Chapter 2: Polynimials

Exercise 2.1 (Page 32 of Grade 9 NCERT Textbook)

Q1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$ (ii) $y^2 + \sqrt{2}$ (iii) $3\sqrt{t} + t\sqrt{2}$

(iv) $y + \frac{2}{y}$ (v) $x^{10} + y^3 + t^{50}$

Difficulty level: Easy

Solution:

(i) $4x^2 - 3x + 7 \rightarrow$ Polynomial in one variable x.

(ii) $y^2 + \sqrt{2} \rightarrow$ Polynomial in one variable y.

(iii) $3\sqrt{t} + t\sqrt{2} \rightarrow$ Not a polynomial, since the power of the variable in the first term is $\frac{1}{2}$ which is not a whole number.

(iv) $y + \frac{2}{y} \rightarrow$ Not a polynomial since the power of the variable in the second term is – 1 which is not a whole number.

(v) $x^{10} + y^3 + t^{50} \rightarrow$ Not a polynomial in one variable since there are 3 variables x, y, t.

Q2. Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$ (ii) $2 - x^2 + x^3$ (iii) $\frac{\pi}{2}x^2 + x$ (iv) $\sqrt{2}x - 1$

Difficulty level: Easy

Solution:

(i) $2 + x^2 + x$

Coefficient of $x^2 = 1$

(ii) $2 - x^2 + x^3$

Coefficient of $x^2 = -1$

(iii) $\frac{\pi}{2}x^2 + x$

Coefficient of $x^2 = \frac{\pi}{2}$

(iv) $\sqrt{2}x - 1$

Coefficient of $x^2 = 0$, since there is no term of x^2.

Q3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Difficulty level: Easy

Solution:

(i)A binomial of degree 35

Binomial means polynomial having only 2 terms. Here the highest degree should be 35. So, the binomial will look like $ax^{35} - bx^c$ where $a \neq 0$, $b \neq 0$ and $0 \leq c < 35$.
Example: $3x^{35} - 5$

(ii) A monomial of degree 100

Monomial means polynomial having only 1 term. Here the highest degree should be 100. So, the monomial will look like ax^{100} where $a \neq 0$.
Example: $5x^{100}$

Q4. Write the degree of each of the following polynomials:
(i) $5x^3 + 4x^2 + 7x$ (ii) $4 - y^2$ (iii) $5t - \sqrt{7}$ (iv) 3

Difficulty level: Easy

Reasoning:
The highest power of the variable in a polynomial is called as the degree of the polynomial.

Solution:

(i) Degree of $5x^3+4x^2+7x$ is 3(the highest power of the variable *x*)

(ii) Degree of $4-y^2$ is 2(the highest power of the variable *y*)

(iii) Degree of $5t-\sqrt{7}$ is 1(the highest power of the variable *t*)

(iv) Degree of 3 is 0 (degree of a constant polynomial is 0. Here $3=3x^0$)

Q5. Classify the following as linear, quadratic and cubic polynomials:

(i) x^2+x (ii) $x-x^3$ (iii) $y+y^2+4$ (iv) $1+x$

(v) $3t$ (vi) r^2 (vii) $7x^3$

Difficulty level: Easy

Reasoning:

A polynomial of degree one is called a linear polynomial.

A polynomial of degree two is called a quadratic polynomial.

A polynomial of degree three is called a cubic polynomial.

Solution:

(i) x^2+x →Quadratic polynomial since the degree is 2.

(ii) $x-x^3$ →Cubic polynomial since the degree is 3.

(iii) $y+y^2+4$ →Quadratic polynomial since the degree is 2.

(iv)$1+x$→Linear polynomial since the degree is 1.

(v) 3t → Liner polynomial since the degree is 1.

(vi) r^2 → Quadratic polynomial since the degree is 2.

(vii) $7x^3$ → Cubic polynomial since the degree is 3.

Chapter 2: Polynimials

Exercise 2.2(Page 34 of Grade 9 NCERT Textbook)

Q1. Find the value of the polynomial $5x-4x^2+3$at
(i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

Difficulty level: Easy

Solution:

Let $p(x)=5x-4x^2+3$

(i) $p(0)=5(0)-4(0)^2+3$
$=3$

(ii) $p(-1)=5(-1)-4(-1)^2+3$
$=-5-4+3$
$=-6$

(iii) $p(2)=5(2)-4(2)^2+3$
$=10-16+3$
$=-3$

Q2. Find p (0), p (1) and p (2) for each of the following polynomials:

(i) $p(y)=y^2-y+1$ (ii) $p(t)=2+t+2t^2-t^3$

(iii) $p(x)=x^3$ (iv) $p(x)=(x-1)(x+1)$

Difficulty level: Easy

Solution:

(i)

$p(y)=y^2-y+1$
$p(0)=(0)^2-(0)+1=1$
$p(1)=(1)^2-(1)+1=1$
$p(2)=(2)^2-2+1=3$

(ii)

$$p(t) = 2 + t + 2(t^2) - t^3$$
$$p(0) = 2 + 0 + 2(0)^2 - (0)^3$$
$$= 2 + 0 + 0 - 0 = 2$$
$$p(1) = 2 + 1 + 2(1)^2 - (1)^3$$
$$= 2 + 1 + 2 - 1 = 4$$
$$p(2) = 2 + 2 + 2(2)^2 - (2)^3$$
$$= 2 + 2 + 8 - 8 = 4$$

(iii)

$$p(x) = x^3$$
$$p(0) = (0)^3 = 0$$
$$p(1) = (1)^3 = 1$$
$$p(2) = (2)^3 = 8$$

(iv)

$$p(x) = (x-1)(x+1)$$
$$p(x) = x^2 - 1$$
$$p(0) = (0)^2 - 1 = -1$$
$$p(1) = (1)^2 - 1 = 0$$
$$p(2) = (2)^2 - 1 = 3$$

Q3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x + 1, \ x = -\frac{1}{3}$

(ii) $p(x) = 5x - \pi, \ x = \frac{4}{5}$

(iii) $p(x) = x^2 - 1, \ x = 1, -1$

(iv) $p(x) = (x+1)(x-2), \ x = -1, 2$

(v) $p(x) = x^2, \ x = 0$

(vi) $p(x) = lx + m, \ x = \frac{-m}{l}$

(vii) $p(x) = 3x^2 - 1, \ x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(viii) $p(x) = 2x + 1, \ x = \frac{1}{2}$

Difficulty level: Easy

Reasoning:

In general, we say that a zero of a polynomial $p(x)$ is a number c such that $p(c) = 0$.

Solution:

(i)

$$p(x)=3x+1\ \ ,x=\frac{-1}{3}$$

$$p\left(\frac{-1}{3}\right)=3\times\left(\frac{-1}{3}\right)+1=-1+1=0$$

$$\therefore \frac{-1}{3} \text{ is a zero of } p(x).$$

(ii)

$$p(x)=5x-\pi, x=\frac{4}{5}$$

$$p\left(\frac{4}{5}\right)=5\times\frac{4}{5}-\pi=4-\pi\neq 0$$

$$\therefore \frac{4}{5} \text{ is not a zero of } p(x).$$

(iii)

$$p(x)=x^2-1, x=1,-1$$

$$p(1)=1^2-1=0$$

$$p(-1)=(-1)^2-1=1-1=0$$

$$\therefore 1 \text{ and } -1 \text{ are zeroes of } p(x).$$

(iv)

$$p(x)=(x+1)(x-2), x=-1,2$$

$$p(-1)=(-1+1)(-1-2)=0\times(-3)=0$$

$$p(2)=(2-1)(2-2)=1\times 0=0$$

$$\therefore -1 \text{ and } 2 \text{ are zeroes of } p(x).$$

(v)

$$p(x)=x^2, x=0$$

$$p(0)=0^2=0$$

$$\therefore 0 \text{ is a zero of } p(x).$$

(vi)

$$p(x)=lx+m, x=\frac{-m}{l}$$

$$p\left(\frac{-m}{l}\right)=l\times\left(\frac{-m}{l}\right)+m$$

$$=-m+m=0$$

$$\therefore \frac{-m}{l} \text{ is a zero of } p(x)$$

(vii)

$$p(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$$

$$p\left(\frac{-1}{\sqrt{3}}\right) = 3\times\left(\frac{-1}{\sqrt{3}}\right)^2 - 1$$

$$= 3\times\frac{1}{3} - 1 = 1 - 1 = 0$$

$$\therefore \quad \frac{-1}{\sqrt{3}} \text{ is a zero of } p(x)$$

$$p\left(\frac{2}{\sqrt{3}}\right) = 3\times\left(\frac{2}{\sqrt{3}}\right)^2 - 1$$

$$= 3\times\frac{4}{3} - 1$$

$$= 4 - 1$$

$$= 3 \neq 0$$

$$\therefore \frac{2}{\sqrt{3}} \text{ is not a zero of } p(x)$$

(viii)

$$p(x) = 2x + 1, x = \frac{1}{2}$$

$$p\left(\frac{1}{2}\right) = 2\times\frac{1}{2} + 1$$

$$= 1 + 1$$

$$= 2 \neq 0$$

$$\therefore \frac{1}{2} \text{ is not a zero of } p(x).$$

Q4. Find the zero of the polynomials in each of the following cases:

(i) $p(x) = x + 5$ (ii) $p(x) = x - 5$ (iii) $p(x) = 2x + 5$

(iv) $p(x) = 3x - 2$ (v) $p(x) = 3x$ (vi) $p(x) = ax, a \neq 0$

(vii) $p(x) = c\,x + d$, $c \neq 0$, c, d are real numbers.

Difficulty level: Easy

Reasoning:

In general, we say that a zero of a polynomial $p(x)$ is a number c such that $p(c) = 0$.

Solution:

(i)

$$p(x) = x + 5$$
$$p(x) = 0 \rightarrow x + 5 = 0$$
$$\Rightarrow x = -5$$
$$\therefore -5 \text{ is the zero of } p(x)$$

(ii)

$$p(x) = x - 5$$
$$p(x) = 0 \rightarrow x - 5 = 0$$
$$\Rightarrow x = 5$$
$$\therefore 5 \text{ is the zero of } p(x)$$

(iii)

$$p(x) = 2x + 5$$
$$p(x) = 0 \rightarrow 2x + 5 = 0$$
$$\Rightarrow 2x = -5$$
$$\Rightarrow x = \frac{-5}{2}$$
$$\therefore \frac{-5}{2} \text{ is the zero of } p(x)$$

(iv)

$$p(x) = 3x - 2$$
$$p(x) = 0 \rightarrow 3x - 2 = 0$$
$$\Rightarrow 3x = 2$$
$$\Rightarrow x = \frac{2}{3}$$
$$\therefore \frac{2}{3} \text{ is the zero of } p(x)$$

(v)

$$p(x) = 3x$$
$$p(x) = 0 \rightarrow 3x = 0$$
$$x = 0$$
$$\therefore 0 \text{ is the zero of } p(x)$$

(vi)

$$p(x) = ax, a \neq 0$$

$$p(x) = 0 \rightarrow ax = 0$$

$$x = 0$$

$\therefore 0$ is the zero of $p(x)$

(vii)

$p(x) = cx + d, c \neq 0, c, d$ are real numbers.

$$p(x) = 0 \rightarrow cx + d = 0$$

$$\Rightarrow cx = -d$$

$$\Rightarrow x = \frac{-d}{c}$$

$\therefore \frac{-d}{c}$ is the zero of $p(x)$.

Chapter 2: Polynimials

Exercise 2.3 (Page 40 of Grade 9 NCERT Textbook)

Q1. Find the remainder when x^3+3x^2+3x+1 is divided by

(i) $x+1$ (ii) $x-\frac{1}{2}$ (iii) x (iv) $x+\pi$ (v) $5+2x$

Difficulty Level: Medium

Reasoning:

Let $p(x)$ be any polynomial of degree greater than or equal to one and let a be any real number. If a polynomial $p(x)$ is divided by $x-a$ then the remainder is $p(a)$.

Solution:

Let $p(x)=x^3+3x^2+3x+1$

(i) The root of $x+1=0$ is -1

$$\begin{aligned} p(-1) &= (-1)^3+3(-1)^2+3(-1)+1 \\ &= -1+3-3+1 \\ &= 0 \end{aligned}$$

Hence by the remainder theorem, 0 is the remainder when x^3+3x^2+3x+1 is divided by $x+1$. We can also say that $x+1$ is a factor of x^3+3x^2+3x+1.

(ii) The root of $x-\frac{1}{2}=0$ is $\frac{1}{2}$

$$\begin{aligned} p\left(\frac{1}{2}\right) &= \left(\frac{1}{2}\right)^3+3\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)+1 \\ &= \frac{1}{8}+\frac{3}{4}+\frac{3}{2}+1 \\ &= \frac{1+6+12+8}{8} \\ &= \frac{27}{8} \end{aligned}$$

Hence by the remainder theorem, $\frac{27}{8}$ is the remainder when x^3+3x^2+3x+1 is divided by $x-\frac{1}{2}$.

(iii) The root of $x=0$ is 0

$$p(0)=(0)^3+3(0)^2+3(0)+1$$
$$=0+0-0+1$$
$$=1$$

Hence by the remainder theorem, 1 is the remainder when x^3+3x^2+3x+1 is divided by x.

(iv) The root of $x+\pi=0$ is $-\pi$

$$p(-\pi)=(-\pi)^3+3(-\pi)^2+3(-\pi)+1$$
$$=-\pi^3+3\pi^2-3\pi+1$$

Hence by the remainder theorem, $-\pi^3+3\pi^2-3\pi+1$ is the remainder when x^3+3x^2+3x+1 is divided by $x+\pi$.

(v) The root of $5+2x=0$ is $\frac{-5}{2}$

$$p\left(\frac{-5}{2}\right)=\left(\frac{-5}{2}\right)^3+3\left(\frac{-5}{2}\right)^2+3\left(\frac{-5}{2}\right)+1$$
$$=\frac{-125}{8}+\frac{75}{4}+\frac{-15}{2}+1$$
$$=\frac{-125+150-60+8}{8}$$
$$=\frac{-185+158}{8}$$
$$=\frac{-27}{8}$$

Hence by remainder theorem, $\frac{-27}{8}$ is the remainder when x^3+3x^2+3x+1 is divided by $5+2x$.

Q2. Find the remainder when x^3-ax^2+6x-a is divided by $x-a$.

Difficulty Level: Medium

Reasoning:

Let $p(x)$ be any polynomial of degree greater than or equal to one and let a be any real number. If a polynomial $p(x)$ is divided by $x-a$ then the remainder is $p(a)$.

Solution:

Let $p(x) = x^3 - ax^2 + 6x - a$

The root of $x - a = 0$ is a.

$$p(a) = (a)^3 - a(a)^2 + 6(a) - a$$
$$= a^3 - a^3 + 5a$$
$$= 5a$$

Hence by remainder theorem, $5a$ is the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Q3. Check whether $7 + 3x$ is a factor of $p(x) = 3x^3 + 7x$.

Difficulty Level: Medium

Reasoning:

When a polynomial *p (x)* is divided by *x-a* and by the remainder theorem if *p(a)* = 0 then *x – a* is a factor of *p(x)*.

Solution:

Let $p(x) = 3x^3 + 7x$

The root of $7 + 3x = 0$ is $\frac{-7}{3}$

$$p\left(\frac{-7}{3}\right) = 3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right)$$
$$= \frac{3 \times (-343)}{27} + \frac{-49}{3}$$
$$= \frac{-343 - 147}{9}$$
$$= \frac{-490}{9} \neq 0$$

Since the remainder of $p\left(\frac{-7}{3}\right) \neq 0$, $7 + 3x$ is not a factor of $3x^3 + 7x$.

Chapter 2: Polynimials

Exercise 2.4 (Page 43 of Grade 9 NCERT Textbook)

Q1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1 (ii) $x^4+x^3+x^2+x+1$

(iii) $x^4+3x^3+3x^2+x+1$ (iv) $x^3-x^2-(2+\sqrt{2})x+\sqrt{2}$

Difficulty Level: Medium

Reasoning:

When a polynomial $p(x)$ is divided by $x-a$ and if $p(a)=0$ then $(x-a)$ is a factor of $p(x)$. The root of $x+1=0$ is -1.

Solution:

(i) Let $p(x)=x^3+x^2+x+1$

$$\therefore p(-1)=(-1)^3+(-1)^2+(-1)+1$$
$$=-1+1-1+1=0$$

Since the remainder of $p(-1)=0$, we conclude that x+1 is a factor of x^3+x^2+x+1.

(ii) Let $p(x)=x^4+x^3+x^2+x+1$

$$\therefore p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$$
$$=1-1+1-1+1$$
$$=1\neq 0$$

Since the remainder of $p(-1)\neq 0$, we conclude that x+1 in not a factor of $x^4+x^3+x^2+x+1$.

(iii) Let $p(x)=x^4+3x^3+3x^2+x+1$

$$\therefore p(-1)=(-1)^4+3(-1)^3+3(-1)^2+(-1)+1$$
$$=1-3+3-1+1$$
$$=1\neq 0$$

Since the remainder of $p(-1)\neq 0$, x+1 is not a factor of $x^4+3x^3+3x^2+x+1$.

(iv) Let $p(x)=x^3-x^2-(2+\sqrt{2})x+\sqrt{2}$

$$\therefore p(-1)=(-1)^3-(-1)^2-(2+\sqrt{2})(-1)+\sqrt{2}$$
$$=-1-1+2+\sqrt{2}+\sqrt{2}$$
$$=2\sqrt{2}$$

Since the remainder of $p(-1)\neq 0$, (x+1) is not a factor of $x^3-x^2-(2+\sqrt{2})x+\sqrt{2}$.

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Q2. Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x)=2x^3+x^2-2x-1,\ g(x)=x+1$

(ii) $p(x)=x^3+3x^2+3x+1, g(x)=x+2$

(iii) $p(x)=x^3-4x^2+x+6, g(x)=x-3$

Difficulty Level: Medium

Reasoning:

By factor theorem, $(x-a)$ is a factor of a polynomial $p(x)$ if $p(a)=0$.

To find if $g(x)=x+a$ is a factor of $p(x)$, we need to find the root of $g(x)$.

$x+a=0 \rightarrow x=-a$

Solution:

(i) Let $p(x)=2x^3+x^2-2x-1, g(x)=x+1$

$$x+1=0 \rightarrow x=-1$$

Now,

$$\begin{aligned}p(-1)&=2(-1)^3+(-1)^2-2(-1)-1\\&=-2+1+2-1\\&=0\end{aligned}$$

Since the remainder of $p(-1)=0$, by factor theorem we can say $g(x)=x+1$ is a factor of $p(x)=2x^3+x^2-2x-1$.

(ii) Let $p(x)=x^3+3x^2+3x+1,\ g(x)=x+2$

$$x+2=0 \rightarrow x=-2$$

Now,

$$\begin{aligned}p(-2)&=(-2)^3+3(-2)^2+3(-2)+1\\&=-8+12-6+1\\&=-1\neq 0\end{aligned}$$

Since the remainder of $p(-2)\neq 0$, by factor theorem we can say $g(x)=x+2$ is not a factor of $p(x)=x^3+3x^2+3x+1$.

(iii) Let $p(x)=x^3-4x^2+x+6,\ g(x)=x-3$

$$x-3=0 \rightarrow x=3$$

Now,

$$\begin{aligned}p(3)&=(3)^3-4(3)^2+3+6\\&=27-36+3+6\\&=0\end{aligned}$$

Since the remainder of $p(3)=0$, by factor theorem we can say *g(x)* = *x*-3 is a factor of $p(x)=x^3-4x^2+x+6.$

Q3. Find the value of *k*, if *x* – 1 is a factor of *p*(*x*) in each of the following cases:

Difficulty Level: Medium

(i) $p(x)=x^2+x+k$ (ii) $p(x)=2x^2+kx+\sqrt{2}$

(iii) $p(x)=kx^2-\sqrt{2x}+1$ (iv) $p(x)=kx^2-3x+k$

Reasoning:

By factor theorem, if *x*-1 is a factor of *p*(x), then $p(1)=0$.

Solution:

(i)

$$p(x)=x^2+x+k$$
$$p(1)=(1)^2+(1)+k$$
$$0=2+k$$
$$\Rightarrow k=-2$$

(ii)

$$p(x)=2x^2+kx+\sqrt{2}$$
$$p(1)=2(1)^2+k(1)+\sqrt{2}$$
$$0=2+k+\sqrt{2}$$
$$\Rightarrow k=-(2+\sqrt{2})$$

(iii)

$$p(x)=kx^2-\sqrt{2x}+1$$
$$p(1)=k(1)^2-\sqrt{2(1)}+1$$
$$0=k-\sqrt{2}+1$$
$$\Rightarrow k=\sqrt{2}-1$$

(iv)

$$p(x)=kx^2-3x-k$$
$$p(1)=k(1^2)-3(1)-k$$
$$0=2k-3$$
$$\Rightarrow k=\frac{3}{2}$$

Q4. Factorise:

(i) $12x^2 - 7x + 1$ (ii) $2x^2 + 7x + 3$

(iii) $6x^2 + 5x - 6$ (iv) $3x^2 - x - 4$

Reasoning:

By splitting method, we can find factors using the following method.
Find 2 numbers *p*, *q* such that:

i. $p + q$ = co-efficient of x
ii. pq = co-efficient of x^2 and the constant term.

Solution:

(i) $12x^2 - 7x + 1$

$p + q = -7$ (co-efficient of x)

$pq = 12 \times 1 = 12$ (co-efficient of x^2 and the constant term.)

By trial and error method, we get $p = -4, \quad q = -3.$

Now splitting the middle term of the given polynomial,

$$\begin{aligned} 12x^2 - 7x + 1 &= 12x^2 - 4x - 3x + 1 \\ &= 4x(3x-1) - 1(3x-1) \\ &= (3x-1)(4x-1) \quad \text{(taking } (3x-1) \text{ as common)} \end{aligned}$$

(ii) $2x^2 + 7x + 3$

$p + q = 7$ (co-efficient of x)

$pq = 2 \times 3 = 6$ (co-efficient of x^2 and the constant term.)

By trial and error method, we get $p = 6$, $q = 1$.

Now splitting the middle term of the given polynomial,

$$\begin{aligned} 2x^2 + 7x + 3 &= 2x^2 + 6x + x + 3 \\ &= 2x(x+3) + 1(x+3) \\ &= (2x+1)(x+3) \end{aligned}$$

(iii) $6x^2 + 5x - 6$

$p + q = 5$ (co-efficient of x)

$pq = 6 \times (-6) = -36$ (co-efficient of x^2 and the constant term.)

By trial and error method, we get $p = 9, \quad q = -4.$

Now splitting the middle term of the given polynomial,

$$\begin{aligned} 6x^2 + 5x - 6 &= 6x^2 + 9x - 4x - 6 \\ &= 3x(2x+3) - 2(2x+3) \\ &= (3x-2)(2x+3) \end{aligned}$$

(iv) $3x^2 - x - 4$

$$p + q = -1 \text{ (co-efficient of } x)$$

$$pq = 3 \times (-4) = -12 \text{ (co-efficient of } x^2 \text{ and the constant term.)}$$

By trial and error method, we get $p = -4, \quad q = 3.$

Now splitting the middle term of the given polynomial,

$$\begin{aligned} 3x^2 - x - 4 &= 3x^2 - 4x + 3x - 4 \\ &= 3x^2 + 3x - 4x - 4 \\ &= 3x(x+1) - 4(x+1) \\ &= (3x-4)(x+1) \end{aligned}$$

Q5. Factorise:

(i) $x^3 - 2x^2 - x + 2$ (ii) $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + 32x + 20$ (iv) $2y^3 + y^2 - 2y - 1$

Solution:

(i) Let $p(x) = x^3 - 2x^2 - x + 2$

By the factor theorem we know that *x-a* is a factor of $p(x)$ if $p(a) = 0.$

We shall find a factor of *p(x)* by using some trial value of x, say $x = 1.$

$$\begin{aligned} p(1) &= (1)^3 - 2(1)^2 - 1 + 2 \\ &= 1 - 2 - 1 + 2 \\ &= 0 \end{aligned}$$

Since the remainder of $p(1) = 0$, by factor theorem we can say $x = 1$ is a factor of $p(x) = x^3 - 2x^2 - x + 2.$

Now divide *p(x)* by *x*-1 using long division,

$$\begin{array}{r|l} & x^2 - x - 2 \\ \hline x-1 & x^3 - 2x^2 - x + 2 \\ & x^3 - x^2 \\ \hline & -x^2 - x \\ & -x^2 + x \\ \hline & -2x + 2 \\ & -2x + 2 \\ \hline & 0 \end{array}$$

Hence $x^3 - 2x^2 - x + 2 = (x-1)(x^2 - x - 2)$

Now taking $x^2 - x - 2$, find 2 numbers *p*, *q* such that:

i. $p + q$ = co-efficient of x

ii. pq = co-efficient of x^2 and the constant term.

$$p + q = -1 \text{ (co-efficient of } x)$$

$$pq = 1 \times (-2) = -2 \text{ (co-efficient of } x^2 \text{ and the constant term.)}$$

By trial and error method, we get $p=-2, \quad q=1$.

Now splitting the middle term of the given polynomial,

$$\begin{aligned} x^2-x-2 &= x^2-2x+x-2 \\ &= x(x-2)+1(x-2) \\ &= (x+1)(x-2) \end{aligned}$$

$\therefore x^3-2x^2-x+2=(x-1)(x-2)(x+1)$

Method 2:

$$\begin{aligned} x^3-2x^2-x+2 &= (x^3-2x^2)-(x-2) \\ &= x^2(x-2)-1(x-2) \\ &= (x-2)(x^2-1) \\ &= (x-2)(x+1)(x-1) \end{aligned}$$

(By using $a^2-b^2=(a+b)(a-b)$)

(ii) Let $p(x)=x^3-3x^2-9x-5$

By the factor theorem we know that $x-a$ is a factor of $p(x)$ if $p(a)=0$.

We shall find a factor of *p(x)* by using some trial value of *x*, say *x* = 1.

$$\begin{aligned} p(1) &= (1)^3-3(1)^2-9(1)-5 \\ &= 1-3-9-5 \\ &= -16 \neq 0 \end{aligned}$$

Since the remainder of $p(1)\neq 0$, by factor theorem we can say x=1 is not a factor of $p(x)=x^3-3x^2-9x-5$.

Now say *x* = -1.

$$\begin{aligned} p(-1) &= (-1)^3-3(-1)^2-9(-1)-5 \\ &= -1-3+9-5 \\ &= -9+9=0 \end{aligned}$$

Since the remainder of $p(-1)=0$, by factor theorem we can say x=-1 is a factor of $p(x)=x^3-3x^2-9x-5$.

Now dividing *p(x)* by *x*+1using long division.

$$\begin{array}{r|l} & x^2-4x-5 \\ x+1 & x^3-3x^2-9x-5 \\ & x^3+x^2 \\ \hline & -4x^2-9x \\ & -4x^2-4x \\ \hline & -5x-5 \\ & -5x-5 \\ \hline & 0 \end{array}$$

Hence $x^3-3x^2-9x-5=(x+1)(x^2-4x-5)$

Now taking $x^2-4x-5,$ find 2 numbers p, q such that:

i. $p + q$ = co-efficient of x

ii. pq = co-efficient of x^2 and the constant term.

$$p+q=-4 \text{ (co-efficient of } x)$$
$$pq=1\times-5=-5 \text{ (co-efficient of } x^2 \text{ and the constant term.)}$$

By trial and error method, we get $p=-5, \quad q=1.$

Now splitting the middle term of the given polynomial,

$$\begin{aligned} x^2-4x-5 &= x^2-5x+x-5 \\ &= x(x-5)+1(x-5) \\ &= (x+1)(x-5) \end{aligned}$$

$$\begin{aligned} \therefore x^3-2x^2-x+2 &= (x+1)(x-5)(x+1) \\ &= (x+1)^2(x-5) \end{aligned}$$

(iii) Let $p(x)=x^3+13x^2+32x+20$

By the factor theorem we know that x-a is a factor of $p(x)$ if $p(a)$ = 0.

We shall find a factor of $p(x)$ by using some trial value of x, say x = -1. (Since all the terms are positive.)

$$\begin{aligned} p(-1) &= (-1)^3+13(-1)^2+32(-1)+20 \\ &= -1+13-32+20 \\ &= 0 \end{aligned}$$

Since the remainder of $p(-1)=0$, by factor theorem we can say x=-1 is a factor of $p(x)=x^3+13x^2+32x+20.$

Now dividing $p(x)$ by x+1 using long division,

$$\begin{array}{r|l} & x^2+12x+20 \\ \hline x+1 & x^3+13x^2+32x+20 \\ & x^3+x^2 \\ \hline & 12x^2+32x \\ & 12x^2+12x \\ \hline & 20x+20 \\ & 20x+20 \\ \hline & 0 \end{array}$$

$$\therefore x^3+13x^2+32x+20=(x+1)(x^2+12x+20)$$

Now taking $x^2+12x+20$, find 2 numbers *p*, *q* such that:

i. $p + q$ = co-efficient of x

ii. pq = co-efficient of x^2 and the constant term.

$$p+q=12 \text{ (co-efficient of } x\text{)}$$
$$pq=1\times 20=20 \text{ (co-efficient of } x^2 \text{ and the constant term.)}$$

By trial and error method, we get $p=10, \quad q=2.$

Now splitting the middle term of the given polynomial,

$$\begin{aligned} x^2+12x+20 &= x^2+10x+2x+20 \\ &= x(x+10)+2(x+10) \\ &= (x+10)(x+2) \end{aligned}$$

$$\therefore x^3+13x^2+32x+20=(x+1)(x+10)(x+2)$$

Method 2:

$$\begin{aligned} x^3+13x^2+32x+20 &= x^3+10x^2+3x^2+30x+2x+20 \\ &= x^2(x+10)+3x(x+10)+2(x+10) \\ &= (x+10)(x^2+3x+2) \\ &= (x+10)(x^2+2x+x+2) \\ &= (x+10)[x(x+2)+1(x+2)] \\ &= (x+10)(x+2)(x+1) \end{aligned}$$

(iv) Let $p(y)=2y^3+y^2-2y-1$

By the factor theorem we know that *(y-a)* is a factor of *p*(*y*) if *p(a)* = 0.

We shall find a factor of *p(y)* by using some trial value of *y*, say *y* = 1.

$$\begin{aligned} p(1) &= 2(1)^3+(1)^2-2(1)-1 \\ &= 2+1-2-1 \\ &= 0 \end{aligned}$$

Since the remainder of $p(1)=0$, by factor theorem we can say *y*-1 is a factor of $p(y)=2y^3+y^2-2y-1$

Now dividing *p(y)* by *y*-1 using long division,

$$\begin{array}{r|l} & 2y^2+3y+1 \\ y-1 & 2y^3+y^2-2y-1 \\ & 2y^3-2y^2 \\ \hline & 3y^2-2y \\ & 3y^2-3y \\ \hline & y-1 \\ & y-1 \\ \hline & 0 \end{array}$$

$$\therefore 2y^3+y^2-2y-1=(y-1)(2y^2+3y+1)$$

Now taking $2y^2+3y+1$, find 2 numbers *p*, *q* such that:

i. $p+q=$ co-efficient of y

ii. $pq=$ co-efficient of y^2 and the constant term.

$$p+q=3 \text{ (co-efficient of } y\text{)}$$

$$pq=2\times1=2 \text{ (co-efficient of } y^2 \text{ and the constant term.)}$$

By trial and error method, we get $p=2,\quad q=1.$

Now splitting the middle term of the given polynomial,

$$\begin{aligned}2y^2+3y+1&=2y^2+2y+y+1\\&=2y(y+1)+1(y+1)\\&=(2y+1)(y+1)\end{aligned}$$

$$\therefore 2y^3+y^2-2y-1=(y-1)(2y+1)(y+1)$$

Chapter 2: Polynimials

Exercise 2.5 (Page 40 of Grade 9 NCERT Textbook)

Q1. Use suitable identities to find the following products:

(i) $(x + 4)(x + 10)$ (ii) $(x + 8)(x – 10)$ (iii) $(3x + 4)(3x – 5)$

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$ (v) $(3- 2x)(3 +2x)$

Difficulty Level: Easy

Reasoning:
Identities:

$$(x+a)(x+b)=x^2+(a+b)x+ab$$
$$(a+b)(a-b)=a^2-b^2$$

Solution:

(i) $(x+4)(x+10)$

Identity: $(x+a)(x+b)=x^2+(a+b)x+ab$

Here $a=4,\quad b=10$

$$\begin{aligned}(x+4)(x+10)&=x^2+(4+10)x+4\times 10\\&=x^2+14x+40\end{aligned}$$

(ii) $(x+8)(x-10)$

Identity: $(x+a)(x+b)=x^2+(a+b)x+ab$

Here $a=8,\quad b=-10$

$$\begin{aligned}(x+8)(x-10)&=x^2+(8-10)x+(8)(-10)\\&=x^2-2x-80\end{aligned}$$

(iii) $(3x+4)(3x-5)$

Identity: $(x+a)(x+b)=x^2+(a+b)x+ab$

Here $x\to 3x, a=4, b=-5$

$$\begin{aligned}(3x+4)(3x-5)&=(3x)^2+(4-5)(3x)+(4)(-5)\\&=9x^2-3x-20\end{aligned}$$

(iv) $(y^2 + \frac{3}{2})(y^2 - \frac{3}{2})$

Identity: $(a+b)(a-b) = a^2 - b^2$

Here $a = y^2, \quad b = \frac{3}{2}$

$$\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right) = \left(y^2\right)^2 - \left(\frac{3}{2}\right)^2$$

$$= y^4 - \frac{9}{4}$$

(v) (3- 2x) (3 +2x)

Identity: $(a+b)(a-b) = a^2 - b^2$

Here $a = 3, \quad b = 2x$

$$(3-2x)(3+2x) = (3)^2 - (2x)^2$$

$$= 9 - 4x^2$$

Q2. Evaluate the following products without multiplying directly:

(i) 103×107 (ii) 95×96 (iii) 104×96

Difficulty Level: Easy

Reasoning:

Identities:

$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

$$(a+b)(a-b) = a^2 - b^2$$

Solution:

(i) 103×107

Identity: $(x+a)(x+b) = x^2 + (a+b)x + ab$

$$103 \times 107 = (100+3)(100+7)$$

$$= (100)^2 + (3+7)(100) + (3)(7)$$

(taking $x = 100, \ a = 3, \ b = 7$)

$$= 10000 + 1000 + 21$$

$$= 11021$$

(ii) 95×96

Identity: $(x+a)(x+b) = x^2 + (a+b)x + ab$

$$95 \times 96 = (100-5)(100-4)$$

$$= (100)^2 + (-5-4)(100) + (-5)(-4)$$

(Taking $x = 100, \quad a = -5, \quad b = -4$)

= 10000 - 900+20

= 9120

(iii) 104×96

Identity: $(a+b)(a-b)=a^2-b^2$

$104 \times 96=(100+4)(100-4)$

$=(100)^2-(4)^2$

(Taking a = 100, b = 4)

= 10000-16

= 9984

Q3. Factorise the following using appropriate identities:

(i) $9x^2+6xy+y^2$ (ii) $4y^2-4y+1$ (iii) $x^2-\frac{y^2}{100}$

Difficulty Level: Easy

Reasoning:

Identities: $(a+b)^2=a^2+2ab+b^2$

$(a-b)^2=a^2-2ab+b^2$

$(a+b)(a-b)=a^2-b^2$

Solution:

(i) $9x^2+6xy+y^2=(3x)^2+2(3x)(y)+(y)^2$

Identity: $(a+b)^2=a^2+2ab+b^2$

Here $a=3x, \quad b=y$

Hence $9x^2+6xy+y^2=(3x+y)^2$

(ii) $4y^2-4y+1=(2y)^2-2(2y)(1)+(1)^2$

Identity: $(a-b)^2=a^2-2ab+b^2$

Here $a=2y, \quad b=1$

Hence $4y^2-4y+1=(2y-1)^2$

(iii) $x^2-\frac{y^2}{100}=x^2-\left(\frac{y}{10}\right)^2$

Identity: $(a+b)(a-b)=a^2-b^2$

Here $a=x, \quad b=\frac{y}{10}$

Hence $x^2-\frac{y^2}{100}=\left(x+\frac{y}{10}\right)\left(x-\frac{y}{10}\right)$

Q4. Expand each of the following, using suitable identities:

(i) $(x+2y+4z)^2$ (ii) $(2x-y+z)^2$ (iii) $(-2x+3y+2z)^2$

(iv) $(3a-7b-c)^2$ (v) $(-2x+5y-3z)^2$ (vi) $\left[\frac{1}{4}a-\frac{1}{2}b+1\right]^2$

Difficulty Level: Easy

Reasoning:

Identity: $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

Solution:

(i) $(x+2y+4z)^2$

Identity: $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

Taking $a=x,\ b=2y,\ c=4z$

$$\begin{aligned}(x+2y+4z)^2 &= x^2+(2y)^2+(4z)^2+2(x)(2y)+2(2y)(4z)+2(4z)(x)\\ &= x^2+4y^2+16z^2+4xy+16yz+8zx\end{aligned}$$

(ii) $(2x-y+z)^2$

Identity: $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

Taking $a=2x,\ b=-y,\ c=z$

$$\begin{aligned}(2x-y+z)^2 &= (2x)^2+(-y)^2+(z)^2+2(2x)(-y)+2(-y)(z)+2(z)(2x)\\ &= 4x^2+y^2+z^2-4xy-2yz+4zx\end{aligned}$$

(iii) $(-2x+3y+2z)^2$

Identity: $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

Taking $a=-2x,\ b=3y,\ c=2z$

$$\begin{aligned}(-2x+3y+2z)^2 &= (-2x)^2+(3y)^2+(2z)^2+2(-2x)(3y)+2(3y)(2z)+2(2z)(-2x)\\ &= 4x^2+9y^2+4z^2-12xy+12yz-8zx\end{aligned}$$

(iv) $(3a-7b-c)^2$

Identity: $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

Taking $a=3a,\ b=-7b,\ c=-c$

$$\begin{aligned}(3a-7b-c)^2 &= (3a)^2+(-7b)^2+(-c)^2+2(3a)(-7b)+2(-7b)(-c)+2(-c)(3a)\\ &= 9a^2+49b^2+c^2-42ab+14bc-6ca\end{aligned}$$

(v) $(-2x+5y-3z)^2$

Identity: $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

Taking $a=-2x,\ b=5y,\ c=-3z$

$$\begin{aligned}(-2x+5y-3z)^2 &= (-2x)^2+(5y)^2+(-3z)^2+2(-2x)(5y)+2(5y)(-3z)+2(-3z)(-2x)\\ &= 4x^2+25y^2+9z^2-20xy-30yz+12zx\end{aligned}$$

(vi) $\left[\frac{1}{4}a-\frac{1}{2}b+1\right]^2$

Identity: $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

Taking $a=\frac{1}{4}a,\ \ b=\frac{-1}{2}b,\ \ c=1$

$$\left[\frac{1}{4}a-\frac{1}{2}b+1\right]^2=(\frac{1}{4}a)^2+(\frac{-1}{2}b)^2+(1)^2+2(\frac{1}{4}a)(\frac{-1}{2}b)+2(\frac{-1}{2}b)(1)+2(1)(\frac{1}{4}a)$$

$$=\frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{1}{4}ab-b+\frac{1}{2}a$$

Q5. Factorise:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Difficulty Level: Easy

Reasoning:

Identity: $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

Solution:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

This can be re-written as:

$$(2x)^2+(3y)^2+(-4z)^2+2(2x)(3y)+2(3y)(-4z)+2(-4z)(2x)+2(2x)(-4z)$$

Which is of the form: $a^2+b^2+c^2+2ab+2bc+2ca=(a+b+c)^2$

Here $a=2x,\ b=3y,\ c=-4z$

Hence $4x^2+9y^2+16z^2+12xy-24yz-16xz=(2x+3y-4z)^2$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

This can be re-written as:

$$\left(-\sqrt{2}x\right)^2+(y)^2+\left(2\sqrt{2}z\right)^2+2\left(-\sqrt{2}x\right)(y)+2(y)\left(2\sqrt{2}z\right)+2\left(2\sqrt{2}z\right)\left(-\sqrt{2}x\right)$$

Which is of the form: $a^2+b^2+c^2+2ab+2bc+2ca=(a+b+c)^2$

Here $a=-2\sqrt{2}x,\ \ b=y,\ \ c=2\sqrt{2}z$

Hence $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz=(-\sqrt{2}x+y+2\sqrt{2}z)^2$

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Q6. Write the following cubes in expanded form:

(i) $(2x+1)^3$ (ii) $(2a-3b)^3$ (iii) $\left(\frac{3}{2}x+1\right)^3$ (iv) $\left(x-\frac{2}{3}y\right)^3$

Difficulty Level: Easy

Reasoning:

Identities: $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

Solution:

(i) $(2x+1)^3$

Identity: $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

Here $x = 2x, \quad y = 1$

$$\begin{aligned}(2x+1)^3 &= (2x)^3 + (1)^3 + 3(2x)(1)(2x+1)\\ &= 8x^3 + 1 + 6x(2x+1)\\ &= 8x^3 + 1 + 12x^2 + 6x\\ &= 8x^3 + 12x^2 + 6x + 1\end{aligned}$$

(ii) $(2a-3b)^3$

Identity: $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

Here $x = 2a, \quad y = 3b$

$$\begin{aligned}(2a-3b)^3 &= (2a)^3 - (3b)^3 - 3(2a)(3b)(2a-3b)\\ &= 8a^3 - 27b^3 - 18ab(2a-3b)\\ &= 8a^3 - 27b^3 - 36a^2b + 54ab^2\\ &= 8a^3 - 36a^2b + 54ab^2 - 27b^3\end{aligned}$$

(iii) $\left[\frac{3}{2}x+1\right]^3$

Identity: $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

Here $x = \frac{3}{2}x, \quad y = 1$

$$\begin{aligned}\left(\frac{3}{2}x+1\right)^3 &= \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x+1\right)\\ &= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x+1\right)\\ &= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x\\ &= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1\end{aligned}$$

(iv) $(x-\frac{2}{3}y)^3$

Identity: $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

Here $x = x, \quad y = \frac{2}{3}y$

$$\left(x-\frac{2}{3}y\right)^3 = x^3 - \left(\frac{2}{3}y\right)^3 - 3(x)\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$$
$$= x^3 - \frac{8}{27}y^3 - 2xy\left(x-\frac{2}{3}y\right)$$
$$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$$
$$= x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3$$

Q7. Evaluate the following using suitable identities:

(i) $(99)^3$ (ii) $(102)^3$ (iii) $(998)^3$

Difficulty Level: Easy

Reasoning:

Identities: $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

Solution:

(i) $(99)^3 = (100-1)^3$

Identity: $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

Take $x = 100, \quad y = 1$

$$(99)^3 = (100)^3 - (1)^3 - 3(100)(1)(100-1)$$
$$= 1000000 - 1 - 300 \times 99$$
$$= 999999 - 29700$$
$$= 9,70,299$$

(ii) $(102)^3 = (100+2)^3$

Identity: $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

Take $x = 100, \quad y = 2$

$$(102)^3 = (100)^3 + (2)^3 + 3(100)(2)(100+2)$$
$$= 1000000 + 8 + 600 \times 102$$
$$= 1000008 + 61200$$
$$= 10,61,208$$

(iii) $(998)^3 = (1000-2)^3$

Identity: $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

Take $x = 1000, \quad y = 2$

$$\begin{aligned}(998)^3 &= (1000)^3 - (2)^3 - 3(1000)(2)(1000-2)\\ &= 1000000000 - 8 - 6000 \times 998\\ &= 999999992 + 5988000\\ &= 99,40,11,992\end{aligned}$$

Q8. Factorise each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Difficulty Level: Medium

Reasoning:

Identities: $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

Solution:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

This can be re-written as: $(2a)^3 + (b)^3 + 3(2a)^2(b) + 3(2a)(b)^2$

Which is of the form: $x^3 + y^3 + 3xy(x+y) = (x+y)^3$

Hence $8a^3 + b^3 + 12a^2b + 6ab^2 = (2a+b)^3$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

This can be re-written as: $(2a)^3 - (b)^3 - 3(2a)^2(b) + 3(2a)(b)^2$

Which is of the form: $x^3 - y^3 - 3x^2y + 3xy^2 = (x-y)^3$

Hence $8a^3 - b^3 - 12a^2b + 6ab^2 = (2a-b)^3$

(iii) $27 - 125a^3 - 135a + 225a^2$

This can be re-written as: $(3)^3 - (5a)^3 - 3(3)^2(5a) + 3(3)(5a)^2$

$(3)^3 - (5a)^3 - 3(3)(5a)(3-5a)$

Which is of the form: $x^3 - y^3 - 3xy(x-y) = (x-y)^3$

Hence $27 - 125a^3 - 135a + 225a^2 = (3-5a)^3$

(iv) $64a^3-27b^3-144a^2b+108ab^2$

This can be re-written as: $(4a)^3-(3b)^3-3(4a)^2(3b)+3(4a)(3b)^2$

$(4a)^3-(3b)^3-3(4a)(3b)(4a-3b)$

Which is of the form: $x^3-y^3-3xy(x-y)=(x-y)^3$

Hence $64a^3-27b^3-144a^2b+108ab^2=(4a-3b)^3$

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

This can be re-written as: $(3p)^3-\left(\frac{1}{6}\right)^3-3(3p)^2\frac{1}{6}+3(3p)\left(\frac{1}{6}\right)^2$

$(3p)^3-\left(\frac{1}{6}\right)^3-3\frac{1}{6}(3p)\frac{1}{6}\left(3p-\frac{1}{6}\right)$

Which is of the form: $a^3-b^3-3ab(a-b)=(a-b)^3$

Hence $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p=\left(3p-\frac{1}{6}\right)^3$

Q9. Verify:

(i) $(x^3+y^3)=(x+y)(x^2-xy+y^2)$ (ii) $(x^3-y^3)=(x-y)(x^2+xy+y^2)$

Difficulty Level: Easy

Solution:

(i)

$$\begin{aligned}(x+y)(x^2-xy+y^2)&=x(x^2-xy+y^2)+y(x^2-xy+y^2)\\&=x^3-x^2y+xy^2+x^2y-xy^2+y^3\\&=x^3+y^3\end{aligned}$$

(ii)

$$\begin{aligned}(x-y)(x^2+xy+y^2)&=x(x^2+xy+y^2)-y(x^2+xy+y^2)\\&=x^3+x^2y+xy^2-x^2y-xy^2-y^3\\&=x^3-y^3\end{aligned}$$

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Q10. Factorise each of the following:

(i) $27y^3 + 125z^3$ (ii) $64m^3 - 343n^3 = (4m)^3 - (7n)^3$

[Hint: See Question 9.]

Difficulty Level: Medium

Solution:

(i) $27y^3 + 125z^3 = (3y)^3 + (5z)^3$

Using factorization: $(x^3 + y^3) = (x + y)(x^2 - xy + y^2)$

We can write: $(3y)^3 + (5z)^3 = (3y + 5z)[(3y)^2 - (3y) = x^3 - y^3(5z) + (5z)^2]$

$$27y^3 + 125z^3 = (3y + 5z)(9y^2 - 15yz + 25z^2)$$

(ii) $64m^3 - 343n^3 = (4m)^3 - (7n)^3$

Using factorization: $(x^3 - y^3) = (x - y)(x^2 + xy + y^2)$

We can write: $(4m)^3 - (7n)^3 = (4m - 7n)[(4m)^2 + (4m)(7n) + (7n)^2]$

$$64m^3 - 343n^3 = (4m - 7n)(16m^2 + 28mn + 49n^2)$$

Q11. Factorise: $27x^3 + y^3 + z^3 - 9xyz$

Difficulty Level: Easy

Reasoning:

Identity: $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Solution:

The above expression can be written as: $(3x)^3 + (y)^3 + (z)^2 - 3(3x)(y)(z)$

By using the identity $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

We can write $(3x)^3 + (y)^3 + (z)^2 - 3(3x)(y)(z)$

$$= (3x + y + z)[(3x)^2 + (y)^2 + (z)^2 - (3x)(y) - yz - (z)(3x)]$$

Hence $27x^3 + y^3 + z^3 - 9xyz = (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$

Q12. Verify that:

$$x^3 + y^3 + z^3 - 3xy = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

Difficulty Level: Medium

Reasoning:

Identity: $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Solution:

Taking

$$\text{R.H.S.} = \frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$$
$$= \frac{1}{2}(x+y+z)\left[(x^2-2xy+y^2)+(y^2-2yz+z^2)+(z^2-2zx+x^2)\right]$$
$$= \frac{1}{2}(x+y+z)\left[2x^2+2y^2+2z^2-2xy-2yz-2zx\right]$$
$$= \frac{1}{2}(x+y+z)(2)\left[x^2+y^2+z^2-xy-yz-zx\right]$$

$$= x[x^2+y^2+z^2-xy-yz-zx]+y[x^2+y^2+z^2-xy-yz-zx]+z[x^2+y^2+z^2-xy-yz-zx]$$
$$= [x^3+xy^2+xz^2-x^2y-xyz-x^2z+x^2y+y^3+yz^2-xy^2-y^2z-xyz$$
$$+zx^2+y^2z+z^3-xyz-yz^2-xz^2]$$
$$= x^3+y^3+z^3-3xyz = \text{LHS}$$

Q13. If $x+y+z=0$, show that $x^3+y^3+z^3=3xyz$

Difficulty Level: Easy

Reasoning:

Identity: $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

Solution:

By the identity: $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

If $x+y+z=0$ then the entire RHS becomes 0 and hence the LHS $x^3+y^3+z^3-3xyz=0$

Hence $x^3+y^3+z^3=3xyz$

Q14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$ (ii) $(28)^3+(-15)^3+(-13)^3$

Difficulty Level: Easy

Reasoning:

If $x+y+z=0$ then $x^3+y^3+z^3=3xyz$

Solution:

(i) Let $x=-12,\ y=7,\ z=5$

Then $x+y+z=-12+7+5=0$

So, by using the identity,

$$(-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5)$$
$$= -1260$$

(ii) Let $x = 28, \quad y = -15, \quad z = -13$

Then $x + y + z = 28 - 15 - 13 = 0$

So, by using the identity,

$$(28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$$
$$= 16380$$

Q15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area: $25a^2 - 35a + 12$ (ii) Area: $35y^2 + 13y - 12$

Difficulty Level: Hard

Reasoning:

Area of rectangle $=$ length $\times$ breadth

What is the known/given?

Area of rectangle

What is the unknown?

Length and breadth of the rectangle.

Solution:

i)

Area of rectangle $=$ length $\times$ breadth

Hence, we shall factorise the given expression $25a^2 - 35a + 12$

Now taking $25a^2 - 35a + 12$, find 2 numbers p, q such that:

i. $p + q$ = co-efficient of a
ii. pq = co-efficient of a^2 and the constant term.

$p + q = -35$ (co-efficient of a)

$pq = 25 \times 12 = 300$ (co-efficient of a^2 and the constant term.)

By trial and error method, we get $p = -20, \quad q = -15.$

Now splitting the middle term of the given polynomial,

$$25a^2 - 35a + 12 = 25a^2 - 20a - 15a + 12$$
$$= 25a^2 - 15a - 20a + 12$$
$$= 5a(5a - 3) - 4(5a - 3)$$
$$= (5a - 4)(5a - 3)$$

$\therefore 25a^2 - 35a + 12 = (5a - 4)(5a - 3)$

Length $= 5a - 3$ Breadth $= 5a - 4$

Length $= 5a - 4$ Breadth $= 5a - 3$

What is the known/given?
Area of rectangle.

What is the unknown?
Length and breadth of the rectangle.

Solution:
Area of rectangle $=$ length $\times$ breadth
Hence, we shall factorise the given expression: $35y^2 + 13y - 12$

Now taking $35y^2 + 13y - 12$, find 2 numbers p, q such that:

i. $p + q$ = co-efficient of y
ii. pq = co-efficient of y^2 and the constant term.

$p + q = -13$ (co-efficient of y)
$pq = 35 \times (-12) = -420$ (co-efficient of y^2 and the constant term.)

By trial and error method, we get $p = -28, \quad q = 15.$
Now splitting the middle term of the given polynomial,

$$\begin{aligned} 35y^2 + 13y - 12 &= 35y^2 + 28y - 15y - 12 \\ &= 7y(5y+4) - 3(5y+4) \\ &= (5y+4)(7y-3) \end{aligned}$$

$\therefore 35y^2 + 13y - 12 = (5y+4)(7y-3)$

Length $= 5y + 4$ Breadth $= 7y - 3$

Length $= 7y - 3$ Breadth $= 5y + 4$

Q16. What are the possible expressions for the dimensions of the cuboids whose volume are given below?

i) Volume: $3x^2 - 12x$ ii) Volume: $12ky^2 + 8ky - 20k$

Difficulty Level: Hard

Reasoning:
Volume of Cuboid = length x breadth x height

i)

What is the known/given?

Volume of cuboid.

What is the unknown?

Length, breadth and height of the cuboid.

Solution:

Volume of Cuboid = length x breadth x height

Hence, we shall express the given polynomial as product of three expression.

$$3x^2 - 12x = 3x(x-4)$$

$$\text{Length} = 3, \quad \text{breadth} = x, \quad \text{height} = x-4$$

$$\text{Length} = 3, \quad \text{breadth} = x-4, \quad \text{height} = x$$

$$\text{Length} = x, \quad \text{breadth} = 3, \quad \text{height} = x-4$$

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$$\text{Length} = x-4, \quad \text{breadth} = 3, \quad \text{height} = x$$

ii)

What is the known/given?

Volume of cuboid.

What is the unknown?

Length, breadth and height of the cuboid.

Solution:

Volume of Cuboid = length x breadth x height

Hence, we shall express the given polynomial as product of three factors.

$$12ky^2 + 8ky - 20k = 4k(3y^2 + 2y - 5)$$

Now taking $3y^2 + 2y - 5$, find 2 numbers *p*, *q* such that:

i. $p + q$ = co-efficient of y

ii. pq = co-efficient of y^2 and the constant term.

$p + q = 2$ (co-efficient of y)

$pq = 3 \times (-5) = -15$ (co-efficient of y^2 and the constant term.)

By trial and error method, we get $p=5, \quad q=-3.$

Now splitting the middle term of the given polynomial,

$$\begin{aligned} 3y^2+2y-5 &= 3y^2+5y-3y-5 \\ &= 3y^2-3y+5y-5 \\ &= 3y(y-1)+5(y-1) \\ &= (3y+5)(y-1) \end{aligned}$$

Volume $=4k(y-1)(3y+5)$

Length $=4k$, breadth $=y-1$, height $=3y+5$.

Length $=4k$, breadth $=3y+5$, height $=y-1$.

Length $=y-1$, breadth $=4k$, height $=3y+5$.

Length $=y-1$, breadth $=3y+5$, height $=4k$.

Length $=3y+5$, breadth $=4k$, height $=y-1$.

Length $=3y+5$, breadth $=y-1$, height $=4k$.