Chapter 10: Circles

Exercise10.1 (Page 171 of Grade 9 NCERT Textbook)

Q1. Fill in the blanks:

Difficulty level: Easy

(i) The center of the circle lies in interior of the circle. (exterior / interior)

Reasoning:

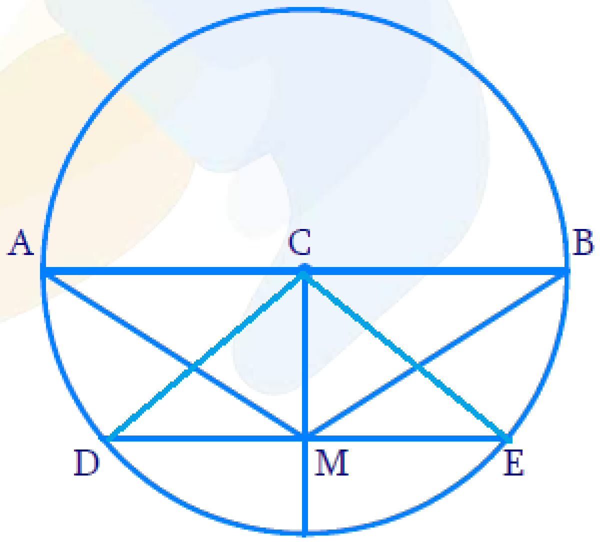
The collection of all points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle. The fixed point is the center of the circle.

(ii) A point, whose distance from the center of circle is greater than its radius lies in exterior of the circle. (exterior / interior)

Reasoning:

The collection of all points in a plane, which are at a fixed distance from a fixed point in the plane is called a circle. The fixed point is the center of the circle. Fixed distance is the radius of the circle. Any point outside the circle will have a greater distance compared to the radius.

(iii) The longest chord of the circle is a diameter of the circle.



Reasoning:

Let us check by drawing a random chord DE and diameter AB in the circle.

AC = CD = CE = BC = radius

AB = 2$\times$ radius.

In ΔDCE, DE < DC+CE (sum of two sides should be greater than the third side)

$$DE < r+r$$
$$DE < 2r$$
$$DE < AB$$
$$DE < \text{diameter}$$

Thus, we know that any chord drawn randomly (without passing through the centre) will be shorter than the diameter. Thus, Diameter is the longest chord in the circle.

(iv) An arc is a semicircle when its ends are the ends of a diameter.

Reasoning:

We know that diameter is the longest chord in the circle. And diameter divides the circle into 2 equal halves or arcs. When two arcs are equal, each is a semicircle.

(v) Segment of a circle is the region between an arc and chord of the circle.

Reasoning:

The region between a chord and either of its arcs is called a segment of the circular region or simply a segment of the circle.

(vi) A circle divides the plane, on which it lies, in three parts.

Reasoning:

A circle divides the plane on which it lies into three parts. They are: (i) inside the circle, which is also called the interior of the circle; (ii) the circle and (iii) outside the circle, which is also called the exterior of the circle.

Q2. Write True or False: Give reasons for your answers.

Difficulty level: Easy

(i) Line segment joining the center to any point on the circle is the radius of the circle.

Solution:

True

Reasoning:

The collection of all points in a plane, which are at a fixed distance from a fixed point in the plane is called a circle. The fixed point is the center of the circle. Fixed distance is the radius of the circle.

(ii) A circle has only finite number of equal chords.

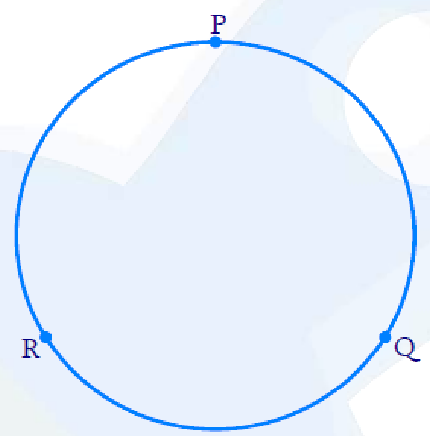
Solution:
False

Reasoning:
There are infinite points on the circle and hence infinite chords can be drawn between these points.

(iii) If a circle is divided into three equal arcs, each is a major arc.

Solution:
False

Reasoning:



If PQ is a minor arc, then QRP is a major arc and it should be greater than semicircular arc. If there are three arcs none of it can be a major arc.

(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

Solution:
True

Reasoning:
Draw a chord that passes through the centre of the circle. We can see that this chord is twice the length of the radius of the circle. This is called the diameter of the circle.

(v) Sector is the region between the chord and its corresponding arc.

Solution:
False

Reasoning:

The region between an arc and the two radii, joining the centre to the end points of the arc is called a sector.

(vi) A circle is a plane figure.

Solution:

True

Reasoning:

A circle is a 2-dimensional figure. So, Circle is a plane figure.

Chapter 10: Circles

Exercise10.2 (Page 173 of Grade 9 NCERT Textbook)

Q1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centers.

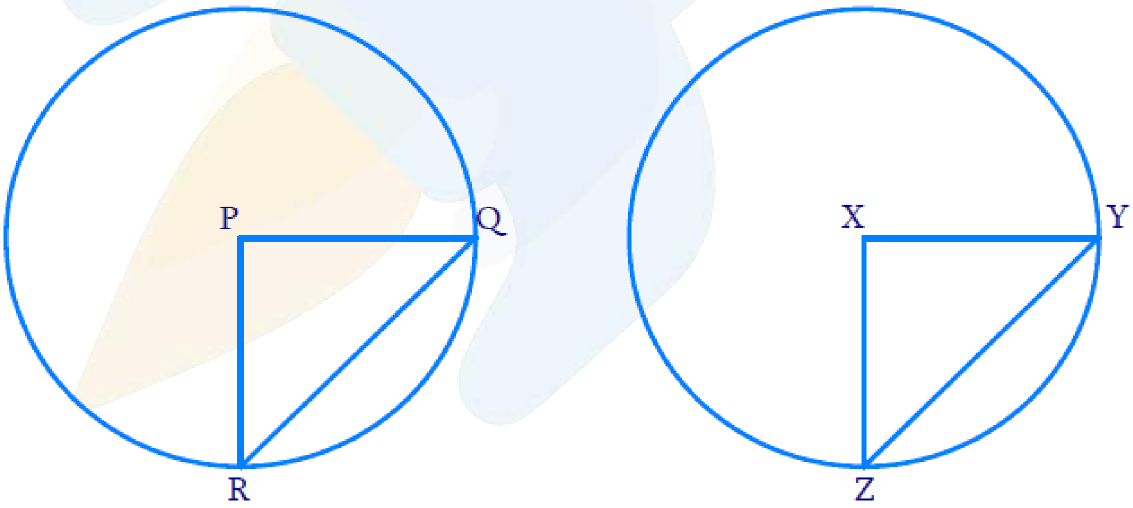
Difficulty level:
Easy

Known/given:
Two circles are congruent if they have same radii.

To prove:
Equal chords of congruent circles subtend equal angles at their centers.

Reasoning:
Using chords are equal and the fact that circles are congruent, we prove the statement using Side-Side-Side (SSS criteria) and Corresponding parts of congruent triangles (CPCT).

Solution:



Let QR and YZ be the equal chords of 2 congruent circles.

$QR = YZ$

We need to prove that they subtend equal angles at centre. i.e. $\angle QPR = \angle YXZ$

We know that the radii of both the circles are equal. So we get:

$PR = PQ = XZ = XY$

Consider the 2 triangles ΔPQR and ΔXYZ.

PQ = XY (Radii are equal)
PR = XZ (Radii are equal)
QR = YZ (Chords are equal)

By SSS criteria, ΔPQR is congruent to ΔXYZ.

So, by CPCT (Corresponding parts of congruent triangles) we get ∠QPR =∠YXZ.

Hence proved that equal chords of congruent circles subtend equal angles at their centers.

Q2. Prove that if chords of congruent circles subtend equal angles at their centers, then the chords are equal.

Difficulty level:

Easy

Known/given:

Two circles are congruent, and their chords subtend equal angles at their centers.

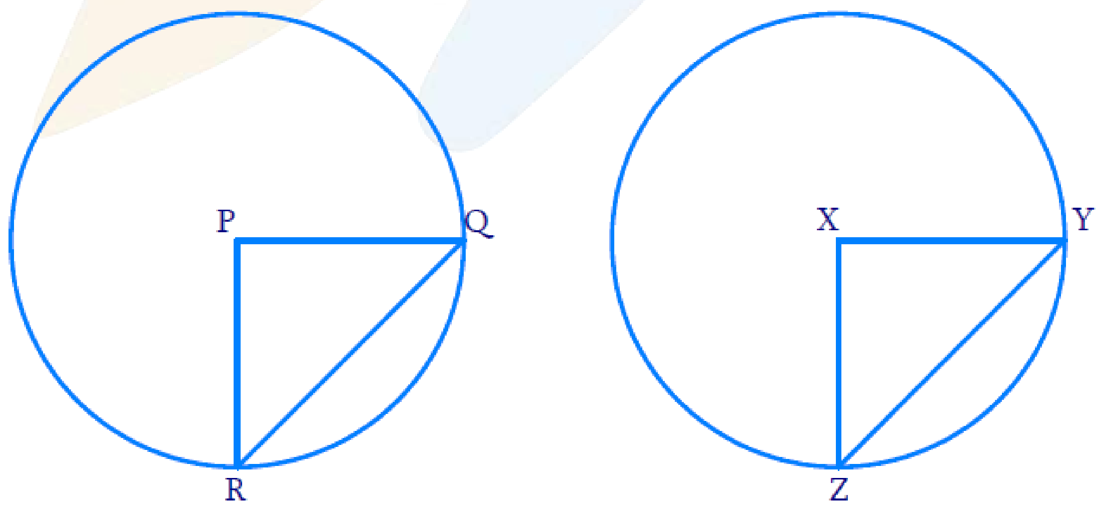
To prove:

The chords are equal.

Reasoning:

Using equal angles at the centers and the fact that circles are congruent, we prove the statement using Side-Angle-Side (SAS criteria) and Corresponding parts of congruent triangles (CPCT).

Solution:



Draw chords QR and YZ in 2 congruent circles respectively. Join the radii PR, PQ and XY, XZ respectively.

Given that chords subtend equal angles at center. So

$$\angle QPR = \angle YXZ.$$

We need to prove that chords are equal. i.e QR = YZ

Since the circles are congruent, their radii will be equal.

$$PR = PQ = XZ = XY$$

Consider the 2 triangles ΔPQR and ΔXYZ.

PQ = XY	(Radii are equal)
∠QPR = ∠YXZ	(Chords subtend equal angles at center)
PR = XZ	(Radii are equal)

By SAS criteria ΔPQR is congruent to ΔXYZ.

So, by CPCT (Corresponding parts of congruent triangles) QR = YZ

Hence proved if chords of congruent circles subtend equal angles at their center then the chords are equal.

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Chapter 10: Circles

Exercise10.3 (Page 176 of Grade 9 NCERT Book)

Q1. Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

Difficulty level:
Easy

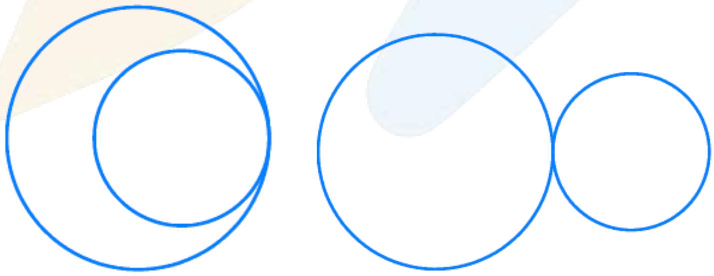
Known/given:
Pair of circles.

Unknown:
Number of points does each pair have in common and the maximum number of common points.

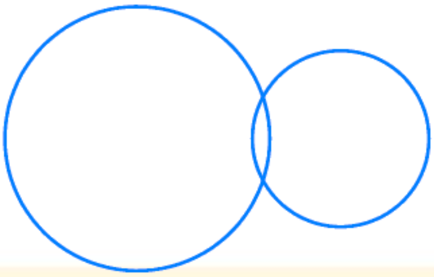
Reasoning:
We can draw number of possible pairs and can observe common points in each case.

Solution:
In this there are no common points.

In this there is only one common point.



In this there are two common points.



In a situation where two congruent circles are superimposed on each other, it can be understood to as if we are drawing the same circle two times.

It is therefore concluded, there can be a maximum of two points for different pairs of circles.

Q2. Suppose you are given a circle. Give a construction to find its center

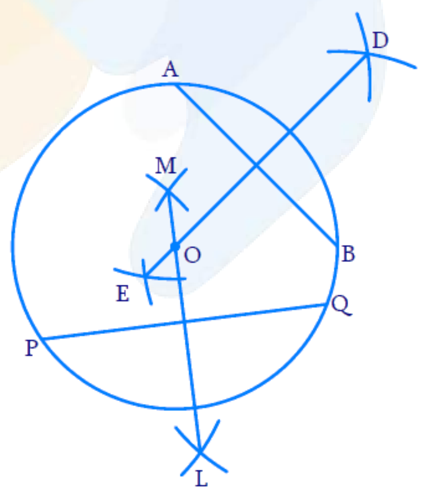
Difficulty level:
Medium

Known/given:
A circle.

Unknown:
Centre of the circle.

Reasoning:
By Theorem 10.3 (The perpendicular from the center of a circle to a chord bisects the chord.) and by Theorem 10.4 (The line drawn through the center of a circle to bisect a chord is perpendicular to the chord), we see that if a perpendicular bisector is drawn for a chord, it passes through the center. As it is not possible to find the center with the help of only one chord, two chords can be drawn. The intersection point of two perpendicular bisectors is the center of the circle. Because the center of the circle should lie on both the perpendicular bisectors of the two chords.

Solution:



Construction:

Step 1: Draw a circle with convenient radius.

Step 2: Draw 2 chords AB and PQ of any length.

Step 3: With A as center and radii as more than half the length of AB, draw two arcs on opposite sides of chord AB. With the same radius and with B as center, draw two arcs cutting the former arcs. Join the line. Now DE is the perpendicular bisector of AB. Centre lies on DE.

Step 4: With P as center and radii as more than half the length of PQ, draw two arcs on opposite sides of chord PQ. With the same radius and with Q as center, draw two arcs cutting the former arcs. Join the line. Now LM is the perpendicular bisector of PQ. Centre lies on LM also.

Step 5: As the center of the circle should lie both on DE and LM, it is obvious that the intersection points of DE and LM is the center. Mark the intersection points as O.

Step 6: O is the required centre of the circle.

Q3. If two circles intersect at two points, prove that their centers lie on the perpendicular bisector of the common chord.

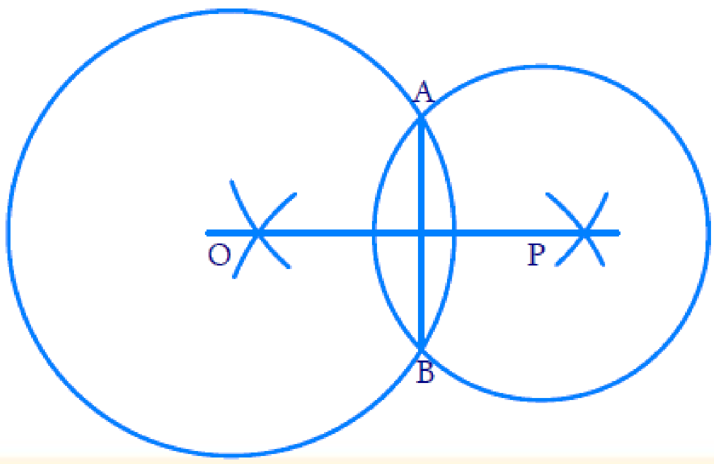
Difficulty level:
Medium

Known/given:
Two circles intersecting at two points.

To prove:
Their centers lie on the perpendicular bisector of the common chord.

Reasoning:
By theorem 10.4, it is known that perpendicular bisector of a chord passes through its center. Since the chord is common to both the circles, the center should lie on the perpendicular bisector of the chord.

Solution:



Let the two circles intersect at point A and B. Join AB. AB is the common chord.
By theorem 10.4, it is known that perpendicular bisector of a chord passes through its center.

As the chord AB is common to both circles, the perpendicular bisector of the common chord should pass through their centers. So, center lies on the perpendicular bisector of the common chords.

Chapter 10: Circles

Exercise10.4 (Page 179 of Grade 9 NCERT Book)

Q1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centers is 4 cm. Find the length of the common chord.

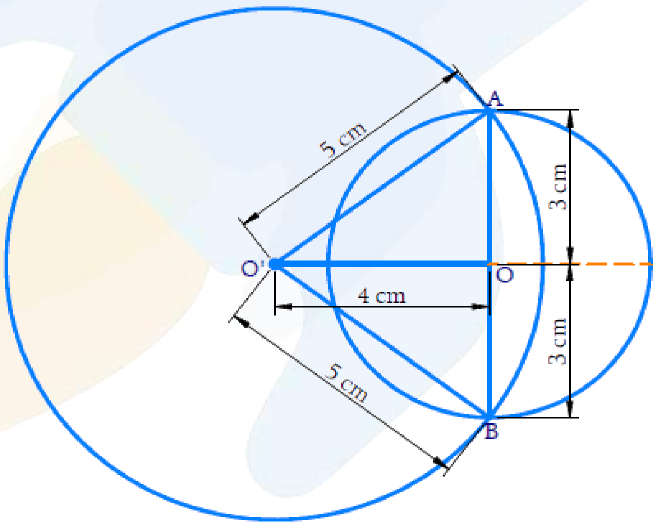
Difficulty level:
Medium

Known/given:
Radii of two circles and distance between the centers of the circles.

Unknown:
Length of common chord.

Reasoning:
Perpendicular bisector of the common chord passes through the centers of both the circles.

Solution:



Given that the circles intersect at 2 points, so we can draw the above figure. Let AB be the common chord. Let O and O' be the centers of the circles respectively.

O'A = 5 cm, OA = 3 cm, OO' = 4 cm.
Since the radius of the bigger circle is more than the distance between the 2 centers, we can say that the center of smaller circle lies inside the bigger circle itself.

OO' is the perpendicular bisector of AB.

So, OA = OB = 3 cm

AB = 3+3 = 6 cm

Length of the common chord is 6 cm.

It is also evident that common chord is the diameter of the smaller circle.

Q2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

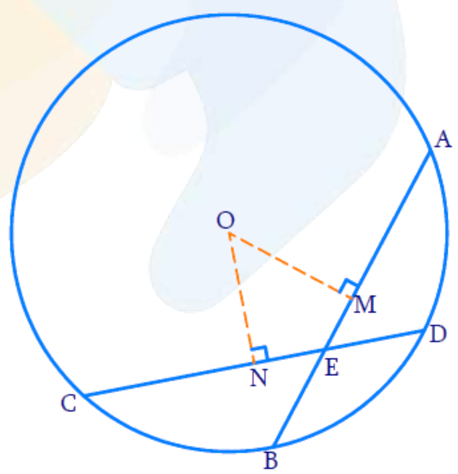
Difficulty level:
Medium

Known/given:
Two equal chords of a circle intersect at a point within the circle.

To prove:
Proof of corresponding segments of equal chords are equal.

Reasoning:
Equal chords are equidistant from the center. Using this and Right-Hypotenuse-Side (RHS) criteria and Corresponding parts of congruent triangles (CPCT) we prove the statement.

Solution:



Let AB and CD be the 2 equal chords. AB = CD.

Let the chords intersect at point E. Join OE.
To prove AE = CE and BE = DE.
Draw perpendiculars from the center to the chords. Perpendicular bisects the chord AB at M and CD at N. AM = MB = CN = DN ……(1)

In ΔOME and ΔONE

$\angle M = \angle N = 90^0$

OE = OE

OM = ON (Equal chords are equidistant from the centre.)

By RHS criteria, ΔOME and ΔONE are congruent.

So by CPCT, ME = NE ….. (2)

We know that: CE = CN + NE and AE = AM + ME

From (1) and (2), it is evident CE = AE

DE = CD - CE and BE = AB - AE

AB and CD are equal, CE and AE are equal. So, DE and BE are also equal.

It is proved corresponding segments of equal chords are equal.

Q3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the center makes equal angles with the chords.

Difficulty level:

Medium

Known/given:

Two equal chords of a circle intersect at a point within the circle.

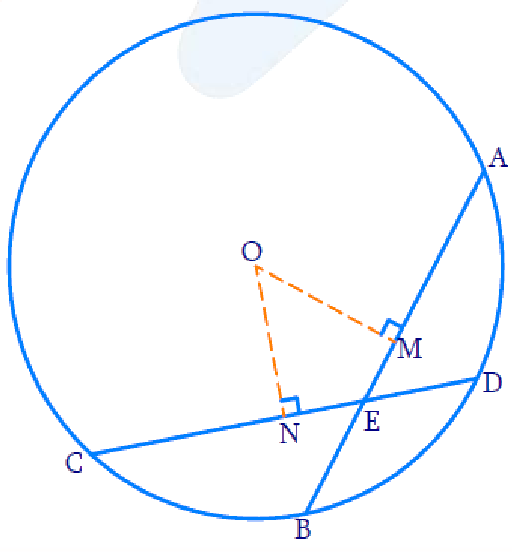
To prove:

Line joining the point of intersection to the center makes equal angles with the chords.

Reasoning:

Equal chords are equidistant from the center. Using this and Right-Hypotenuse-Side (RHS) criteria and Corresponding parts of congruent triangles (CPCT) we prove the statement.

Solution:



Let AB and CD be the 2 equal chords. AB = CD.
Let the chords intersect at point E. Join OE.

Draw perpendiculars from the centre to the chords. Perpendicular bisects the chord AB at M and CD at N.

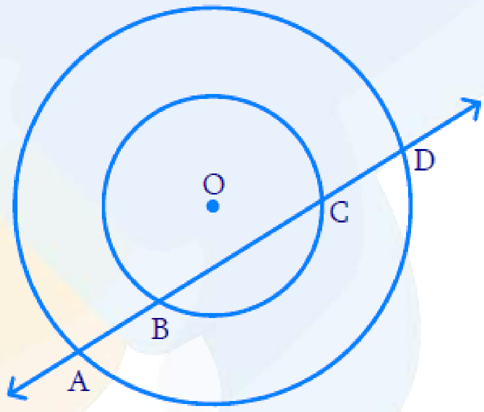
To prove $\angle OEN = \angle OEM$.
In ΔOME and ΔONE

$\angle M = \angle N = 90^0$
OE = OE
OM = ON (Equal chords are equidistant from the centre.)

By RHS criteria, ΔOME and ΔONE are congruent.
So, by CPCT, $\angle OEN = \angle OEM$

Hence proved that line joining the point of intersection of 2 equal chords to the center makes equal angles with the chords.

Q4. If a line intersects two concentric circles (circles with the same center) with center O at A, B, C and D, prove that AB = CD.



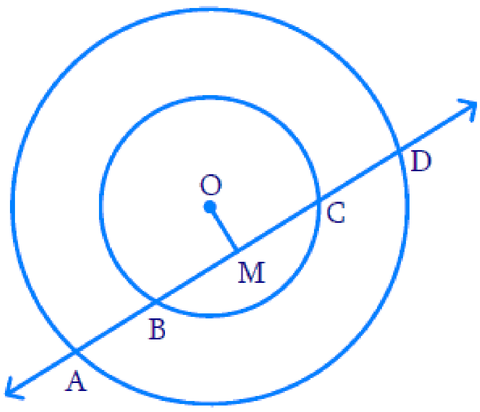
Difficulty level:
Easy

Known/given:
Two concentric circles with center O.

To prove:
AB = CD

Reasoning:
Perpendicular drawn from the center of the circle bisects the chord.

Solution:



Draw a perpendicular from the center of the circle OM to the line AD.
We can see that BC is the chord of the smaller circle and AD is the chord of the bigger circle.
We know that perpendicular drawn from the center of the circle bisects the chord.

$\therefore$ BM = MC ... (1)

And, AM = MD ... (2)

Subtracting (2) from (1), we obtain

AM − BM = MD − MC

$\therefore$ AB = CD

Q5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

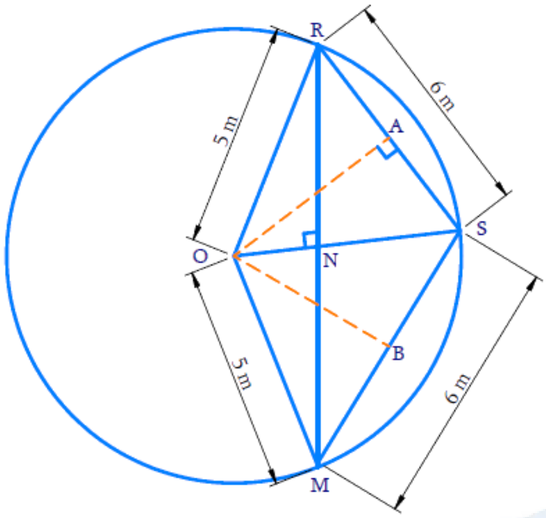
Difficulty level:
Medium

Known/given:
Three players are standing in a circle. Distance between two pairs is given. Radius of the circle is given.

Unknown:
Distance between Reshma and Mandip

Reasoning:
Perpendicular from center to either of the chord bisects the chord. Using this fact and by Pythagoras theorem we can find value of OA. After that we can find area of ΔORS by two ways like RS as base and OA as height or OS as base and RN as height. From this we will get the value of RN and double of value RN will give the distance between Reshma and Mandip.

Solution:



Let O be the center of the circle and R, M and S denote Reshma, Mandip and Salma respectively.

Draw a perpendicular OA to RS from O. Then RA = AS = 3 m.

Using Pythagoras theorem, we get, OA = 4 m.

We can see that quadrilateral ORSM takes that shape of a kite.
(Because OR = OM and RS = SM).
We know that the diagonals of a kite are perpendicular, and the main diagonal bisects the other diagonal.

$\angle$RNS will be 90° and RN = NM

Area of ΔORS = $\frac{1}{2}\times RS\times OA$ $= \frac{1}{2}\times 6\times 4$

= 12 ________________(1)

Also

Area of ΔORS = $\frac{1}{2}\times OS\times RN$

$= \frac{1}{2}\times 5\times RN$ _________(2)

From equation (1) and (2)

$$\frac{1}{2}\times 5\times RN = 12$$

$$RN = \frac{24}{5} = 4.8m$$

RM = 2 × RN = 2 × 4.8 = 9.6 m

Distance between Reshma and Salma is 9.6 m.

Q6. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. Find the length of string of each phone.

Difficulty level:

Medium

Known/given:

Three boys are on boundary of a circular park. Distance between them is equal. Radius of circular park is given.

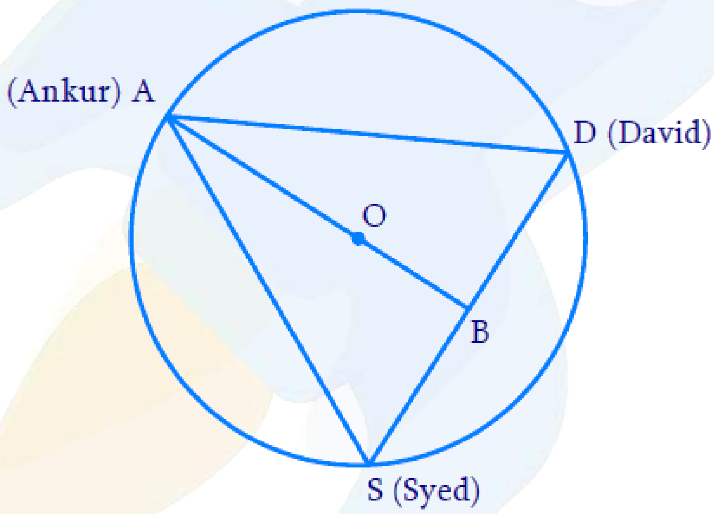
Unknown:

Length of string of each phone.

Reasoning:

Centre and Centroid are the same for an equilateral triangle and it divides the median in the ratio 2:1. The median is also the perpendicular bisector for the opposite side.

Solution:



Let A, D, S denote the positions of Ankur, David and Syed respectively.

ΔADS is an equilateral triangle since all the 3 boys are on equidistant from one another.

Let B denote the mid-point of DS and hence AB is the median and perpendicular bisector of DS. Hence ΔABS is a right-angled triangle with $\angle ABS = 90^0$.

O (centroid) divides the line AB in the ratio 2:1. So OA : OB = 2 : 1.

$$\frac{OA}{OB} = \frac{2}{1}$$

Since OA = 20 ~~then~~ –

OB = 10m

AB = OA + OB = 20 + 10 = 30 m ….(1)

Let the side of equilateral triangle ΔADS be $2x$.

$$AD = DS = SA = 2x \;\ldots.\; (2)$$

Since B is the mid-point of DS, we get BS = BD = x …. (3)

Applying Pythagoras theorem to ΔABS, we get:

$$AD^2 = AB^2 + BD^2$$
$$(2x)^2 = 30^2 + x^2$$
$$4x^2 = 900 + x^2$$
$$3x^2 = 900$$
$$x^2 = 300$$
$$x = 10\sqrt{3}$$
$$x = 17.32$$

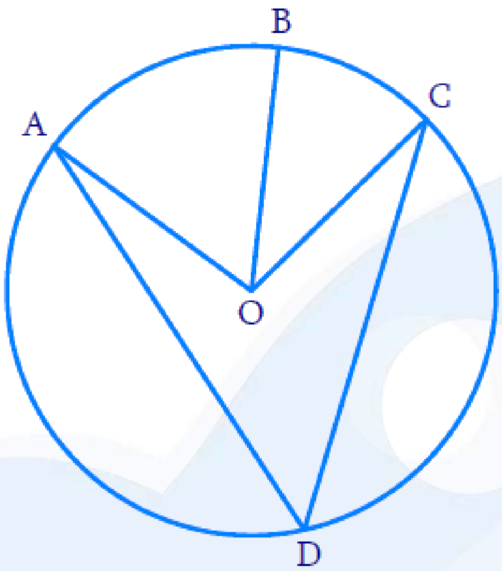
$$AD = DS = SA = 2x = 34.64 \text{ m}$$

Length of the string = Distance between them = AD or DS or SA = 34.64 m.

Chapter 10: Circles

Exercise10.5 (Page 184 of Grade 9 NCERT Book)

Q1. In the given figure A, B and C are three points on a circle with center O such that $\angle BOC = 30^o$ and $\angle AOB = 60^o$. If D is a point on the circle other than the arc ABC, find $\angle ADC$.



Difficulty level:
Easy

Known/given:
Two angles subtended by arcs at the center.

Unknown:
Value of $\angle ADC$

Reasoning:
The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

Solution:

$$\angle AOC = \angle AOB + \angle BOC$$
$$= 90°$$
$$\angle AOC = 2\angle ADC \quad \text{(By Theorem 10.8)}$$
$$\angle ADC = \frac{1}{2}\angle AOC$$
$$\angle ADC = \frac{1}{2}\times 90 = 45°$$
$$\therefore \angle ADC = 45°$$

Q2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Difficulty level:

Easy

Known/given:

Chord's length is equal to the radius.

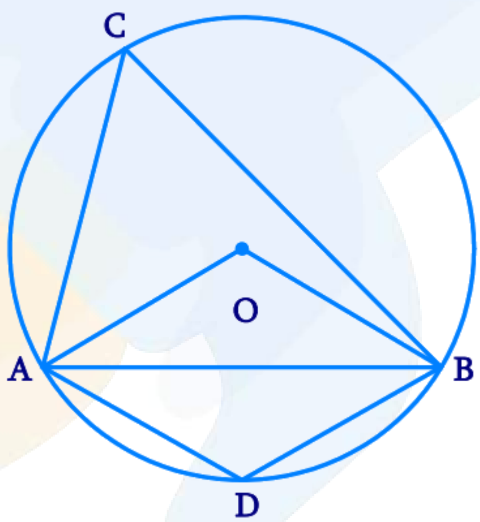
Unknown:

Angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Reasoning:

- The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.
- A quadrilateral ABCD is called cyclic if all the four vertices of it lie on a circle.
- The sum of either pair of opposite angles of a cyclic quadrilateral is 180^o.

Solution:



Draw a circle with any radius and center O. Let AO and BO be the 2 radii of the circle and let AB be the chord equal to the length of radius. Join them to form a triangle.

$$OA = OB = AB$$

Hence ΔABO becomes an equilateral triangle.

Draw 2 points C and D on the circle such that they lie on major arc and minor arc respectively.

Since ΔABO is an equilateral triangle, we get $\angle AOB = 60^0$.

For the arc AB, $\angle AOB = \angle 2\, ACB$ as we know that the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

$$\angle ACB = \frac{1}{2}\angle AOB$$
$$= \frac{1}{2}\times 60 = 30°$$

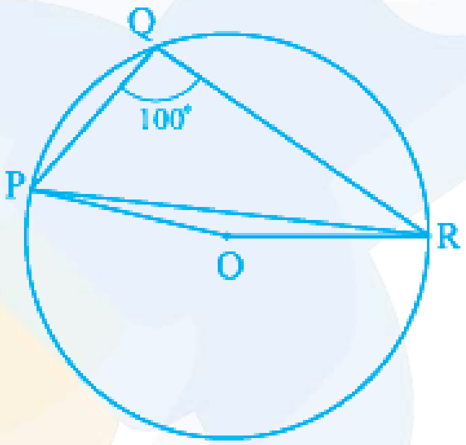
As you can notice the points A, B, C and D lie on the circle. Hence ABCD is a cyclic quadrilateral.

We know that, the sum of either pair of opposite angles of a cyclic quadrilateral is 180º. Therefore,

$$\angle ACB + \angle ADB = 180°$$
$$30 + \angle ADB = 180$$
$$\angle ADB = 150°$$

So, when the chord of a circle is equal to the radius of the circle, the angle subtended by the chord at a point on the minor arc is 150^0 and also at a point on the major arc is 30^0.

Q3. In the given figure, $\angle PQR = 100°$ where P, Q and R are points on a circle with center O. Find $\angle$OPR.



Difficulty level:

Easy

Known/given:

3 points on the circle and $\angle PQR = 100°$

Unknown:

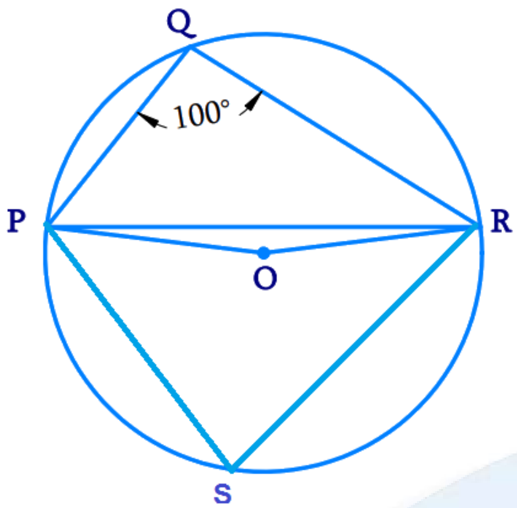
Value of $\angle$OPR

Reasoning:

- The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.
- A quadrilateral ABCD is called cyclic if all the four vertices of it lie on a circle.
- The sum of either pair of opposite angles of a cyclic quadrilateral is 180º.

Solution:

Mark any point on the major arc side (opposite side to point Q) as S.



Since all points P, Q, R, S lie on the circle, PQRS becomes a cyclic quadrilateral.

We know that, the sum of either pair of opposite angles of a cyclic quadrilateral is 180º.
Therefore,

$$\begin{aligned} \angle PQR + \angle PSR &= 180^\circ \\ 100^\circ + \angle PSR &= 180^\circ \\ \angle PSR &= 180^\circ - 100^\circ \\ &= 80^\circ \end{aligned}$$

We know that, the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.
Therefore,

$$\begin{aligned} \angle POR &= 2\angle PSR \\ &= 2 \times 80^\circ \\ &= 160^\circ \end{aligned}$$

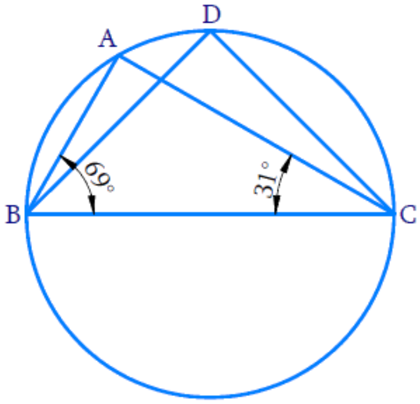
Consider the ΔOPR. It is an isosceles triangle as OP = OR = Radius of the circle.

$$\angle OPR = \angle ORP$$

Sum of all angles in a triangle is 180º.
Therefore,

$$\begin{aligned} \angle OPR + \angle POR + \angle ORP &= 180^\circ \\ \angle OPR + 160^\circ + \angle OPR &= 180^\circ \\ 2\angle ORP &= 180^\circ - 160^\circ \\ \angle OPR &= 10^\circ \end{aligned}$$

Q4. In the given figure, ∠ABC = 69º and ∠ACB = 31º, find ∠BDC.



Difficulty level:

Easy

Known/given:

Two angles in a triangle.

Unknown:

Value of ∠BDC

Reasoning:

- Sum of angles in a triangle is 180º.
- Angles in the same segment are equal.

Solution:

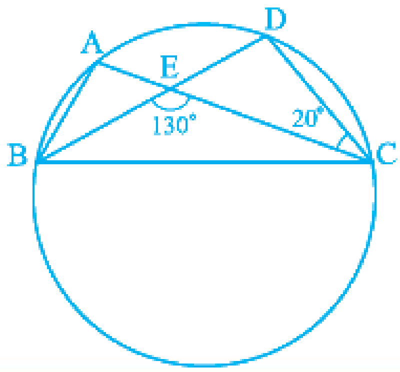
Consider the ΔABC, the sum of all angles will be 180º.

$$\angle ABC + \angle BAC + \angle ACB = 180^\circ$$
$$69^\circ + \angle BAC + 31^\circ = 180^\circ$$
$$\angle BAC = 180^\circ - 100^\circ$$
$$= 80^\circ$$

We know that, angles in the same segment of a circle are equal.

$$\angle BDC = \angle BAC = 80^\circ$$

Q5. In the given figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠BEC = 130º and ∠ECD = 20º. Find ∠BAC.



Difficulty level:
Easy

Known/given:
Two angles ∠BEC=130°and ∠ECD =20°.

Unknown:
Value of ∠BAC

Reasoning:
- Sum of angles in a triangle is 180°.
- Angles in the same segment are equal.

Solution:
Consider the straight-line BD. As the line AC intersects with the line BD, then the sum of two adjacent angles so formed is 180°.
Therefore,

$$\angle BEC + \angle DEC = 180^\circ$$
$$130^\circ + \angle DECC = 180^\circ$$
$$\angle DEC = 180^\circ - 130^\circ$$
$$= 50^\circ$$

Consider the ΔDEC, the sum of all angles will be 180°.

$$\angle DEC + \angle EDC + \angle ECD = 180^\circ$$
$$50^\circ + \angle EDC + 20^\circ = 180^\circ$$
$$\angle EDC = 180^\circ - 70^\circ$$
$$= 110^\circ$$
$$\therefore \angle BDC = \angle EDC = 110^\circ$$

We know that, angles in the same segment of a circle are equal.

$$\therefore \angle BAC = \angle BDC = 110^\circ$$

Q6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠DBC = 70°, ∠BAC = 30° find ∠BCD. Further if AB=BC, find ∠ECD.

Difficulty level:
Medium

Known/ given:
ABCD is cyclic quadrilateral. ∠DBC =70° and ∠BAC =30°. AB = BC.

Unknown:

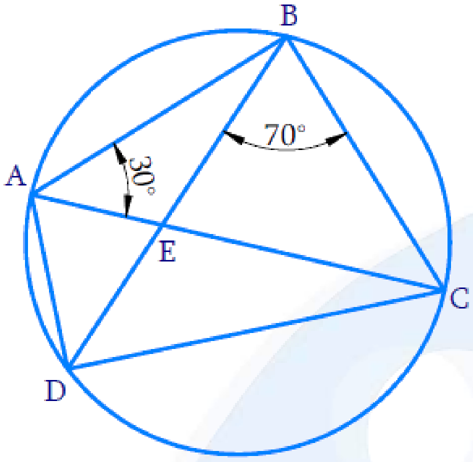
Value of ∠BCD and ∠ECD

Reasoning:

- A quadrilateral ABCD is called cyclic if all the four vertices of it lie on a circle.
- The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.
- Sum of angles in a triangle is 180°.
- Angles in the same segment are equal.

Solution:

Based on the data given, draw the figure.



In the triangles, ABD and BCD, ∠CAD = ∠CBD = 70°. (Angles in the same segment)

So,

$$\angle BAD = 30^\circ + 70^\circ$$
$$= 100^\circ$$

Since ABCD is a cyclic quadrilateral, the sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

$$\angle BAD + \angle BCD = 180^\circ$$
$$\angle BCD = 180^\circ - 100^\circ \ \angle BAD + \angle BCD = 180^\circ$$
$$= 80^\circ$$

Also given AB = BC.

So, $\angle BCA = \angle BAC = 30^\circ$ (Base angles of isosceles triangle are equal)

$$\angle ECD = \angle BCD - \angle BCA$$
$$= 80^\circ - 30^\circ$$
$$= 50^\circ$$

Q7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Difficulty level:

Medium

Known/given:

Diagonals of a cyclic quadrilateral are diameters of a circle passing through the vertices.

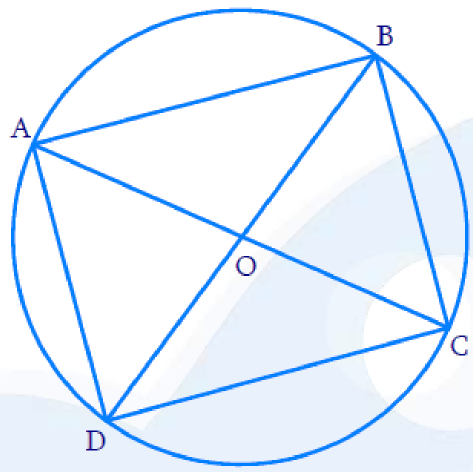
To prove:

Lines joining the vertices is a rectangle.

Reasoning:

- The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.
- The sum of either pair of opposite angles of a cyclic quadrilateral is 180º.
- Diameter is a chord.

Solution:



Let DB be the diameter of the circle which is also a chord.

Then ∠BOD = 180º

We know that, the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\therefore \quad \angle BAD = \frac{1}{2} \times \angle BOD$$

$$= 90^\circ$$

Similarly, ∠BCD = 90º

Now considering AC as the diameter of the circle, we get ∠AOC = 180º

We know that, the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

$$\angle \text{ABC} = \frac{1}{2} \times \angle \text{AOC}$$

$$= 90^\circ$$

Similarly, $\angle \text{ADC} = 90^\circ$

As you can see, all the angles at the corners are 90º we can say that the shape joining the vertices is a rectangle.

This problem can also be solved by using the property of cyclic quadrilaterals.

Q8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

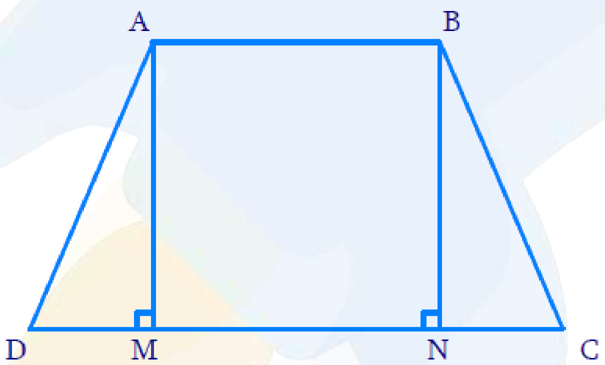
Difficulty level:
Hard

Known/given:
Non-parallel sides of trapezium are equal.

To prove:
The trapezium is cyclic.

Reasoning:
If the sum of a pair of opposite angles of a quadrilateral is 180º, the quadrilateral is cyclic. Using Right-Hypotenuse-Side (RHS) criteria and Corresponding parts of congruent triangles (CPCT) we prove the statement.

Solution:
Draw a trapezium ABCD with AB || CD.
AD and BC are the non-parallel sides which are equal. AD = BC.
Draw AM ⊥ CD and BN ⊥ CD.



Consider ΔAMD and ΔBNC

$$AD = BC \text{ (Given)}$$
$$\angle AMD = \angle BNC \text{ } (90^\circ)$$
$$AM = BN \text{ (Perpendicular distance between two parallel lines is same)}$$

By RHS congruence, $\Delta AMD \cong \Delta BNC$.
Using CPCT,

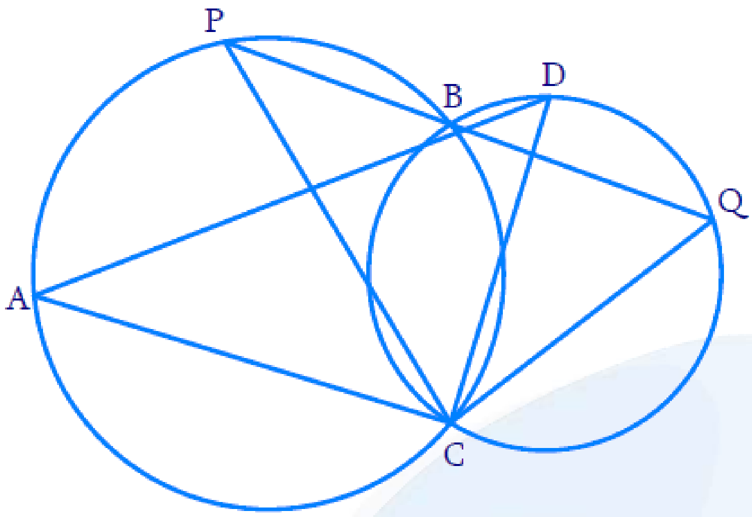
$$\angle ADC = \angle BCD \quad \ldots\ldots(1)$$

$\angle BAD$ and $\angle ADC$ are on the same side of transversal AD.

$$\angle BAD + \angle ADC = 180^\circ$$
$$\angle BAD + \angle BCD = 180^\circ \quad \ldots\ldots\ldots\ldots [\text{Using } (1)]$$

This equation proves that the opposite angles are supplementary. Hence, ABCD is a cyclic quadrilateral.

Q9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D, P and Q respectively. Prove that ∠ACP = ∠QCD



Difficulty level:

Hard

Known/given:

Two circles intersect at two points

To prove:

∠ACP = ∠QCD

Reasoning:

∠ACP and ∠ABP lie on the same segment. Similarly, ∠DCQ and ∠DBQ lie on the same segment. Angles in the same segment of a circle are equal.

Solution:

We know that, angles in the same segment of a circle are equal.

So, we get ∠ACP = ∠ABP and ∠QCD = ∠QBD

Also, ∠QBD = ∠ABP (Vertically opposite angles)

Therefore ∠ACP = ∠QCD

Q10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Difficulty level:

Medium

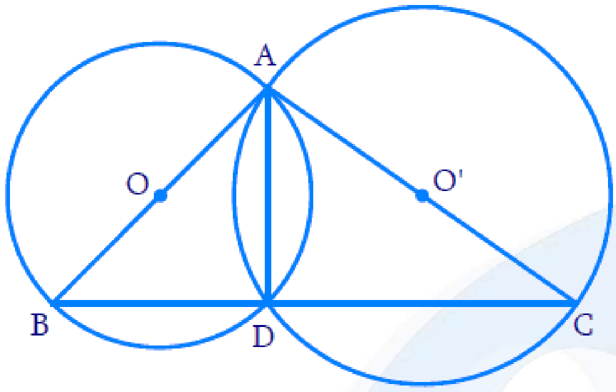
Known/given:

Two circles are drawn taking two sides of a triangle as diameters.

To prove:
That point of intersection of the 2 circles lie on the third side.

Reasoning:
Angle in a semicircle is a right angle. By using this fact, we can show that BDC is a line which will lead to the proof that point of intersection lie on the third side.

Solution:



Since angle in a semicircle is a right angle, we get:

$$\angle ADB = 90° \text{ and } \angle ADC = 90°$$
$$\angle ADB + \angle ADC = 90° + 90°$$
$$\Rightarrow \angle ADB + \angle ADC = 180°$$
$$\Rightarrow BDC \text{ is a straight line.}$$

$\therefore$ D lies on BC

Hence, point of intersection of circles lie on the third side BC.

Q11. ABC and ADC are two right triangles with common hypotenuse AC.
Prove that $\angle CAD = \angle CBD$

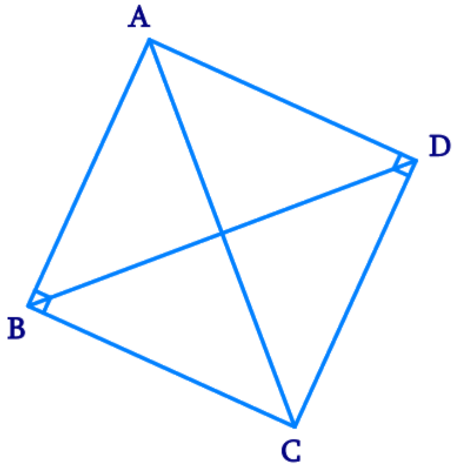
Difficulty level:
Hard

Known/given:
ABC and ADC are two right triangles with common hypotenuse AC.

To prove:
$\angle CAD = \angle CBD$

Reasoning:
Sum of all angles in a triangle is 180°.
If the sum of pair of opposite angles in a quadrilateral is 180° then it is cyclic quadrilateral
Angles in the same segment of a circle are equal.

Solution:



Consider ΔABC,

$$\angle ABC + \angle BCA + \angle CAB = 180^\circ \quad (\text{Angle sum property of a triangle})$$
$$90^\circ + \angle BCA + \angle CAB = 180^\circ$$
$$\angle BCA + \angle CAB = 90^\circ \text{ .... } (1)$$

Consider ΔADC,

$$\angle CDA + \angle ACD + \angle DAC = 180^\circ \quad (\text{Angle sum property of a triangle})$$
$$90^\circ + \angle ACD + \angle DAC = 180^\circ$$
$$\angle ACD + \angle DAC = 90^\circ \text{ .... } (2)$$

Adding Equations (1) and (2), we obtain

$$\angle BCA + \angle CAB + \angle ACD + \angle DAC = 180^\circ$$
$$(\angle BCA + \angle ACD) + (\angle CAB + \angle DAC) = 180^\circ$$
$$\angle BCD + \angle DAB = 180^\circ \text{ .... } (3)$$

However, it is given that

$$\angle B + \angle D = 90^\circ + 90^\circ = 180^\circ \text{ .... } (4)$$

From Equations (3) and (4), it can be observed that the sum of the measures of opposite angles of quadrilateral ABCD is 180°. Therefore, it is a cyclic quadrilateral.

Consider chord CD.

∠CAD = ∠CBD (Angles in the same segment)

Q12. Prove that cyclic parallelogram is a rectangle.

Difficulty level:

Hard

Known/given:

Cyclic quadrilateral is a parallelogram.

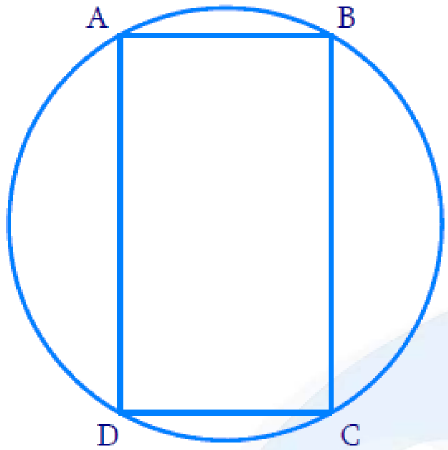
To prove:

The cyclic parallelogram is a rectangle.

Reasoning:

The sum of either pair of opposite angles of a cyclic quadrilateral is 180º. By using this fact, we can show each angle of a cyclic parallelogram as 90º which will prove the statement it is a rectangle.

Solution:



Let ABCD be the cyclic parallelogram.

We know that opposite angles of a parallelogram are equal.

$$\angle A = \angle C \text{ and } \angle B = \angle D \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

We know that the sum of either pair of opposite angles of a cyclic quadrilateral is 180º.

$$\angle A + \angle C = 180° \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Substituting (1) in (2),

$$\angle A + \angle C = 180°$$
$$\angle A + \angle A = 180°$$
$$2\angle A = 180°$$
$$\angle A = 90°$$

We know that if one of the interior angles of a parallelogram is 90°, all the other angles will also be equal to 90°.

Since all the angles in the parallelogram is 90°, we can say that parallelogram ABCD is a rectangle.

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Chapter 10: Circles

Exercise10.6 (Page 186 of Grade 9 NCERT Book)

Q1. Prove that the line of centers of two intersecting circles subtends equal angles at the two points of intersection.

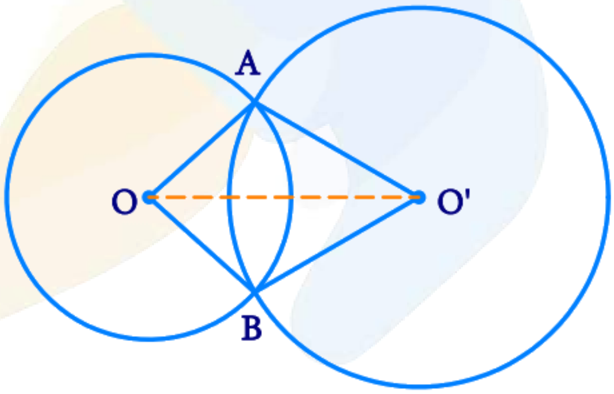
Difficulty level:
Medium

Known/given:
Two intersecting circles.

To prove:
The line of centers of two intersecting circles subtends equal angles at the two points of intersection.

Reasoning:
Using Side-Side-Side (SSS criteria) and Corresponding parts of congruent triangles (CPCT) we prove the statement.

Solution:
Draw 2 intersecting circles with centers O and O’ respectively. Join these 2 centers. Let the points of intersection be A and B.



We need to prove that ∠OAO'= ∠OBO'

Consider ΔOAO’ and ΔOBO’

OA = OB (Radii of circle with center O)
O’A = O’B (Radii of circle with center O’)
OO’= OO’ (Common)

Therefore, by SSS criteria, ΔOAO’ and ΔOBO’ are congruent to each other.
By CPCT, ∠OAO'= ∠OBO'
Hence it is proved that the line of centers of two intersecting circles subtends equal angles at the two points of intersection.

Q2. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its center. If the distance between AB and CD is 6 cm, find the radius of the circle.

Difficulty level:

Medium

Known/given:

Two parallel chords of lengths 5 cm and 11 cm and the distance between parallel chords is 6 cm.

Unknown:

Radius of the circle.

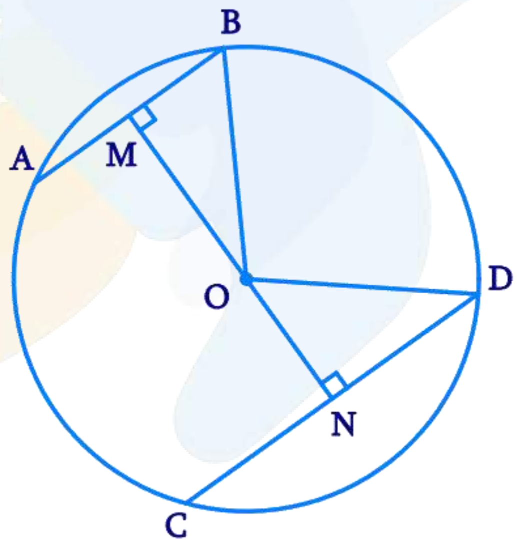
Reasoning:

The perpendicular drawn from the center of the circle to the chords, bisects it.

Pythagoras theorem states that $(Side1)^2 + (Side\,2)^2 = (Hypotenuse)^2$

Solution:

Draw 2 parallel chords AB and CD of lengths 5 cm and 11 cm. Let the center of the circle be O. Join one end of each chord to the center. Draw 2 perpendiculars OM and ON to AB and CD respectively which bisects the chords.



AB = 5 cm

CD = 11 cm

MB = 2.5 cm

ND = 5.5 cm

Let OM = x cm and ON = 6 – x cm

Consider ΔOMB

By Pythagoras theorem,

$$OM^2 + MB^2 = OB^2$$
$$x^2 + 2.5^2 = OB^2$$
$$x^2 + 6.25 = OB^2 \quad .... \quad (1)$$

Consider ΔOND

By Pythagoras theorem,

$$ON^2 + ND^2 = OD^2$$
$$(6-x)^2 + 5.5^2 = OD^2$$
$$36 + x^2 - 12x + 30.25 = OD^2$$
$$x^2 - 12x + 66.25 = OD^2 ....(2)$$

OB and OD are the radii of the circle. Therefore OB = OD.

Equating (1) and (2) we get,

$$x^2 + 6.25 = x^2 - 12x + 66.25$$
$$12x = 60$$
$$x = 5$$

Substituting the value of x in (1) or (2) we get the radius of circle = 5.59 cm.

Q3. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the center, what is the distance of the other chord from the center?

Difficulty level:

Medium

Known/given:

Two parallel chords of lengths 6 cm and 8 cm and distance of smaller chord from center is 4cm.

Unknown:

Radius of the circle and distance of one chord from center.

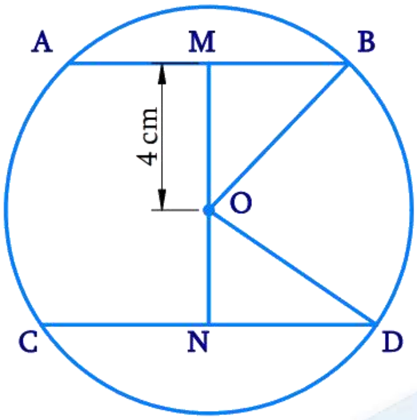
Reasoning:

The perpendicular drawn from the center of the circle to the chords, bisects it.

Pythagoras theorem states that $(Side1)^2 + (Side\,2)^2 = (Hypotenuse)^2$

Solution:

Draw 2 parallel chords AB and CD of lengths 6 cm and 8 cm. Let the center of the circle be O. Join one end of each chord to the center. Draw 2 perpendiculars OM and ON to AB and CD respectively which bisects the chords.



AB = 6 cm
CD = 8 cm
MB = 3 cm
ND = 4 cm

Given OM = 4 cm and let ON = x cm

Consider ΔOMB
By Pythagoras theorem,

$$OM^2 + MB^2 = OB^2$$
$$4^2 + 3^2 = OB^2$$
$$OB^2 = 25$$

OB = 5 cm

OB and OD are the radii of the circle. Therefore OD = OB = 5 cm.

Consider ΔOND
By Pythagoras theorem,

$$ON^2 + ND^2 = OD^2$$
$$x^2 + 4^2 = 5^2$$
$$x^2 = 25 - 16$$
$$x^2 = 9$$
$$x = 3$$

The distance of the chord CD from the center is 3 cm.

Q4. Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the center.

Difficulty level:

Hard

Known/given:

Point B lies outside the circle also AD and CE are equal chords.

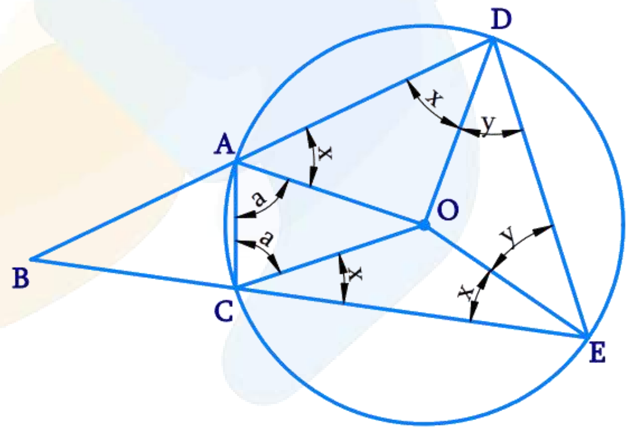
To prove:

To prove that ∠ABC is equal to half the difference of the angles subtended by the chords AC and DE at the center means $\angle ABC = \frac{1}{2}(\angle DOE - \angle AOC)$.

Reasoning:

- The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.
- A quadrilateral ACED is called cyclic if all the four vertices of it lie on a circle.
- The sum of either pair of opposite angles of a cyclic quadrilateral is 180º.
- Using Side-Side-Side (SSS criteria) and Corresponding parts of congruent triangles (CPCT) we prove the statement.

Solution:



Consider ΔAOD and ΔCOE,

OA = OC	(Radii of the circle)
OD = OE	(Radii of the circle)
AD = CE	(Given)

$\therefore \Delta AOD \cong \Delta COE$ (SSS Congruence Rule)

$\angle OAD = \angle OCE$ (By CPCT) ... (1)

$\angle ODA = \angle OEC$ (By CPCT) ... (2)

Also,

$$\angle OAD = \angle ODA \quad (\text{As } OA = OD) \ldots (3)$$

From Equations (1), (2), and (3), we obtain

$$\angle OAD = \angle OCE = \angle ODA = \angle OEC$$

Let $\angle OAD = \angle OCE = \angle ODA = \angle OEC = x$

In ΔOAC,

$$OA = OC$$

$$\therefore \angle OCA = \angle OAC \text{ (Let } a)$$

In ΔODE,

$$OD = OE$$

$$\angle OED = \angle ODE \text{ (Let } y)$$

ADEC is a cyclic quadrilateral.

$\therefore \angle CAD + \angle DEC = 180°$ (Opposite angles are supplementary)

$$x + a + x + y = 180°$$

$$2x + a + y = 180°$$

$$y = 180° - 2x - a \ldots (4)$$

However, $\angle DOE = 180° - 2y$

And, $\angle AOC = 180° - 2a$

$$\angle DOE - \angle AOC = 2a - 2y = 2a - 2\,(180° - 2x - a)$$

$$= 4a + 4x - 360° \ldots (5)$$

$$\angle BAC + \angle CAD = 180° \text{ (Linear pair)}$$

$\therefore \angle BAC = 180° - \angle CAD = 180° - (a + x)$

Similarly, $\angle ACB = 180° - (a + x)$

In ΔABC,

$$\angle ABC + \angle BAC + \angle ACB = 180° \text{ (Angle sum property of a triangle)}$$

$$\angle ABC = 180° - \angle BAC - \angle ACB$$

$$= 180° - (180° - a - x) - (180 - a - x)$$

$$= 2a + 2x - 180°$$

$$= \frac{1}{2}[4a + 4x - 360°]$$

Using Equation (5)

$$\angle ABC = \frac{1}{2}(\angle DOE - \angle AOC)$$

Hence it is proved that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Q5. Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.

Difficulty level:

Medium

Known/given:

Diameter of the circle is the length of one side of rhombus.

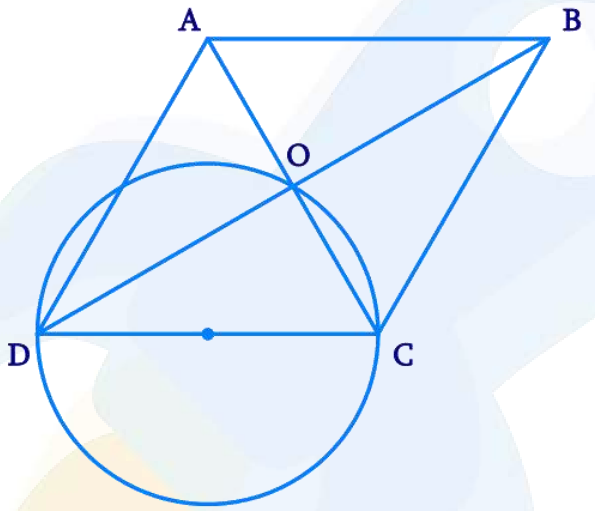
To prove:

The circle passes through the point of intersection of rhombus' diagonals.

Reasoning:

The diagonals of rhombus intersect at 90°.

Solution:



Let ABCD be a rhombus in which diagonals are intersecting at point O and a circle is drawn while taking side CD as its diameter. We know that a diameter subtends 90° on the arc.

∴ ∠COD = 90°

Also, in rhombus, the diagonals intersect each other at 90°.

$$\angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ$$

Clearly, point O has to lie on the circle.

Q6. ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.

Difficulty level:

Medium

Known/given:

ABCD is a parallelogram.

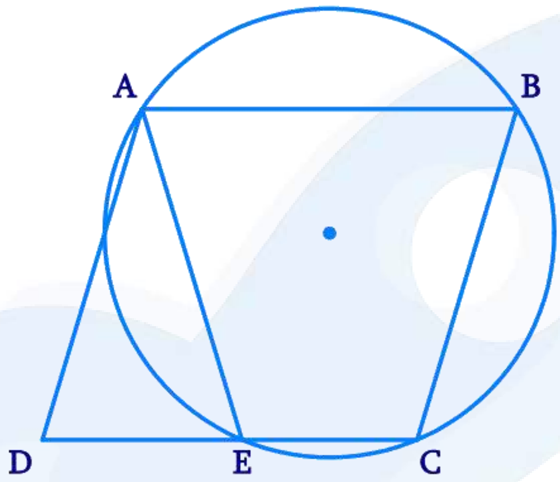
To prove:

AE = AD

Reasoning:

- A quadrilateral ABCD is called cyclic if all the four vertices of it lie on a circle.
- The sum of either pair of opposite angles of a cyclic quadrilateral is 180º.
- Opposite angles in a parallelogram are equal.

Solution:



We can see that ABCE is a cyclic quadrilateral.

We know that in a cyclic quadrilateral, the sum of the opposite angles is 180°.

$$\angle AEC + \angle CBA = 180$$
$$\angle AEC + \angle AED = 180 \quad \text{(Linear pair)}$$
$$\angle AED = \angle CBA \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

We know that in a parallelogram, opposite angles are equal.

$$\angle ADE = \angle CBA \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

From (1) and (2),

$$\angle AED = \angle ADE$$

AD = AE (sides opposite to equal Angles of a triangle are equal).

Q7. AC and BD are chords of a circle which bisect each other. Prove that
(i) AC and BD are diameters, (ii) ABCD is a rectangle.

Difficulty level:

Medium

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Known/given:

AC and BD are 2 chords of a circle which bisect each other.

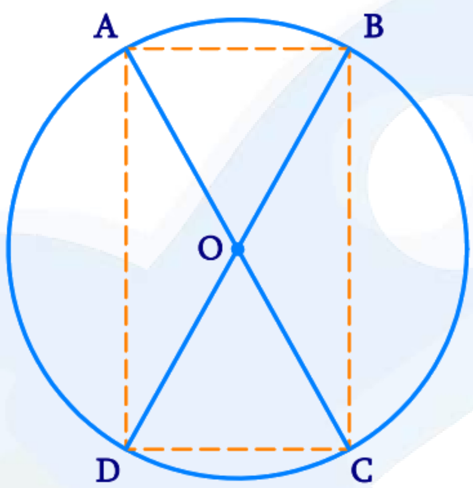
To prove:

AC and BD are diameters of the circle and ABCD is a rectangle.

Reasoning:

- A quadrilateral ABCD is called cyclic if all the four vertices of it lie on a circle.
- The sum of either pair of opposite angles of a cyclic quadrilateral is 180^o.
- Opposite angles in a parallelogram are equal.
- Using Side-Angle-Side (SAS criteria) and Corresponding parts of congruent triangles (CPCT) we prove the statement.

Solution:



Let AC and BD be 2 chords intersecting at O.

In ΔAOB and ΔCOD,

OA = OC (Given)

OB = OD (Given)

∠AOB = ∠COD (Vertically opposite angles)

∴ ΔAOB ≅ ΔCOD (SAS congruence rule)

AB = CD (By CPCT)

Similarly, it can be proved that ΔAOD ≅ ΔCOB

∴ AD = CB (By CPCT)

Since in quadrilateral ABCD, opposite sides are equal in length, ABCD is a parallelogram.

We know that opposite angles of a parallelogram are equal.

∴ ∠A = ∠C

However, ∠A + ∠C = 180° (ABCD is a cyclic quadrilateral)

∠A + ∠A = 180°

2∠A = 180°

∴ ∠A = 90°

As ABCD is a parallelogram and one of its interior angles is 90°, therefore, it is a rectangle.

∠A is the angle subtended by chord BD.

And as ∠A = 90°, therefore, BD should be the diameter of the circle. Similarly, AC is the diameter of the circle.

Q8. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are

$$90^\circ - \frac{1}{2}A,\ 90^\circ - \frac{1}{2}B, 90^\circ - \frac{1}{2}C.$$

Difficulty level:

Medium

Known/given:

Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively.

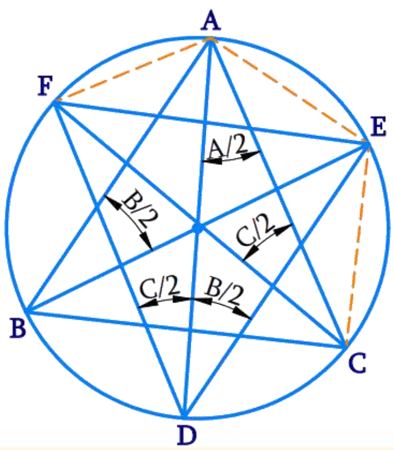
To prove:

The angles of the triangle DEF are $90^\circ - \frac{1}{2}A,\ 90^\circ - \frac{1}{2}B, 90^\circ - \frac{1}{2}C.$

Reasoning:

Angles in the same segment are equal.

Solution:



It is given that BE is the bisector of ∠B.

$$\angle ABE = \frac{\angle B}{2}$$

However, ∠ADE = ∠ABE (Angles in the same segment for chord AE)

$$\angle ADE = \frac{\angle B}{2}$$

Similarly, $\angle ADF = \angle ACF = \dfrac{\angle C}{2}$ (Angle in the same segment for chord AF)

$$\begin{aligned}\angle D &= \angle ADE + \angle ADF \\ &= \frac{\angle B}{2} + \frac{\angle C}{2} \\ &= \frac{1}{2}(\angle B + \angle C) \\ &= \frac{1}{2}(180° - \angle A)\end{aligned}$$

Similarly, it can be proved for

$$\angle E = \frac{1}{2}(180° - \angle B)$$

$$\angle F = \frac{1}{2}(180° - \angle C)$$

Q9. Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Difficulty level:

Hard

Known/given:

Two congruent circles intersect each other at points A and B.

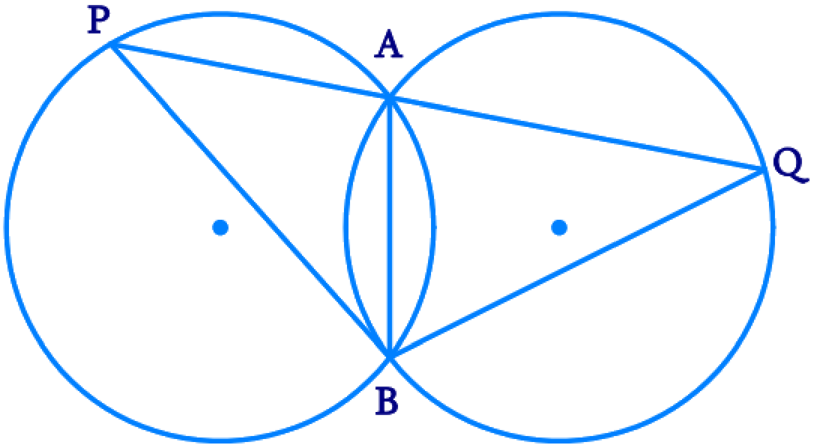
To prove:

BP = BQ.

Reasoning:

Angles in the same segment are equal.

Solution:



AB is the common chord to both circles.

Since the circles are congruent, AB is a common chord and we know that angles in the same segment are equal, we get:

$\angle APB = \angle AQB$

Consider the ΔBPQ,

$\angle APB = \angle AQB$

This implies that ΔBPQ is an isosceles triangle as base angles are equal.
Therefore, we get BP = BQ.

Q10. In any triangle ABC, if the angle bisector of ∠A and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

Difficulty level:
Hard

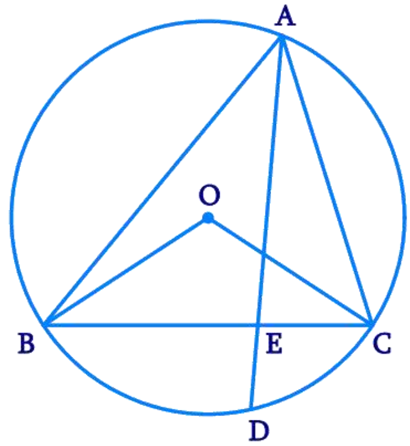
Known/given:
Bisectors of angle A and perpendicular bisector of BC intersect.

To prove:
The intersecting points is circumcircle of the triangle ABC.

Reasoning:

- Angles in the same segment are equal.
- The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

Solution:



Let perpendicular bisector of side BC and angle bisector of ∠A meet at point D.
Let the perpendicular bisector of side BC intersect it at E.

Perpendicular bisector of side BC will pass through circumcenter O of the circle.

∠BOC and ∠BAC are the angles subtended by arc BC at the center and a point A on the remaining part of the circle respectively. We also know that the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.

∠BOC = 2∠BAC = 2∠A ... (1)

Consider ΔBOE and ΔCOE,

OE = OE (Common)

OB = OC (Radii of same circle)

∠OEB = ∠OEC (Each 90° as OD ⊥ BC)

∴ ΔBOE ≅ ∠COE (RHS congruence rule)

∠BOE = ∠COE (By CPCT) ... (2)

However, ∠BOE + ∠COE = ∠BOC

∴ ∠BOE +∠BOE = 2 ∠A [Using Equations (1) and (2)]

2∠BOE = 2∠A

∠BOE = ∠A

∠BOE = ∠COE = ∠A

The perpendicular bisector of side BC and angle bisector of ∠A meet at point D.

∴ ∠BOD = ∠BOE = ∠A ... (3)

Since AD is the bisector of angle ∠A,
∠BAD = Half of angle A.

∴ 2∠BAD = ∠A ... (4)

From Equations (3) and (4), we obtain

$$\angle BOD = 2\angle BAD$$

This can be possible only when point BD will be a chord of the circle. For this, the point D lies on the circumcircle.

Therefore, the perpendicular bisector of side BC and the angle bisector of ∠A meet on the circumcircle of triangle ABC.