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Chapter - 13: Surface Area and Volumes

Exercise 13.1 (Page 213 of Grade 9 NCERT Textbook)

Q1. Plastic box 1.5 m long, 1.25 m wide and 65 cm deep, is to be made. It is opened at the top. Ignoring the thickness of the plastic sheet, determine:

i) The area of the sheet required for making the box.

ii) The cost of sheet for it, if a sheet measuring $1m^2$ costs ₹20.

Difficulty Level:

Medium

Known:

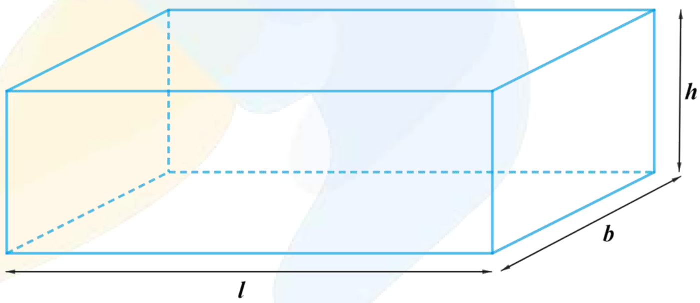
The length, breadth and depth of the plastic bag to be made opened at the top. Cost of 1 m^2 sheet is ₹20.

Unknown:

Area of the sheet required for making the box and its cost.

Reasoning:

Since box is opened at the top, it has only 5 surfaces, including the 4 walls and the base.
Area of the sheet required for making the cuboidal box includes the 4 walls of the box and the base.
Hence, area of the sheet can be obtained by adding area of the base to the lateral surface area of the cuboidal box.
Lateral surface area of cuboid $= 2(l+b)h$



The cost of the sheet to create the box will be equal to area of the sheet multiplied by rate of the sheet.

Solution:

Length, $l = 1.5\text{ m}$

Breadth, $b = 1.25\text{ m}$

Height,

$$\begin{aligned} h &= 65\text{ cm} \\ &= \frac{65}{100}\text{ m} \\ &= 0.65\text{ m} \end{aligned}$$

The area of the sheet required to make the box,

$$= lb + 2(l+b)h$$
$$= (1.5\text{ m}\times 1.25)\text{ m} + 2\times(1.25\text{ m}+1.5\text{ m})\times 0.65\text{ m}$$
$$= 1.875\text{ m}^2 + 2\times 2.75\text{ m}\times 0.65\text{ m}$$
$$= 1.875\text{ m}^2 + 3.575\text{ m}^2$$
$$= 5.45\text{ m}^2$$

Therefore, the cost of the sheet = Rate of the sheet $\times$ Area of the sheet

$$= ₹20/\text{m}^2 \times 5.45\text{ m}^2$$
$$= ₹109$$

Answer:

The area of the sheet required for making the open box is 5.45 m^2 and Cost of the sheet is ₹109

Q2. The length, breadth and height of a room are 5 m, 4 m, and 3 m respectively. Find the cost of white washing the walls of the room and ceiling at the rate of ₹7.50 per m^2.

Difficulty Level:

Medium

Known:

The length, breadth and height of a room are 5 m, 4 m, and 3 m respectively.

Unknown:

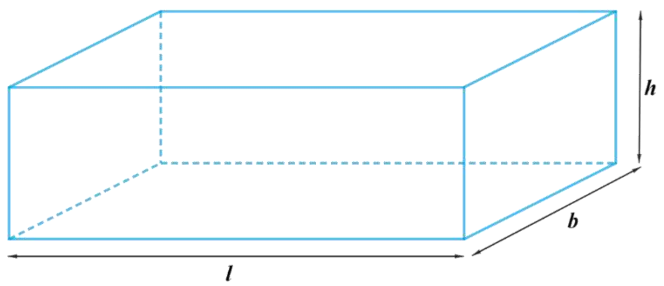
The cost of white washing the walls of the room and ceiling.

Reasoning:

Since the four walls and ceiling are to be whitewashed. So, it has 5 faces only, excluding the base.

Hence, area of the room to be whitewashed can be obtained by adding area of the ceiling to the lateral surface area of the cuboidal room.

Lateral surface area of cuboid $= 2(l+b)h$



The cost of white washing the walls of the room and ceiling will be equal to area of the room to be whitewashed multiplied by rate of the whitewashing.

Solution:

Length, $l = 5\text{ m}$
Breadth, $b = 4\text{ m}$
Height, $h = 3\text{ m}$

Surface area of 5 faces = Area of the 4 walls and ceiling $= lb + 2(l+b)h$

$$\begin{aligned} lb + 2(l+b)h &= (5\text{ m} \times 4\text{ m}) + 2\times(5\text{ m} + 4\text{ m}) \times 3\text{ m} \\ &= 20\text{ m}^2 + 2\times 9\text{ m}\times 3\text{ m} \\ &= 20\text{ m}^2 + 54\text{ m}^2 \\ &= 74\text{ m}^2 \end{aligned}$$

The cost of white washing the walls of the room and ceiling $=$ Rate $\times$ Area

$$\begin{aligned} ₹\text{ Rate} \times \text{Area} &= 7.50 / \text{m}^2 \times 74\text{ m}^2 \\ &= ₹555 \end{aligned}$$

Answer:

Cost of white washing the walls of the room and the ceiling is ₹555.

Q3. The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of ₹10 per m^2 is ₹15000, find the height of the hall.
[Hint: Area of the four walls = Lateral surface area.]

Difficulty Level:

Medium

Known:

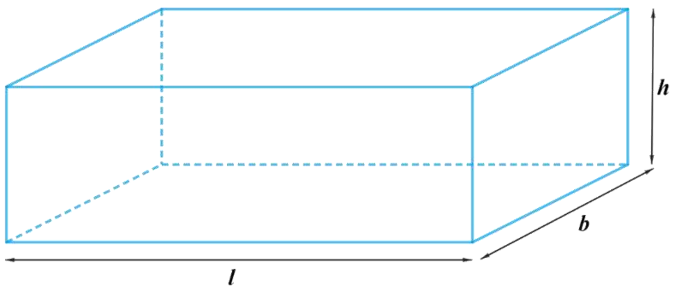
Perimeter of the floor of the rectangular hall is 250m and the cost of painting the four walls at the rate of ₹10 per m^2 is ₹15000.

Unknown:

Height of the hall.

Reasoning:

The area of 4 walls of the cuboidal room will be Lateral surface area of the cuboid.
Lateral surface area of cuboid $= 2(l+b)h$



Area of the 4 walls can also be obtained by dividing the total cost of painting by the rate of painting.

Solution:

Let the length, breadth and height of the room are *l*, *b* and *h* respectively.
The cost of painting the four walls is ₹15000.
The rate of painting is ₹$10/m^2$
Perimeter of the floor $= 250m$

Therefore, $2(l+b) = 250m$

Now,

$$\begin{aligned} \text{Area of four walls} &= \frac{15000}{10}\ \text{m}^2 \\ &= 1500\ \text{m}^2 \\ 2(l+b)h &= 1500\ \text{m}^2 \\ 250\ \text{m} \times h &= 1500\ \text{m}^2 \\ h &= \frac{1500\ \text{m}^2}{250\ \text{m}} \\ &= 6\ \text{m} \end{aligned}$$

Answer:

The height of the hall is 6 m.

Q4. The paint in a certain container is sufficient to paint an area equal to $9.375m^2$. How many bricks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?

Difficulty Level:

Medium

Known:

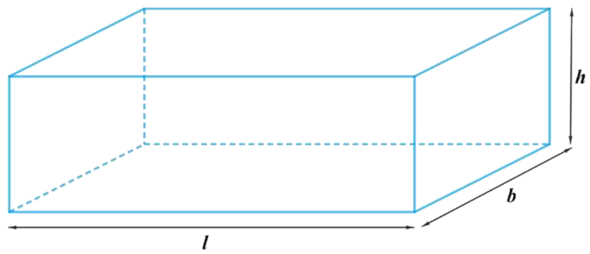
A container with paint for painting an area equal to $9.375m^2$. and bricks of dimensions $22.5cm \times 10cm \times 7.5cm$.

Unknown:

Number of bricks can be painted out of the container

Reasoning:

Since a brick is cuboidal in shape, surface area of the brick will be total surface area of the cuboid.
Hence, area of each brick to be painted will be total surface area of the cuboid
Total surface area of cuboid $= 2(lb + bh + hl)$



Number of bricks which can be painted out of the container can be calculated by dividing the area which can be painted with paint available in the container by the area of each brick.

Solution:

The area which can be painted with paint available in the container $= 9.375 m^2$

Let the length, breadth and height of the bricks are *l*, *b* and *h* respectively.

$$l = 22.5 \text{ cm}$$
$$b = 10 \text{ cm}$$
$$h = 7.5 \text{ cm}$$

The area of each brick to be painted $= 2(lb + bh + hl)$

$$\begin{aligned} 2(lb + bh + hl) &= 2 \times (22.5 \text{ cm} \times 10 \text{ cm} + 10 \text{ cm} \times 7.5 \text{ cm} \times 7.5 \text{ cm} \times 22.5 \text{ cm}) \\ &= 2 \times (225 \text{ cm}^2 + 75 \text{ cm}^2 + 168.75 \text{ cm}^2) \\ &= 2 \times 468.75 \text{ cm}^2 \\ &= 937.5 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Number of bricks canbe painted} &= \frac{9.375 \text{ m}^2}{937.5 \text{ cm}^2} \\ &= \frac{9.375 \times 10000 \text{ cm}^2}{937.5 \text{ cm}^2} \\ &= 100 \text{ cm}^2 \end{aligned}$$

Answer:

Number of bricks can be painted out of the container is 100.

Q5. A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.

i) Which box has the greater lateral surface area and by how much?

ii) Which box has the smaller total surface area and by how much?

Difficulty Level:

Medium

Known:

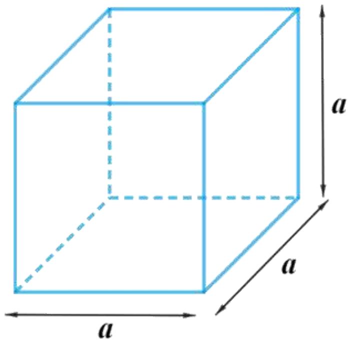
The length of edge of cubical box is 10cm and the length, breadth and height of the cuboidal box are 12.5 cm, 10 cm and 8 cm respectively.

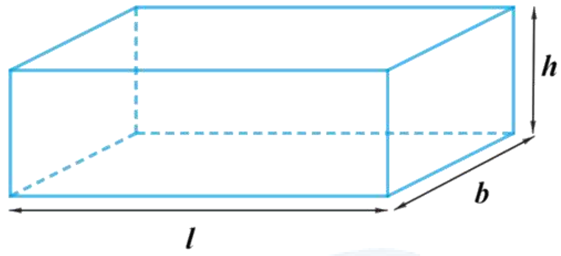
Unknown:

Box having greater lateral surface area and box having smaller total surface area and by how much.

Reasoning:

A cube is cuboid whose length, breadth and height and equal. A cuboid has six faces and the total surface area is the sum of the surface area of the 6 faces and Lateral surface area is the sum of the area of the four faces.





Total surface area of cube $= 6a^2$

Total surface area of cuboid $= 2(lb + bh + hl)$

Lateral surface area of cuboid $= 4a^2$

Lateral surface area of cuboid $= 2(l + b)h$

Solution:

Length of edge of the cube, $a = 10cm$

Length of the cuboid, $l = 12.5\ cm$

Breadth of the cuboid, $b = 10\ cm$

Height of the cuboid, $h = 8\ cm$

$$\begin{aligned} \text{Lateral surface area of the cube} &= 4a^2 \\ &= 4 \times (10\text{ cm})^2 \\ &= 4 \times 100\text{ cm}^2 \\ &= 400\text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Lateral surface area of the cuboid} &= 2(l + b)h \\ &= 2 \times (12.5\text{ cm} + 10\text{ cm}) \times 8\text{ cm} \\ &= 2 \times 22.5\text{ cm} \times 8\text{ cm} \\ &= 360\text{ cm}^2 \end{aligned}$$

Cubical box has the greater lateral surface area by $(400cm^2 - 360cm^2) = 40\ cm^2$

$$\begin{aligned} \text{Total surface area ofthe cube} &= 6a^2 \\ &= 6 \times (10cm)^2 \\ &= 6 \times 100cm^2 \\ &= 600cm^2 \end{aligned}$$

Total surface area of the cuboid $= 2(lb + bh + hl)$

$$
\begin{aligned}
2(lb + bh + hl) &= 2 \times (12.5cm \times 10cm + 10cm \times 8cm + 8cm \times 12.5cm) \\
&= 2 \times (125cm^2 + 80cm^2 + 100cm^2) \\
&= 2 \times 305cm^2 \\
&= 610cm^2
\end{aligned}
$$

Cubical box has the smaller total surface area by $(610cm^2 - 600cm^2) = 10cm^2$

Answer:

Cubical box has the greater lateral surface area by $40\ cm^2$

Cubical box has the smaller total surface area by $10cm^2$

Q6. A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high.

i. What is the area of the glass?
ii. How much of tape is needed for all the 12 edges?

Difficulty Level:

Medium

Known:

Herbarium of dimensions $30cm \times 25cm \times 25cm$.

Unknown:

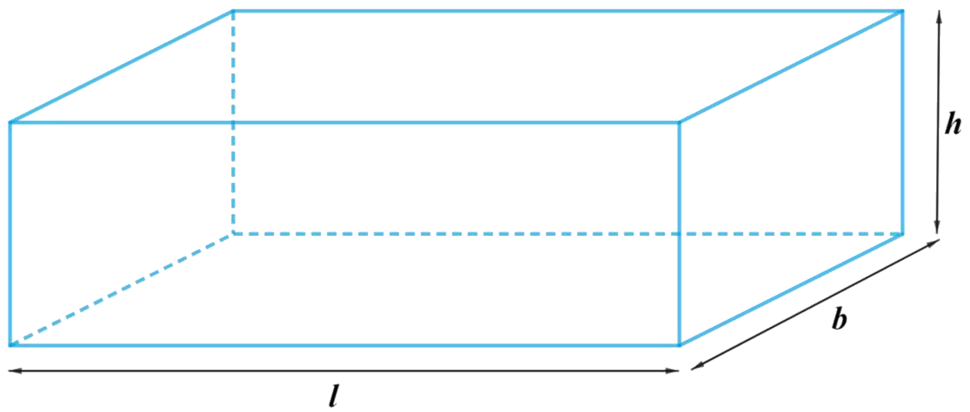
The area of the glass and the length of the tape.

Reasoning:

Since the herbarium is a cuboid is shape which is enclosed by six rectangle regions called faces and it has 12 edges.

Then the area of the glass used to make the herbarium will be equal to total surface area of the cuboid.

Total surface area of cuboid $= 2(lb + bh + hl)$



Since the herbarium is made entirely of glass panes (including base) held together with tape.
Then the length of the tape which is needed for all the 12 edges will be same as the sum of the lengths of all the 12 edges.

Solution:
Let the length, breadth and height of the herbarium are *l*, *b* and *h* respectively.
Length, $l = 30\ cm$

Breadth, $b = 25\ cm$

Height, $h = 25\ cm$

$$\begin{aligned}\text{The area of the glass} &= 2(lb + bh + hl)\\ &= 2 \times (30cm \times 25cm + 25cm \times 25cm + 25cm \times 30cm)\\ &= 2 \times (750cm^2 + 625cm^2 + 750cm^2)\\ &= 2 \times 2125\,cm^2\\ &= 4250\,cm^2\end{aligned}$$

$$\begin{aligned}\text{Length of the tape needed for all the 12 edges} &= 4(l + b + h)\\ &= 4 \times (30cm + 25cm + 25cm)\\ &= 4 \times 80cm\\ &= 320\,cm\end{aligned}$$

Answer:
The area of the glass is $4250\,cm^2$ and
Tape needed for all the 12 edges is $320\,cm$.

Q7. Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm × 20 cm × 5 cm and the smaller of dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is ₹4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind.

Difficulty Level:
Hard

Known:
The smaller boxes with dimensions 15 cm × 12 cm × 5 cm and bigger boxes with dimensions 25 cm × 20 cm × 5 cm. The cost of the cardboard is ₹4 for 1000 cm^2.
For all the overlaps, 5% of the total surface area is required extra.

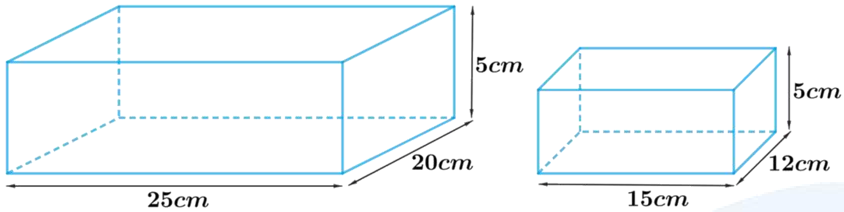
Unknown:
The cost of cardboard required for supplying 250 boxes of each kind.

Reasoning:

Since the cardboard boxes are cuboidal in shape, the total area of the cardboard is same as the total surface area of the cuboid and for all the overlaps, 5% of the total surface area is required extra.

Hence, the area of each box can be obtained by adding 5% of the total surface area to the total surface area of the cuboid.

Total surface area of cuboid $= 2(lb + bh + hl)$



So, we can find the area of cardboard for each bigger as well as smaller boxes.
Then we can find the area of such 250 boxes and the total cost of the cardboard at ₹4 per 1000 cm^2.

Solution:

For bigger box:

Let the length, breadth and height of the bigger box are *L*, *B* and *H* respectively.

Length, $L = 25\ cm$

Breadth, $B = 20\ cm$

Height, $H = 5\ cm$

$$\begin{aligned}\text{The area of the card board} &= 2(LB + BH + HL)\\ &= 2\times(25cm\times 20cm + 20cm\times 5cm + 5cm\times 25cm)\\ &= 2\times(500cm^2 + 100cm^2 + 125cm^2)\\ &= 2\times 725cm^2\\ &= 1450cm^2\end{aligned}$$

For all the overlaps, 5% of the total surface area is required extra.
Therefore,

$$\begin{aligned}\text{Overlap area} &= 5\% \text{ of } 1450cm^2\\ &= \frac{5}{100}\times 1450cm^2\\ &= 72.5cm^2\end{aligned}$$

$$\begin{aligned}\text{Net card board required for each bigger box} &= 1450cm^2 + 72.5cm^2\\ &= 1522.5\ cm^2\end{aligned}$$

$$\begin{aligned}\text{Area of card board required for 250 such boxes} &= 250\times 1522.5\ cm^2\\ &= 380625cm^2\end{aligned}$$

$$\begin{aligned}\text{The total cost of the cardboard at ₹4 per 1000 cm}^2 &= \frac{4}{1000}\times 380625\\ &= ₹1525.50\end{aligned}$$

For smaller box:
Let the length, breadth and height of the smaller box are *l*, *b* and *h* respectively.
Length, $l = 15\ cm$
Breadth, $b = 12\ cm$
Height, $h = 5\ cm$

$$\begin{aligned}\text{The area of the cardboard} &= 2(lb + bh + hl)\\ &= 2 \times (15cm \times 12cm + 12cm \times 5cm + 5cm \times 15cm)\\ &= 2 \times (180cm^2 + 60cm^2 + 75cm^2)\\ &= 2 \times 315cm^2\\ &= 630cm^2\end{aligned}$$

For all the overlaps, 5% of the total surface area is required extra.
Therefore,

$$\begin{aligned}\text{Overlap area} &= 5\% \text{ of } 630cm^2\\ &= \frac{5}{100} \times 630cm^2\\ &= 31.5cm^2\end{aligned}$$

$$\begin{aligned}\text{Net cardboardrequired for bigger box} &= 630cm^2 + 31.5cm^2\\ &= 631.5cm^2\end{aligned}$$

Area of cardboard required for 250 such boxes $= 250 \times 631.5\ cm^2 = 165375cm^2$

The total cost of the cardboard at ₹4 per 1000 $cm^2 = \frac{4}{1000} \times 165375$
= ₹661.50

Answer:
Cost of cardboard required for supplying 250 boxes of each kind
= ₹1522.50 + ₹661.50
= ₹2184

Q8. Parveen wanted to make a temporary shelter for her car, by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m, with base dimensions 4 m × 3 m?

Difficulty Level:
Easy

Known:
The shelter of height 2.5 m, with base dimensions 4 m by 3 m.

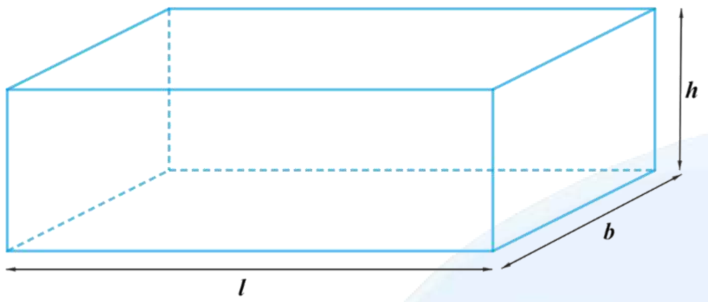
Unknown:

Area of the tarpaulin required to make the shelter.

Reasoning:

Since the shelter is a box-like structure (cuboid) with tarpaulin that covers all the four sides and the top of the car, the surface area of the shelter is the sum of the lateral surface area of the cuboid and area of the top.

Lateral surface area of cuboid $= 2(l+b)h$



Then the area of the tarpaulin required to make the shelter $= lb + 2(l+b)h$

Solution:

Let the length, breadth and height of the shelter are l, b and h respectively.

Length, $l = 4m$

Breadth, $b = 3m$

Height, $h = 2.5\ m$

The area of the tarpaulin required to make the shelter $= lb + 2(l+b)h$

$$= (4m \times 3m) + 2 \times (4m + 3m) \times 2.5m$$
$$= 12m^2 + 2 \times 7m \times 2.5m$$
$$= 12m^2 + 35m^2$$
$$= 47m^2$$

Answer:

Hence $47m^2$ of tarpaulin will be required.

Chapter - 13: Surface Area and Volumes

Exercise 13.2 (Page 216 of Grade 9 NCERT Textbook)

Assume $\pi = \dfrac{22}{7}$ unless stated otherwise

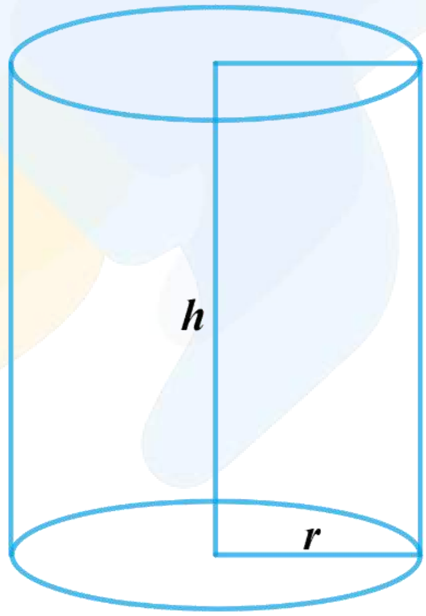
Q1. The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder.

Difficulty Level:
Easy

Known:
Curved surface area of the cylinder and its height.

Unknown:
Diameter of the base of the cylinder.

Reasoning:
Let's draw a diagram of the cylinder to visualize it better. The radius and height of the cylinder are *r* and *h* respectively



CSA of the right circular cylinder $= 2\pi rh$
Diameter $= 2 \times \text{radius}$

Solution:
Let the radius and height of the cylinder be *r* and *h* respectively.
Height of the cylinder, $h = 14cm$
CSA of the cylinder $= 88cm^2$

$$2\pi rh = 88cm^2$$

$$2 \times \frac{22}{7} \times r \times 14cm = 88cm^2$$

$$r = \frac{88cm^2 \times 7}{2 \times 22 \times 14cm}$$

$$= 1cm$$

$$\text{Diameter} = 2 \times \text{radius}$$

$$= 2 \times 1cm$$

$$= 2cm$$

Answer:

The diameter of the base of the cylinder is 2 cm.

Q2. It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square metres of the sheet are required for the same?

Difficulty Level:

Easy

Known:

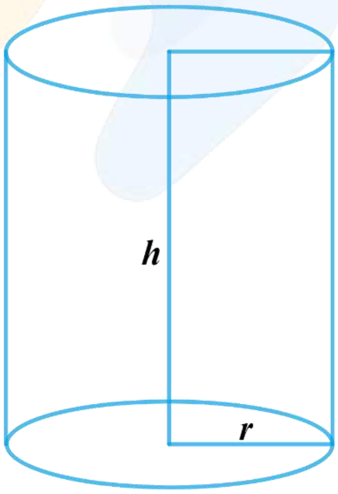
A closed cylindrical tank of height 1 m and base diameter 140 cm.

Unknown:

Area of the sheet required for making the cylindrical tank in square metres.

Reasoning:

Let's draw a diagram of the cylinder to visualize it better. The radius and height of the cylinder are r and h respectively.



TSA of the right circular cylinder $= 2\pi r(r + h)$

Diameter $= 2 \times \text{radius}$

Solution:

Let the radius and height of the cylinder be r and h respectively.

Height of the tank, $h = 1m$

Radius of the tank, $r = \frac{140cm}{2} = \frac{70}{100}m = 0.7m$

Total surface area of the closed cylindrical tank $= 2\pi r(r+h)$

$$= 2 \times \frac{22}{7} \times 0.7m \times (0.7m + 1m)$$
$$= 4.4m \times 1.7m$$
$$= 7.48m^2$$

Answer:

$7.48m^2$ of the sheet is required for making the cylindrical tank.

Q3. A metal pipe is 77 cm long. The inner diameter of a cross section is 4 cm, the outer diameter being 4.4 cm (see Fig. 13.11). Find its

i. Inner curved surface area
ii. Outer curved surface area
iii. Total surface area

Difficulty Level:

Medium

Known:

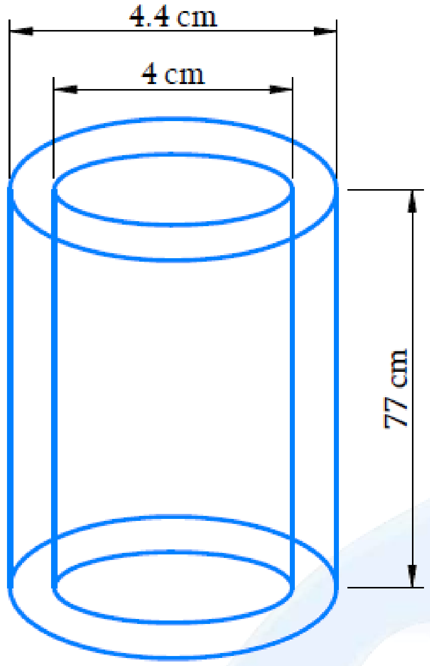
Length of the metal pipe is 77cm and inner diameter of cross section is 4cm and the outer diameter is 4.4cm

Unknown:

Inner curved surface area, outer curved surface area and total surface area of the pipe.

Reasoning:

Let's draw a diagram of the metal pipe to visualize it better.



The inner radius, outer radius and height of the cylinder are r, R and h respectively

Inner curved surface area $= 2\pi rh$

Outer curved surface area $= 2\pi Rh$

Total surface area of the pipe can be obtained by adding CSA of inner and outer surfaces with the area of both the circular ends

We can find the area of circular ends by subtracting area of inner circle from the area of the outer circle.

TSA of pipe = CSA of inner surface + CSA of outer surface + Area of both the circular ends of the pipe

Hence, TSA of the pipe $= 2\pi rh + 2\pi Rh + 2\pi\left(R^2 - r^2\right)$

Solution:

Length of the pipe, $h = 77\ cm$

Inner radius of the pipe, $r = \dfrac{4cm}{2} = 2\ cm$

Outer radius of the pipe, $R = \dfrac{4.4cm}{2} = 2.2\ cm$

Inner curved surface area $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 2cm \times 77cm$$

$$= 968\ cm^2$$

Outer curved surface area $= 2\pi Rh$

$$= 2 \times \frac{22}{7} \times 2.2cm \times 77cm$$

$$= 1064.8\ cm^2$$

Total surface area $= 2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$

$$= 1064.8cm^2 + 968cm^2 + 2\times\frac{22}{7}\times\left[(2.2cm)^2 - (2cm)^2\right]$$

$$= 1064.8cm^2 + 968cm^2 + 2\times\frac{22}{7}\times\left[4.84cm^2 - 4cm^2\right]$$

$$= 1064.8cm^2 + 968cm^2 + 2\times\frac{22}{7}\times 0.84cm^2$$

$$= 1064.8cm^2 + 968cm^2 + 5.28cm^2$$

$$= 2038.08\ cm^2$$

Answer:

Inner curved surface area $= 968\ cm^2$

Outer curved surface area $= 1064.8\ cm^2$

Total surface area $= 2038.08\ cm^2$

Q4. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2.

Difficulty Level:

Medium

Known:

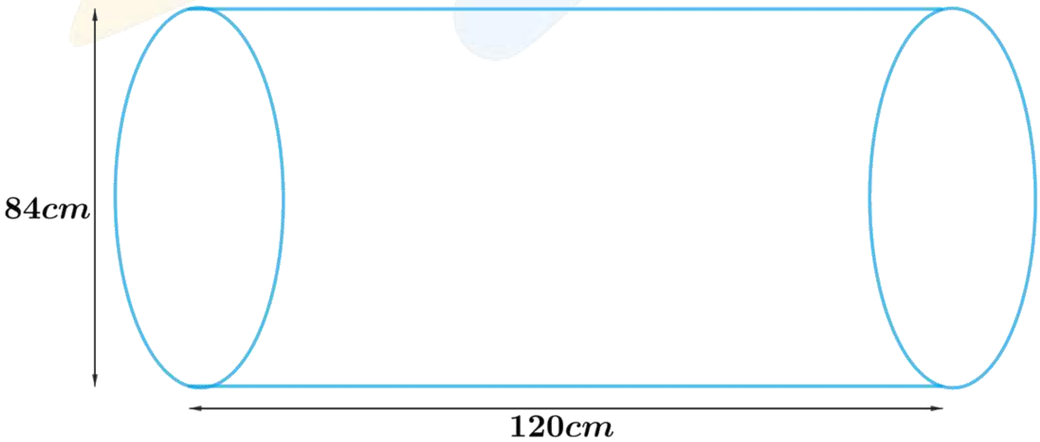
The diameter of the roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions.

Unknown:

The area of the playground in m^2.

Reasoning:

Let's draw a diagram of the roller to visualize it better.



The roller is cylindrical in shape and hence it is considered as a right circular cylinder and in one revolution it covers the CSA of the roller. It takes 500 complete revolutions to move once over to level a playground.

Then the area of the playground will be equal to the 500 times the CSA of the roller.

The radius and height of the cylinder are r and h respectively.
CSA of the cylinder $= 2\pi rh$

Solution:

Length of the roller, $h = 120\ cm$

Radius of the roller, $r = \dfrac{84cm}{2} = 42\ cm$

CSA of the roller = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 42cm \times 120cm$$

$$= 31680\ cm^2$$

Area of the playground = Area levelled by the cylinder in 500 revolutions.

$$= 500 \times 31680 cm^2$$

$$= 15840000\ cm^2$$

$$= \frac{15840000}{10000} m^2$$

$$= 1584\ m^2$$

Answer:

Area of the playground $= 1584\ m^2$

Q5. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of ₹12.50 per m^2.

Difficulty Level:

Medium

Known:

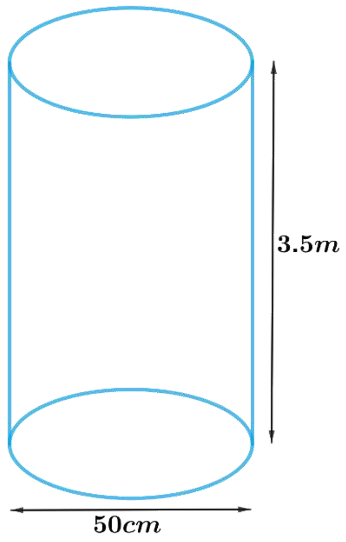
The diameter of the cylindrical pillar is 50 cm and its height is 3.5 m and the rate of painting is ₹12.50 per m^2

Unknown:

The cost of painting the curved surface of the pillar.

Reasoning:

Let's draw a diagram of the cylindrical pillar to visualize it better.



Since the pillar is cylindrical its curved surface area will be equal to CSA of cylinder.

The radius and height of the cylinder are r and h respectively.

CSA of the cylinder $= 2\pi rh$

We can calculate the cost of painting by multiplying CSA of the pillar and rate of painting.

Solution:

Length of the pillar, $h = 3.5\ m$

Radius of the pillar, $r = \frac{50cm}{2} = \frac{25m}{100} = 0.25m$

CSA of the pillar $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 0.25m \times 3.5m$$

$$= 5.5m^2$$

Cost of painting the curved surface area at ₹12.50 per $m^2 = 12.50 \times 5.5 = 68.75$

Answer:

Cost of painting the curved surface of the pillar is ₹68.75.

Q6. Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height.

Difficulty Level:

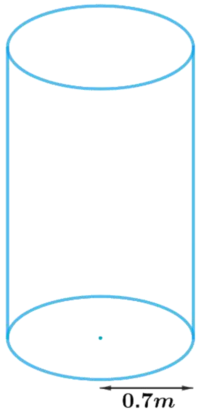
Medium

Known:

CSA of the cylindrical is 4.4 m^2 and its radius is 0.7 m.

Unknown:
The height of the cylinder.

Reasoning:
Let's draw a diagram of the cylinder to visualize it better.



The radius and height of the cylinder are r and h respectively.
CSA of the cylinder $= 2\pi rh$

We can calculate the cost of painting by multiplying CSA of the pillar and rate of painting.

Solution:
Let the radius and height of the cylinder are r and h respectively.
Radius of the cylinder, $r = 0.7m$
CSA of the pillar $= 4.4\ m^2$

$$2\pi rh = 4.4\ m^2$$

$$2 \times \frac{22}{7} \times 0.7m \times h = 4.4\ m^2$$

$$h = \frac{7 \times 4.4m^2}{2 \times 22 \times 0.7m}$$

$$h = 1m$$

Answer:
The height of the right circular cylinder is 1m.

Q7. The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find

(i) Its inner curved surface area

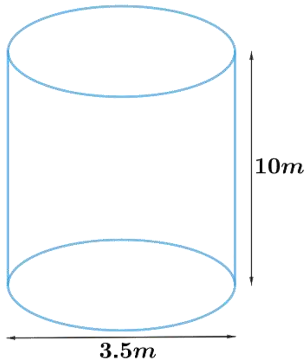
(ii) The cost of plastering this curved surface at the rate of ₹40 per m^2 .

Difficulty Level:
Medium

Known:
The inner diameter of the circular well is 3.5 m and depth is 10 m. Rate of plastering is ₹40 per m^2.

Unknown:
The inner curved surface area and cost of its plastering.

Reasoning:
Let's draw a diagram of the circular well to visualize it better.



Since the well is cylindrical its curved surface area will be equal to CSA of cylinder.
The radius and height of the cylinder are r and h respectively.
CSA of the cylinder $= 2\pi rh$

We can calculate the cost of plastering by multiplying CSA of the well and rate of plastering.

Solution:
Diameter of the well, $d = 3.5m$

Radius of the well, $r = \frac{d}{2} = \frac{3.5m}{2} = 1.75m$

Depth of the well, $h = 10m$

Inner CSA of the well $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 1.75m \times 10m$$

$$= 110\ m^2$$

Cost of plastering the curved surface area at ₹40 per $m^2 = 110 \times 40 = 4400$
= ₹4400

Answer:
The inner curved surface area is 110 m^2
Cost of plastering the curved surface is ₹4400

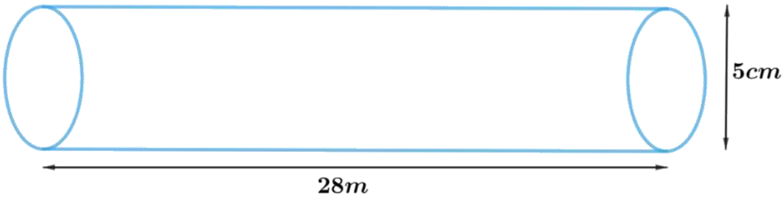
Q8. In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system.

Difficulty Level:
Medium

Known:
The length is 28 m and diameter is 5cm of the cylindrical pipe.

Unknown:
Total radiating surface area in the system.

Reasoning:
Let's draw a diagram of the circular well to visualize it better.



Since the pipe is cylindrical its total radiating surface area will be equal to CSA of cylinder.

The radius and height of the cylinder are r and h respectively.
CSA of the cylinder $= 2\pi rh$

Solution:
Diameter of the pipe, $d = 5cm$

Radius of the pipe, $r = \frac{d}{2} = \frac{5cm}{2} = \frac{2.5}{100}m = 0.025m$

Length of the pipe, $h = 28m$

Total radiating surface area of the pipe $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 0.025m \times 28m$$

$$= 4.4\ m^2$$

Answer:
The total radiating surface is 4.4 m^2

Q9. Find

i. The lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.

ii. How much steel was actually used, if $\frac{1}{12}$ of the steel actually used was wasted in making the tank?

Difficulty Level:
Medium

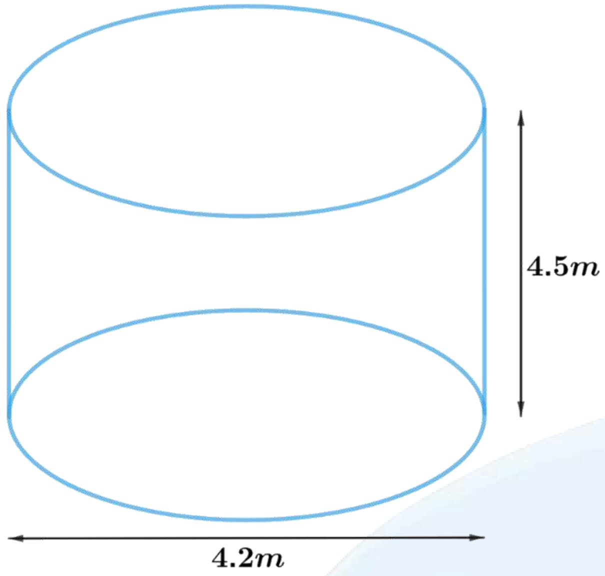
Known:
The diameter is 4.2 m and height is 4.5 m of the closed cylindrical petrol storage tank.

$\frac{1}{12}$ of the steel used was wasted in making the tank.

Unknown:
Lateral or curved surface area of the storage and area of steel used in making the tank.

Reasoning:

Let's draw a diagram of the cylindrical tank to visualize it better.



Since the storage tank is a closed cylinder its curved surface area will be equal to CSA of cylinder and area of steel in the tank will be equal to the TSA of the cylinder.

The radius and height of the cylinder are r and h respectively.
CSA of the cylinder $= 2\pi rh$
TSA of the cylinder $= 2\pi r(h+r)$

Since $\frac{1}{12}$ of the steel used was wasted in making the tank hence $\left(1-\frac{1}{12}\right)=\frac{11}{12}$ of the steel is present in the tank

Then the area of steel used in making the tank $=\frac{12}{11}\times$ TSA of the tank

Solution:

Diameter of the tank, $d = 4.2m$

Radius of the tank, $r = \frac{d}{2} = \frac{4.2m}{2} = 2.1m$

Height of the tank, $h = 4.5m$

Lateral or curved surface area of the tank $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 2.1m \times 4.5m$$

$$= 59.4\ m^2$$

TSA of the tank $= 2\pi r(h+r)$

$$= 2 \times \frac{22}{7} \times 2.1m \times (4.5m + 2.1m)$$

$$= 2 \times \frac{22}{7} \times 2.1m \times 6.6m$$

$$= 87.12\ m^2$$

The area of steel used in making the tank $= \frac{12}{11} \times$ TSA of the tank

$$= \frac{12}{11} \times 87.12\ m^2$$

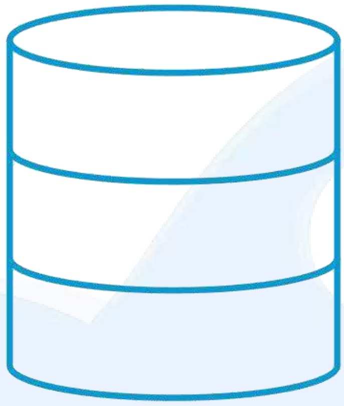
$$= 95.04\ m^2$$

Answer:

Curve surface area $= 59.4 m^2$

Steel actually used $= 95.04 m^2$

Q10. In Fig. 13.12, you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade.



Difficulty Level:

Medium

Known:

The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame

Unknown:

Cloth required for covering the lamp shade.

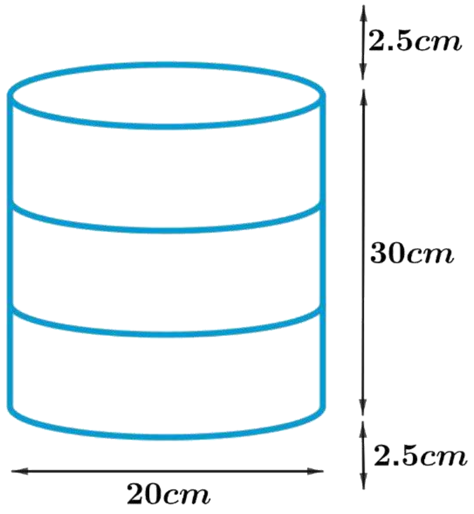
Reasoning:

Since the frame is cylindrical in shape the cloth required to cover the lampshade will be equal to CSA of cylinder.

As a margin of 2.5 cm is to be given for folding it over the top and bottom of the frame then the height of the cloth used in frame will be equal to the height of cylinder plus 2.5 cm margin on both sides

The radius and height of the cylinder are r and h respectively.

CSA of the cylinder $= 2\pi rh$



Solution:

Diameter of the frame, $d = 20cm$

Radius of the frame, $r = \frac{d}{2} = \frac{20cm}{2} = 10cm$

Height of the frame $= 30cm$

Height of the cloth, $h = 30cm + 2.5cm + 2.5cm = 35cm$

Cloth required for covering the lampshade $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 10cm \times 35cm$$

$$= 2200\ cm^2$$

Answer:

Cloth required $= 2200\ cm^2$

Q11. The students of a Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition?

Difficulty Level:

Medium

Known:

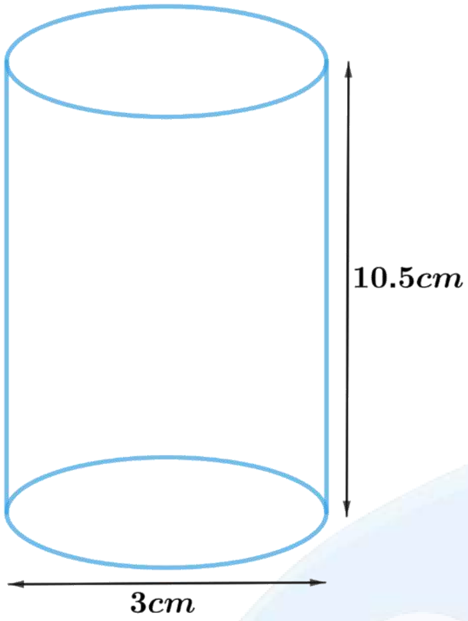
Cylindrical penholder of radius 3 cm and height 10.5 cm. There were 35 competitors.

Unknown:

Cardboard required to be bought for the competition.

Reasoning:

Let's draw a diagram of the cylindrical penholder to visualize it better.



Since each penholder is cylindrical in shape and opened at top, the cardboard required for making and decorating penholder will be equal to CSA of cylinder and area of base.
As there were 35 competitors then the total area of cardboard can be calculated by multiplying CSA of cylinder and number of students.

The radius and height of the cylinder are r and h respectively.
CSA of the cylinder $= 2\pi rh$

Area of cardboard required for each penholder $= 2\pi rh + \pi r^2$

$$= \pi r(2h + r)$$

Area of cardboard required for 35 penholders $= 35 \times \pi r(2h + r)$

Solution:

Radius of the penholder, $r = 3cm$
Height of the penholder, $h = 10.5cm$
Area of cardboard required for 35 penholders $= 35 \times \pi r(2h + r)$

$$= 35 \times \pi r(2h + r)$$

$$= 35 \times \frac{22}{7} \times 3cm \times (2 \times 10.5cm + 3cm)$$

$$= 330cm \times 24cm$$

$$= 7920cm^2$$

Answer:

$7920\ cm^2$ of cardboard was required to be bought for the competition.

Chapter - 13: Surface Area and Volumes

Exercise 13.3 (Page 221 of Grade 9 NCERT Textbook)

Assume $\pi = \frac{22}{7}$ unless stated otherwise

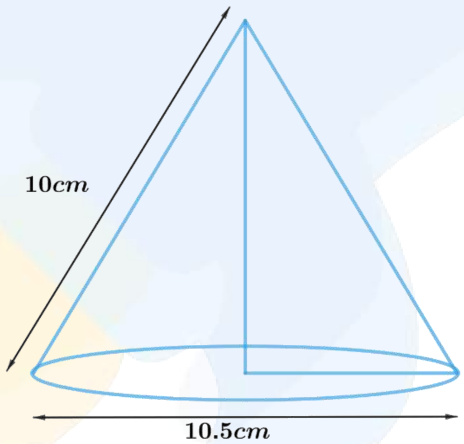
Q1. Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.

Difficulty Level:
Easy

Known:
Diameter of the base of the cone is 10.5 cm and slant height is 10 cm.

Unknown:
Curved surface area.

Reasoning:
Let's draw a diagram of the cone to visualize it better.



Curved surface area of a right circular cone of base radius, r and slant height, l is πrl

Solution:
Diameter, $d = 10.5 cm$

Radius, $r = \frac{10.5}{2} cm$

Slant height, $l = 10\ cm$

Curved surface area $= \pi rl$

$= \frac{22}{7} \times \frac{10.5}{2} \times 10$

$= 165\ cm^2$

Answer:

Curved surface area of the cone $= 165\ cm^2$

Q2. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

Difficulty Level:

Easy

Known:

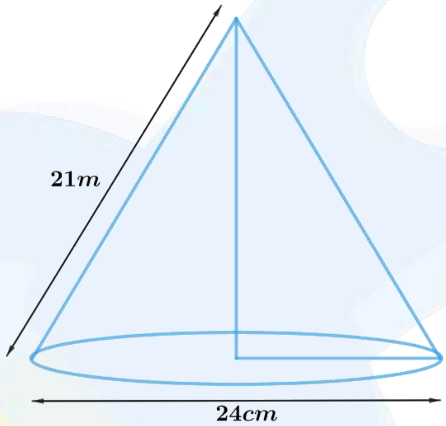
Diameter of the box and slant height of the cone.

Unknown:

Total surface area of the cone.

Reasoning:

Let's draw a diagram of the cone to visualize it better.



The total surface area of the cone is the sum of the curved surface area and area of the base which is a circle.

Curved surface area of a right circular cone of base radius, r and slant height, l is πrl

Total surface area of the cone $= \pi rl + \pi r^2 = \pi r(l + r)$

Solution:

Diameter, $d = 24m$

Radius, $r = \frac{24m}{2} = 12m$

Slant height, $l = 21\,m$

Total surface area of the cone $= \pi r(l + r)$

$$= \frac{22}{7} \times 12m \times (12m + 21m)$$

$$= \frac{22}{7} \times 12m \times 33m$$

$$= \frac{8712}{7} m^2$$

$$= 1244.57\ m^2$$

Answer:

Total surface area of the cone $= 1244.57\, m^2$

Q3. Curved surface area of a cone is $308\ cm^2$ and its slant height is 14 cm. Find

(i) Radius of the base

(ii) Total surface area of the cone.

Difficulty Level:

Easy

Known:

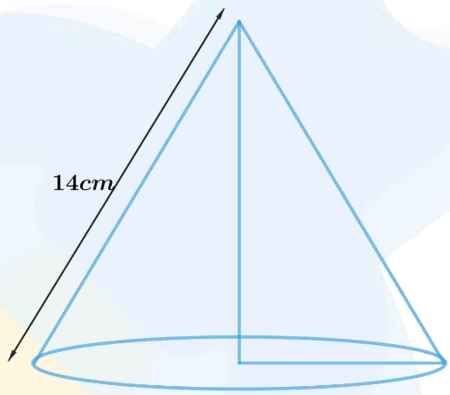
Curved surface area of the cone and its slant height.

Unknown:

Radius of the base.

Reasoning:

Let's draw a diagram of the cone to visualize it better.



The total surface area of the cone is the sum of the curved surface area and area of the base which is a circle.

Curved surface area of a right circular cone of base radius, r and slant height, l is πrl

Total surface area of the cone $= \pi rl + \pi r^2 = \pi r(l + r)$

Solution:

Let the radius be r

Slant height, $l = 14\ cm$

Curved surface area $= 308 cm^2$

$$\pi rl = 308$$

$$r = \frac{308cm^2}{\pi l}$$

$$r = \frac{308cm^2}{14cm} \times \frac{7}{22}$$

$$= 7\ cm$$

Total surface area $= \pi r(l+r)$

$$= \frac{22}{7} \times 7cm \times (7cm + 14cm)$$

$$= 22cm \times 21cm$$

$$= 462\ cm^2$$

Answer:

Radius of the cone is 7 cm.

Total surface area of the cone is 462 cm^2.

Q4. A conical tent is 10 m high and the radius of its base is 24 m. Find

(i). Slant height of the tent.

(ii). Cost of the canvas required to make the tent, if the cost of $1m^2$ canvas is ₹70.

Difficulty Level:

Easy

Known:

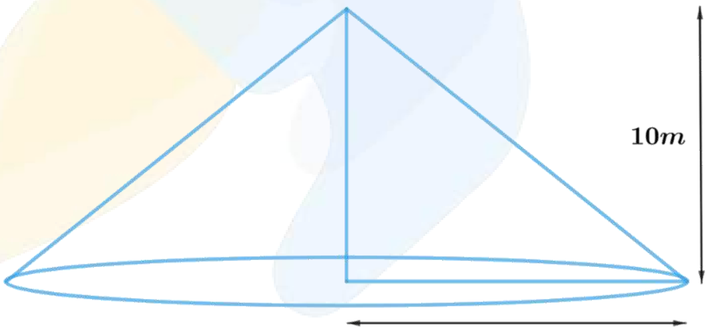
Height of the cone and its base radius.

Unknown:

Slant height of the tent.

Reasoning:

Let’s draw a diagram of the cone to visualize it better.



The total surface area of the cone is the sum of the curved surface area and area of the base which is a circle.

Curved surface area of a right circular cone of base radius, *r* and slant height, *l* is πrl

Slant height, $l = \sqrt{r^2 + h^2}$ where *h* is the height of the cone.

We can calculate the cost of the canvas required to make the tent, at ₹70 per m^2 by multiplying CSA and rate.

Solution:

Radius, $r = 24m$

Height, $h = 10m$

Slant height, $l = \sqrt{r^2 + h^2}$

$$= \sqrt{(24m)^2 + (10m)^2}$$
$$= \sqrt{576m^2 + 100m^2}$$
$$= \sqrt{676m^2}$$
$$= 26\ m$$

Curved surface area of the cone $= \pi rl$

$$= \frac{22}{7} \times 24m \times 26m$$
$$= \frac{13728}{7} m^2$$

The cost of the canvas required to make the tent, at ₹70 per m^2 $= 70 \times \frac{13728}{7}$

= ₹137280

Answer:

Slant height of the tent is 26 m.
The cost of the canvas is ₹137280.

Q5. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm (Use $\pi = 3.14$)

Difficulty Level:

Medium

Known:

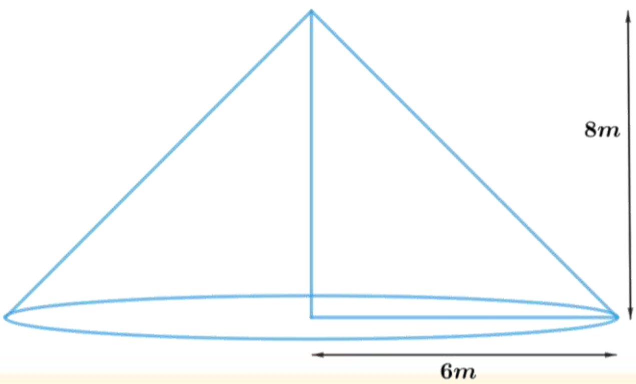
3 m wide tarpaulin will be used to make a conical tent of height 8m and radius 6 m with a 20 cm of stitching margins and wastage.

Unknown:

Length of the tarpaulin.

Reasoning:

Let's draw a diagram of the conical tent to visualize it better.



Since, the tent is conical in shape the area of tarpaulin will be required to make the tent will be same as the curved surface area of the cone.
Curved surface area of a right circular cone of base radius, r and slant height, l is πrl
Slant height, $l=\sqrt{r^2+h^2}$ where h is the height of the cone.

Length of the tarpaulin can be calculated by dividing its area by its breadth.
Since, the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm then actual length of the tarpaulin can be obtained by adding 20cm to the length of the tarpaulin.

Solution:
Radius, $r=6m$
Height, $h=8m$
Slant height, $l=\sqrt{r^2+h^2}$

$$=\sqrt{(6m)^2+(8m)^2}$$
$$=\sqrt{36m^2+64m^2}$$
$$=\sqrt{100m^2}$$
$$=10\ m$$

Curved surface area $=\pi rl$

$$=3.14\times 6m\times 10m$$
$$=188.4\ m^2$$

Width of the tarpaulin is 3m
Area of the tarpaulin $=188.4\ m^2$
Length of the tarpaulin $=\dfrac{188.4\ m^2}{3m}=62.8m$
Extra length of the material $=20cm=\dfrac{20}{100}m=0.2m$
Actual length required $=62.8m+0.2m=63m$

Answer:
Length of the tarpaulin required is 63 m

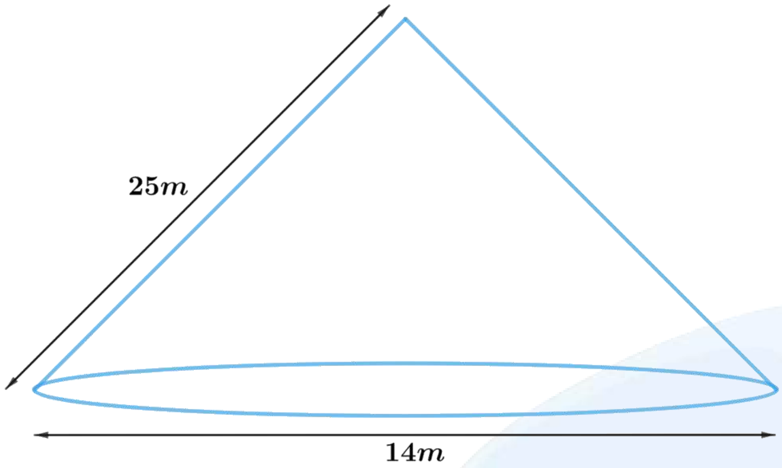
Q6. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white washing its curved surface at the rate of ₹210 per 100 m^2.

Difficulty Level:
Medium

Known:
The slant height of the conical tomb is 25 m and base diameter is 14 m. Rate of white washing is ₹210 per 100 m^2.

Unknown:
Cost of white washing the curved surface area of the tomb.

Reasoning:
Let's draw a diagram of the conical tomb to visualize it better.



Since the curved surface area of the tomb is to be whitewashed.
Curved surface area of a right circular cone of base radius, *r* and slant height, *l* is πrl
Slant height, $l = \sqrt{r^2 + h^2}$ where *h* is the height of the cone.

We can calculate the cost of the whitewashing, at ₹210 per 100 m^2 by multiplying CSA and rate.

Solution:
Diameter, $d = 14\ m$

Radius, $r = \frac{14m}{2} = 7m$

Slant height, $l = 25\ m$

Curved surface area $= \pi rl$

$$= \frac{22}{7} \times 7m \times 25m$$

$$= 550m^2$$

Cost of the whitewashing at ₹210 per 100 m^2 $= \frac{210}{100} \times 550 = 1155$

Answer:
Cost of white washing the conical tomb is ₹1155.

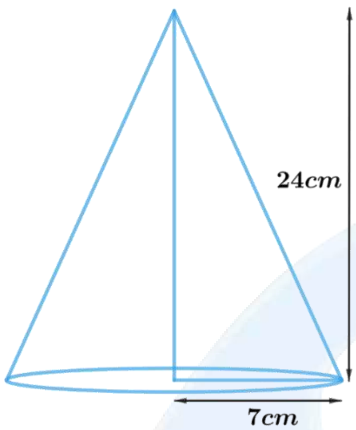
Q7. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Difficulty Level:
Medium

Known:
Caps box radius and height.
Number of caps.

Unknown:
Area of the sheet required to 10 caps.

Reasoning:
Let's draw a diagram of the joker's cap to visualize it better.



Since the cap is conical in shape the area of the sheet required to make each cap will be equal to the curved surface area of the cone.
Curved surface area of a right circular cone of base radius, r and slant height, l is πrl
Slant height, $l = \sqrt{r^2 + h^2}$ where h is the height of the cone.

We can calculate the area of the sheet required to make 10 such caps by multiplying CSA and number of caps.

Solution:
Radius, $r = 7cm$
Height, $h = 24cm$
Slant height, $l = \sqrt{r^2 + h^2}$

$$= \sqrt{(7cm)^2 + (24cm)^2}$$
$$= \sqrt{49cm^2 + 576cm^2}$$
$$= \sqrt{625cm^2}$$
$$= 25cm$$

Area of the sheet required to make each cap $= \pi rl$

$$= \frac{22}{7} \times 7cm \times 25cm$$
$$= 550\ cm^2$$

Area of the sheet required to make 10 such caps $= 10 \times 550cm^2 = 5500cm^2$

Answer:
The area of the sheet required to make 10 such caps is 5500 cm^2.

Q8. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is ₹12 per m^2. What will be the cost of painting all these cones?

(Use $\pi = 3.14$ and take $\sqrt{1.04} = 1.02$)

Difficulty Level:

Medium

Known:

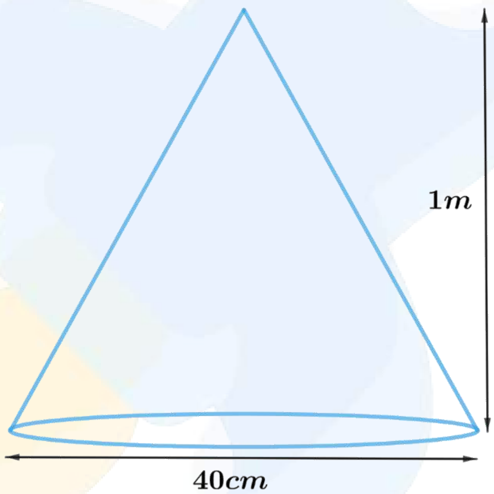
Base diameter of the cone is 40 cm and its height is 1m. There are 50 cones and the rate of painting the cones is ₹12 per m^2.

Unknown:

The cost of painting the 50 cones.

Reasoning:

Let's draw a diagram of the cone to visualize it better.



Since the outer side of each cone is to be painted then the area to be painted will be equal to the curved surface area of the cone.

Curved surface area of a right circular cone of base radius, r and slant height, l is πrl

Slant height, $l = \sqrt{r^2 + h^2}$ where h is the height of the cone.

We can calculate the cost of painting of such 50 cones at ₹12 per m^2 by multiplying CSA of 50 cones and rate of cones.

Solution:

Diameter, $d = 40cm = \frac{40}{100}m = 0.4m$

Radius, $r = \frac{0.4m}{2} = 0.2m$

Height, $h = 1\,m$

Slant height, $l = \sqrt{r^2 + h^2}$

$$= \sqrt{(0.2m)^2 + (1m)^2}$$

$$= \sqrt{0.04m^2 + 1m^2}$$

$$= \sqrt{1.04m^2}$$

$$= 1.02m \quad \text{(approx.)}$$

Curved surface area $= \pi r l$

$$= 3.14 \times 0.2m \times 1.02m$$

$$= 0.64056\ m^2$$

Curved surface area of 50 cones $= 50 \times 0.64056\ m^2 = 32.028\ m^2$

Cost of painting of 50 cones at ₹12 per m^2 $= 32.028 \times 12$

= ₹384.34

Answer:

Cost of painting all the cones is ₹384.34 (approx.)

Chapter - 13: Surface Area and Volumes

Exercise 13.4 (Page 225 of Grade 9 NCERT Textbook)

Assume $\pi = \frac{22}{7}$ unless stated otherwise

Q1. Find the surface area of a sphere of radius:

(i) 10.5 cm (ii) 5.6 cm (iii) 14 cm

Difficulty Level:

Easy

Known:

Radius of the sphere:

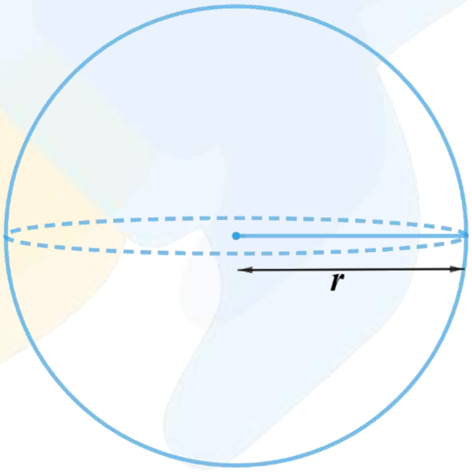
(i) 10.5 cm
(ii) 5.6 cm
(iii) 14 cm

Unknown:

Surface area of the sphere

Reasoning:

Let's draw a diagram of the sphere to visualize it better.



Surface area of a sphere of radius, $r = 4\pi r^2$

Solution:

(i) Radius, $r = 10.5 cm$

Surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (10.5)^2$$
$$= 1386\ cm^2$$

(ii) Radius, $r = 5.6cm$

Surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (5.6)^2$$

$$= 394.24\ cm^2$$

(iii) Radius, $r = 14cm$

Surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (14)^2$$

$$= 2464\ cm^2$$

Answer:

(i) $1386\ cm^2$

(ii) $394.24\ cm^2$

(iii) $2464\ cm^2$

Q2. Find the surface area of a sphere of diameter:

(i) 14 cm (ii) 21 cm (iii) 3.5 m

Difficulty Level:

Medium

Known:

Diameter of the sphere.

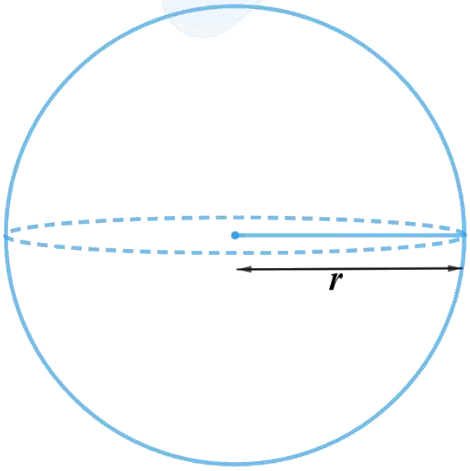
(i) 14 cm
(ii) 21 cm
(iii) 3.5 cm

Unknown:

Surface area of the sphere.

Reasoning:

Let's draw a diagram of the sphere to visualize it better.



Surface area of a sphere of radius, $r = 4\pi r^2$

Solution:

(i) Diameter, $d = 14cm$

Radius, $r = \frac{14cm}{2} = 7cm$

Surface area $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 7cm \times 7cm$

$= 616cm^2$

(ii) Diameter, $d = 21cm$

Radius, $r = \frac{21}{2}cm$

Surface area $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times \frac{21}{2}cm \times \frac{21}{2}cm$

$= 1386cm^2$

(iii) Diameter, $d = 3.5cm$

Radius, $r = \frac{3.5cm}{2} = 1.75cm$

Surface area $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 1.75cm \times 1.75cm$

$= 38.5cm^2$

Answer:

(i) $616\ cm^2$

(ii) $1386\ cm^2$

(iii) $38.5\ m^2$

Q3. Find the total surface area of a hemisphere of radius 10 cm. (Use π = 3.14)

Difficulty Level:

Medium

Known:

Radius of the hemisphere is 10 cm.

Unknown:

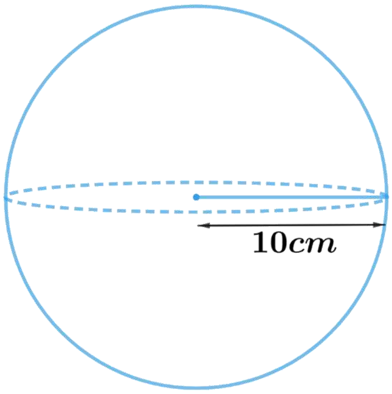
Total surface area of the hemisphere.

Reasoning:

A hemisphere is half of a sphere having one circular surface at the top.

Total surface area of a hemisphere is the half surface area of sphere and the top circular area.

TSA of hemisphere $= 3\pi r^2$



Solution:

Radius of the hemisphere $r = 10cm$

Total Surface area $= 3\pi r^2$

$= 3 \times 3.14 \times 10cm \times 10cm$

$= 942\ cm^2$

Answer:

Total surface area of the hemisphere is 942 cm^2.

Q4. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Difficulty Level:

Medium

Known:

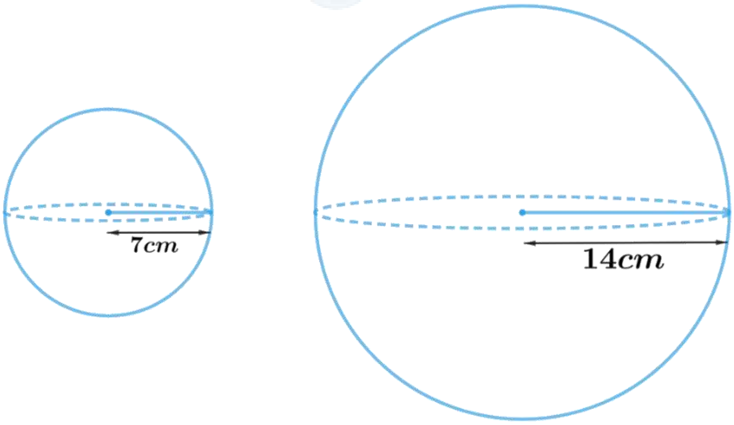
The radius of a spherical balloon is 7 cm and becomes 14 cm as air is being pumped into it.

Unknown:

Ratio of the surface areas of the balloon.

Reasoning:

Let's draw a diagram of the balloons to visualize it better.



Surface area of a sphere of radius, $r = 4\pi r^2$

Solution:

Radius of the balloon before pumping air, $r_1 = 7cm$

Radius of the balloon after pumping air, $r_2 = 14cm$

Surface area of the balloon before pumping air, $CSA_1 = 4\pi r_1^2$

Surface area of the balloon after pumping air, $CSA_2 = 3\pi r_2^2$

Ratio of the surface areas of the balloon, $= \frac{CSA_1}{CSA_2} = \frac{4\pi r_1^2}{4\pi r_2^2}$

$$= \frac{r_1^2}{r_2^2}$$

$$= \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{7}{14}\right)^2$$

$$= \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{4}$$

Answer:

The ratio of the surface areas of the balloons $= 1:4$

Q5. A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tinplating it on the inside at the rate of ₹16 per 100 cm^2.

Difficulty Level:

Medium

Known:

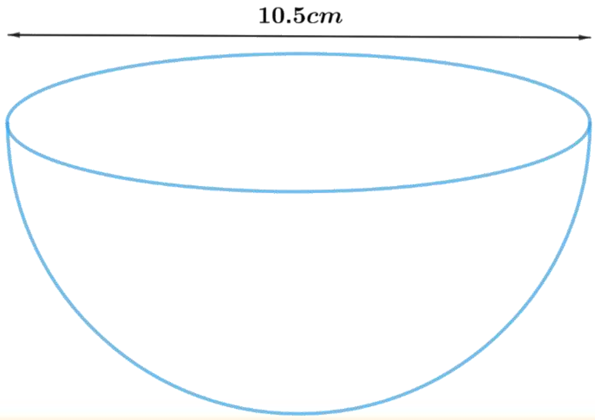
Inner diameter of hemispherical ball is 10.5 cm. Rate of tinplating is ₹16 per 100 cm^2.

Unknown:

Cost of tinplating.

Reasoning:

Let's draw a diagram of the bowl to visualize it better.



Hemisphere is half of a sphere, so curved surface area is half of surface area of sphere.
CSA of hemisphere of radius, $r = 2\pi r^2$.

Since the bowl is to be tin plated from inside then the area to be tin plated will be equal to the CSA of the hemisphere.

We can find the cost of tinplating the bowl by multiplying its inner CSA and the rate.

Solution:

Inner diameter, $d = 10.5cm$

Inner radius, $r = \dfrac{10.5cm}{2} = 5.25cm$

CSA of hemispherical bowl $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times 5.25cm \times 5.25cm$$

$$= 173.25\ cm^2$$

The cost of tinplating the bowl at ₹16 per 100 cm^2 $= \dfrac{173.25}{100} \times 16 = 27.72$

Answer:

The cost of tin plating is ₹27.72.

Q6. Find the radius of a sphere whose surface area is 154 cm^2.

Difficulty Level:

Medium

Known:

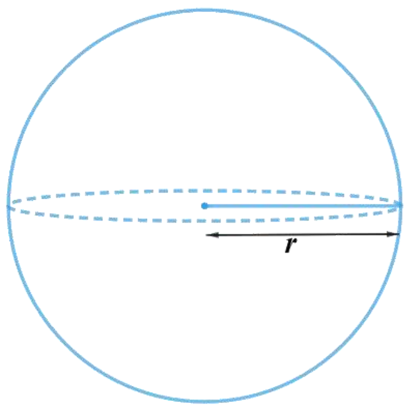
Surface area of the sphere is 154 cm^2.

Unknown:

Radius of the sphere.

Reasoning:

Let's draw a diagram of the sphere to visualize it better.



Surface area of sphere of radius, $r = 4\pi r^2$

Solution:

Surface area of the sphere $= 4\pi r^2 = 154cm^2$

$$r^2 = \frac{154cm^2}{4\pi}$$

$$r^2 = \frac{154cm^2}{4} \times \frac{7}{22}$$

$$r^2 = \frac{49}{4} cm^2$$

$$r = \sqrt{\frac{49}{4} cm^2}$$

$$r = \frac{7}{2} cm$$

$$= 3.5\ cm$$

Answer:

Radius of the sphere is 3.5 cm.

Q7. The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Difficulty Level:

Medium

Known:

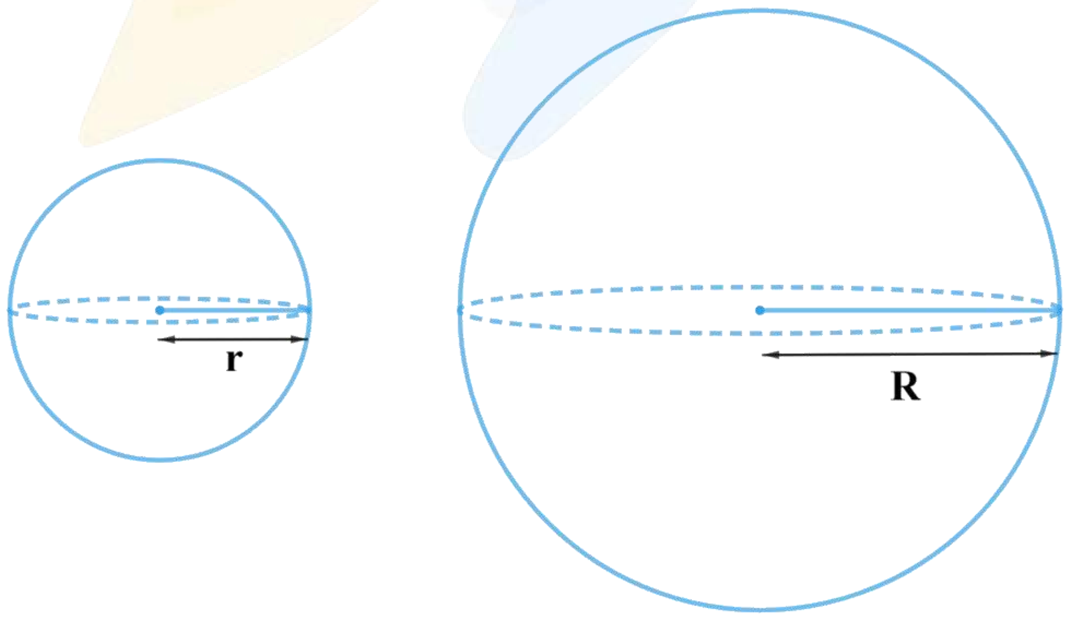
The diameter of the moon is approximately one fourth of the diameter of the earth.

Unknown:

The ratio of their surface areas.

Reasoning:

Let's draw a diagram of the moon and earth to visualize it better.



Since the moon and earth are almost spherical in shape.
Surface area of sphere of radius, $r = 4\pi r^2$

Solution:

Let the radius of the earth be R
and the radius of the moon be r

Diameter of the moon $= \frac{1}{4} \times$ diameter of the earth

Radius of the moon $= \frac{1}{4} \times$ radius of the earth

$$r = \frac{1}{4} \times R$$

$$\frac{r}{R} = \frac{1}{4}$$

Surface area of the earth $= 4\pi R^2$
Surface area of the moon $= 4\pi r^2$

Ratio of their surface areas $= \frac{4\pi r^2}{4\pi R^2}$

$$= \frac{r^2}{R^2}$$

$$= \left(\frac{r}{R}\right)^2$$

$$= \left(\frac{1}{4}\right)^2$$

$$= \frac{1}{16}$$

Answer:

Ratio of their surface areas $= 1:16$

Q8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.

Difficulty Level:

Medium

Known:

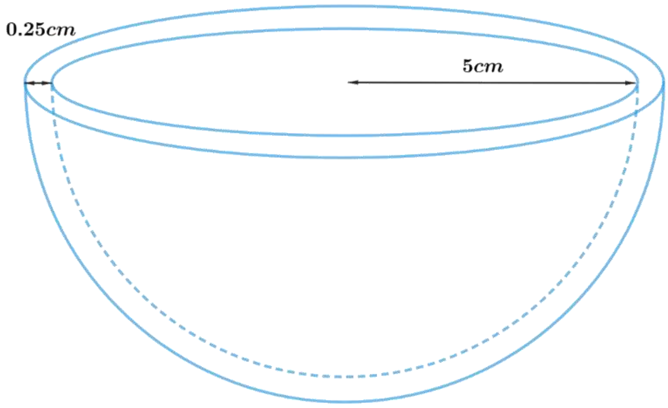
Inner radius of the bowl is 5 cm and the thickness of the steel is 0.25cm.

Unknown:

The outer curved surface area of the bowl.

Reasoning:

Let's draw a diagram of the bowl to visualize it better.



Since the hemispherical bowl is made of 0.25cm thick steel, we can find the outer radius of the bowl by adding thickness to the inner radius.

Then outer CSA can be calculated easily.

Curved Surface area of hemisphere of radius, $r = 2\pi r^2$

Solution:
Inner radius of the bowl, $r = 5cm$
Thickness of steel $= 0.25cm$
Outer radius of the bowl, $R = 5cm + 0.25cm = 5.25cm$

Outer CSA of the hemisphere $= 2\pi R^2$

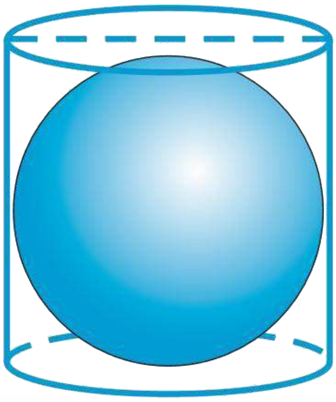
$$= 2 \times \frac{22}{7} \times 5.25cm \times 5.25cm$$

$$= 173.25\ cm^2$$

Answer:
The outer curved surface area of the hemisphere $= 173.25\ cm^2$

Q9. A right circular cylinder just encloses a sphere of radius *r* (see Fig. 13.22). Find

(i) Surface area of the sphere
(ii) Curved surface area of the cylinder
(iii) Ratio of the areas obtained in (i) and (ii).

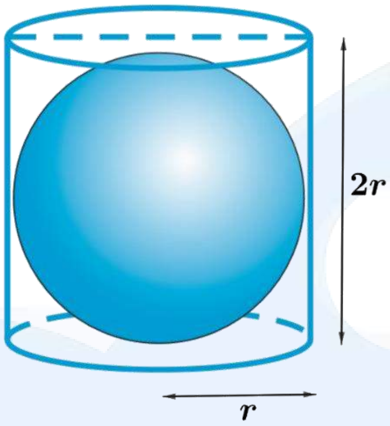


Difficulty Level:
Medium

Known:
A right circular cylinder just encloses a sphere of radius r

Unknown:
Surface area of the cylinder, the CSA of the cylinder and their ratio.

Reasoning:
Since the cylinder just encloses the sphere as we can see in figure, radius of the cylinder will be equal to the radius of the sphere and height of the cylinder will be equal to the diameter of the sphere.



Surface area of sphere of radius, $r = 4\pi r^2$
CSA of cylinder of radius, r and height, $h = 2\pi rh$
Then we can easily find their ratios.

Solution:
Radius of the sphere = Radius of the cylinder = r
Height of the cylinder, h = diameter of the sphere = $2r$

(i) Surface area of the sphere $= 4\pi r^2$

(ii) Curved surface area of the cylinder $= 2\pi rh$

$$= 2\pi r \times 2r$$

$$= 4\pi r^2$$

(iii) Ratio of the areas obtained in (i) and (ii) $= \dfrac{4\pi r^2}{4\pi r^2} = \dfrac{1}{1}$

Answer:
Surface area of the sphere $= 4\pi r^2$
Curved surface area of cylinder $= 4\pi r^2$
Ratio between their area $= 1:1$

Chapter - 13: Surface Area and Volumes

Exercise 13.5 (Page 228 of Grade 9 NCERT Textbook)

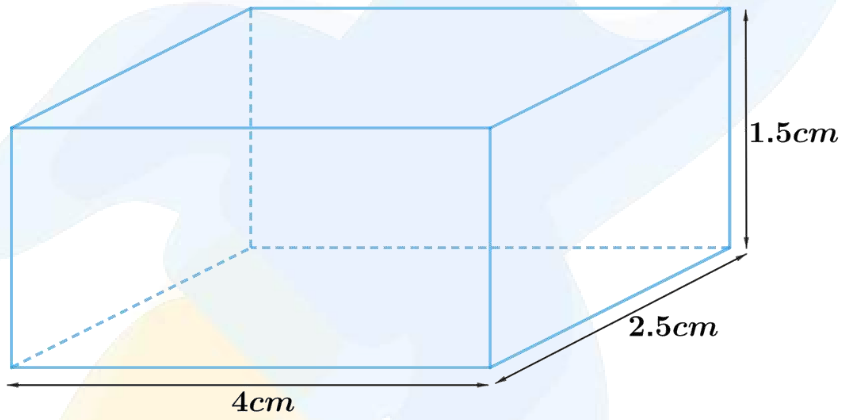
Q1. A matchbox measures 4 cm × 2.5 cm × 1.5 cm. What will be the volume of a packet containing 12 such boxes?

Difficulty Level:
Medium

Known:
Dimensions of matchbox $4cm \times 2.5cm \times 1.5cm$

Unknown:
The volume of a packet containing 12 matchboxes.

Reasoning:
Since the matchbox is cuboidal in shape, volume of each matchbox will be equal to volume of the cuboid.



Volume of the cuboid of length, l breadth, b and height, $h = lbh$

Solution:
Length of the matchbox, $l = 4\ cm$
Breadth of the matchbox, $b = 2.5\ cm$
Height of the matchbox, $h = 1.5\ cm$

Volume of each matchbox $= l \times b \times h$
$= 4cm \times 2.5cm \times 1.5cm$
$= 15cm^3$

Volume of 12 matchboxes $= 12 \times 15cm^3 = 180cm^3$

Answer:
Volume of a packet containing 12 such boxes is 180 cm^3.

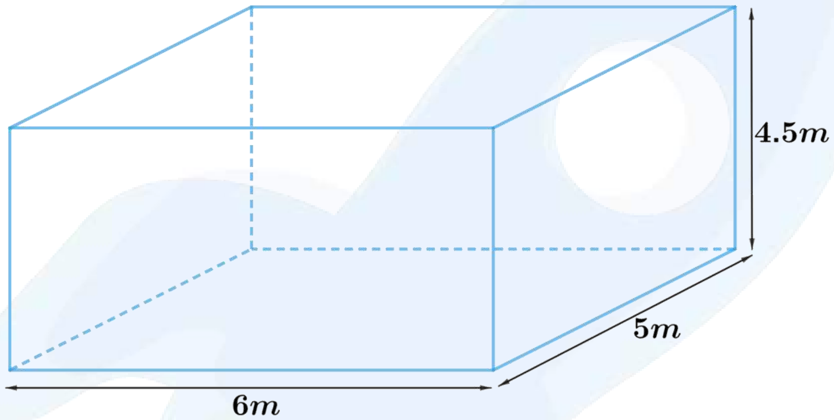
Q2. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many liters of water can it hold? ($1\ m^3 = 1000\ l$)

Difficulty Level:
Medium

Known:
Length, breadth and depth of the cuboidal water tank are 6 m, 5 m and 4.5 m respectively.

Unknown:
The volume of water in litres tank can hold.

Reasoning:
Since the water tank is cuboidal in shape, volume of water in the tank will be equal to volume of the cuboid.



Volume of the cuboid of length, l breadth, b and height, $h = lbh$

Solution:
Length of the cuboidal tank, $l = 6\ m$
Breadth of the cuboidal tank, $b = 5\ m$
Height of the cuboidal tank, $h = 4.5\ m$

Volume of the cuboidal tank $= lbh$

$$= 6m \times 5m \times 4.5m$$
$$= 135\ m^3$$
$$= 135 \times 1000l \qquad (\because 1m^3 = 1000l)$$
$$= 135000l$$

Answer:
The cuboidal water tank can hold 135000 litres of the water.

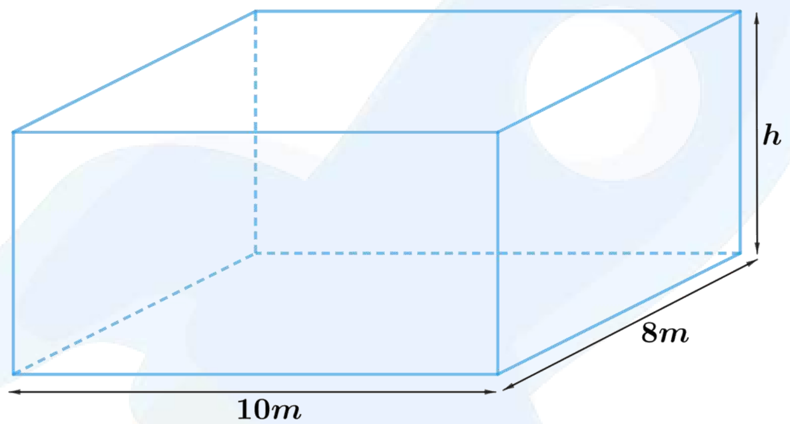
Q3. A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?

Difficulty Level:
Medium

Known:
Length and breadth of the cuboidal vessel are 10 m and 8 m respectively. It must hold 380 m^3 of a liquid.

Unknown:
The height of the vessel.

Reasoning:
Since the vessel is cuboidal in shape, volume of liquid in the vessel will be equal to volume of the cuboid.



Volume of the cuboid of length, l breadth, b and height, h $= lbh$

Solution:
Let the height of the cuboidal vessel be h.
Length of the cuboidal vessel, $l = 10$ m
Breadth of the cuboidal vessel, $b = 8$ m
Capacity of the cuboidal vessel = 380 m^3

Volume of the liquid in the cuboidal vessel $= lbh$

$$lbh = 380m^3$$
$$h = \frac{380m^3}{lb}$$
$$h = \frac{380m^3}{10m \times 8m}$$
$$h = 4.75m$$

Answer:
The cuboidal vessel must be made 4.75 m high.

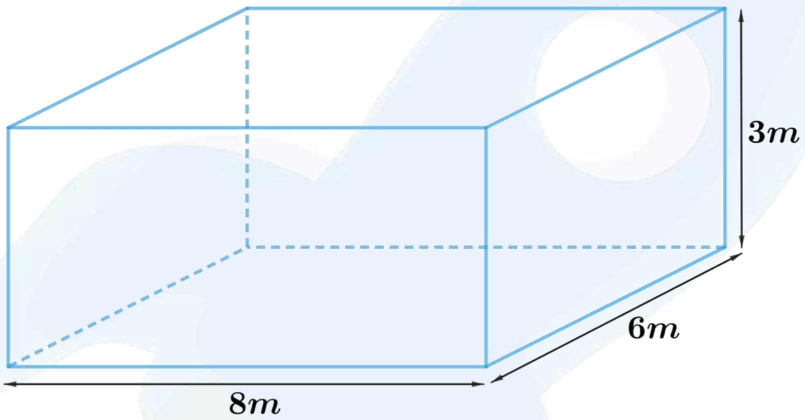
Q4. Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of ₹30 per m^3.

Difficulty Level:
Medium

Known:
Length, breadth and depth of the cuboidal pit are 8 m, 6 m and 3 m respectively. Rate of digging the pit is ₹30 per m^3.

Unknown:
The cost of digging the cuboidal pit.

Reasoning:
Since the pit is cuboidal in shape, volume of pit will be equal to volume of the cuboid.



Volume of the cuboid of length, l breadth, b and height, $h = lbh$

We can find the cost of digging the pit by multiplying volume of the pit and rate of digging.

Solution:
Length of the cuboidal pit, $l = 8\ m$
Breadth of the cuboidal pit, $b = 6\ m$
Height of the cuboidal pit, $h = 3\ m$

Volume of the cuboidal tank $= lbh$

$$= 8m \times 6m \times 3m$$
$$= 144\ m^3$$

Cost of the digging the pit at ₹30 per m^3 $= 30 \times 144 = 4320$

Answer:
Cost of digging the cuboidal pit is ₹4320.

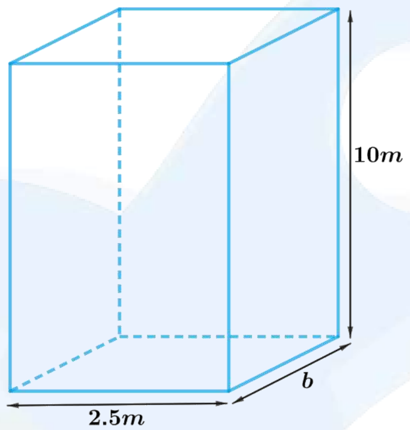
Q5. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

Difficulty Level:
Medium

Known:
Length and depth of the cuboidal tank are 2.5 m and 10 m respectively. The capacity of the tank is 50000 litres.

Unknown:
The breadth of the cuboidal tank.

Reasoning:
Since the tank is cuboidal in shape, volume of tank will be equal to volume of the cuboid.



Volume of the cuboid of length, l breadth, b and height, h $= lbh$
First, we will change volume in cubic metres because all the measurements are in metres.

Solution:
Capacity of the tank $= 50000l$

$$= \frac{50000}{1000} m^3 \qquad (\because 1m^3 = 1000l)$$

$$= 50\ m^3$$

Length of the cuboidal tank, $l = 2.5\ m$
Height of the cuboidal tank, $h = 10\ m$
Let the breadth of the cuboidal tank be b
Volume of the cuboidal tank $= lbh$

$$lbh = 50m^3$$

$$b = \frac{50m^3}{lh}$$

$$b = \frac{50m^3}{2.5m \times 10m}$$

$$= 2m$$

Answer:
The breadth of the cuboidal tank is 2 m.

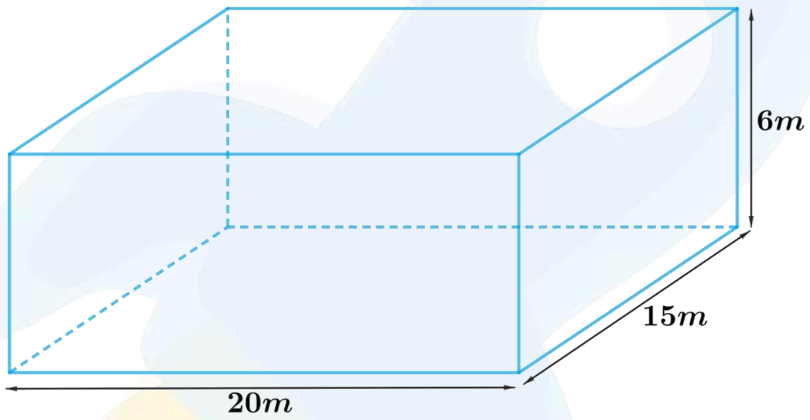
Q6. A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?

Difficulty Level:
Medium

Known:
150 litres of water requires per head per day in a village of 4000 population. Dimensions of tank is $20m \times 15m \times 6m$

Unknown:
The number of days, the water of this tank will last.

Reasoning:
Since the tank is cuboidal in shape, volume of water in the tank will be equal to volume of the cuboid.



Volume of the cuboid of length, l breadth, b and height, $h = lbh$
First, we will find the requirement of water per day for a total of 4000 population in cubic metres and volume of the water in the tank.
The number of days for which the water of tank will last can be obtained by dividing the volume of the water in tank by requirement of water per day for total population.

Solution:
Requirement of water per head per day = 150 litres.
Requirement of water per day for 4000 population $= 4000 \times 150 l$

$$= 600000 l$$

$$= \frac{600000}{1000} m^3 \qquad (\because 1000 l = 1 \, m^3)$$

$$= 600 \, m^3$$

Length of the tank, $l = 20 \, m$
Breadth of the tank, $b = 15 \, m$
Height of the tank, $h = 6 \, m$

Volume of the water in the tank $= lbh$

$= 20m \times 15m \times 6m$

$= 1800\ m^3$

Number of days for which the water of the tank will last $= \dfrac{1800m^3}{600m^3} = 3$

Answer:
The water of the tank will last for 3 days.

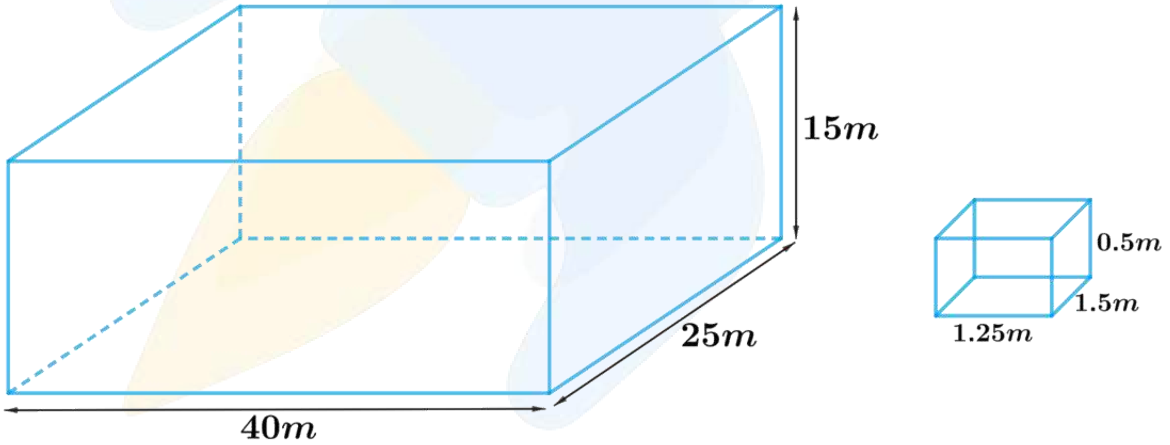
Q7. A godown measures 40m × 25m × 15m. Find the maximum number of wooden crates each measuring 1.5m × 1.25m × 0.5m that can be stored in the godown.

Difficulty Level:
Medium

Known:
Dimensions of godown is $40m \times 25m \times 15m$ and of crates $1.5m \times 1.25m \times 0.5m$

Unknown:
The number of crates, that can be stored in the godown.

Reasoning:
Since the godown and crates are cuboidal in shape, their volume will be equal to volume of the cuboid.



Volume of the cuboid of length, l breadth, b and height, h $= lbh$

The maximum number of wooden crates that can be stored in the godown will be the ratio of volume of the godown to the volume of wooden crate.

Solution:
Length of the godown, $L = 40\ m$

Breadth of the godown, $B = 25\ m$

Height of the godown, $H = 15\ m$

Capacity of the godown $= LBH$

$$= 40m \times 25m \times 15m$$

$$= 1500\ m^3$$

Length of the crate, $l = 1.5\ m$

Breadth of the crate, $b = 1.25\ m$

Height of the crate, $h = 0.5\ m$

Volume of each crate $= lbh$

$$= 1.5m \times 1.25m \times 0.5m$$

$$= 0.9375\, m^3$$

Number of crates $= \dfrac{1500m^3}{0.9375m^3} = 1600$

Answer:

The maximum number of wooden crates that can be stored in the godown is 1600.

Q8. A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.

Difficulty Level:

Medium

Known:

A solid cube of side 12 cm is cut into eight cubes of equal volume.

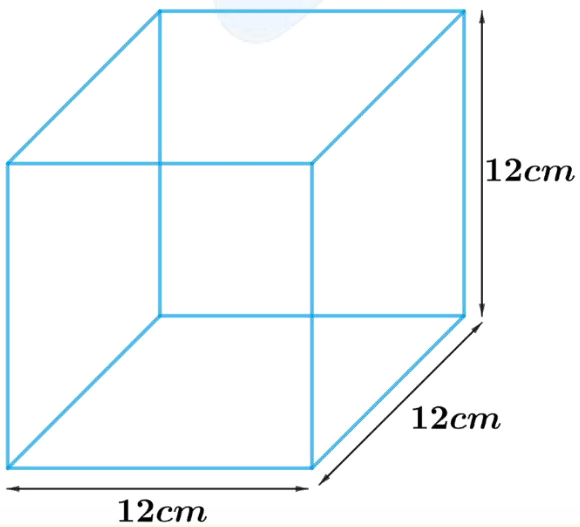
Unknown:

Side of the new cube and ratio between their surface areas.

Reasoning:

Since the solid cube is cut into eight cubes of equal volume, each smaller cube so obtained will have one-eighth volume of the solid cube of side 12cm.

For the ratio of this surface area we will find the surface area of the two cubes.



Volume of the cube of edge, $a = a^3$
Surface area of the cube of edge, $a = 6a^2$

Solution:
Edge of the solid cube, $a = 12cm$
Volume of the solid cube $= a^3$

$$= (12cm)^3$$
$$= 1728cm^3$$

It is cut into 8 equal cubes of same volume
Volume of each small cube $= \frac{1}{8} \times 1728cm^3 = 216cm^3$

Let x be side of each small cube.
Volume of each small cube $= x^3 = 216cm^3$

$$x^3 = (6cm)^3$$
$$x = 6\ cm$$

Surface area of the solid cube $= 6a^2$
Surface area of the small cube $= 6x^2$
Ratio between their surface areas $= \frac{6a^2}{6x^2}$

$$= \frac{a^2}{x^2}$$
$$= \left(\frac{a}{x}\right)^2$$
$$= \left(\frac{12cm}{6cm}\right)^2$$
$$= \left(\frac{2}{1}\right)^2$$
$$= \frac{4}{1}$$

Answer:
The side of the new cube is 6 cm.
Ratio between the surface area is 4:1

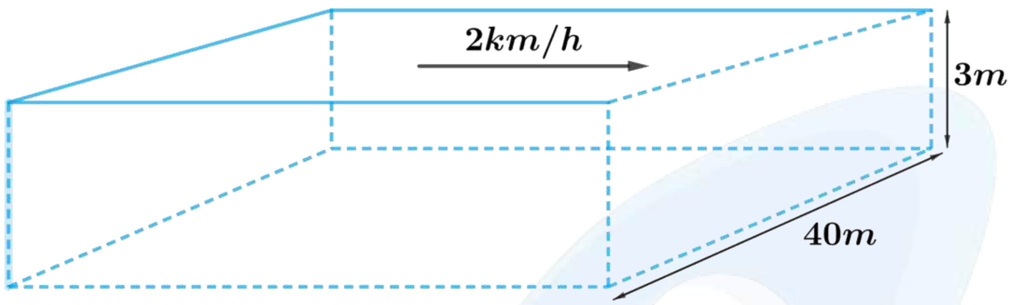
Q9. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

Difficulty Level:
Hard

Known:
Depth and width of the river are 3m and 40m respectively. Water is flowing at the rate of 2 km per hour.

Unknown:
The amount of water which will fall into the sea in a minute.

Reasoning:
Since the water in the river flowing in cuboidal shape and volume of the water that falls into the sea is nothing but the volume of the cuboid.



Volume of the cuboid of length l, breadth b, and height h, $= lbh$

Water is flowing at the rate of 2 km per hour, we need to change this into metre per minute so that we can obtain length of the flowing water in a minute.

Hence, we can easily find the volume of water that falls into the sea in a minute by calculating volume of the cuboid.

Solution:
Width of the river, $b = 40\ m$
Depth of the river, $h = 3\ m$
Flowing rate of water $= 2km/h$

$$= \frac{2000m}{60\min}$$

$$= \frac{100}{3} m/\min$$

Length of the water flowing in 1 minute, $l = \frac{100}{3} m$

volume of the water that falls into the sea in 1 minute $= lbh$

$$= \frac{100}{3} m \times 40m \times 3m$$

$$= 4000m^3$$

Answer:
4000 m^3 of water will fall into the sea in a minute.

Chapter - 13: Surface Area and Volumes

Exercise 13.6 (Page 230 of Grade 9 NCERT Textbook)

Assume $\pi = \dfrac{22}{7}$ unless stated otherwise

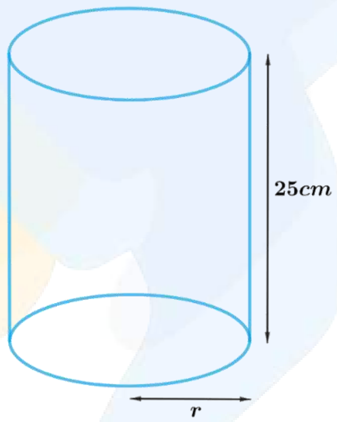
Q1. The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold? $(1000\ cm^3 = 1\ l)$

Difficulty Level:
Medium

Known:
The height of the cylindrical vessel is 132 cm and circumference of its base is 132 cm.

Unknown:
The amount of water in litres the vessel can hold.

Reasoning:
Since the base of a cylindrical vessel is circle so its radius can easily be obtained using circumference $= 2\pi r$
Then capacity of the vessel can be calculated.



Volume of a cylinder of base radius, r and height, $h = \pi r^2 h$

Solution:
Let the radius of the base be r
Height of the cylinder, $h = 25cm$
Circumference of the base $= 132\,cm$

$$2\pi r = 132\,cm$$
$$r = \frac{132\,cm}{2\pi}$$
$$= \frac{132cm}{2} \times \frac{7}{22}$$
$$= 21\,cm$$

Capacity of the cylindrical vessel $= \pi r^2 h$

$$= \frac{22}{7} \times 21cm \times 21cm \times 25cm$$

$$= 34650 \ cm^3$$

$$= \frac{34650}{1000} l \qquad (\because 1000 \ cm^3 = 1 \ l)$$

$$= 34.65 l$$

Answer:

Capacity of the cylindrical vessel is 34.65 litres.

Q2. The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm^3 of wood has a mass of 0.6 g.

Difficulty Level:

Medium

Known:

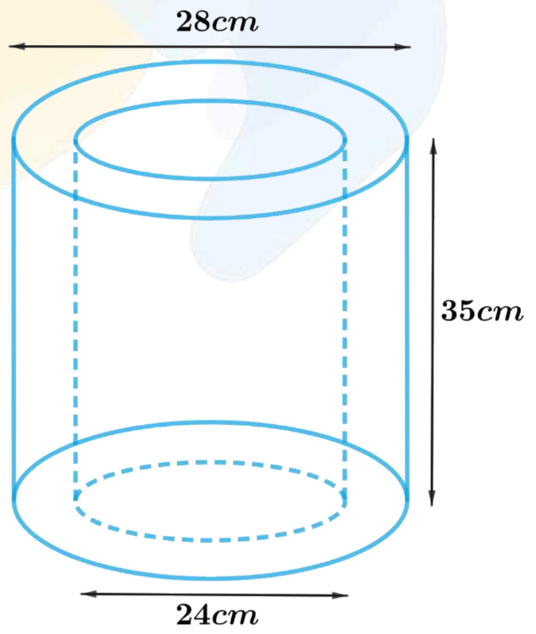
The inner and outer diameter of the cylindrical wooden pipe are 24 cm and 28 cm respectively and its length is 35 cm. Mass of 1 cm^3 wood is 0.6g.

Unknown:

The mass of the pipe.

Reasoning:

Since the cylindrical wooden pipe is made up of two concentric circles at the top and bottom, we will find the volume of both the cylinders.



Volume of a cylinder of base radius, r and height, $h = \pi r^2 h$

The volume of wood can be obtained by finding the difference between the volumes of both the outer and inner cylinders.

To find the mass of the wood we will multiply the volume of wood and its density.

Solution:

Outer diameter of the pipe = 28 *cm*

Outer radius of the pipe, $R = \frac{28cm}{2} = 14cm$

Inner diameter of the pipe = 24 *cm*

Inner radius of the pipe, $r = \frac{24cm}{2} = 12cm$

Length of the pipe, $h = 35cm$

Volume of the outer cylinder, $V_1 = \pi R^2 h$

$$V_1 = \frac{22}{7} \times 14cm \times 14cm \times 35cm$$

$$= 21560\ cm^3$$

Volume of the inner cylinder, $V_2 = \pi r^2 h$

$$V_2 = \frac{22}{7} \times 12cm \times 12cm \times 35cm$$

$$= 15840\ cm^3$$

Volume of the wood used = Volume of the outer cylinder – Volume of the inner cylinder

$$= 21560\ cm^3 - 15840\ cm^3$$

$$= 5720\ cm^3$$

Mass of 1 cm^3 wood is 0.6 g

Mass of 5720 cm^3 wood $= 5720 \times 0.6g$

$$= 3432\ g$$

$$= \frac{3432}{1000} kg$$

$$= 3.432\ kg$$

Answer:

Mass of the wooden pipe is 3.432 kg

Q3. A soft drink is available in two packs – (i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and (ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?

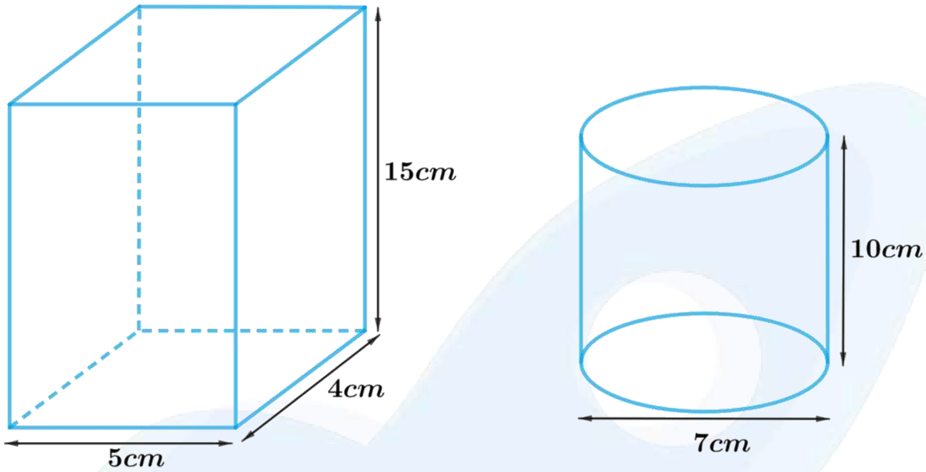
Difficulty Level:

Medium

Known:
The inner and outer diameter of the cylindrical wooden pipe are 24 cm and 28 cm respectively and its length is 35 cm. Mass of 1 cm^3 wood is 0.6g.

Unknown:
The mass of the pipe.

Reasoning:
Since the tin can is cuboidal in shape while other is cylindrical, we will find the volume of both the containers.



Volume of a cylinder of base radius, r and height, $h = \pi r^2 h$
Volume of a cuboid of length, l breadth, b and height, $h = lbh$

Solution:
Tin can with a rectangular base
Length of the cuboidal tin can, $l = 5cm$
Breadth of the cuboidal tin can, $b = 4cm$
Height of the cuboidal tin can, $h = 15cm$

Volume of the cuboidal tin can $= lbh$

$$= 5cm \times 4cm \times 15cm$$
$$= 300\ cm^3$$

A plastic cylinder with circular base
Diameter of the cylindrical plastic can = 7 cm
Radius of the cylindrical plastic can, $r = \frac{7}{2} cm$
Height of the cylindrical plastic can, $h = 10cm$

Volume of the cylindrical plastic can $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{2} cm \times \frac{7}{2} cm \times 10cm$$
$$= 385\ cm^3$$

Clearly, the plastic cylinder with circular base has greater capacity than the tin container.

Difference $= 385cm^3 - 300cm^3 = 85cm^3$

Answer:
The plastic cylindrical can have more capacity than the tin can by 85 cm^3.

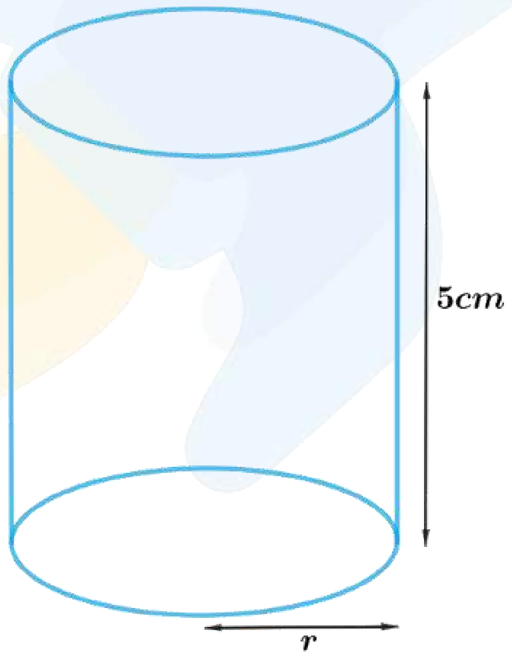
Q4. If the lateral surface of a cylinder is 92.4 cm^2 and its height is 5 cm, then find
(i) radius of its base (ii) its volume. (Use π = 3.14)

Difficulty Level:
Medium

Known:
The lateral surface of a cylinder is 92.4 cm^2 and its height is 5 cm.

Unknown:
The radius of its base and volume.

Reasoning:
Since the lateral surface area and height is known, we can easily obtain the radius and its volume.



Lateral surface area (CSA) of a cylinder of base radius r, and height h, $= 2\pi rh$

Volume of a cylinder of base radius r, and height h, $= \pi r^2 h$

Solution:
Let the radius of the cylinder be r.
Height of the cylinder, $h = 5\ cm$

$$\text{Lateral surface area } = 92.4\ cm^2$$
$$2\pi rh = 92.4\ cm^2$$
$$r = \frac{94.2\ cm^2}{2\pi h}$$
$$r = \frac{94.2\ cm^2}{2 \times 3.14 \times 5cm}$$
$$= 3\ cm$$

$$\text{Volume of cylinder } = \pi r^2 h$$
$$= 3.14 \times 3cm \times 3cm \times 5cm$$
$$= 141.3 cm^3$$

Answer:

Radius of the base is 3cm

Volume is 141.3 cm^3

Q5. It costs ₹2200 to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate of ₹20 per m^2, find.

(i) inner curved surface area of the vessel,

(ii) radius of the base,

(iii) capacity of the vessel.

Difficulty Level:

Medium

Known:

The depth of the cylindrical vessel is 10 m. The cost to paint the inner curved surface is ₹2200 at the rate of ₹20 per m^2.

Unknown:

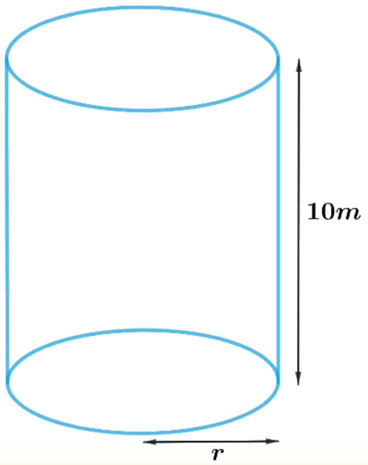
The inner curved surface area, radius of the base and capacity of the vessel.

Reasoning:

Since the cost to paint the inner curved surface and rate in known, we can obtain the inner CSA.

The ratio between the total cost and the rate per m^2 will give the inner CSA in m^2.

Now radius and volume can be obtained easily.



CSA of a cylinder of base radius r, and height h, $= 2\pi rh$
Volume of a cylinder of base radius r, and height h, $= \pi r^2 h$

Solution:

Total cost to paint inner CSA = ₹2200
Rae of painting = ₹20 per m^2

Inner CSA of the vessel $= \frac{2200}{200} = 110\ m^2$

Height of the vessel, $h = 10m$

Inner CSA of the vessel $= 110\ m^2$

$$2\pi rh = 110\ m^2$$
$$r = \frac{110\ m^2}{2\pi h}$$
$$= \frac{110\ m^2}{2 \times 10m} \times \frac{7}{22}$$
$$= \frac{7}{4} m$$
$$= 1.75m$$

Volume of the vessel = $\pi r^2 h$

$$= \frac{22}{7} \times 1.75m \times 1.75m \times 10m$$
$$= 96.25\ m^3$$

Answer:

Inner curved surface area is 110 m^2
Radius of the base is 1.75 m
Capacity of the vessel is 96.25 m^3

Q6. The capacity of a closed cylindrical vessel of height 1 m is 15.4 litres. How many square metres of metal sheet would be needed to make it?

Difficulty Level:

Medium

Known:

The height of the cylindrical vessel is 1 m. and capacity is 15.4 litres.

Unknown:

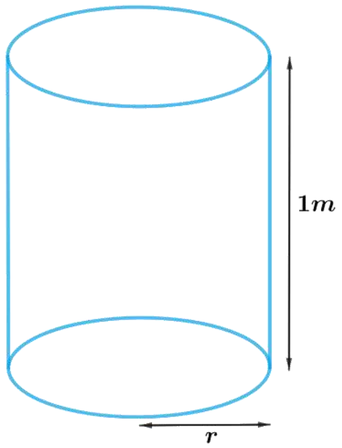
The area of metal sheet would be needed to make the vessel.

Reasoning:

Since the cylinder is closed, metal sheet would be needed for curved surface and area of two bases, top and bottom, i.e. TSA of the cylinder.

Hence, area of the metal sheet will be equal to TSA of the cylinder.

Volume (Capacity) of the vessel and height are known we can easily obtain its radius.



TSA of a cylinder of base radius r, and height h, $= 2\pi r\left(r + h\right)$

Volume of a cylinder of base radius r, and height h, $= \pi r^2 h$

Solution:

Capacity of the vessel = 15.4 litres

$$= \frac{15.4}{1000} m^3 \qquad \left(1000l = 1m^3\right)$$

$$= 0.0154\ m^3$$

Let the radius of the vessel be r

Height of the vessel, $h = 1\ m^3$

Volume of the vessel $= 0.0154\ m^3$

$$\pi r^2 h = 0.0154 m^3$$

$$r^2 = \frac{0.0154 m^3}{\pi h}$$

$$r^2 = \frac{0.0154 m^3}{1m} \times \frac{7}{22}$$

$$r^2 = 0.0049 m^2$$

$$r = \sqrt{0.0049 m^2}$$

$$r = 0.07\ m$$

TSA of the cylinder $= 2\pi r(r + h)$

$$= 2 \times \frac{22}{7} \times 0.07m \times \left(0.07m + 1m\right)$$

$$= 0.44m \times 1.07m$$

$$= 0.4708\ m^2$$

Answer:

0.4708 m^2 of metal sheet would be needed to make the vessel.

Q7. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.

Difficulty Level:

Medium

Known:

The diameter and length of the pencil are 7 mm and 14 cm respectively. The diameter of the graphite is 1 mm.

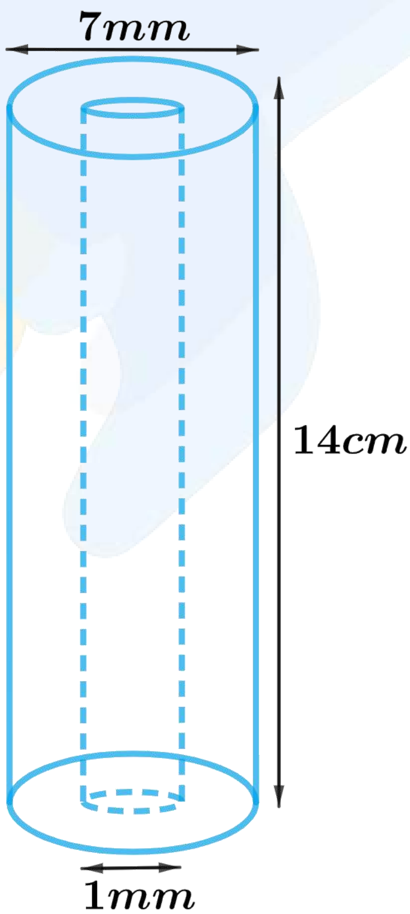
Unknown:

The volume of the wood and the graphite.

Reasoning:

Since, the lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior, the height of the graphite and wood will be as the height of the pencil.

Volume of the wood can be calculated by subtracting volume of graphite from volume of the pencil.



Volume of a cylinder of base radius, r and height, $h = \pi r^2 h$

Solution:

For cylinder of graphite:

Diameter of the graphite = 1 mm

Radius of the graphite, $r = \frac{1mm}{2} = \frac{0.5}{10}cm = 0.05cm$

Length of the graphite, $h = 14cm$

Volume of the graphite $= \pi r^2 h$

$$= \frac{22}{7} \times 0.05cm \times 0.05cm \times 14cm$$

$$= 0.11cm^3$$

For cylinder of wood:

Diameter of the pencil =71 mm

Radius of the pencil, $R = \frac{7mm}{2} = \frac{3.5}{10}cm = 0.35cm$

Length of the pencil, $h = 14cm$

Volume of the pencil $= \pi R^2 h$

$$= \frac{22}{7} \times 0.35cm \times 0.35cm \times 14cm$$

$$= 5.39cm^3$$

Volume of wood = Volume of the pencil – Volume of the graphite

$$= 5.39cm^3 - 0.11cm^3$$

$$= 5.28cm^3$$

Answer:

Volume of the wood is 5.28 cm^3

Volume of graphite is 0.11 cm^3

Q8. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients?

Difficulty Level:

Medium

Known:

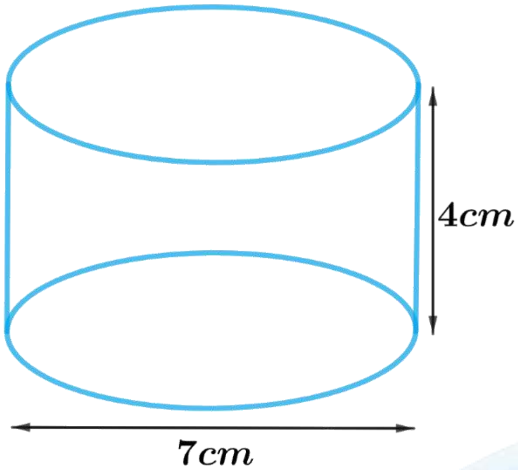
A cylindrical bowl of diameter 7 cm and height of 4 cm.

Unknown:

The amount of soup to be prepared daily by the hospital to serve 250 patients.

Reasoning:

Since, the cylindrical bowl is filled with soup, volume of the soup will be equal to the volume of cylindrical bowl.



Volume of a cylinder of base radius r, and height h, $= \pi r^2 h$

The amount of soup to be prepared will be the product of volume of soup in each bowl and total number of patients.

Solution:

Diameter of the bowl = 7 cm

Radius of the bowl, $r = \frac{7cm}{2} = 3.5cm$

Height of the bowl, $h = 4cm$

Volume of soup in each bowl $= \pi r^2 h$

$= \frac{22}{7} \times 3.5cm \times 3.5cm \times 4cm$

$= 154\ cm^3$

Volume of soup for 1 patient $= 154\ cm^3$

Volume of soup for 250 patients $= 250 \times 154 cm^3$

$= 38500 cm^3$

$= \frac{38500}{1000} l \qquad (1000cm^3 = 1l)$

$= 38.5l$

Answer:

The hospital has to prepare 38.5 litres of soup daily to serve 250 patients.

Chapter - 13: Surface Area and Volumes

Exercise 13.7 (Page 233 of Grade 9 NCERT Textbook)

Assume $\pi = \frac{22}{7}$ unless stated otherwise

Q1. Find the volume of the right circular cone with

(i) radius 6 cm, height 7 cm (ii) radius 3.5 cm, height 12 cm

Difficulty Level:

Medium

Known:

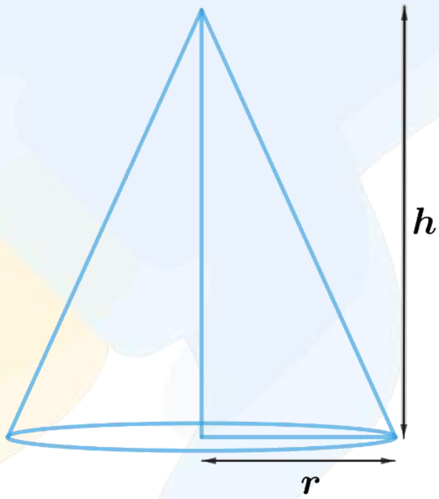
Radius and height of the right circular cones.

Unknown:

The volume of the right circular cones.

Reasoning:

Let's draw a diagram of the cone to visualize it better.



Volume of a cone of base radius r, and height h, $= \frac{1}{3}\pi r^2 h$

Solution:

(i) Radius of the cone, $r = 6cm$

Height of the cone, $h = 7cm$

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 6cm \times 6cm \times 7cm$$

$$= 264\ cm^3$$

(ii) Radius of the cone, $r = 3.5cm$
Height of the cone, $h = 12cm$

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 3.5cm \times 3.5cm \times 12cm$

$= 154\ cm^3$

Answer:

(i) Volume of the cone $= 264\ cm^3$

(ii) Volume of the cone $= 154\ cm^3$

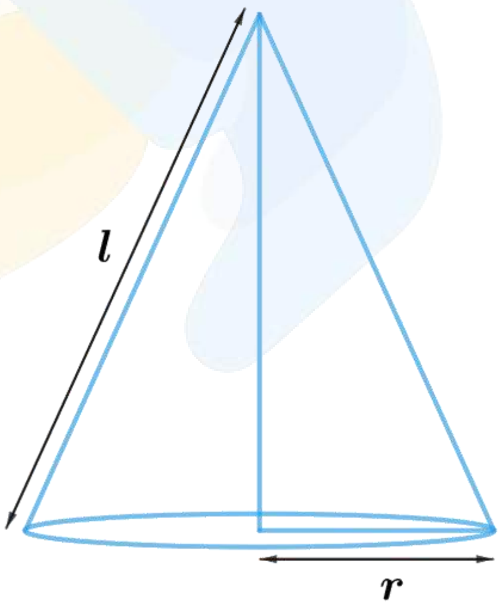
Q2. Find the capacity in litres of a conical vessel with
(i) radius 7 cm, slant height 25 cm (ii) height 12 cm, slant height 13 cm

Difficulty Level:
Medium

Known:
Radius and slant height of the conical vessels.

Unknown:
The capacity of the conical vessels.

Reasoning:
Let's draw a diagram of the conical vessel to visualize it better.



Capacity of a conical vessel is nothing but the volume of the cone.

Volume of a cone of base radius r, and height h, $= \frac{1}{3}\pi r^2 h$

Slant height of the cone, $l = \sqrt{r^2 + h^2}$

Solution:

(i) Radius of the conical vessel, $r = 7cm$

Slant height of the conical vessel, $l = 25cm$

Height of the conical vessel, $h = \sqrt{l^2 - r^2}$

$$= \sqrt{(25cm)^2 - (7cm)^2}$$

$$= \sqrt{625cm^2 - 49cm^2}$$

$$= \sqrt{576cm^2}$$

$$h = 24cm$$

Capacity of the conical vessel $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 7cm \times 7cm \times 24cm$$

$$= 1232\ cm^3$$

$$= 1232 \times \frac{1}{1000} l \qquad [\because 1000\ cm^3 = 1l]$$

$$= 1.232l$$

(ii) Height of the conical vessel, $h = 7cm$

Slant height of the conical vessel, $l = 13cm$

Radius of the conical vessel, $r = \sqrt{l^2 - h^2}$

$$= \sqrt{(13cm)^2 - (12cm)^2}$$

$$= \sqrt{169cm^2 - 144cm^2}$$

$$= \sqrt{25cm^2}$$

$$r = 5cm$$

Capacity of the conical vessel $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 5cm \times 5cm \times 12cm$$

$$= \frac{2200}{7}\ cm^3$$

$$= \frac{2200}{7} \times \frac{1}{1000} l \qquad [\because 1000\ cm^3 = 1l]$$

$$= \frac{11}{35} l$$

Answer:

(i) Capacity of the conical vessel is 1.232 l

(ii) Capacity of the conical vessel is $\frac{11}{35} l$

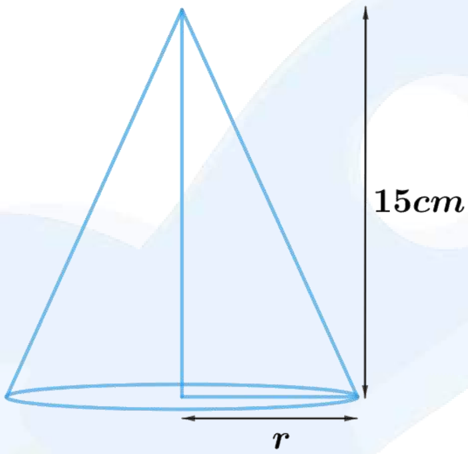
Q3. The height of a cone is 15 cm. If its volume is 1570 cm^3, find the radius of the base. (Use $\pi = 3.14$)

Difficulty Level:
Medium

Known:
The height of the cone is 15 cm and its volume is 1570 cm^3.

Unknown:
The radius of the base.

Reasoning:
Let's draw a diagram of the cone to visualize it better.



Volume of a cone of base radius r, and height h, $= \frac{1}{3}\pi r^2 h$

Solution:
Let the radius of the cone be r
Height of the cone, $h = 15cm$
Volume of the cone $= 1570\ cm^3$

$$\frac{1}{3}\pi r^2 h = 1570 cm^3$$

$$r^2 = 1570 cm^3 \times \frac{3}{\pi h}$$

$$r^2 = \frac{1570 cm^3 \times 3}{3.14 \times 15 cm}$$

$$= 100 cm^2$$

$$r = \sqrt{100 cm^2}$$

$$r = 10 cm$$

Answer:
Radius of the base = 10 cm

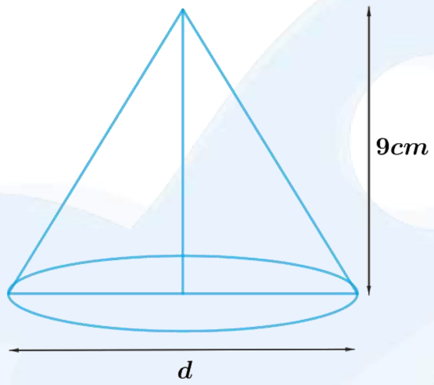
Q4. If the volume of a right circular cone of height 9 cm is $48\pi\ cm^3$, find the diameter of its base.

Difficulty Level:
Medium

Known:
The height of the cone is 9 cm and volume is $48\pi\ cm^3$.

Unknown:
The diameter of the base.

Reasoning:
Let's draw a diagram of the cone to visualize it better.



Volume of a cone of base radius *r*, and height *h*, $= \frac{1}{3}\pi r^2 h$

Solution:
Let the radius of the cone be r
Height of the cone, $h = 9cm$
Volume of the cone $= 48\pi\ cm^3$

$$\frac{1}{3}\pi r^2 h = 48\pi cm^3$$

$$r^2 = 48cm^3 \times \frac{3}{h}$$

$$r^2 = 48cm^3 \times \frac{3}{9cm}$$

$$r^2 = 16cm^2$$

$$r = \sqrt{16cm^2}$$

$$r = 4\ cm$$

Base diameter, $d = 2r$

$$= 2 \times 4cm$$

$$= 8\ cm$$

Answer:
The diameter of the box of the night circular cone is 8 cm.

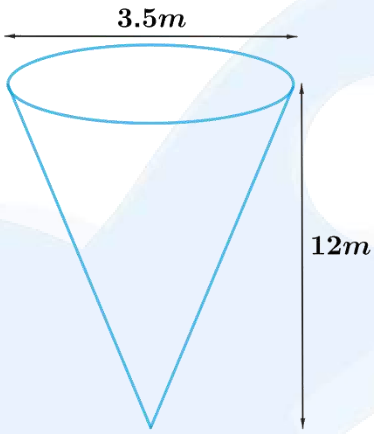
Q5. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?

Difficulty Level:
Medium

Known:
The diameter and depth of the conical pit are 3.5 m and 12 m.

Unknown:
The capacity of the pit in kilolitres.

Reasoning:
Let's draw a diagram of the conical pit to visualize it better.



Volume of a cone of base radius r, and height h, $= \frac{1}{3}\pi r^2 h$

Solution:
Diameter of the conical pit, $d = 3.5m$

Radius of the conical pit, $r = \frac{3.5m}{2} = 1.75m$

Depth of the conical pit, $h = 12m$

Volume of conical pit $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 1.75m \times 1.75m \times 12m$$

$$= 38.5\ m^3$$

$$= 38.5 \times 1kl \qquad (1m^3 = 1000l = 1kl)$$

$$= 38.5kl$$

Answer:
Capacity of the conical pit is 38.5 kilolitres.

Q6. The volume of a right circular cone is 9856 cm^3. If the diameter of the base is 28 cm, find

(i) height of the cone (ii) slant height of the cone

(iii) curved surface area of the cone

Difficulty Level:

Medium

Known:

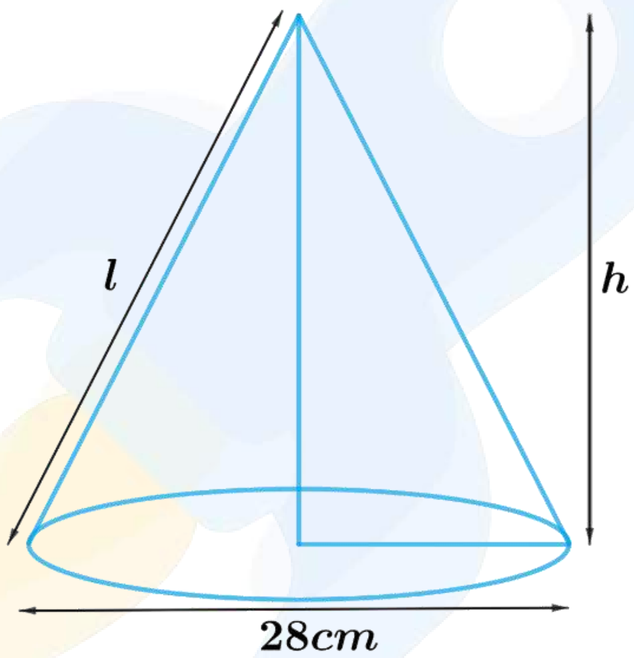
The diameter of the base is 28cm and volume is 9856 cm^3.

Unknown:

The height, slant height and CSA of the cone.

Reasoning:

Let's draw a diagram of the cone to visualize it better.



Volume of a cone of base radius r, and height h, $= \frac{1}{3}\pi r^2 h$

Curved surface area of the cone of base radius, r and slant height, $l = \pi rl$

Slant height of the cone, $l = \sqrt{r^2 + h^2}$

Solution:

Diameter of the cone, $d = 28cm$

Radius of the cone, $r = \frac{28m}{2} = 14cm$

Volume of the cone $= 9856cm^3$

Let the height of the cone be h

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$\frac{1}{3}\pi r^2 h = 9856cm^3$$

$$h = 9856cm^3 \times \frac{3}{\pi r^2}$$

$$= 9856cm^3 \times \frac{3}{14cm \times 14cm} \times \frac{7}{22}$$

$$= 48cm$$

Slant height of the cone, $l = \sqrt{r^2 + h^2}$

$$= \sqrt{(14cm)^2 + (48cm)^2}$$

$$= \sqrt{196cm^2 + 2304cm^2}$$

$$= \sqrt{2500cm^2}$$

$$= 50cm$$

Curved surface area of the cone $= \pi rl$

$$= \frac{22}{7} \times 14cm \times 50cm$$

$$= 2200cm^2$$

Answer:

Height of the cone is 48 cm

Slant height of the cone is 50 cm

Surface area of the cone is 2200 cm^2

Q7. A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Difficulty Level:

Medium

Known:

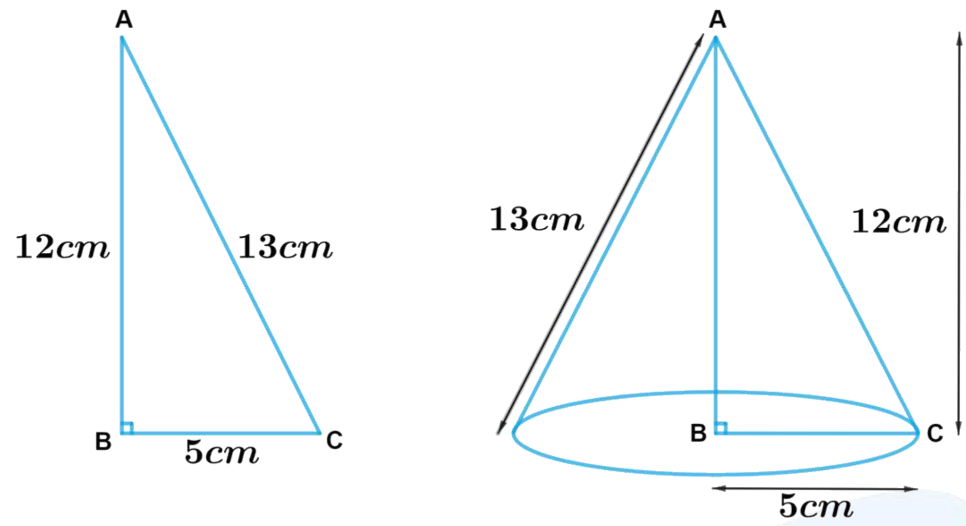
A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm.

Unknown:

The volume of the solid so obtained.

Reasoning:

Let's draw a diagram of the triangle and the solid obtained to visualize it better.



Since the triangle is revolved about the side 12 cm a solid cone is formed with the height of 12 cm and radius of base 5 cm.

Volume of a cone of base radius r, and height h, $= \frac{1}{3}\pi r^2 h$

Solution:

Radius of the cone, $r = 5cm$

Height of the cone, $h = 12cm$

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi \times 5cm \times 5cm \times 12cm$$

$$= 100\pi \, cm^3$$

Answer:

Volume of the cone $100\pi \ cm^3$.

Q8. If the triangle ABC in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

Difficulty Level:

Medium

Known:

A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 5 cm.

Unknown:

The volume of the solid so obtained and ratio of the volumes of two solids which are revolved about the sides 12 cm and 5 cm.

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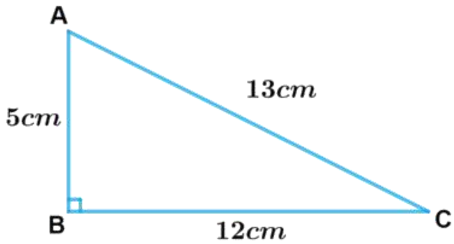
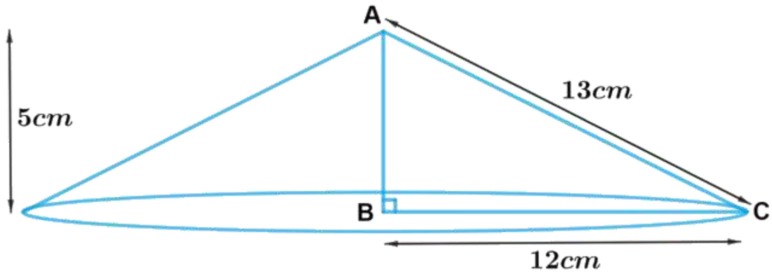
Reasoning:

Let's draw a diagram of the triangle and the solid obtained to visualize it better.





Since the triangle is revolved about the side 12 cm a solid cone is formed with the height of 5 cm and radius of base 12 cm.

Volume of a cone of base radius r, and height h, $= \frac{1}{3}\pi r^2 h$

Curved surface area of the cone of base radius, r and slant height, $l = \pi r l$

Slant height of the cone, $l = \sqrt{r^2 + h^2}$

Solution:

Radius of the cone, $r = 12cm$

Height of the cone, $h = 5cm$

$$\text{Volume of the cone } = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 12cm \times 12cm \times 5cm$$

$$= 240\pi cm^3$$

Volume of the cone in question $7 = 100\pi \ cm^3$

$$\text{Ratio of the volumes of the cones } = \frac{100\pi cm^3}{240\pi cm^3} = \frac{5}{12}$$

Answer:

Volume of the cone is $240\pi \ cm^3$

Ratio between the volume $5:12$

Q9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

Difficulty Level:

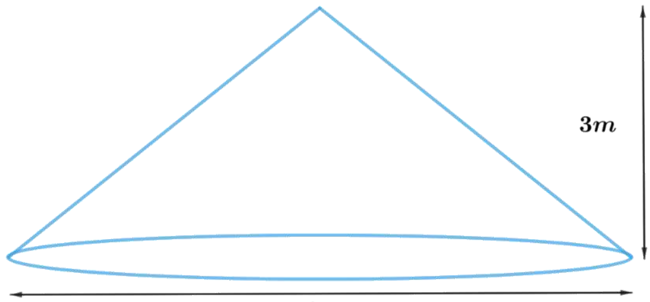
Medium

Known:

A conical heap whose diameter is 10.5 m and height is 3 m.

Unknown:
The volume of the conical heap and area of the canvas required to cover the heap.

Reasoning:
Let's draw a diagram of the conical heap to visualize it better.



Since heap of wheat is in the form of a cone and the canvas required to cover the heap will be required for CSA of the cone.

Volume of a cone of base radius, r and height, $h = \frac{1}{3}\pi r^2 h$

CSA of the cone of base radius, r and slant height, $l = \pi rl$

Slant height of the cone, $l = \sqrt{r^2 + h^2}$

Solution:
Diameter of the conical heap, $d = 10.5m$

Radius of the conical heap, $r = \frac{10.5m}{2} = 5.25m$

Height of the conical heap, $h = 3m$

Volume of the conical heap $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 5.25m \times 5.25m \times 3m$$

$$= 86.625m^3$$

Slant height, $l = \sqrt{r^2 + h^2}$

$$= \sqrt{(5.25m)^2 + (3m)^2}$$

$$= \sqrt{27.5625m^2 + 9m^2}$$

$$= \sqrt{36.5625m^2}$$

$$= 6.05m$$

The area of the canvas required to cover the heap of wheat $= \pi rl$

$$= \frac{22}{7} \times 5.25m \times 6.05m$$

$$= 98.825m^2$$

Answer:
Volume of the conical heap is 86.625 m^3
Area of the canvas required is 98.825 m^2

Chapter - 13: Surface Area and Volumes

Exercise 13.8 (Page 236 of Grade 9 NCERT Textbook)

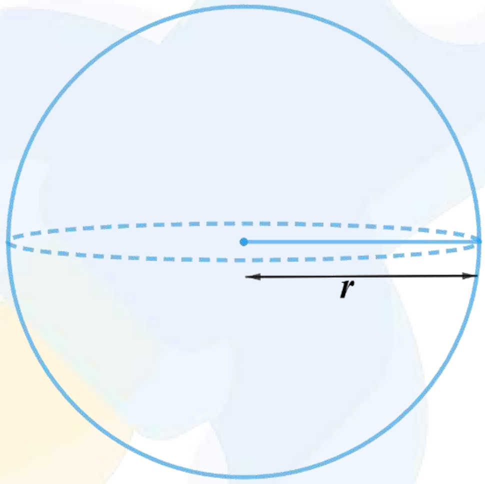
Q1. Find the volume of a sphere whose radius is
(i) 7 cm (ii) 0.63 m

Difficulty Level:
Medium

Known:
Radius of the spheres.

Unknown:
Volume of the spheres.

Reasoning:
Let's draw a diagram of the sphere to visualize it better.



Volume of a sphere of base radius, $r = \frac{4}{3}\pi r^3$

Solution:

(i) Radius of the sphere, $r = 7cm$

Volume of the sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 7cm \times 7cm \times 7cm$$

$$= \frac{4312}{3} cm^3$$

$$= 1437\frac{1}{3} cm^3$$

(ii) Radius of the sphere, $r = 0.63m$

Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 0.63m \times 0.63m \times 0.63m$

$= 1.047816m^3$

$= 1.05m^3$ (approx.)

Answer:

(i) Volume of the sphere is $1437\frac{1}{3}cm^3$

(ii) Volume of the sphere is $1.05m^3$

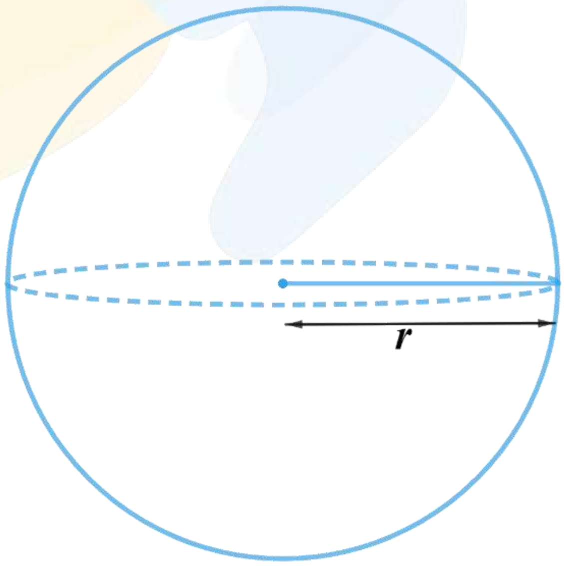
Q2. Find the amount of water displaced by a solid spherical ball of diameter.
(i) 28 cm (ii) 0.21 m

Difficulty Level:
Medium

Known:
Diameter of the solid spherical balls.

Unknown:
Amount of the water displayed by the solid spherical balls.

Reasoning:
Let's draw a diagram of the spherical ball to visualize it better.



The amount of water displaced by a solid spherical ball is nothing but its volume.

Volume of a sphere of base radius r, $= \frac{4}{3}\pi r^3$

Solution:

(i) Diameter of the spherical ball, $d = 28cm$

Radius of the spherical ball, $r = \frac{28cm}{2} = 14cm$

Amount of water displaced by the solid spherical ball

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 14cm \times 14cm \times 14cm$$

$$= \frac{34496}{3} cm^3$$

$$= 11498\frac{2}{3}\ cm^3$$

(ii) Diameter of the spherical ball, $d = 0.21m$

Radius of the spherical ball, $r = \frac{0.21m}{2} = 0.105cm$

Amount of water displaced by the solid spherical ball

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 0.105m \times 0.105m \times 0.105m$$

$$= 0.004851\ m^3$$

Answer:

(i) $11498\frac{2}{3}\ cm^3$ of water is displaced.

(ii) $0.004851\ m^3$ of water is displaced.

Q3. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3?

Difficulty Level:

Medium

Known:

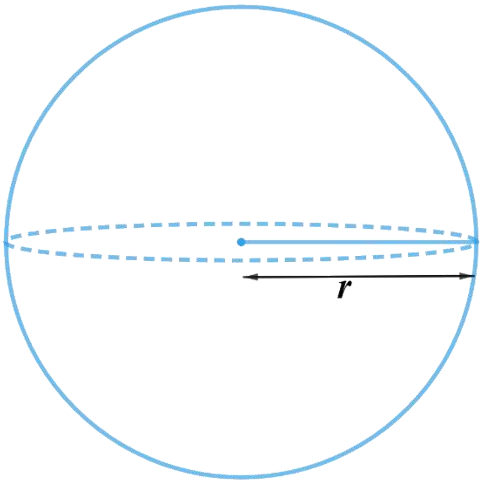
Diameter of the metallic ball is 4.2 cm and density of the metal is 8.9 g per cm^3.

Unknown:

Mass of the metallic ball.

Reasoning:

Let's draw a diagram of the metallic ball to visualize it better.



Since mass of a solid can be obtained by multiplying its volume and density. First, we will find volume of the metallic ball.

Volume of a sphere of base radius r, $= \frac{4}{3}\pi r^3$

Solution:

Diameter of the metallic ball, $d = 4.2cm$

Radius of the metallic ball, $r = \frac{4.2cm}{2} = 2.1cm$

Volume of the metallic ball $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 2.1cm \times 2.1cm \times 2.1cm$$

$$= 38.808\ cm^3$$

$\text{Mass} = \text{Volume} \times \text{Density}$

Mass of the metallic ball $= 38.808\ cm^3 \times 8.9g / cm^3$

$$= 345.3912\ g$$

$$= 345.39\ g \qquad (\text{approx.})$$

Answer:

Mass of the ball is $345.39\ g$

Q4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Difficulty Level:

Medium

Known:

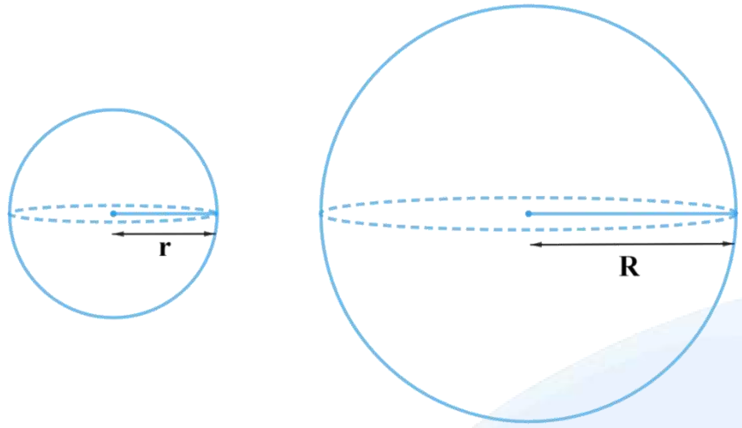
Diameter of the moon is approximately one-fourth of the diameter of the earth

Unknown:

Fraction of the volume of the earth is the volume of the moon.

Reasoning:

Let's draw a diagram of the moon and earth to visualize it better.



Volume of a sphere of base radius r, $= \frac{4}{3}\pi r^3$

Solution:

Let the radius of the earth be R
and the radius of the moon be r

Diameter of the moon $= \frac{1}{4} \times$ diameter of the earth

Radius of the moon $= \frac{1}{4} \times$ radius of the earth

$$r = \frac{1}{4} \times R$$

$$r = \frac{R}{4} \qquad \ldots(\text{i})$$

Volume of the earth $= \frac{4}{3}\pi R^3$

Volume of the moon $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi \left(\frac{R}{4}\right)^3 \qquad [\text{from (i)}]$$

$$= \frac{4}{3}\pi \times \frac{R^3}{64}$$

$$= \frac{1}{64} \times \frac{4}{3}\pi R^3$$

Volume of the moon $= \frac{1}{64} \times$ Volume of the earth

Answer:

Hence the volume of the moon is $\frac{1}{64}$ th fraction of the volume of the earth.

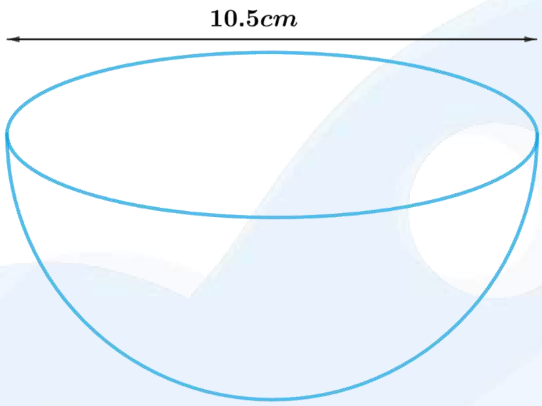
Q5. How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?

Difficulty Level:
Medium

Known:
Diameter of the hemispherical bowl is 10.5 cm.

Unknown:
Quantity of milk in litres which the hemispherical bowl can hold.

Reasoning:
Let's draw a diagram of the hemispherical bowl to visualize it better.



Quantity of milk which the hemispherical bowl can hold is nothing but the volume of the hemispherical bowl.

Volume of a hemisphere of base radius, $r = \frac{2}{3}\pi r^3$

Solution:
Diameter of the hemispherical ball, $d = 10.5cm$

Radius of the hemispherical ball, $r = \frac{10.5cm}{2} = 5.25cm$

Volume of the hemispherical ball $= \frac{4}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times 5.25cm \times 5.25cm \times 5.25cm$$

$$= 303.1875cm^3$$

$$= \frac{303.1875}{1000} l \qquad (\because 1000cm^3 = 1l)$$

$$= 0.3031875l$$

$$= 0.303l \qquad \text{(approx.)}$$

Answer:
0.303 litres of milk can be held in the bowl.

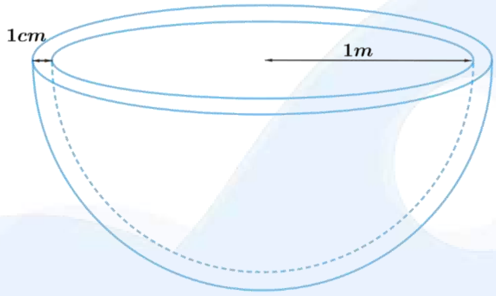
Q6. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1m, then find the volume of the iron used to make the tank.

Difficulty Level:
Medium

Known:
Inner radius of the hemispherical tank is 1m and the thickness of the iron sheet is 1 cm.

Unknown:
The volume of the iron used to make the tank

Reasoning:
Let’s draw a diagram of the hemispherical tank to visualize it better.



Since the hemispherical bowl is made of 1 cm thick iron, we can find the outer radius of the bowl by adding thickness to the inner radius.

Volume of the iron used to make the tank can be calculated by subtracting volume of bowl with inner radius from volume of the bowl with outer radius.

Volume of a hemisphere of base radius, $r = \frac{2}{3}\pi r^3$

Solution:
Inner radius of the bowl, $r = 1m$

Thickness of iron $= 1cm = \frac{1}{100}m = 0.01m$

Outer radius of the bowl, $R = 1m + 0.01m = 1.01m$

Volume of the iron used to make the tank $= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3$

$$= \frac{2}{3}\pi\left(R^3 - r^3\right)$$

$$= \frac{2}{3} \times \frac{22}{7} \times \left[(1.01m)^3 - (1m)^3\right]$$

$$= \frac{2}{3} \times \frac{22}{7} \times \left[1.030301m^3 - 1m^3\right]$$

$$= \frac{2}{3} \times \frac{22}{7} \times 0.030301m^3$$

$$= 0.06348\ m^3 \qquad \text{(approx.)}$$

Answer:
0.06348 m^3 of the iron used to make the tank.

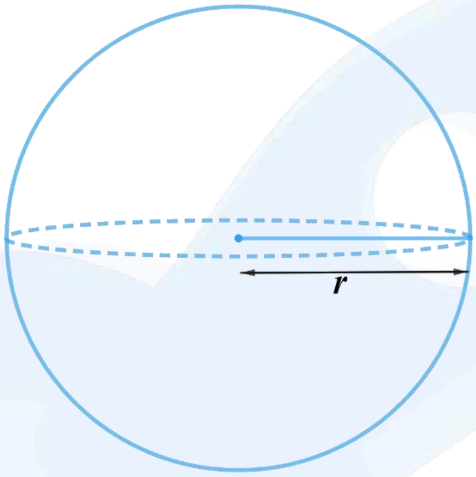
Q7. Find the volume of a sphere whose surface area is 154 cm^2.

Difficulty Level:
Medium

Known:
Surface area of the sphere is 154 cm^2.

Unknown:
Volume of the sphere.

Reasoning:
Let's draw a diagram of the sphere to visualize it better.



Since the surface area of the sphere is given, we can obtain the radius easily using the formula of surface area then its volume.
Surface area of the sphere of base radius r, $= 4\pi r^2$

Volume of a sphere of base radius r, $= \frac{4}{3}\pi r^3$

Solution:
Let the radius of the sphere be r.
Surface area of the sphere $= 4\pi r^2 = 154cm^2$

$$r^2 = \frac{154cm^2}{4\pi}$$

$$r^2 = \frac{154cm^2}{4} \times \frac{7}{22}$$

$$r^2 = \frac{49}{4}cm^2$$

$$r = \frac{7}{2}cm$$

Volume of the sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} cm \times \frac{7}{2} cm \times \frac{7}{2} cm$$

$$= \frac{539}{3}\ cm^3$$

$$= 179\frac{2}{3} cm^3$$

Answer:

Volume of the sphere is $179\frac{2}{3}\ cm^3$

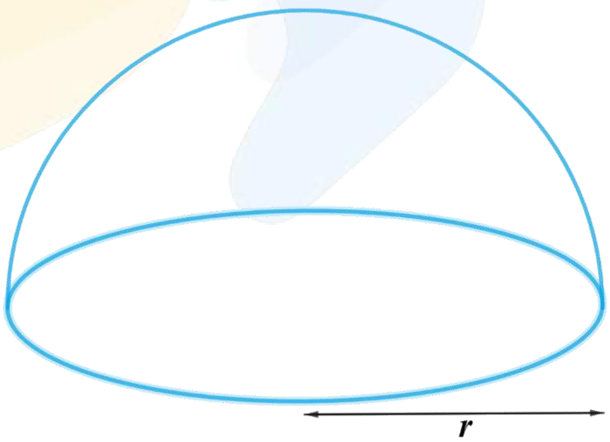
Q8. A dome of a building is in the form of a hemisphere. From inside, it was whitewashed at the cost of ₹4989.60. If the cost of white washing is ₹20 per square meter, find the
(i) inside surface area of the dome, (ii) Volume of the air inside the dome.

Difficulty Level:
Medium

Known:
Cost of whitewashing is ₹4989.60, and rate of whitewashing is ₹20 per m^2.

Unknown:
Inside surface area and volume of the hemispherical dome.

Reasoning:
Let's draw a diagram of the hemispherical dome to visualize it better.



Since the total cost of whitewashing the dome from inside, which is a CSA of the hemisphere and rate are known we can calculate inner CSA of the hemispherical dome, by dividing total cost of whitewashing by rate of whitewashing.

Volume of the air inside the dome will be same as the volume of the hemisphere.

Surface area of the hemisphere of base radius r, $= 2\pi r^2$

Volume of a hemisphere of base radius r, $= \frac{2}{3}\pi r^3$

Solution:

Total cost for whitewashing the dome from inside = ₹4989.60
Rate of whitewashing = ₹20 per m^2

Inside surface area of the dome $= \frac{4989.6}{20} = 249.48\ m^2$

Let the radius of the hemispherical dome be r.
Inside surface area of the dome $= 2\pi r^2 = 2494.8\ m^2$

$$r^2 = \frac{249.48\ m^2}{2\pi}$$

$$r^2 = \frac{249.48m^2}{2} \times \frac{7}{22}$$

$$r^2 = 39.69m^2$$

$$r = \sqrt{39.69m^2}$$

$$r = 6.3m$$

Volume of the air inside the dome $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times 6.3m \times 6.3m \times 6.3m$$

$$= 523.9\ m^3 \qquad \text{(approx.)}$$

Answer:

Inner surface area of the dome is 249.48 m^2
Volume of the air inside the dome is 523.9 m^3

Q9. Twenty-seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S′. Find the

(i) Radius r' of the new sphere, (ii) Ratio of S and S′.

Difficulty Level:

Medium

Known:

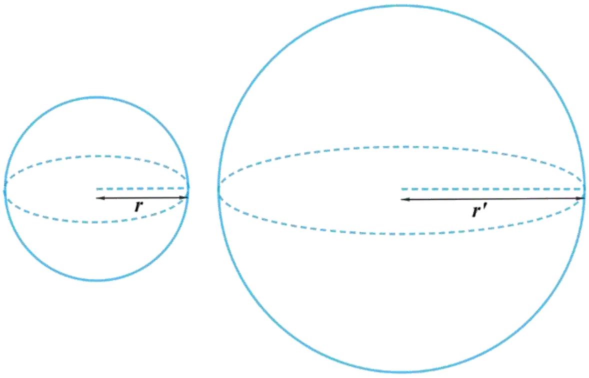
27 solid iron spheres, each of radius r and surface area S are melted to form a sphere.

Unknown:

Radius r’ of new sphere and ratio of S and S’.

Reasoning:

Let’s draw a diagram of the spheres to visualize it better.



Since 27 solid iron spheres are melted to form a single solid sphere then the volume of new formed sphere will be equal to the volume of 27 solid iron spheres together.
Surface area of the sphere of base radius $r, = 4\pi r^2$

Volume of a sphere of base radius $r, = \frac{4}{3}\pi r^3$

Solution:
Radius of each solid iron sphere $= r$

Volume of each solid iron sphere $= \frac{4}{3}\pi r^3$

Volume of 27 solid spheres $= 27 \times \left(\frac{4}{3}\pi r^3\right) = 36\pi r'^3$

Let the radius of the new sphere $= r$

Volume of the new sphere $= \frac{4}{3}\pi r'^3$

Volume of the new sphere = Volume of 27 solid spheres

$$\frac{4}{3}\pi r'^3 = 36\pi r^3$$

$$r'^3 = 36\pi r^3 \times \frac{3}{4\pi}$$

$$r'^3 = 27r^3$$

$$r' = \sqrt[3]{27r^3}$$

$$r' = 3r$$

Radius of the new sphere, $r' = 3r$

Now,
Surface area of each iron sphere, $S = 4\pi r^2$
Surface area of the new sphere, $S' = 4\pi r'^2 = 4\pi(3r)^2 = 36\pi r^2$

Ratio of the S and S’ $= \frac{4\pi r^2}{36\pi r^2} = \frac{1}{9}$

Answer:
Radius r of new sphere is $3r$.
Ratio of S and S’ is 1:9

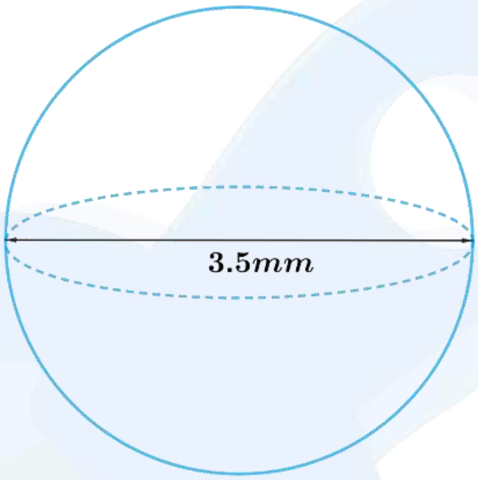
Q10. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much Medicine (in mm^3) is needed to fill this capsule?

Difficulty Level:
Medium

Known:
Diameter of a spherical capsule is 3.5 mm.

Unknown:
Amount of the medicine needed to fill the capsule.

Reasoning:
Let's draw a diagram of the spherical capsule to visualize it better.



Since the capsule is spherical in shape, the amount of medicine needed to fill the capsule is the volume of sphere.

Volume of a sphere of base radius r, $= \frac{4}{3}\pi r^3$

Solution:
Diameter of the spherical capsule, $d = 3.5mm$

Radius of the spherical capsule, $r = \frac{3.5mm}{2} = 1.75mm$

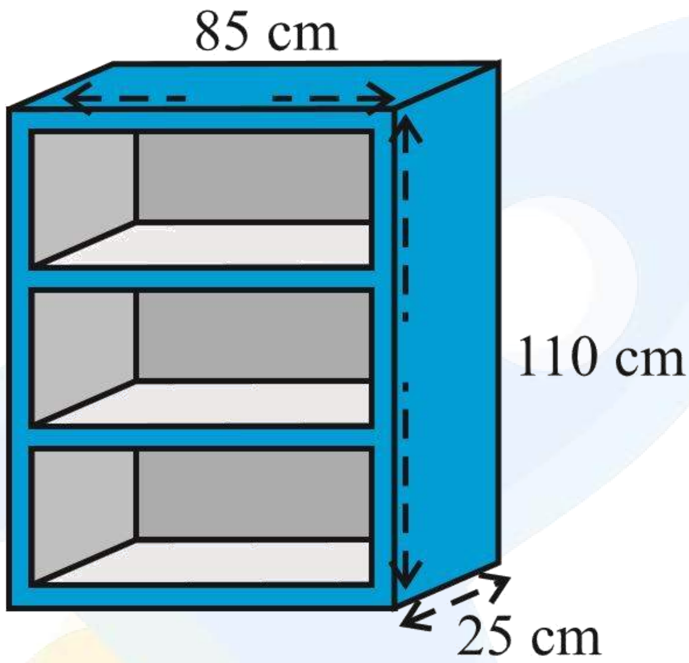
$$\text{Volume of the capsule} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times 1.75mm \times 1.75mm \times 1.75mm$$
$$= 22.46mm^3 \quad \text{(approx.)}$$

Answer:
22.46 mm^3 of medicine is needed to fill this capsule.

Chapter - 13: Surface Area and Volumes

Exercise 13.9(Page 236 of Grade 9 NCERT Textbook)

Q1. A wooden bookshelf has external dimensions as follows: Height = 110 cm, Depth = 25 cm, Breadth = 85 cm (see Fig. 13.31). The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2. Find the total expenses required for polishing and painting the surface of the bookshelf.



Difficulty Level:
Hard

Known:
Height 110 cm, depth 25 cm, and breadth 85 cm are external dimensions of the bookshelf and the thickness of the plank is 5 cm everywhere. The rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2.

Unknown:
The total expenses required for polishing and painting the surface of the bookshelf.

Reasoning:
Since the bookshelf is cuboidal in shape and opened at front with three shelves with same dimensions.

Area to be polished will be 5 surfaces of cuboidal bookshelf and front border with plank's thickness.

Area to be painted will be 3 shelves of the bookshelf with internal dimensions and area of each shelf will be 5 surfaces of the cuboidal shelf.

We can calculate the total cost of polishing and painting by multiplying rate and their respective area.

Volume of a cuboid of length l, breadth b, and height $h, = lbh$

Solution:

External measures of the bookshelf;
Breadth, $L = 85cm$
Depth, $B = 25cm$
Height, $H = 110cm$
Thickness of the plank, $t = 5cm$

Internal measures of the bookshelf;
Breadth, $l = 85cm - 2\times 5cm = 75cm$
Depth, $b = 25cm - 5cm = 20cm$
Height of each shelf, $h = \dfrac{110cm - 4\times 5cm}{3} = \dfrac{90cm}{3} = 30cm$

Surface area to be polished = External 5 surfaces of the bookshelf + border of the shelf

$$= 2(L+H)B + LH + 2Ht + 4lt$$
$$= 2\times(85cm + 110cm)\times 25cm + 85cm\times 110cm + 2\times 110cm\times 5cm + 4\times 75cm\times 5cm$$
$$= 2\times 195cm\times 25cm + 85cm\times 110cm + 2\times 110cm\times 5cm + 4\times 75cm\times 5cm$$
$$= 9750cm^2 + 9350cm^2 + 1100cm^2 + 1500cm^2$$
$$= 21700cm^2$$

Cost of polishing at the rate of 20 paise per cm^2 $= 21700cm^2 \times ₹\dfrac{20}{100}/cm^2 = ₹4380$

Surface area to be painted = Internal 5 surfaces of 3 shelves

$$= 3\left[2(l+h)b + lh\right]$$
$$= 3\left[2\times(75cm + 30cm)\times 20cm + 75cm\times 30cm\right]$$
$$= 3\left[4200cm^2 + 2250cm^2\right]$$
$$= 3\times 6450cm^2$$
$$= 19350cm^2$$

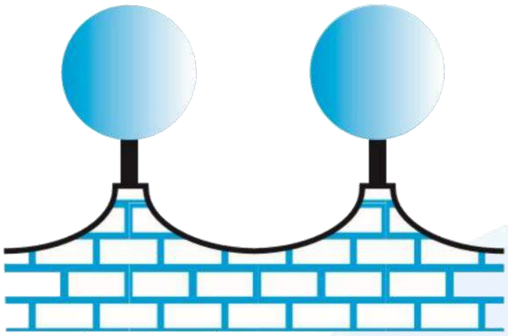
Cost of painting at the rate of 10 paise per cm^2 $= 19350cm^2 \times ₹\dfrac{10}{100}/cm^2 = ₹1935$

Total expense required for polishing and painting = ₹4380 + ₹1935
= ₹6275

Answer:

Total expense required for polishing and painting the surface of the bookshelf is ₹6275.

Q2. The front compound wall of a house is decorated by wooden spheres of diameter 21cm, placed on small supports as shown in Fig 13.32. Eight such spheres are used for this purpose and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.



Difficulty level:

Hard

Known:

8 wooden spheres of diameter 21 cm placed on small supports which are cylinder of radius 1.5 cm and height 7 cm each. The silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.

Unknown:

The cost of paint required.

Reasoning:

Since each sphere is placed on the cylinder, the area which is to be painted silver will be calculated by subtracting top circular area of the cylinder from the surface area of the sphere.

Area of cylinder which is to be painted black is the CSA of the cylinder.

Surface area of a sphere of base radius, $r = 4\pi r^2$

CSA of the cylinder of base radius, r and height, $h = 2\pi rh$

Solution:

Diameter of the wooden sphere, $d = 21cm$

Radius of the wooden sphere, $R = \frac{21}{2}cm$

Surface area for wooden sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times \frac{21}{2}cm \times \frac{21}{2}cm$$

$$= 1386\ cm^2$$

Since the support is a cylinder of radius, $r = 1.5cm$

Area of the circular end of the cylinder $= \pi r^2$

$$= \frac{22}{7} \times 1.5cm \times 1.5cm$$

$$= 7.07cm^2$$

So, the area of each wooden sphere to be painted $= 1386cm^2 - 7.07cm^2 = 1378.93cm^2$

Total area of the 8 spheres to be painted $= 8 \times 1378.93cm^2 = 11031.44cm^2$

Cost of silver painting the wooden spheres at the rate of 25 paise per cm^2

$$= 11031.44 \times \frac{25}{100}$$

= ₹2757.86

Now,

Radius of the cylinder, $r = 1.5cm$

Height of the cylinder, $h = 7cm$

CSA of the cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 1.5cm \times 7cm$$

$$= 66cm^2$$

CSA of 8 cylindrical support to be painted $= 8 \times 66cm^2 = 528cm^2$

Cost of black painting the cylindrical support at 5 paise per cm^2

$$= 528 \times \frac{5}{100}$$

= ₹26.40

Hence the cost of paint required = ₹2757.86 + ₹26.40

= ₹2784.26

Answer:

The cost of paint required is ₹2784.26

Q3. The diameter of a sphere is decreased by 25%. By what percent does its curved surface area decrease?

Difficulty level:

Hard

Known:

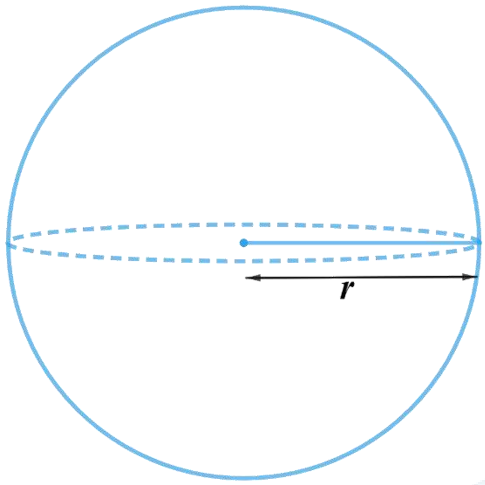
The diameter of a sphere is decreased by 25%.

Unknown:

The percent by which CSA decreases.

Reasoning:

Let's draw a diagram of the sphere to visualize it better.



Surface area of a sphere of base radius, $r = 4\pi r^2$

Solution:

Let the radius of the sphere be r

Then its diameter be $2r$

Curved surface area of the sphere $= 4\pi r^2$

Diameter of the sphere is decreased by 25% hence a new sphere is formed.

Therefore, diameter of the new sphere $= 2r - 2r \times \frac{25}{100}$

$$= 2r - \frac{r}{2}$$

$$= \frac{3r}{2}$$

Radius of the new sphere $= \frac{1}{2} \times \frac{3r}{2} = \frac{3r}{4}$

Curved surface area of the new sphere $= 4\pi \left(\frac{3r}{4}\right)^2$

$$= 4\pi \times \frac{9r^2}{16}$$

$$= \frac{9\pi r^2}{4}$$

Decrease in the original curved surface area $= 4\pi r^2 - \frac{9\pi r^2}{4}$

$$= \frac{16\pi r^2 - 9\pi r^2}{4}$$

$$= \frac{7\pi r^2}{4}$$

Percentage of decrease in the original curved surface area

$$= \left(\frac{7\pi r^2 / 4}{4\pi r^2} \right) \times 100\%$$

$$= \frac{7\pi r^2}{4} \times \frac{1}{4\pi r^2} \times 100\%$$

$$= 43.75\%$$

Answer:

Hence the original curved surface area decreases by 43.75%

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