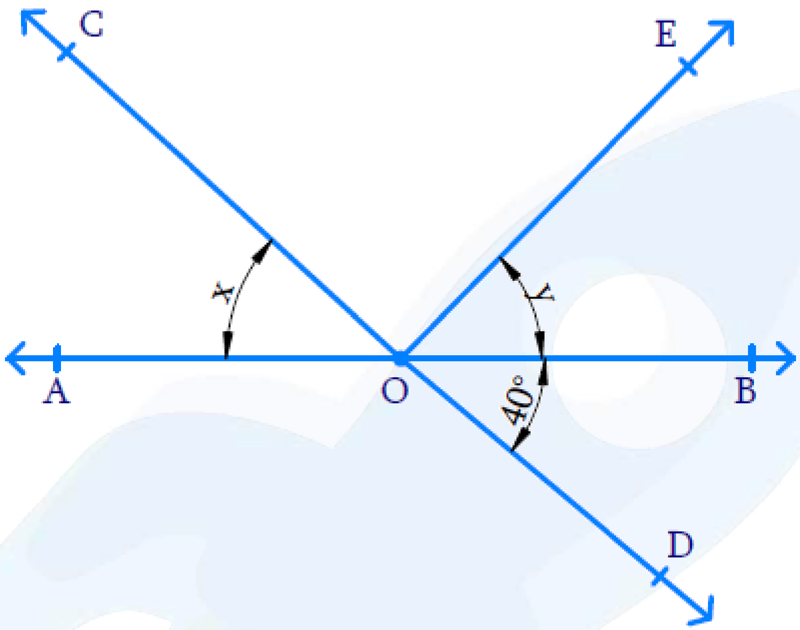
Chapter - 6: Lines and Angles

Exercise 6.1 (Page 96 of Grade 9 NCERT Textbook)

Q1. In the given figure lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70^0$ and $\angle BOD = 40^0$, find $\angle BOE$ and reflex $\angle COE$.

Diagram



What is known/given?

$\angle AOC + \angle BOE = 70^0$ and $\angle BOD = 40^0$

What is unknown?

$\angle BOE$, and Reflex $\angle COE$

Reasoning:

We know that vertically opposite angles formed when two lines intersect are equal. Also, sum of the adjacent angles is 180 degrees.

Solution:

Let $\angle AOC = x$ and $\angle BOE = y$.

Then $x + y = 70^0$ $\quad (\because \angle AOC + \angle BOE = 70^0)$

Let Reflex $\angle COE = z$

We can see that AB and CD are two intersecting lines, so the pair of angles formed are vertically opposite angles and they are equal.

i.e.: $\angle AOD$ and $\angle BOC$ and $\angle AOC = \angle BOD$.

Since $\angle AOC = x$ and $\angle AOC = \angle BOD = 40^0$, we can say that $x = 40^0$.

Also we know that,

$$x + y = 70^0$$
$$40^0 + y = 70^0$$
$$y = 70^0 - 40^0 = 30^0$$
$$\angle BOE = 30^0$$

If we consider line AB and ray OD on it, then $\angle AOD$ and $\angle BOD$ are adjacent angles.

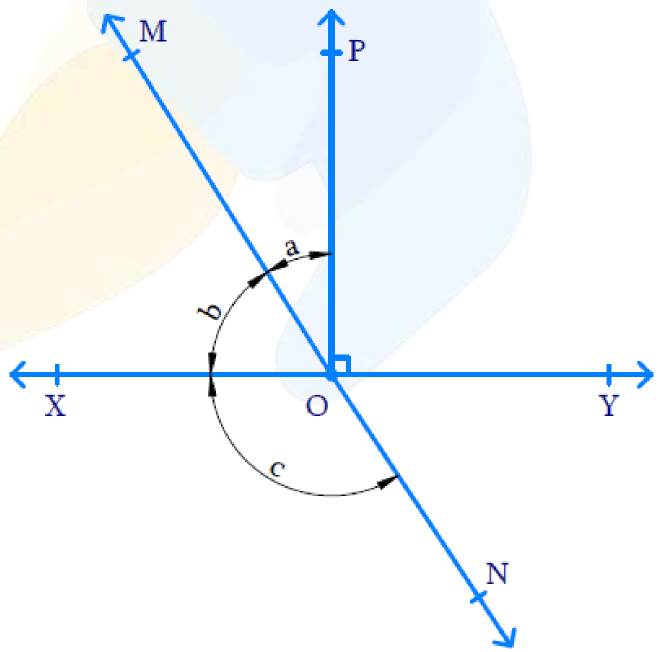
$$\angle AOD + \angle BOD = 180^0$$
$$\angle AOD + 40^0 = 180^0$$
$$\angle AOD = 180^0 - 40^0$$
$$= 140^0$$

Reflex $\angle COE = \angle AOC + \angle AOD + \angle BOD + \angle BOE$

$$= 40^0 + 140^0 + 40^0 + 30^0$$
$$= 250^0$$

Q2. In the given figure, lines XY and MN intersect at O. If $\angle POY = 90^0$ and $a{:}b = 2{:}3$, find c.

Diagram



What is known/given?

$\angle$ POY= 90^0 and $a{:}b = 2{:}3$.

What is unknown?

$\angle$XON or c

Reasoning:

If two lines intersect each other, then the vertically opposite angles formed are equal.

Solution:

Line OP is perpendicular to line XY. Hence $\angle POY = \angle POX = 90^0$

$$\angle POX = \angle POM + \angle MOX$$

$$90^0 = a + b \quad \ldots.(1)$$

Since a and b are in the ratio 2:3 that is, $a = 2x$ and $b = 3x$ ….(2)

Substituting (2) in (1),

$$a + b = 90^0$$

$$2x + 3x = 90^0$$

$$5x = 90^0$$

$$x = \frac{90^0}{5} = 18^0$$

$$a = 2x = 2 \times 18^0$$

$$a = 36^0$$

$$b = 3x = 3 \times 18^0$$

$$b = 54^0$$

Also , $\angle$MOY= $\angle$MOP + $\angle$ POY

$$= a + 90^0$$

$$= 36^0 + 90^0 = 126^0$$

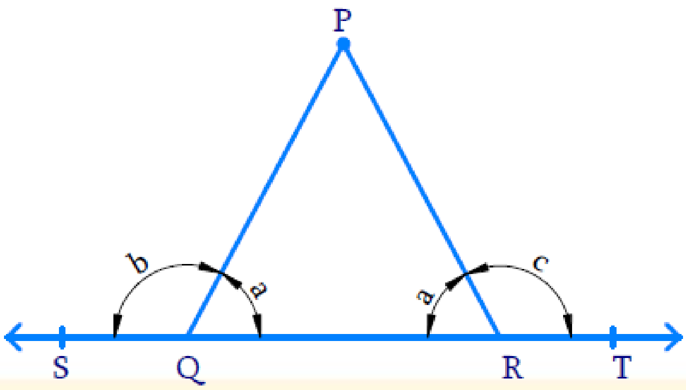
Lines MN an XY intersect at point O and the vertically opposite angles formed are equal.

$$\angle XON = \angle MOY$$

$$c = 126^0$$

Q3. In the given figure, $\angle PQR = \angle PRQ$ then prove that $\angle PQS = \angle PRT$.

Diagram



What is known/given?

$\angle PQR = \angle PRQ$

What is unknown?

To prove $\angle PQS = \angle PRT$

Reasoning:

If a ray stands on a line, then the sum of adjacent angles formed is 180^0.

Solution:

Let $\angle PQR = \angle PRQ = a$.
Let $\angle PQS = b$ and $\angle PRT = c$.

Line ST and PQ intersect at point Q, then the sum of adjacent angles $\angle PQS$ and $\angle PQR$ is 180^0.

$$\angle PQS + \angle PQR = 180^0$$
$$b + a = 180^0$$
$$b = 180^0 - a \quad \ldots.(1)$$

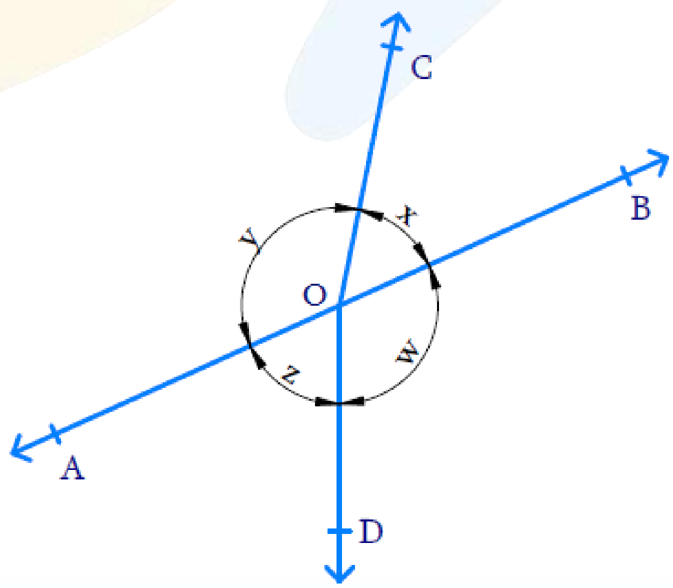
Line ST and PR intersect at point R, then the sum of adjacent angles $\angle PRQ$ and $\angle PRT$ is 180^0.

$$\angle PRQ + \angle PRT = 180^0$$
$$a + c = 180^0$$
$$c = 180^0 - a \quad \ldots.(2)$$

From equations (1) and (2), it is clear that $b = c$. Hence $\angle PQS = \angle PRT$ is proved.

Q4. In the given figure, if $x+y = w+z$, then prove that AOB is a line.

Diagram



What is known/given?

$$x+y = w+z$$

What is unknown?

To prove AOB is a line.

Reasoning:

If the sum of two adjacent angles is 180^0, then the non – common arms of the angles form a line.

Solution:

From the figure we can see that:

$$(x+y)+(w+z)=360^o \qquad (\text{complete angle})$$

It is given that *(x + y) = (w + z)*,

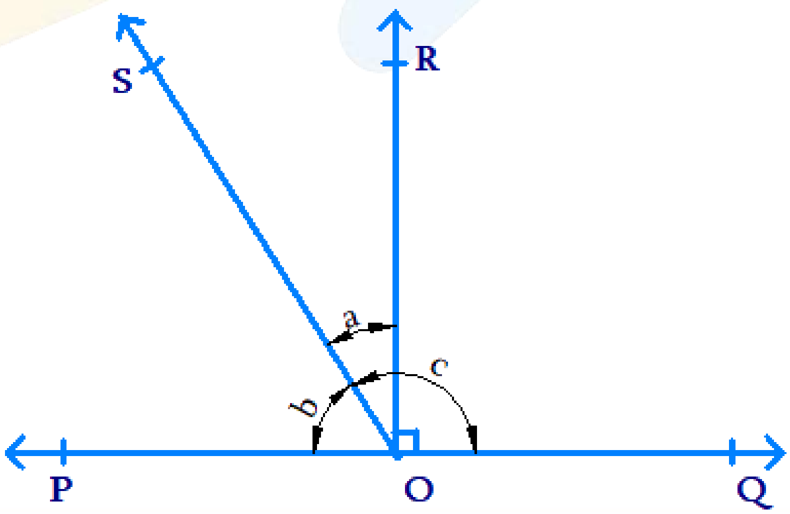
Hence $(x+y)+(w+z)=360^o$ can be written as $(x+y)+(x+y)=360^o$

$$2x+2y=360^o$$
$$2(x+y)=360^o$$
$$x+y=\frac{360^o}{2}=180^o$$

Since sum of adjacent angles *x* and *y* with OA and OB as the non- common arms is 180^0 we can say that AOB is a line.

Q5. In the given figure, POQ is a line. Ray OR, is perpendicular to line PQ. OS another ray lying between rays OP and OR. Prove that $\angle ROS=\frac{1}{2}(\angle QOS-\angle POS)$.

Diagram



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What is known/given?

OR is perpendicular to PQ. $\angle$ROQ = $\angle$ROP = 90^0.

What is unknown?

To prove that: $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

Reasoning:

When a ray intersects a line, then the sum of adjacent angles so formed is 180^0.

Solution:

Let $\angle$ROS = a, $\angle$POS = b and $\angle$SOQ = c.

To prove that: $a = \frac{1}{2}(c - b)$.

Since $\angle$ROQ = $\angle$ROP = 90^0,

We can say, $\angle$POS + $\angle$SOR = $\angle$POR

$$b + a = 90^0 \ldots..(1)$$

Line PQ is intersected by ray OS.

Hence $\angle$POS + $\angle$SOQ = $b + c = 180^0$

$$b + c = 180^0 \ldots.. (2)$$

From equation (1), we get: $a + b = 90^0$

Multiplying by 2 on both sides we get,

$$2(a+b) = 2 \times 90^0$$

$$2(a+b) = 180^0 \ldots..(3)$$

Comparing equations (3) and (2),

$$2(a+b) = b + c$$

$$2a + 2b = b + c$$

$$2a = b + c - 2b$$

$$2a = c - b$$

$$a = \frac{1}{2}(c - b)$$

$$\therefore \angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$

Q6. It is given that $\angle XYZ = 64^0$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

What is known/given?

$\angle XYZ = 64^0$ and Ray YQ bisects $\angle PYZ$.

What is unknown?

$\angle XYQ$ and Reflex $\angle QYP$

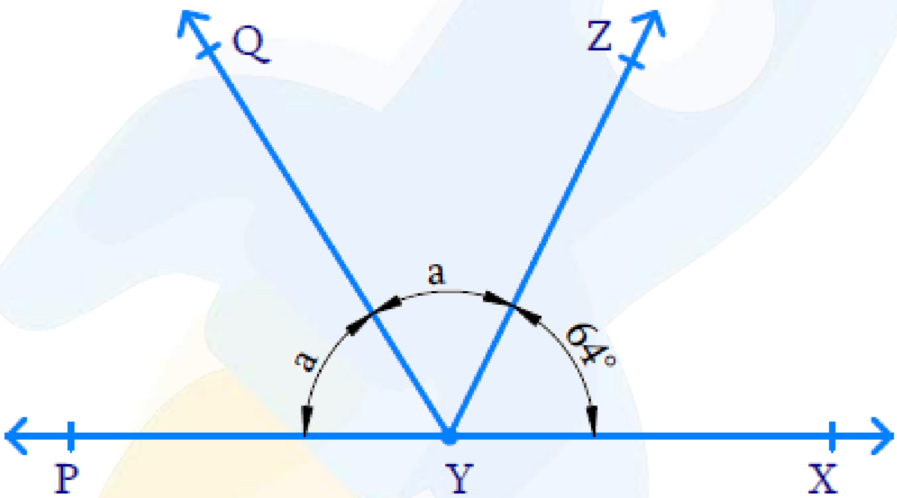
Reasoning:

When a ray intersects a line sum of adjacent angles formed is 180^0.

Solution:

With the given information in the question, we can come up with this diagram.

Diagram



Ray YQ bisects $\angle ZYP$ and let $\angle ZYQ = \angle QYP = a$.

We can see from figure that PX a line and YZ is a ray intersecting at point Y and the sum of adjacent angles so formed is 180^0.

$$\begin{aligned} \text{Hence} \quad \angle ZYP + \angle ZYX &= 180^0 \\ \angle ZYQ + \angle QYP + \angle ZYX &= 180^0 \\ a + a + 64^0 &= 180^0 \\ 2a + 64^0 &= 180^0 \\ 2a &= 180^0 - 64^0 = 116^0 \\ a &= \frac{116}{2} = 58^0 \end{aligned}$$

\Then $\angle XYQ = \angle XYZ + \angle ZYQ$

$\angle XYQ = a + 64^0$

$= 58^0 + 64^0 = 122^0$.

As $\angle QYP = a$, Reflex $\angle QYP = 360^0 - a$

$= 360^0 - 58^0 = 302^0$

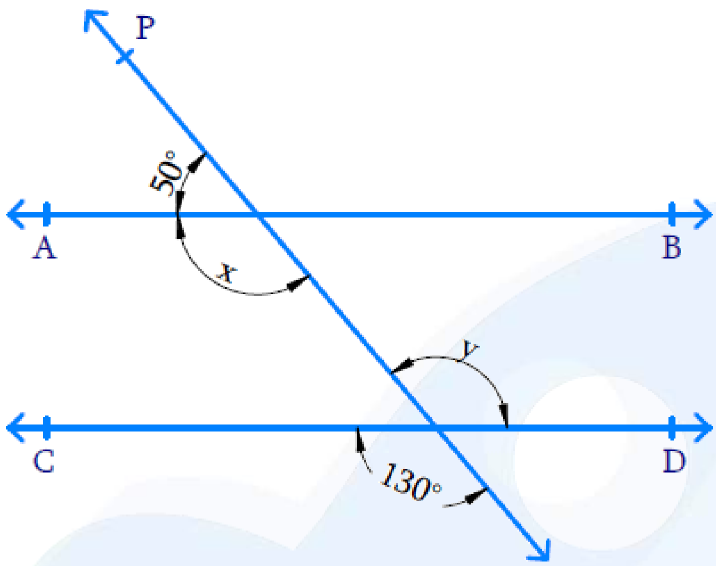
Reflex $\angle QYP = 302^0$.

Chapter - 6: Lines and Angles

Exercise 6.2 (Page 103 of Grade 9 NCERT Textbook)

Q1. In the given figure, find the values of x and y and then show that AB||CD.

Diagram



Reasoning:
When two lines intersect, vertically opposite angles formed are equal.

Also, when a ray intersects a line sum of adjacent angles formed is 180^0.

If a transversal intersects two lines such that a pair of alternate angles is equal, then the two lines are parallel to each other.

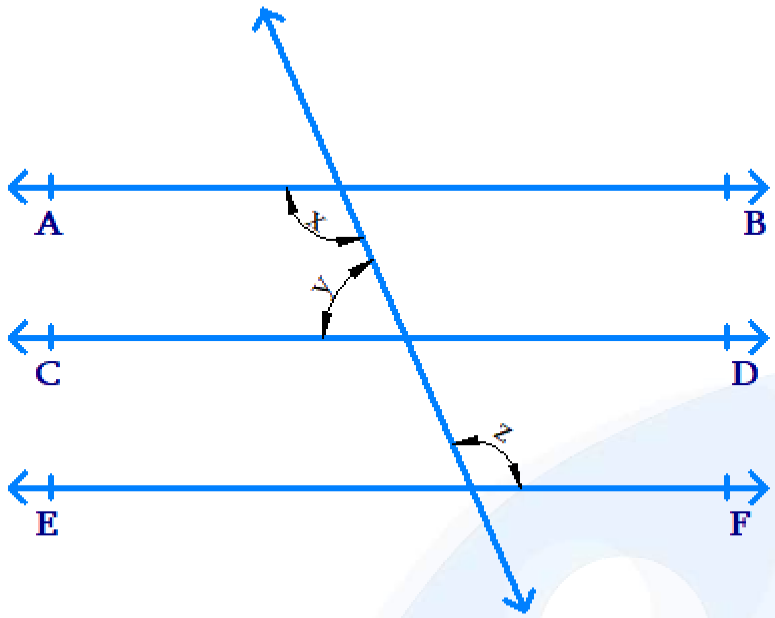
Solution:
Line CD is intersected with line P hence the vertically opposite angles so formed are equal. $y = 130^0$.
Similarly, line AB is intersected by line P hence the sum of adjacent angles formed is 180^0.

$$x + 50^0 = 180^0$$
$$x = 180^0 - 50^0$$
$$x = 130^0$$

We know that, if a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel. Here we can see that the pair of alternate angles formed when lines AB and CD are intersected by transversal P are equal.
$x = y = 130^0$. So we can say the two lines AB and CD are parallel.
Hence AB || CD is proved.

Q2. In the given figure, if AB || CD, CD || EF and y: z = 3:7, find x.

Diagram



What is known/given?

AB || CD

CD || EF

y:z = 3:7

What is unknown?

x

Reasoning:

Lines which are parallel to the same line are parallel to each other.

When two parallel lines are cut by a transversal, co-interior angles formed are supplementary.

Solution:

We know that, lines which are parallel to the same line are parallel to each other.

If AB || CD, CD || EF, we can say AB || EF.

Therefore, the angles x and z are alternate interior angles and hence are equal.

$x = z$ …..(1)

AB and CD are parallel lines cut by transversal. So the co-interior angles formed are supplementary.

$x + y = 180^0$.

Since $x = z$, we get $y + z = 180^0$.

Given y = 3a, z = 7a

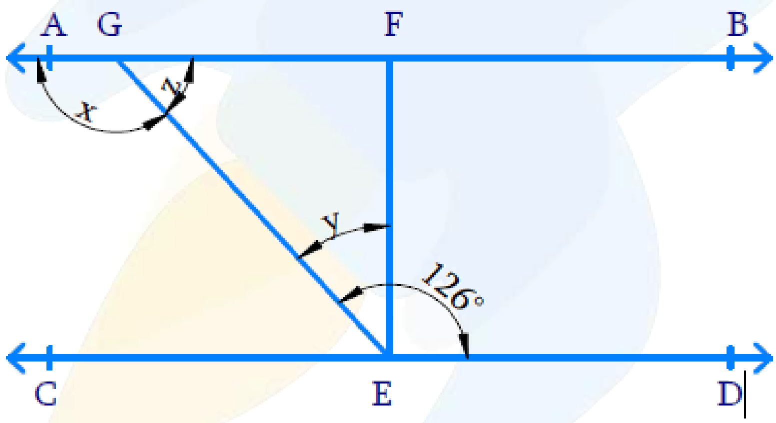
$$3a + 7a = 180^0$$
$$10a = 180^0$$
$$a = \frac{180}{10}$$
$$a = 18^0$$

$$\therefore y = 3a = 3 \times 18 = 54^0$$
$$y = 54^0$$

$$\therefore x + y = 180^0$$
$$x + 54^0 = 180^0$$
$$x = 180^0 - 54^0$$
$$x = 126^0$$

Q3. In the given figure, if AB||CD, EF $\perp$ CD and $\angle$GED $= 126^0$, find $\angle$AGE, $\angle$GEF and $\angle$FGE.

Diagram



What is known/given?

AB||CD, EF $\perp$ CD $\angle$GED $= 126^0$

What is unknown?

$\angle$AGE, $\angle$GEF and $\angle$FGE

Reasoning:

When two lines intersect, adjacent angles formed are supplementary.

When two parallel lines are cut by a transversal, alternate interior angles formed are equal.

Solution:

Let $\angle AGE = x$, $\angle GED = y$ and $\angle FGE = z$.

From the figure, we can see that,

$$\angle GED = \angle GEF + \angle FED$$
$$y = 126^0 - 90^0$$
$$\angle GEF = y = 36^0$$

AB and CD are parallel lines cut by a transversal, the pair of alternative angles formed are equal.

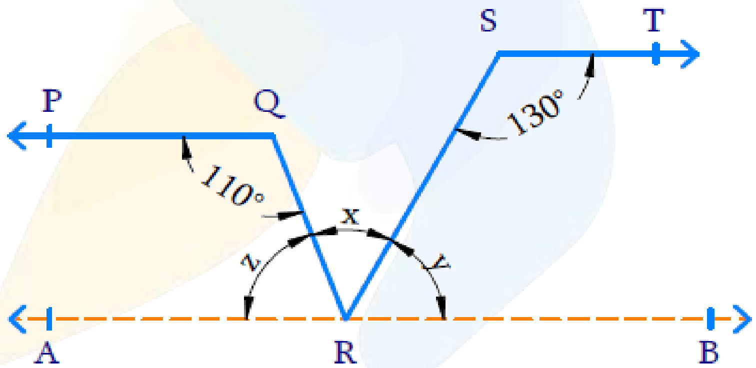
$$\angle AGE = \angle GED$$
$$\angle AGE = x = 126^0$$

Line AB is intersected by line GE hence adjacent angles formed are supplementary.

$$x + z = 180^0$$
$$126^0 + z = 180^0$$
$$z = 180^0 - 126^0 = 54^0$$
$$\angle FGE = z = 54^0$$

Q4. In the given figure, if PQ || ST, $\angle PQR = 110^0$ and $\angle RST = 130^0$, find $\angle$ QRS.
[Hint : Draw a line parallel to ST through point R.]

Diagram



What is known/given?

PQ || ST, $\angle PQR = 110^0$ and $\angle RST = 130^0$.

What is unknown?

$\angle QRS$

Reasoning:

Lines which are parallel to the same line are parallel to each other.

When two parallel lines are cut a transversal, co-interior angles formed are supplementary.

Solution:

Draw a line AB parallel to ST through point R. Since AB || ST and PQ || ST. So, AB || PQ.

Let $\angle SRQ = x, \angle SRB = y$ and $\angle QRA = z$

Lines ST and AB are parallel with transversal SR intersecting. Therefore, the co-interior angles are supplementary.

$$\angle RST + \angle SRB = 180^0$$
$$130^0 + y = 180^0$$
$$y = 180^0 - 130^0 = 50^0$$
$$\angle SRB = y = 50^0$$

Similarly, lines PQ and AB are parallel with transversal QR intersecting. Therefore, the co-interior angles are supplementary.

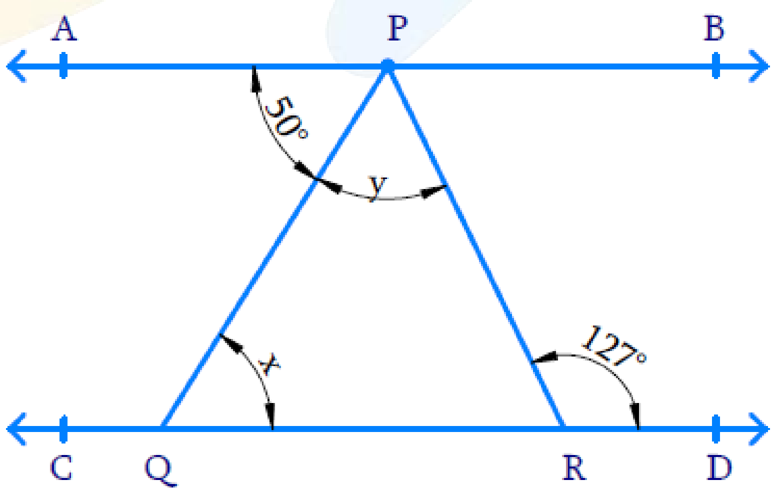
$$\angle PQR + \angle QRA = 180^0$$
$$110^0 + z = 180^0$$
$$z = 180^0 - 110^0 = 70^0$$
$$\angle QRA = z = 70^0$$

AB is a line, RQ and RS are rays on AB. Hence,

$$\angle QRA + \angle QRS + \angle SRB = 180^0$$
$$70^0 + x + 50^0 = 180^0$$
$$120^0 + x = 180^0$$
$$x = 180^0 - 120^0 = 60^0$$
$$\angle QRS = x = 60^0.$$

Q5. In the given figure, if AB || CD, $\angle APQ = 50^0$ and $\angle PRD = 127^0$, find x and y.

Diagram



What is known/given?

$AB \parallel CD$, $\angle APQ = 50^0$ and $\angle PRD = 127^0$

What is unknown?

x and y

Reasoning:

When a ray intersects a line, sum of adjacent angles formed is 180^0.

When two parallel lines are cut by a transversal, alternate interior angles formed are equal.

Solution:

AB and CD are parallel lines cut by transversal PQ hence the alternate interior angles formed are equal.

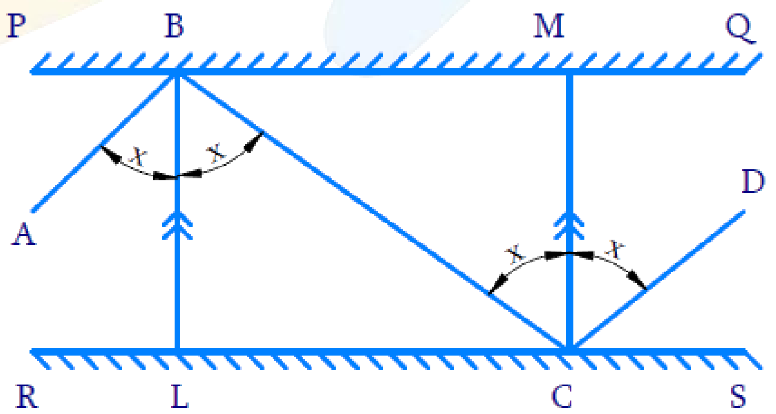
$\angle APQ = \angle PQR$ and hence $x = 50^0$.

Similarly, AB and CD are parallel lines cut by transversal PR hence the alternate angles formed are equal.

$$\angle APR + \angle PRD = 127^0$$
$$\angle APQ + \angle QPR = \angle PRD = 127^0$$
$$50^0 + y = 127^0$$
$$y = 127^0 - 50^0$$
$$y = 77^0.$$

Q6. In the given figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB||CD.

Diagram



What is known/given?

$PQ \parallel RS$

What is unknown?
To prove: AB||CD

Reasoning:
When two parallel lines are cut by a transversal, alternate angles formed are equal.

In optics the angle of incidence (the angle which an incident ray makes with a perpendicular to the surface at the point of incidence) and the angle of reflection (the angle formed by the reflected ray with a perpendicular to the surface at the point of incidence) are equal.

Solution:
Draw perpendicular lines BL and CM at the point of incident on both mirrors since PQ and RS parallel to each other, perpendiculars drawn are parallel BL ∥ CM.

Since BC is a transversal to lines BL and CM, alternate angles are equal so we get

$$\angle LBC = \angle BCM = x(say) \ ......(1)$$

By laws of reflection, at the first point of incidence B, we get:

$$\begin{aligned} \angle ABL &= \angle LBC = x \\ \therefore \angle ABC &= \angle ABL + \angle LBC \\ &= x + x \\ \therefore \angle ABC &= 2x \ \ ....(2) \end{aligned}$$

By laws of reflection, at the first point of incidence C, we get:

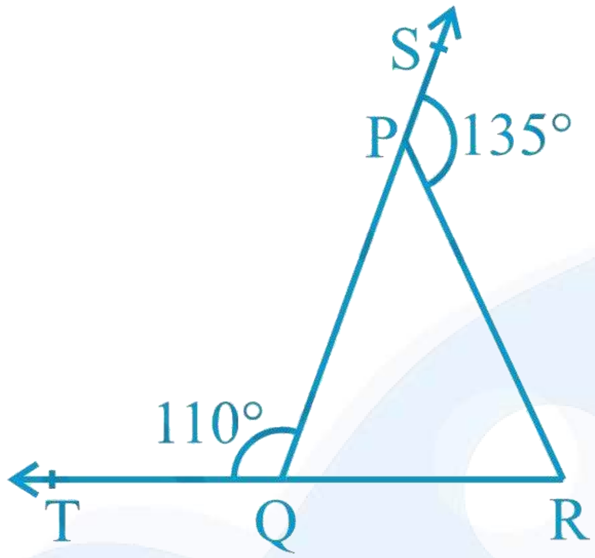
$$\begin{aligned} \angle MCD &= \angle BCM = x \\ \therefore \angle BCD &= \angle BCM + \angle MCD \\ &= x + x \\ \angle BCD &= 2x \quad .....(3) \end{aligned}$$

From equations (2) and (3), we get $\angle ABC = \angle BCD$.
We know that, if a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel. As Alternate angles are equal, we can say AB || CD.

Chapter - 6: Lines and Angles

Exercise 6.3 (Page 107 of Grade 9 NCERT Textbook)

Q1. In Fig. 6.39, sides QP and RQ of Δ PQR are produced to points S and T respectively. If $\angle SPR = 135°$ and $\angle PQT = 110°$, find $\angle PRQ$.



Known/given:
$\angle SPR = 135°$ and $\angle PQT = 110°$

Unknown:
$\angle PRQ$

Reasoning:
As we know the linear pair axioms:
If Non-common arms of two adjacent angles form a line, then these angles are called linear pair of angles and the sum of the linear pair is 180°.
If the sum of two adjacent angles is 180° then the two non-common arms of the angles form a line.

Angle sum property of a triangle:
Sum of the interior angles of a triangle is 360°.

Solution:
Given,

$$\angle SPR = 135° \text{ and } \angle PQT = 110°$$

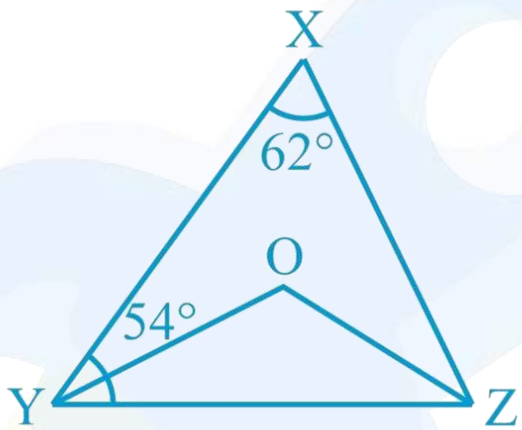
$$\angle SPR + \angle QPR = 180° \qquad (\text{By linear pair axiom})$$
$$135° + \angle QPR = 180°$$
$$\angle QPR = 180° - 135°$$
$$\angle QPR = 45° \qquad (\text{i})$$

$$\angle PQT + \angle PQR = 180° \quad \text{(By linear pair axiom.)}$$
$$110° + \angle PQR = 180°$$
$$\angle PQR = 180° - 110°$$
$$\angle PQR = 70° \quad \text{(ii)}$$

Now,

$$\angle PQR + \angle QPR + \angle PRQ = 180° \quad \text{(Angle sum property of a triangle.)}$$
$$70° + 45° + \angle PRQ = 180° \quad [\text{from (i) and (ii)}]$$
$$\angle PRQ = 180° - 115°$$
$$\angle PRQ = 65°$$

Q2. In Fig. 6.40, $\angle X = 62°$, $\angle XYZ = 54°$. If YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively of ΔXYZ, find $\angle OZY$ and $\angle YOZ$.



Known/given:
$\angle X = 62°$, $\angle XYZ = 54°$ and YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively.

Unknown:
$\angle OZY$ and $\angle YOZ$

Reasoning:
As we know the angle sum property of a triangle:
Sum of the interior angles of a triangle is 360°.

Solution:
Given, in ΔXYZ,

$$\angle X = 62°$$
$$\angle XYZ = 54°$$
$$\angle X + \angle XYZ + \angle Z = 180° \quad \text{(Angle sum property of a triangle.)}$$
$$62° + 54° + \angle Z = 180°$$
$$\angle Z = 180° - 116°$$
$$\angle Z = 64°$$

Now, OZ is angle bisector of $\angle XZY$

$$\angle OZY = \frac{1}{2}\angle XZY = \frac{1}{2}\times 64° = 32° \qquad \text{(i)}$$

Similarly, OY is angle bisector of $\angle XYZ$

$$\angle OYZ = \frac{1}{2}\angle XYZ = \frac{1}{2}\times 54° = 27° \qquad \text{(ii)}$$

Now, in ΔOYZ

$$\angle OYZ + \angle OZY + \angle YOZ = 180° \qquad \text{(Angle sum property of a triangle.)}$$

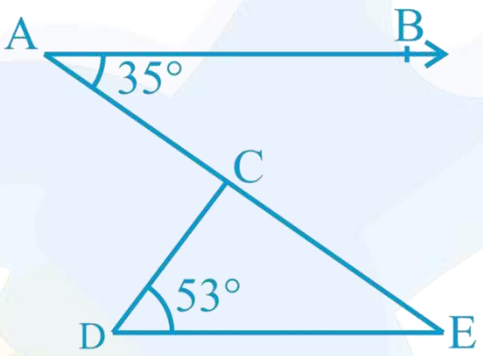
$$27° + 32° + \angle YOZ = 180° \qquad [\text{from (i) and (ii)}]$$

$$\angle YOZ = 180° - 59°$$

$$\angle YOZ = 121°$$

Hence, $\angle OZY = 32°$ and $\angle YOZ = 121°$

Q3. In Fig. 6.41, if AB || DE, $\angle BAC = 35°$ and $\angle CDE = 53°$, find $\angle DCE$.



Known/given:

AB || DE, $\angle BAC = 35°$ and $\angle CDE = 53°$

Unknown:

$\angle DCE$

Reasoning:

As we know when two parallel lines are cut by a transversal, alternate interior angles formed are equal.

Angle sum property of a triangle:
Sum of the interior angles of a triangle is 360°.

Solution:

Given,

$$AB \parallel DE, \ \angle BAC = 35° \text{ and } \angle CDE = 53°$$

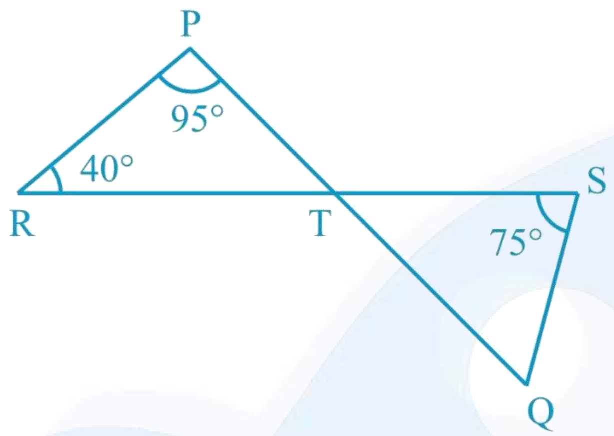
$$\angle DEC = \angle BAC \qquad \text{(Alternate interior angles)}$$

$$\angle DEC = 35°$$

Now, in ΔCDE

$$\angle CDE + \angle DEC + \angle DCE = 180^\circ \quad \text{(Angle sum property of a triangle.)}$$
$$53^\circ + 35^\circ + \angle DCE = 180^\circ$$
$$\angle DCE = 180^\circ - 88^\circ$$
$$\angle DCE = 92^\circ$$

Q4. In Fig. 6.42, if lines PQ and RS intersect at point T, such that $\angle PRT = 40^\circ$, $\angle RPT = 95^\circ$ and $\angle TSQ = 75^\circ$, find $\angle SQT$.



Known/given:
$\angle PRT = 40^\circ$, $\angle RPT = 95^\circ$ and $\angle TSQ = 75^\circ$

Unknown:
$\angle SQT$

Reasoning:
As we know when two line intersect each other at a point then there are two pairs of vertically opposite angles formed are equal.

Angle sum property of a triangle:
Sum of the interior angles of a triangle is 360°.

Solution:
Given,

$$\angle PRT = 40^\circ, \angle RPT = 95^\circ \text{ and } \angle TSQ = 75^\circ$$

In ΔPRT

$$\angle PTR + \angle PRT + \angle RPT = 180^\circ \quad \text{(Angle sum property of a triangle.)}$$
$$\angle PTR + 40^\circ + 95^\circ = 180^\circ$$
$$\angle PTR = 180^\circ - 135^\circ$$
$$\angle PTR = 45^\circ$$

Now,

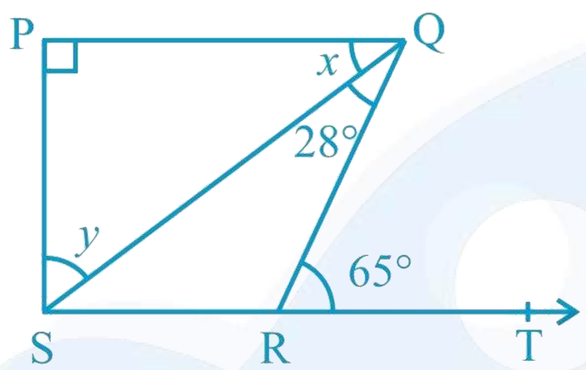
$$\angle QTS = \angle PTR \quad \text{(Vertically opposite angles)}$$
$$\angle QTS = 45^\circ \quad \text{(i)}$$

In ΔTSQ

$$\angle QTS + \angle TSQ + \angle SQT = 180^\circ \quad \text{(Angle sum property of a triangle.)}$$
$$45^\circ + 75^\circ + \angle SQT = 180^\circ \quad \text{[From (i)]}$$
$$\angle SQT = 180^\circ - 120^\circ$$
$$\angle SQT = 60^\circ$$

Hence, $\angle SQT = 60^\circ$

Q5. In Fig. 6.43, if PQ ⊥ PS, PQ || SR, $\angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$, then find the values of x and y.



Known/given:

PQ ⊥ PS, PQ || SR, $\angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$

Unknown:

x and y.

Reasoning:

As we know when two parallel lines are cut by a transversal, alternate interior angles formed are equal.

Angle sum property of a triangle:
Sum of the interior angles of a triangle is 360°.

Solution:

Given,
PQ ⊥ PS, PQ || SR, $\angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$

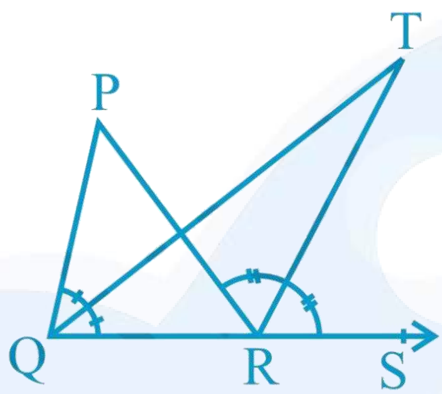
$$\angle PQR = \angle QRT \quad \text{(Alternate interior angles)}$$
$$\angle PQS + \angle SQR = \angle QRT \quad \text{(By figure)}$$
$$x + 28^\circ = 65^\circ$$
$$x = 65^\circ - 28^\circ$$
$$x = 37^\circ$$

Now, in ΔPQS

$$\angle PQS + \angle PSQ + \angle QPS = 180^\circ \qquad \text{(Angle sum property of a triangle.)}$$
$$37^\circ + y + 90^\circ = 180^\circ$$
$$y = 180^\circ - 127^\circ$$
$$y = 53^\circ$$

Hence, $x = 37^\circ$ and $y = 53^\circ$

Q6. In Fig. 6.44, the side QR of ΔPQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that $\angle QTR = \frac{1}{2}\angle QPR$.



Known/given:

The side QR of ΔPQR is produced to a point S and the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T.

To prove:

$\angle QTR = \frac{1}{2}\angle QPR$

Reasoning:

As we know exterior angle of a triangle:
If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

Solution:

Given,
Bisectors of $\angle PQR$ and $\angle PRS$ meet at point T.
Hence, TR is a bisector of $\angle PRS$ and TQ is a bisector of $\angle PQR$

$$\angle PRS = 2\angle TRS \qquad \text{(i)}$$
$$\angle PQR = 2\angle TQR \qquad \text{(ii)}$$

Now, in ΔTQR

$$\angle TRS = \angle TQR + \angle QTR \qquad \text{(Exterior angle of a triangle)}$$
$$\angle QTR = \angle TRS - \angle TQR \qquad \text{(iii)}$$

Similarly, in ΔPQR

$$\angle PRS = \angle QPR + \angle PQR \qquad \text{(Exterior angle of a triangle)}$$

$$2\angle TRS = \angle QPR + 2\angle TQR \qquad \text{[From (i) and (ii)]}$$

$$\angle QPR = 2\angle TRS - 2\angle TQR$$

$$\angle QPR = 2(\angle TRS - \angle TQR)$$

$$\angle QPR = 2\angle QTR \qquad \text{[From (iii)]}$$

$$\angle QTR = \frac{1}{2}\angle QPR$$

Hence proved.