{pq}\right]$$

$$= (5)^{3\times\frac{1}{3}}$$

$$= 5^1$$

$$= 5$$

Q2. Find: (i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$ (iv) $125^{\frac{-1}{3}}$

Difficulty Level: Easy

Solution:

(i) $9^{\frac{3}{2}}$

$$9^{\frac{3}{2}} = \left(3^2\right)^{\frac{3}{2}} \qquad \left[\text{using } (a^p)^q = a^{pq}\right]$$

$$= (3)^{2\times\frac{3}{2}}$$

$$= 3^3$$

$$= 27$$

(ii) $32^{\frac{2}{5}}$

$$32^{\frac{2}{5}} = \left(2^5\right)^{\frac{2}{5}} \qquad \left[\text{using } (a^p)^q = a^{pq}\right]$$

$$= (2)^{5\times\frac{2}{5}}$$

$$= 2^2$$

$$= 4$$

(iii) $16^{\frac{3}{4}}$

$$16^{\frac{3}{4}} = \left(2^4\right)^{\frac{3}{4}} \qquad \left[\text{using } (a^p)^q = a^{pq}\right]$$

$$= (2)^{4\times\frac{3}{4}}$$

$$= 2^3$$

$$= 8$$

(iv) $125^{\frac{-1}{3}}$

$$125^{\frac{-1}{3}} = \left(5^3\right)^{\frac{-1}{3}}$$

$$= (5)^{3\times\frac{-1}{3}} \qquad \left[\text{using } \left(a^p\right)^q = a^{pq}\right]$$

$$= 5^{-1}$$

$$= \frac{1}{5}$$

Q3. Simplify:

(i) $2^{\frac{2}{3}}.2^{\frac{1}{5}}$ (ii) $\left(\frac{1}{3^3}\right)^7$ (iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ (iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

Difficulty Level: Medium

Solution:

(i) $2^{\frac{2}{3}}.2^{\frac{1}{5}}$

$$2^{\frac{2}{3}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} \qquad \because a^p.a^q = a^{p+q}\left(\begin{array}{l}\text{For } a>0,\ p \text{ and } q \text{ are}\\ \text{rational numbers.}\end{array}\right)$$

$$= 2^{\frac{10+3}{15}}$$

$$= 2^{\frac{13}{15}}$$

(ii) $\left(\frac{1}{3^3}\right)^7 = \frac{1^7}{\left(3^3\right)^7}$

$$\left(\frac{1}{3^3}\right)^7 = \frac{1^7}{\left(3^3\right)^7}$$

$$= \frac{1}{3^{21}}$$

$$= 3^{-21}$$

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

$$\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}}$$

$$= 11^{\frac{2-1}{4}}$$

$$= 11^{\frac{1}{4}}$$

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}}$

$$7^{\frac{1}{2}}.8^{\frac{1}{2}} = (7\times 8)^{\frac{1}{2}}$$

$$= (56)^{\frac{1}{2}}$$