Chapter 12: Heron's Formula

Exercise 12.1 (Page 202 of Grade 9 NCERT Textbook)

Q1. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?

Difficulty Level:
Medium

What is known/given?
Dimensions of the traffic signal board (triangle) and its perimeter.

What is Unknown?
Area of the signal board.

Reasoning:
By using Heron's formula, we can calculate the area of triangle.
The formula given by Heron about the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where *a*, *b* and *c* are the sides of the triangle, and

s = Semi-perimeter = Half the Perimeter of the triangle = $\dfrac{(a+b+c)}{2}$

Solution:
Each side of traffic signal board (triangle) = '*a*' cm
Perimeter of traffic signal board (triangle) = sum of all the sides
$= a + a + a$
$= 3a$

Semi Perimeter

$$s = \frac{(a+b+c)}{2} = \frac{a+a+a}{2} = \frac{3a}{2}$$

By using Heron's formula,

Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Area of a (triangle) traffic signal board

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{s(s-a)(s-a)(s-a)}$$
$$= (s-a)\sqrt{s(s-a)} \quad .... \quad (1)$$

We know $s = \dfrac{3a}{2}$ so substituting in equation (1)

$$= \left(\frac{3a}{2} - a\right)\sqrt{\frac{3a}{2}\left(\frac{3a}{2} - a\right)}$$

$$= \left(\frac{a}{2}\right)\sqrt{\frac{3a}{2}\left(\frac{a}{2}\right)}$$

$$= \frac{a}{2} \times \frac{a}{2}\sqrt{3}$$

$$= \frac{a^2}{4}\sqrt{3}$$

Area of the signal board = $\frac{a^2}{4}\sqrt{3}$ sq. units

Now given perimeter = 180 cm

Each side of triangle = $\frac{180}{3}$ cm

$a = 60$ cm

Area of the signal board

$$= \frac{a^2}{4}\sqrt{3}$$

$$= \frac{60^2}{4}\sqrt{3}$$

Area of the signal board = $900\sqrt{3}$ cm^2

Q2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig. 12.9). The advertisements yield an earning of ₹ 5000 per sq. m per year. A company hired one of its walls for 3 months. How much rent did it pay?

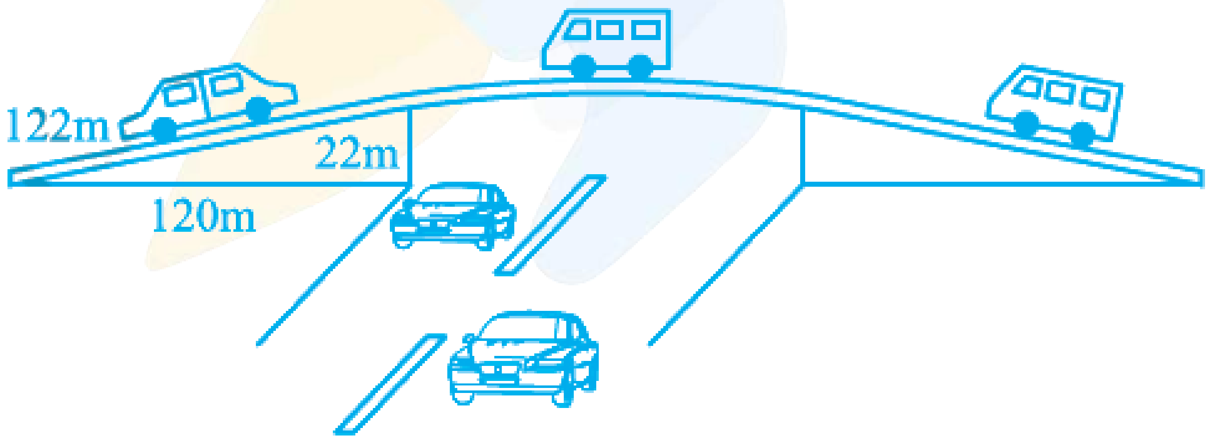

Fig. 12.9

Difficulty Level:

Medium

What is known/given?

Dimensions of the triangular sides of walls.

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What is Unknown?

Rent to pay.

Reasoning:

By using Heron’s formula, we can calculate the area of triangle.

The formula given by Heron about the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where a, b and c are the sides of the triangle, and

s = Semi-perimeter = Half the Perimeter of the triangle = $\frac{(a+b+c)}{2}$

Solution:

Triangular sides of walls are a = 122 m, b = 22 m, c = 120 m

Semi Perimeter,

$$s = \frac{(a+b+c)}{2}$$
$$= \frac{122+22+120}{2}$$
$$= \frac{264}{2}$$
$$= 132 \text{ m}$$

By using Heron’s formula,

$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

Area of triangular wall

$$= \sqrt{s(s-a)\,(s-b)\,(s-c)}$$
$$= \sqrt{132(132-122)\,(132-22)\,(132-120)}$$
$$= \sqrt{132 \times 10 \times 110 \times 12}$$
$$= \sqrt{132 \times 10 \times 132 \times 10}$$
$$= 1320 \text{ m}$$

Rent of $1\,\text{m}^2$ area per year = Rs 5000

Rent of $1\,\text{m}^2$ area per month = Rs $\frac{5000}{12}$

Rent of $1320\,\text{m}^2$ area for 3 months = Rs $\frac{5000}{12} \times 3 \times 1320$

= Rs. 1650000

Therefore, the rent to pay is Rs. 1650000.

Q3. There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" (see Fig. 12.10). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.

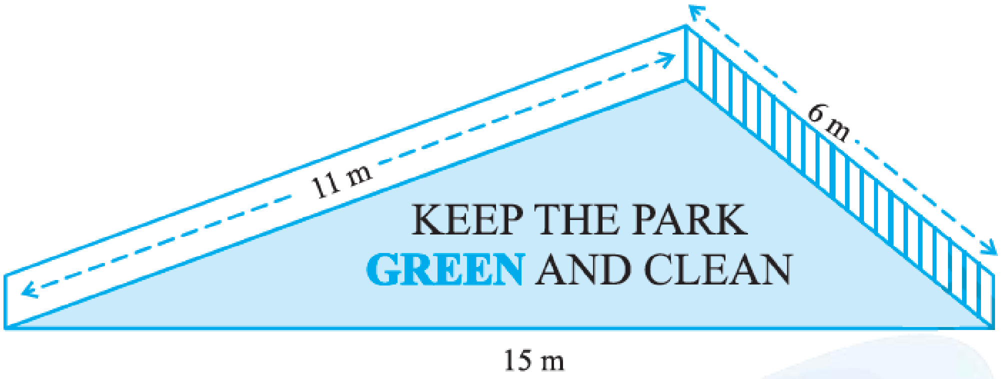


Fig. 12.10

Difficulty Level:
Medium

What is known/given?
Sides of the wall i.e. Dimensions of the triangle.

What is unknown?
Area of the (triangle) i.e. area of slope painted in colour.

Reasoning:
By using Heron's formula, we can calculate the area of triangle.
The formula given by Heron about the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
Where a, b and c are the sides of the triangle, and
s = Semi-perimeter = Half the Perimeter of the triangle = $\dfrac{(a+b+c)}{2}$

Solution
The sides of the walls (triangle) are a =11 m, b = 6 m and c=15 m.
Semi Perimeter:

$$s=\frac{(a+b+c)}{2}=\frac{11+6+15}{2}=\frac{32}{2}=16\text{ m}$$

By using Heron's formula,

$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

Area of a triangle wall:

$$=\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\sqrt{16(16-11)(16-6)(16-15)}$$
$$=\sqrt{16\times 5\times 10\times 1}$$
$$=\sqrt{800}\text{ m}^2$$
$$=20\sqrt{2}\text{ m}^2$$

Area of the wall of park painted in color = $20\sqrt{2}$ m^2.

Q4. Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.

Difficulty Level:

Easy

What is known/given?

Two sides of the triangle and its perimeter.

What is unknown?

Area of the triangle and one of its side.

Reasoning:

By using Heron's formula, we can calculate the area of triangle.

The formula given by Heron about the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where a, b and c are the sides of the triangle, and

s = Semi-perimeter = Half the Perimeter of the triangle = $\dfrac{(a+b+c)}{2}$

Solution:

The sides of triangle given: a =18 cm, b = 10 cm

Perimeter of the triangle = $a + b + c$

$$42 = 18 + 10 + c$$
$$42 = 28 + c$$
$$c = 42 - 28$$
$$c = 14$$

Semi Perimeter

$$s = \frac{(a+b+c)}{2} = \frac{42}{2} = 21 \text{ cm}$$

By using Heron's formula,

$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{21(21-18)(21-10)(21-14)}$$
$$= \sqrt{21\times3\times11\times7}$$
$$= 21\sqrt{11}\ \text{cm}^2$$

Area of a triangle = $21\sqrt{11}$ cm^2.

Q5. Sides of a triangle are in the ratio of 12:17:25 and its perimeter is 540 cm. Find its area.

Difficulty Level:

Easy

What is known/given?

Ratio of sides of the triangle and its perimeter.

What is unknown?

Area of the triangle.

Reasoning:

By using Heron's formula, we can calculate the area of triangle.

The formula given by Heron about the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where a, b and c are the sides of the triangle, and

s = Semi-perimeter = Half the Perimeter of the triangle = $\dfrac{(a+b+c)}{2}$

Solution:

Suppose the sides are 12x cm, 17x cm and 25x cm.

Perimeter of the triangle = 540 cm

$$12x+17x+25x=540$$
$$54x=540$$
$$x=\frac{540}{54}$$
$$x=10\text{ cm}$$

Therefore, sides of triangle:

$$12x = 12\times10 = 120\text{ cm},\ 17x=17\times10=170\text{ cm},\ 25x=25\times10=250\text{ cm}$$
$$a=120\text{cm},\ b=170\text{ cm},\ c=250\text{ cm}$$

s = Half the Perimeter

s = Half the Perimeter

$$s=\frac{540}{2}$$
$$=270\text{ cm}$$

By using Heron's formula,

$$\text{Area of a triangle}=\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\sqrt{270(270-120)(270-170)(270-250)}$$
$$=\sqrt{270\times150\times100\times20}$$
$$=\sqrt{81000000}$$
$$=9000\text{ cm}^2$$

Area of a triangle $=9000\text{ cm}^2$.

Q6. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Difficulty Level:

Easy

What is known/given?

Equal Sides of the triangle and its perimeter.

What is unknown?

Area of the triangle.

Reasoning:

By using Heron's formula, we can calculate the area of triangle.
The formula given by Heron about the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where a, b and c are the sides of the triangle, and

s = Semi-perimeter = Half the Perimeter of the triangle = $\frac{(a + b + c)}{2}$

Solution

Equal sides: $a = b = 12$ cm

$$\text{Perimeter} = a + b + c$$
$$30 = 12 + 12 + c$$
$$c = 30 - 24$$
$$c = 6 \text{ cm}$$

Semi Perimeter

$$s = \frac{(a + b + c)}{2}$$
$$s = \frac{30}{2}$$
$$s = 15 \text{ cm}$$

By using Heron's formula,

$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{15(15-12)(15-12)(15-6)}$$
$$= \sqrt{15 \times 3 \times 3 \times 9}$$
$$= 9\sqrt{15} \text{ cm}^2$$

$$\text{Area of a triangle} = 9\sqrt{15} \text{ cm}^2$$

Chapter 12: Heron's Formula

Exercise 12.2 (Page 206 of Grade 9 NCERT Textbook)

Q1. A park, in the shape of a quadrilateral ABCD, has ∠C = 90º, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?

Difficulty Level:
Medium

What is known/given?
Dimensions of the quadrilateral AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m.

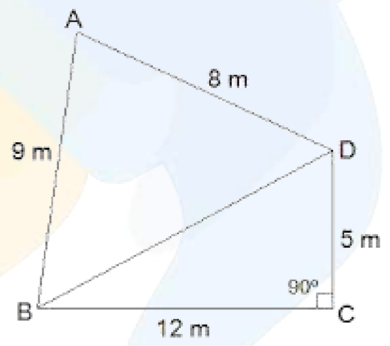
What is unknown?
Area of the park (quadrilateral).

Reasoning:
We can calculate the area of quadrilateral by dividing the quadrilateral into triangular parts and then use the Heron's formula for area of the triangle.
The formula given by Heron about the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where a, b and c are the sides of the triangle, and

s = Semi-perimeter = Half the Perimeter of the triangle = $\dfrac{(a+b+c)}{2}$

Solution:
ABCD is park shown in the figure



We have ∠C = 90º, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. BD is joined

In ΔBDC, we have

$BD^2 = BC^2 + CD^2$ [Pythagoras theorem]

$BD^2 = 12^2 + 5^2$

$BD^2 = 144 + 25$

$BD = \sqrt{169}$

BD = 13 m

Area of ΔBDC $= \dfrac{1}{2} \times \text{base} \times \text{height}$

$= \dfrac{1}{2} \times 12 \times 5$

$= 30 \text{ m}^2$

In ΔABD, AB = a = 9 m, AD = b = 8 m, BD = c = 13 m
Semi Perimeter

$$s = \frac{(a+b+c)}{2}$$
$$= \frac{9+8+13}{2}$$
$$= \frac{30}{2}$$
$$= 15 \text{ m}$$

By using Heron's formula,

$$\text{Area of } \Delta \text{ABD} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{15(15-9)(15-8)(15-13)}$$
$$= \sqrt{15 \times 6 \times 7 \times 2}$$
$$= \sqrt{1260} \text{ m}^2$$
$$\text{Area of } \Delta \text{ABD} = 35.5 \text{ m}^2$$

Therefore,

$$\text{Area of park ABCD} = \text{Area of } \Delta \text{BDC} + \text{Area of } \Delta \text{ABD}$$
$$= 30 \text{ m}^2 + 35.5 \text{ m}^2$$
$$\text{Area of the park} = 65.5 \text{ m}^2$$

Q2. Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.

Difficulty Level:
Medium

What is known/given?
Dimensions of the quadrilateral AB = 3 cm, BC = 4 cm, CD = 4 cm, AD = 5 cm and AC = 5 cm.

What is unknown?
Area of the quadrilateral.

Reasoning:
We can calculate the area of quadrilateral by dividing the quadrilateral into triangular parts and then use the Heron's formula for area of the triangle.

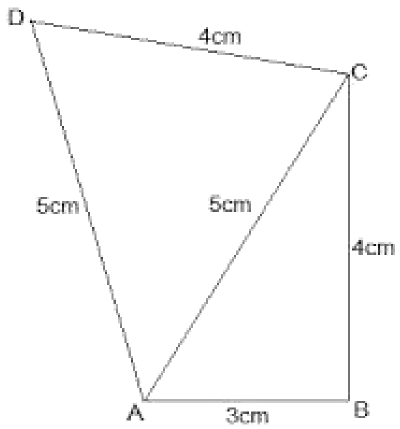
The formula given by Heron about the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where a, b and c are the sides of the triangle, and

s = Semi-perimeter = Half the Perimeter of the triangle = $\frac{(a+b+c)}{2}$

Solution:

ABCD is quadrilateral shown in the figure.



In ΔABC, consider

$$AB^2 + BC^2 = 3^2 + 4^2$$
$$= 5^2$$
$$= AC^2$$

Since it obeys Pythagoras theorem, we can say ΔABC is right angled at B.
Since ΔABC is right angled at B,

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times 3 \times 4$$
$$\text{Area of } \Delta ABC = 6 \text{ cm}^2$$

In ΔADC, we have a = 5 cm, b = 4 cm and c = 5 cm

Semi Perimeter

$$s = \frac{(a+b+c)}{2}$$
$$= \frac{5+4+5}{2}$$
$$= \frac{14}{2}$$
$$= 7 \text{ cm}$$

By using Heron's formula,

$$\text{Area of } \Delta ADC = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{7(7-5)(7-4)(7-5)}$$
$$= \sqrt{7 \times 2 \times 3 \times 2}$$
$$= \sqrt{84} \text{ cm}^2$$
$$= 9.2 \text{ cm}^2$$

$$\text{Area of the quadrilateral ABCD} = \text{Area of } \Delta ADC + \text{Area of } \Delta ABC$$
$$= 9.2 \text{ cm}^2 + 6 \text{ cm}^2$$
$$= 15.2 \text{ cm}^2$$

Q3. Radha made a picture of an aeroplane with coloured paper as shown in Fig 12.15. Find the total area of the paper used.

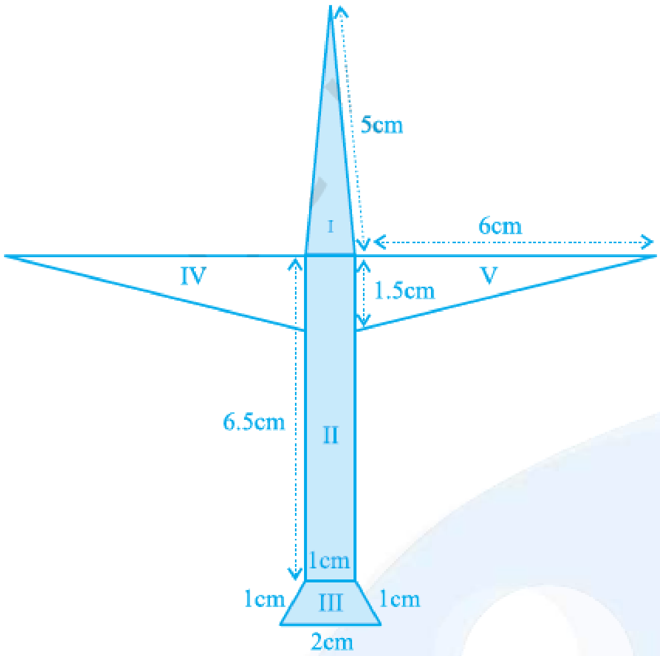


Fig. 12.15

Difficulty Level:
Medium

What is known/given?
Dimensions of the included figures in the given polynomial.

What is unknown?
Area of the triangle, rectangle, trapezium.

Reasoning:
We can calculate the area of polygon by dividing the polygon into triangular and quadrilateral parts and then use the Heron's formula for area of the triangle and general formula for trapezium and rectangle.
The formula given by Heron about the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
Where a, b and c are the sides of the triangle, and
s = Semi-perimeter = Half the Perimeter of the triangle = $\frac{(a + b + c)}{2}$

Solution:
i) For the triangle marked I:

Sides a = 5 cm, b = 5 cm, c = 1 cm
Semi Perimeter

$$s = \frac{(a+b+c)}{2}$$
$$= \frac{5+5+1}{2}$$
$$= \frac{11}{2}$$
$$= 5.5 \text{ cm}$$

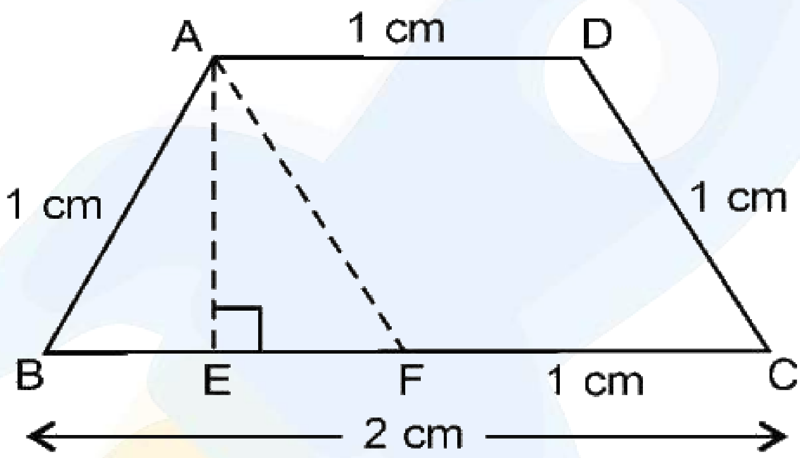
By using Heron's formula,

$$\text{Area of triangle I} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{5.5(5.5-5)(5.5-5)(5.5-1)}$$
$$= \sqrt{5.5 \times 0.5 \times 0.5 \times 4.5}$$
$$= \sqrt{6.1875}\ \text{cm}^2$$
$$= 2.5\ \text{cm}^2$$
$$\text{Area of a triangle} = 2.5\ \text{cm}^2$$

(ii) For the rectangle marked II

$$\text{Area of rectangle} = \text{length} \times \text{breadth}$$
$$= 6.5\ \text{cm} \times 1\ \text{cm}$$
$$= 6.5\ \text{cm}^2$$

(iii) For the trapezium marked III



Draw $AE \perp BC$ and AF parallel to DC

$AD = FC = 1$ cm, $AB = DC = 1$ cm, $BC = 2$ cm, $AF = DC = 1$ cm.

$$BF = BC - FC$$
$$= 2\ \text{cm} - 1\ \text{cm}$$
$$= 1\ \text{cm}$$

Here ΔABF is equilateral triangle, E is the mid-point of BF

So, BE = EF

$$EF = \frac{BF}{2}$$
$$= \frac{1}{2}$$
$$= 0.5\ \text{cm}$$

In ΔAEF,

$$AF^2 = AE^2 + EF^2 \quad [\text{Pythagoras theorem}]$$
$$1^2 = AE^2 + 0.5^2$$
$$AE^2 = \sqrt{1^2 - 0.5^2}$$
$$AE^2 = \sqrt{0.75}$$
$$AE = 0.9 \text{ cm } (\text{approx})$$

$$\text{Area of trapezium} = \frac{1}{2} \times \text{sum of parallel sides} \times \text{distance between them}$$
$$= \frac{1}{2} \times (BC+AD) \times AE$$
$$= \frac{1}{2} \times 3 \times 0.9$$
$$\text{Areaof trapezium} = 1.4 \text{ cm}^2 \ (\text{approx.})$$

(iv)For the triangle marked IV and V

Triangles IV and V are congruent right-angled triangles with base 6 cm and height 1.5 cm.

$$\text{Area of the triangles} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times 6 \times 1.5$$
$$= 4.5 \text{ cm}^2$$

$$\text{Area of two } (\text{IV \& V}) \text{ triangles} = 4.5 \text{ cm}^2 + 4.5 \text{ cm}^2 = 9 \text{ cm}^2$$
$$\text{Total area of the paper used} = 2.5 \text{ cm}^2 + 6.5 \text{ cm}^2 + 1.4 \text{ cm}^2 + 9 \text{ cm}^2$$
$$\text{Total area of the paper used} = 19.4 \text{ cm}^2$$

Q4. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

Difficulty Level:
Medium

What is known/given?
Dimensions of the triangle and base of parallelogram.

What is unknown?
Height of the parallelogram.

Reasoning:
By using Heron's formula calculate the area of triangle and it is given that area of triangle is equal to area of parallelogram.

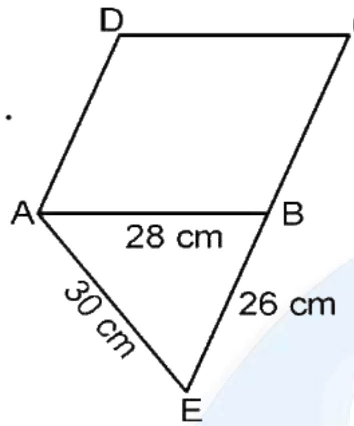
The formula given by Heron about the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where a, b and c are the sides of the triangle, and

s = Semi-perimeter = Half the Perimeter of the triangle = $\dfrac{(a+b+c)}{2}$

Solution:

ABCD is parallelogram.



ABE is a triangle having base common with parallelogram ABCD.

For ΔABE, a = 30 cm, b = 26 cm, c = 28 cm

Semi Perimeter

$$s = \frac{(a+b+c)}{2}$$
$$= \frac{30+26+28}{2}$$
$$= \frac{84}{2}$$
$$= 42 \text{ cm}$$

By using Heron's formula,

$$\text{Area of a } \Delta \text{ABE} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{42(42-30)(42-28)(42-26)}$$
$$= \sqrt{42 \times 12 \times 14 \times 16}$$
$$= 336 \text{ cm}^2$$

$$\text{Area of parallelogram ABCD} = \text{Area of } \Delta \text{ABE}$$
$$\text{Base} \times \text{Height} = 336 \text{ cm}^2$$
$$28 \text{ cm} \times \text{Height} = 336 \text{ cm}^2$$
$$\text{Height} = \frac{336 \text{ cm}^2}{28 \text{ cm}}$$
$$\text{Height of parallelogram} = 12 \text{ cm}$$

Q5. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

Difficulty Level:
Medium

What is known/given?
Dimensions of the rhombus.

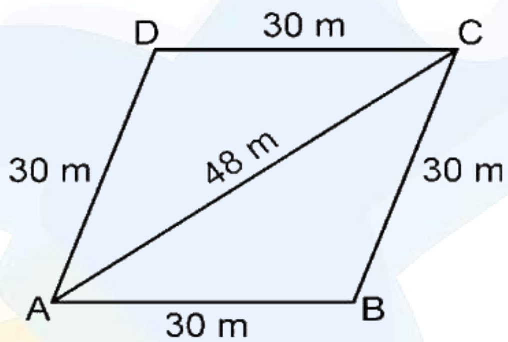
What is unknown?
Area of grass field that each Cow be getting.

Reasoning:
By using Heron's formula calculate the area of triangle and then area of rhombus.
The formula given by Heron about the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where a, b and c are the sides of the triangle, and

s = Semi-perimeter = Half the Perimeter of the triangle = $\dfrac{(a+b+c)}{2}$

Solution:



The diagonal AC divides the rhombus into two congruent triangles

For ΔABC, $a = b = 30$ m, $c = 48$ m
Semi Perimeter

$$s = \frac{(a+b+c)}{2} = \frac{30+30+48}{2} = \frac{108}{2} = 54 \text{ m}$$

By using Heron's formula,

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{54(54-30)(54-30)(54-48)}$$
$$= \sqrt{54 \times 24 \times 24 \times 6}$$
$$= 432 \text{ m}^2$$

Area of rhombus = $2 \times$ Area of a ΔABC

$= 2 \times 432 \text{ m}^2$

$= 864 \text{ m}^2$

Number of cows = 18

Area of grass field will each cow get = $\frac{864 \text{ m}^2}{18}$

Each cow gets an area of grass = 48 m^2.

Q6. An umbrella is made by stitching 10 triangular pieces of cloth of two different colors (See Fig. 12.16), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each color is required for the umbrella?

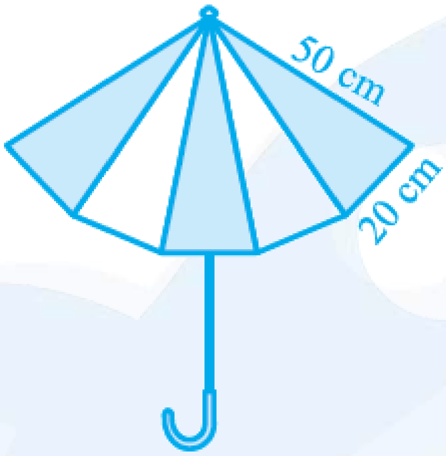


Fig. 12.16

Difficulty Level:
Medium

What is known/given?
Dimensions of each triangular piece used in umbrella.

What is unknown?
Each color of cloth required for the umbrella.

Reasoning:
By using Heron's formula, we can calculate the area of triangle.
The formula given by Heron about the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where a, b and c are the sides of the triangle, and

s = Semi-perimeter = Half the Perimeter of the triangle = $\frac{(a + b + c)}{2}$

Solution:
We know that umbrella is made of 10 triangular pieces of cloth of two different colours.

Let us calculate the area of one triangle.

For ΔABC, $a = b = 50$ cm, $c = 20$ cm

Semi Perimeter

$$s = \frac{(a+b+c)}{2} = \frac{50+50+20}{2} = \frac{120}{2} = 60 \text{ cm}$$

By using Heron's formula,

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{60(60-50)(60-50)(60-20)} = \sqrt{60 \times 10 \times 10 \times 40} = 200\sqrt{6} \text{ cm}^2$$

Therefore,

$$\text{Area of 10 triangular pieces} = 10 \times 200\sqrt{6} \text{ cm}^2 = 2000\sqrt{6} \text{ cm}^2$$

Hence cloth required for each color = $\frac{2000\sqrt{6}}{2} = 1000\sqrt{6}\text{ cm}^2$.

Q7. A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in Fig. 12.17 How much paper of each shade has been used in it?

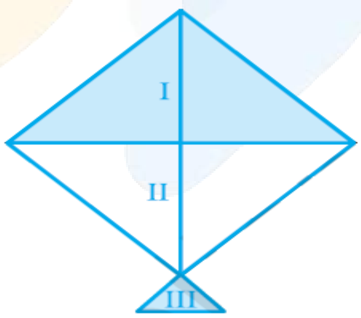


Fig. 12.17

Difficulty Level:
Medium

What is known/given?
Dimensions of the square used to prepare kite.

What is unknown?

Paper used in each shade to make kite.

Reasoning:

By using Heron's formula calculate the area of each triangular shade.

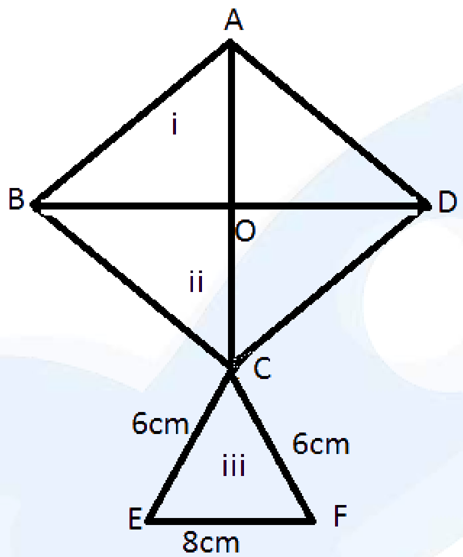
The formula given by Heron about the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where a, b and c are the sides of the triangle, and

s = Semi-perimeter = Half the Perimeter of the triangle = $\dfrac{(a+b+c)}{2}$

Solution:

We know that diagonals of square bisect each other at right angle



Given diagonal BD = AC = 32 cm, then OA = $\frac{1}{2}$ AC = 16 cm.

Isosceles triangle of base 32 cm and sides 16 cm

ΔABD is right angled triangle.

$$\begin{aligned}\text{Area of } \Delta\,\text{ABD} &= \frac{1}{2}\times\text{base}\times\text{height}\\ &= \frac{1}{2}\times 32\times 16\\ &= 256\text{ cm}^2\end{aligned}$$

Since the triangles that divides the square into two parts are equal.

Therefore, Area of ΔABD = Area of ΔBCD = 256 cm^2

For ΔCEF

Semi Perimeter

$$\begin{aligned}s &= \frac{(a+b+c)}{2}\\ &= \frac{6+6+8}{2}\\ &= \frac{20}{2}\\ &= 10\text{ cm}\end{aligned}$$

By using Heron's formula,

$$\text{Area of } \Delta CEF = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{10(10-6)(10-6)(10-8)}$$
$$= \sqrt{10 \times 4 \times 4 \times 2}$$
$$= \sqrt{320}$$
$$= 17.92 \text{ cm}^2$$

Paper used to make shade I = 256 cm^2, shade II = 256 cm^2, shade III = 17.92 cm^2.

Q8. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see Fig. 12.18). Find the cost of polishing the tiles at the rate of 50p per cm^2.

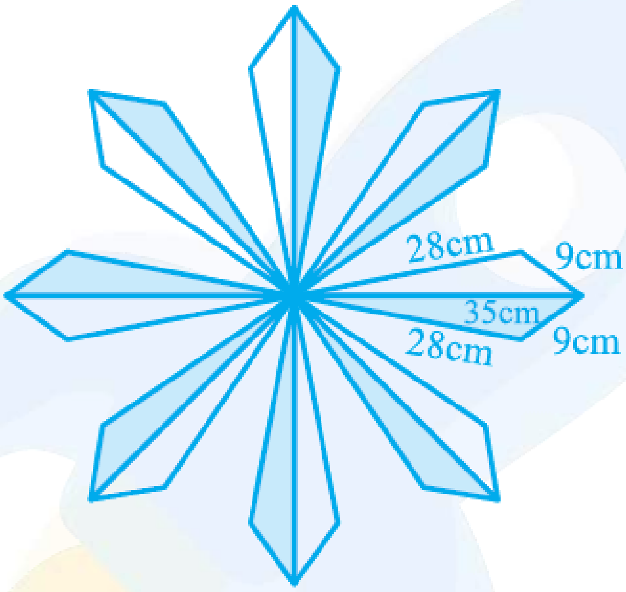


Fig. 12.18

Difficulty Level:
Medium

What is known/given?
Dimensions of the tiles(triangle).

What is unknown?
Total area of floral design and cost of polishing the tiles.

Reasoning:
By using Heron's formula calculate the area of triangle.
The formula given by Heron about the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
Where a, b and c are the sides of the triangle, and
s = Semi-perimeter = Half the Perimeter of the triangle = $\frac{(a+b+c)}{2}$

Solution:

We have the dimensions of sides of one triangular tile.

$a = 35 \text{ cm}, \ b = 28 \text{ cm}, \ c = 9 \text{ cm}$

Semi Perimeter

$$\begin{aligned} s &= \frac{(a+b+c)}{2} \\ &= \frac{35+28+9}{2} \\ &= \frac{72}{2} \\ &= 36 \text{ cm} \end{aligned}$$

By using Heron's formula,

$$\begin{aligned} \text{Area of 1 triangular tile} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{36(36-35)(36-28)(36-9)} \\ &= \sqrt{36 \times 1 \times 8 \times 27} \\ &= 36\sqrt{6} \text{ cm}^2 \end{aligned}$$

Area of a 1 triangular tile = $36\sqrt{6}$ cm^2

$$\begin{aligned} \text{Area of 16 triangular tile} &= 16 \times 36\sqrt{6} \text{ cm}^2 \\ &= 1410.90 \text{ cm}^2 \end{aligned}$$

Cost of polishing 1cm^2 of tile is 50p

Therefore,

$$\begin{aligned} \text{Cost of } 1410.9 \text{ cm}^2 \text{ of tile} &= 1410.90 \text{ cm}^2 \times 0.5 \\ &= \text{Rs } 705.45 \ (\text{approx.}) \end{aligned}$$

Q9. A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

Difficulty Level:

Medium

What is known/given?

Dimensions of the field (trapezium).

What is unknown?

Area of field (trapezium).

Reasoning:

Using Heron's formula find the area of triangle and then using that area, we can find the height of the trapezium and hence its area.

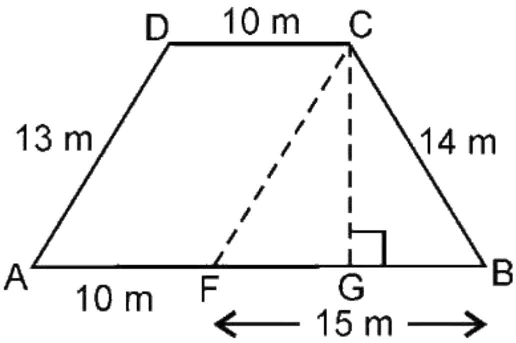
The formula given by Heron about the area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where *a*, *b* and *c* are the sides of the triangle, and

s = Semi-perimeter = Half the Perimeter of the triangle = $\dfrac{(a+b+c)}{2}$

Solution:

Given ABCD is field. Draw $CG \perp AB$ and CF parallel to DA.



$$DC = AF = 10 \text{ m},\ DA = CF = 13 \text{ m},\ \text{So, } FB = 15 \text{ m}$$

In ΔCFB, $a = 15$ m, $b = 14$ m, $c = 13$ m.

Semi Perimeter

$$s = \frac{(a+b+c)}{2} = \frac{15+14+13}{2} = \frac{42}{2} = 21 \text{ m}$$

By using Heron's formula,

$$\text{Area of } \Delta CFB = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21(21-15)(21-14)(21-13)} = \sqrt{21 \times 6 \times 7 \times 8} = 84 \text{ m}^2$$

Also,

$$\text{Area of } \Delta CFB = \frac{1}{2} \times \text{base} \times \text{height}$$

$$84 = \frac{1}{2} \times BF \times CG$$

$$84 = \frac{1}{2} \times 15 \times CG$$

$$CG = \frac{84 \times 2}{15}$$

$$CG = 11.2 \text{ m}$$

$$\text{Area of trapezium} = \frac{1}{2} \times \text{sum of parallel sides} \times \text{distance between them} = \frac{1}{2} \times (25+10) \times 11.2 = 196 \text{ m}^2$$

Hence the area of field is 196 m^2