Triangles

Exercise (7.1)

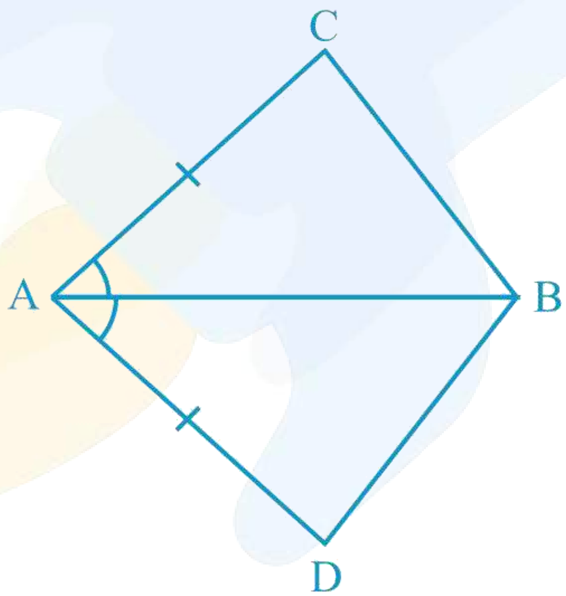
Q1. In quadrilateral ACBD, AC = AD and AB bisects $\angle A$ (See the given figure). Show that $\Delta ABC \cong \Delta ABD$. What can you say about BC and BD?

Difficulty Level:
Easy

Known/given:
AC = AD and AB bisects $\angle A$

To prove:
$\Delta ABC \cong \Delta ABD$ and, what can be said about BC and BD.

Reasoning:
We can show two sides and included angle of ΔABC are equals to corresponding sides and included angle of ΔABD, by using SAS congruency criterion both triangles will be congruent and by CPCT, BC and BD will be equal.



Solution:
In ΔABC and ΔABD,

AC = AD (Given)
$\angle CAB = \angle DAB$(AB bisects $\angle A$)
AB = AB (Common)
$\therefore \Delta ABC \cong \Delta ABD$ (By SAS congruence rule)
$\therefore BC = BD$ (By CPCT)

Therefore, BC and BD are of equal lengths.

Q2. ABCD is a quadrilateral in which $AD = BC$ and $\angle DAB = \angle CBA$ (See the given figure). Prove that

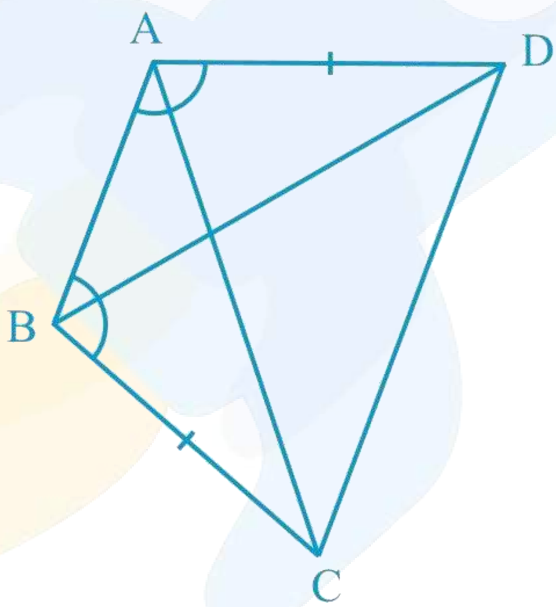
(i) $\Delta ABD \cong \Delta BAC$
(ii) $BD = AC$
(iii) $\angle ABD = \angle BAC$

Difficulty Level:
Easy

Known/given:
$AD = BC$ and $\angle DAB = \angle CBA$

To prove:
(i) $\Delta ABD \cong \Delta BAC$ (ii) $BD = AC$ (iii) $\angle ABD = \angle BAC$

Reasoning:
We can show two sides and included of ΔABD are equals to corresponding sides and included angle of ΔBAC, by using SAS congruency criterion both triangles will be congruent. Then we can say corresponding parts of congruent triangle will be equal.



Solution:
In ΔABD and ΔBAC,

$AD = BC$ (Given)
$\angle DAB = \angle CBA$ (Given)
$AB = BA$ (Common)
$\therefore \Delta ABD \cong \Delta BAC$ (By SAS congruence rule)
$\therefore BD = AC$ (By CPCT)
And, $\angle ABD = \angle BAC$ (By CPCT)

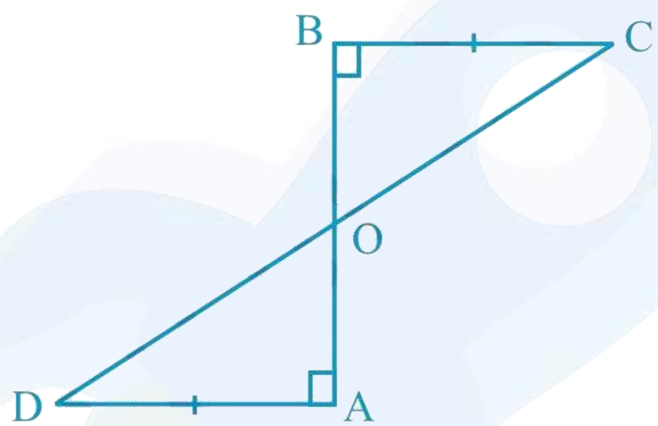
Q3. AD and BC are equal perpendiculars to a line segment AB (See the given figure). Show that CD bisects AB.

Difficulty Level:
Easy

Known/given:
$AD \perp AB$, $BC \perp AB$ and $AD = BC$

To prove:
CD bisects AB or OA = OB

Reasoning:
We can show two triangles OBC and OAD congruent by using AAS congruency rule and then we can say corresponding parts of congruent triangles will be equal.



Solution:
In ΔBOC and ΔAOD,

$$\angle BOC = \angle AOD \text{ (Vertically opposite angles)}$$

$$\angle CBO = \angle DAO \left(\text{Each } 90^\circ\right)$$

$BC = AD$ (Given)

$$\therefore \Delta BOC \cong \therefore \Delta AOD \left(\text{AAS congruence rule}\right)$$

$$\therefore BO = AO \left(\text{By CPCT}\right)$$

$$\Rightarrow \text{CD bisects AB.}$$

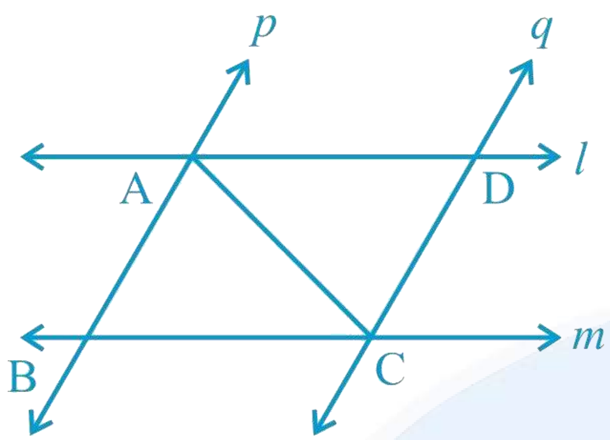
Q4. l and m are two parallel lines intersected by another pair of parallel lines p and q (see the given figure). Show that $\Delta ABC \cong \Delta CDA$.

Difficulty Level:
Easy

Known/given:
$l \parallel m \;\; and \;\; p \parallel q$

To prove:
$\Delta ABC \cong \Delta CDA$.

Reasoning:
We can show both the triangles congruent by using ASA congruency criterion



Solution:
In ΔABC and ΔCDA,

$\angle BAC$ and $\angle DCA$ (Alternate interior angles, as $p \parallel q$)

$AC = CA$ (Common)

$\angle BCA$ and $\angle DAC$ (Alternate interior angles, as $l \parallel m$)

$\therefore \Delta ABC \cong \Delta CDA$ (By ASA congruence rule)

Q5. Line l is the bisector of an angle $\angle A$ and B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle A$ (see the given figure). Show that:

(i) $\Delta APB \cong \Delta AQB$

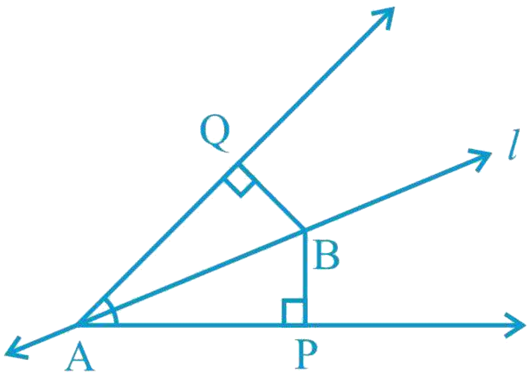
(ii) $BP = BQ$ or B is equidistant from the arms of $\angle A$

Difficulty Level:
Easy

What is known/given?
l is the bisector of an angle $\angle A$ and $BP \perp AP$ and $BQ \perp AQ$

To prove:
$\Delta APB \cong \Delta AQB$ and $BP = BQ$ or B is equidistant from the arms of $\angle A$

Reasoning:
We can show two triangles APB and AQB congruent by using AAS congruency rule and then we can say corresponding parts of congruent triangles will be equal.



Solution:

In ΔAPB and ΔAQB,

$\angle BAP = \angle BAQ$ (l is the angle bisector of $\angle A$)

$\angle APB = \angle AQB$ (Each 90°)

$AB = AB$ (Common)

$\therefore \Delta APB \cong \Delta AQB$ (By AAS congruence rule)

$\therefore BP = BQ$ (By CPCT)

Or, it can be said that B is equidistant from the arms of $\angle A$.

Q6. In the given figure, AC = AE, AB = AD and $\angle BAD = \angle EAC$. Show that BC = DE.

Difficulty Level:

Medium

Known/given:

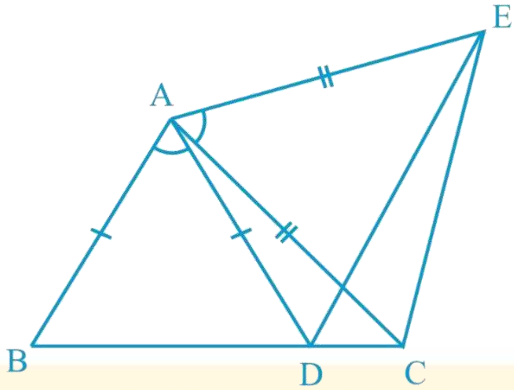
AC = AE, AB = AD and $\angle BAD = \angle EAC$.

To prove:

BC = DE.

Reasoning:

We can show two triangles BAC and DAE congruent by using SAS congruency rule and then we can say corresponding parts of congruent triangles will be equal. To show both triangles congruent two pair of equal sides are given and add angle DAC on both sides in given pair of angles BAD and angle EAC to find the included angle BAC and DAE.



Solution:

It is given that $\angle BAD = \angle EAC$

$$\angle BAD + \angle DAC = \angle EAC + \angle DAC$$
$$\angle BAC = \angle DAE$$

In ΔBAC and ΔDAE,

$AB = AD$ (Given)

$\angle BAC = \angle DAE$ (Prove above)

$AC = AE$ (Given)

$\therefore \Delta BAC \cong \Delta DAE$ (By SAS congruence rule)

$\therefore BC = DE$ (By CPCT)

Q7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$ (See the given figure). Show that

(i) $\Delta DAP \cong \Delta EBP$

(ii) $AD = BE$

Difficulty Level:

Medium

Known/given

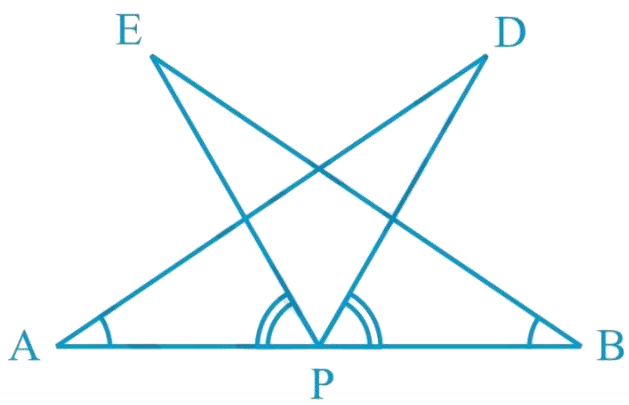
P is its mid-point of AB, $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$

To prove:

(i) $\Delta DAP \cong \Delta EBP$ and (ii) $AD = BE$.

Reasoning:

We can show two triangles DAP and EBP congruent by using ASA congruency rule and then we can say corresponding parts of congruent triangles will be equal. To show both triangles congruent one pair of equal sides and one pair of equal angles are given and add angle EPD on both sides in given pair of angles EPA and angle DPB to find the other pair of angles APD and BPE



Solution:

It is given that $\angle EPA = \angle DPB$

$$\angle EPA + \angle DPE = \angle DPB + \angle DPE$$
$$\therefore \angle DPA = \angle EPB$$

In ΔDAP and ΔEBP,

$$\angle DAP = \angle EBP \text{ (Given)}$$
$$AP = BP \text{ (P is mid-point of AB)}$$
$$\angle DPA = \angle EPB \text{ (From above)}$$
$$\therefore \Delta DAP \cong \Delta EBP \text{ (ASA congruence rule)}$$
$$\therefore AD = BE \text{ (By CPCT)}$$

Q8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see the given figure). Show that:

(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2}AB$

Difficulty Level:

Medium

Known/given:

M is the mid-point of hypotenuse AB, $\angle C = 90°$ and DM = CM

To prove:

(i) $\Delta AMC \cong \Delta BMD$

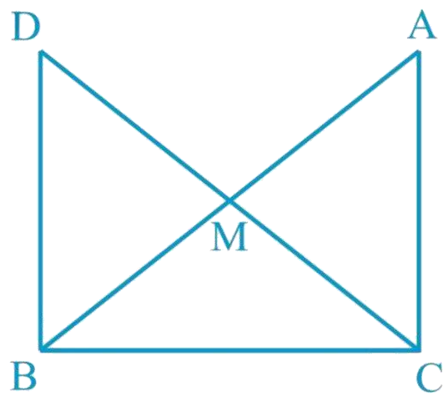
(ii) $\angle DBC$ is a right angle.

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2}AB$

Reasoning:

We can show two triangles AMC and BMD congruent by using SAS congruency rule and then we can say corresponding parts of congruent triangles will be equal means angle ACM will equal to angle BDM, which are alternate interiors angle and can conclude DB is parallel to AC. Now it will help to find angle DBC by co-interior angles. Similarly, triangles DBC and ACB will be congruent by using SAS criterion and CM will be half of AB by using M mid-point.



Solution:

(i) In ΔAMC and ΔBMD,

$$AM = BM \left(M \text{ is the mid} - \text{point of } AB\right)$$

$$\angle AMC = \angle BMD \left(\text{Vertically opposite angles}\right)$$

$$CM = DM \left(\text{Given}\right)$$

$$\therefore \Delta AMC \cong \Delta BMD \left(\text{By SAS congruence rule}\right)$$

$$\therefore AC = BD \left(\text{By CPCT}\right)$$

$$\text{And, } \angle ACM = \angle BDM \left(\text{By CPCT}\right)$$

(ii) $\angle ACM = \angle BDM$

However, $\angle ACM$ and $\angle BDM$ are alternate interior angles.

Since alternate angles are equal,

It can be said that $DB \parallel AC$

$$\angle DBC + \angle ACB = 180^0 \left(\text{Co} - \text{interior angles}\right)$$

$$\angle DBC + 90^0 = 180^0$$

$$\therefore \angle DBC = 90^0$$

(iii) In ΔDBC and ΔACB,

$$DB = AC \ \left(\text{Already proved}\right)$$

$$\angle DBC = \angle ACB \ \left(\text{Each } 90 \ \right)$$

$$BC = CB \left(\text{Common}\right)$$

$$\therefore \Delta DBC \cong \Delta ACB \left(\text{SAS congruence rule}\right)$$

(iv) $\Delta DBC \cong \Delta ACB$

$$AB = DC \left(\text{By CPCT}\right)$$

$$AB = 2 \ CM$$

$$\therefore CM = \frac{1}{2} AB$$

Triangles

Exercise (7.2)

Q1. In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Join A to O. Show that:

(i) OB = OC (ii) AO bisects ∠A

Difficulty Level:

Easy

Known/given:

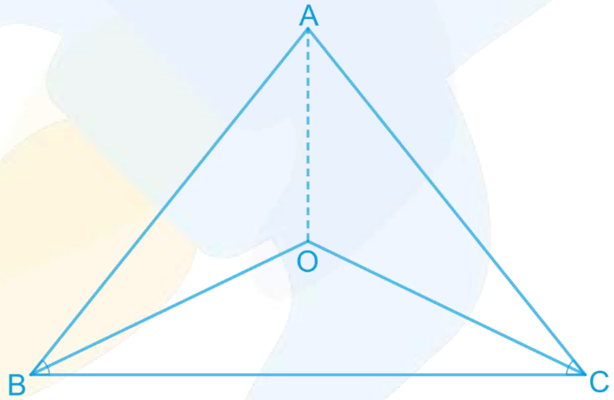
Triangle ABC is isosceles in which AB=AC also OB and OC are bisectors of angle B and angle C

To prove:

(i) OB = OC (ii) AO bisects ∠A

Reasoning:

As OB and OC are bisectors of angle B and angle C means half of angle B will be equal to half of angle c which will help us to conclude OB is equal to OC. Now We can show two triangles OAB and OAC congruent by using SSS congruency rule and then we can say corresponding parts of congruent triangles will be equal.



Solution:

(i) It is given that in triangle ABC, $AB = AC$

$$\angle ACB = \angle ABC \text{ (Angles opposite to equal sides of a triangle are equal)}$$

$$\frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$$

$$\therefore OB = OC \text{ (Sides opposite to equal angles of a triangle are also equal)}$$

(ii) In ΔOAB and ΔOAC,

$$AO = AO \text{ (Common)}$$

$$AB = AC \text{ (Given)}$$

$$OB = OC \text{ (Proved above)}$$

Therefore, $\Delta OAB \cong \Delta OAC$ (By SSS congruence rule)

$\angle BAO = \angle CAO\ (CPCT)$

$\therefore$ AO bisects $\angle A$.

Q2. In ΔABC, AD is the perpendicular bisector of BC (see the given figure). Show that ΔABC is an isosceles triangle in which AB = AC.

Difficulty Level:

Easy

Known/given:

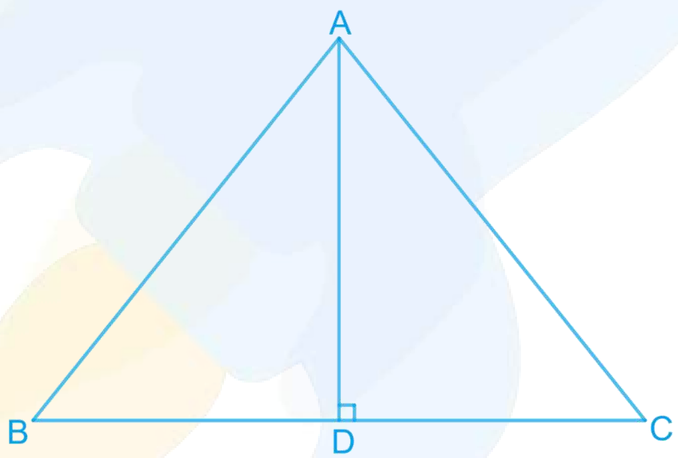
AD is perpendicular bisector of BC means $\angle ADB = \angle ADC = 90°$ and BD=DC

To prove:

ΔABC is an isosceles triangle in which AB = AC.

Reasoning:

We can show two triangles ADB and ADC congruent by using SAS congruency rule and then we can say corresponding parts of congruent triangles will be equal.



Solution:

In ΔADC and ΔADB,

$AD = AD\ (\text{Common})$

$\angle ADC = \angle ADB\ (\text{Each } 90^0)$

$CD = BD\ (\text{AD is the perpendicular bisector of BC})$

$\therefore \Delta ADC \cong \Delta ADB\ (\text{By SAS congruence rule})$

$\therefore AB = AC\ (\text{By CPCT})$

Therefore, ABC is an isosceles triangle in which $AB = AC$.

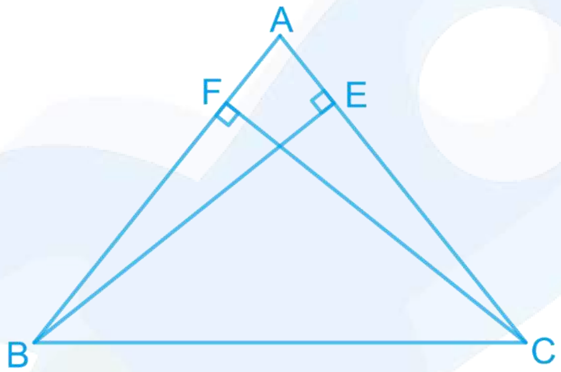
Q3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see the given figure). Show that these altitudes are equal.

Difficulty Level:
Easy

known/given:
Sides AB=AC, $BE \perp AC$ and $CF \perp AB$

To prove:
Altitudes BE and CF are equal or BE = CF

Reasoning:
We can show two triangles ABE and ACF congruent by using AAS congruency rule and then we can say corresponding parts of congruent triangles will be equal.



Solution:

In ΔAEB and ΔAFC,

$\angle AEB$ and $\angle AFC$ $(\text{Each } 90^\circ)$

$\angle A = \angle A$ (Common angle)

$AB = AC$ (Given)

$\therefore \Delta AEB \cong \Delta AFC$ $(\text{By AAS congruence rule})$

$\therefore BE = CF$ (By CPCT)

Q4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see the given figure). Show that

(i) $\Delta ABE \cong \Delta ACF$

(ii) $AB = AC$, i.e., ABC is an isosceles triangle.

Difficulty Level:
Easy

Known/given:
Altitudes, $BE = CF$, $BE \perp AC$ and $CF \perp AB$

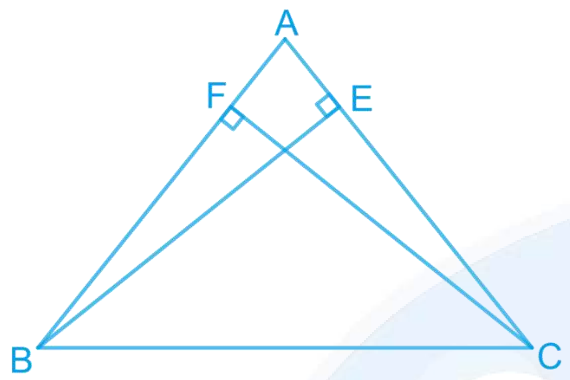
To prove:

(i) $\Delta ABE \cong \Delta ACF$

(ii) $AB = AC,$ i.e., ABC is an isosceles triangle.

Reasoning:

We can show two triangles ABE and ACF congruent by using AAS congruency rule and then we can say corresponding parts of congruent triangles will be equal.



Solution:

(i) In ΔABE and ΔACF,

$\angle AEB$ and $\angle AFC$ (Each 90°)

$\angle A = \angle A$ (Common angle)

$BE = CF$ (Given)

$\therefore \Delta ABE \cong \Delta ACF$ (By AAS congruence rule)

(ii) It has already been proved that

$\Delta ABE \cong \Delta ACF$

$\therefore AB = AC$ (By CPCT)

Q5. ABC and DBC are two isosceles triangles on the same base BC (see the given figure). Show that $\angle ABD = \angle ACD$.

Difficulty Level:

Easy

Known/given:

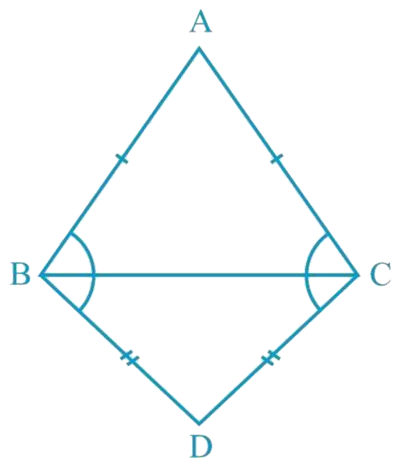
ABC and DBC are two isosceles triangles.

To prove:

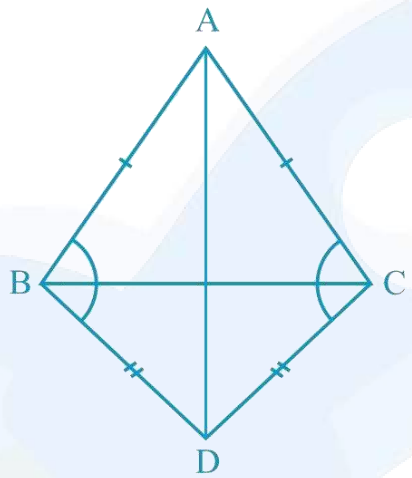
$\angle ABD = \angle ACD$

Reasoning:

First of all, we can join point A and D then we can show two triangles ADB and ADC congruent by using SSS congruency rule after that we can say corresponding parts of congruent triangles will be equal.



Solution:



Let us join AD.

In ΔABD and ΔACD,

$$AB = AC \,(\text{Given})$$
$$BD = CD \,(\text{Given})$$
$$AD = AD \,(\text{Common side})$$
$$\therefore \Delta ABD \cong \Delta ACD \,(\text{By SSS congruence rule})$$
$$\therefore \angle ABD = \angle ACD \,(\text{By CPCT})$$

Q6. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see the given figure). Show that $\angle BCD$ is a right angle.

Difficulty Level:

Easy

Known/given:

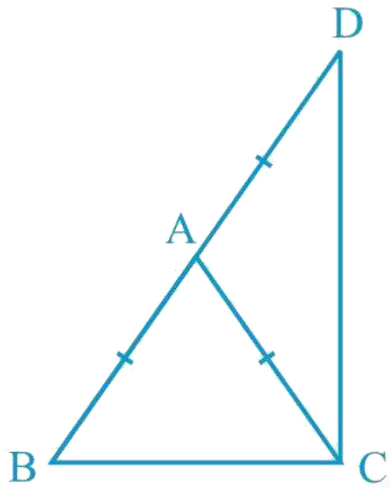
Sides AB=AC and AD=AB

To prove:

$\angle BCD$ is a right angle.

Reasoning:

We can use the property angles opposite to equal sides are equal and then by the angle sum property in triangle BCD we can show the required result.



Solution:

In ΔABC,

$$AB = AC \left(\text{Given}\right)$$
$$\therefore \angle ACB = \angle ABC \left(\text{Angles opposite to equal sides of a triangle are also equal}\right)$$

In ΔACD,

$$AC = AD$$
$$\therefore \angle ADC = \angle ACD \left(\text{Angles opposite to equal sides of a triangle are also equal}\right)$$

In ΔBCD,

$$\angle ABC + \angle BCD + \angle ADC = 180^0 \left(\text{Angle sum property of a triangle}\right)$$
$$\angle ACB + \angle ACB + \angle ACD + \angle ACD = 180^0$$
$$2\left(\angle ACB + \angle ACD\right) = 180^0$$
$$2\left(\angle BCD\right) = 180^0$$
$$\therefore \angle BCD = 90^0$$

Q7. ABC is a right-angled triangle in which $\angle A = 90^0$ and AB = AC. Find $\angle B$ and $\angle C$.

Difficulty Level:

Easy

Known/given:

ABC is right-angled triangle and sides AB=AC

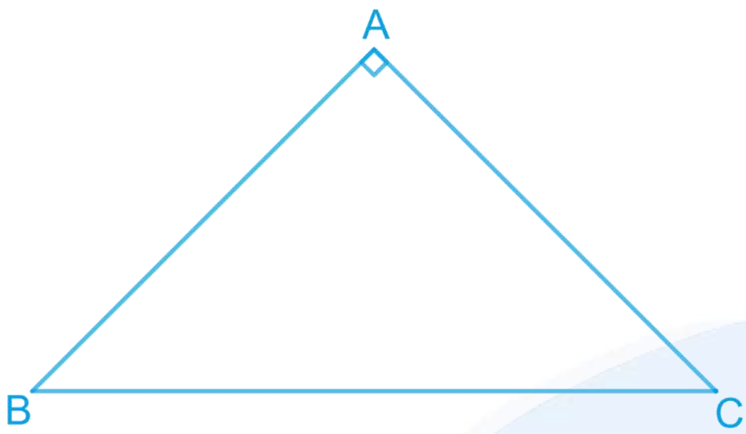
To Find:

Value of $\angle B$ and $\angle C$.

Reasoning:

We can use the property angles opposite to equal sides are equal and then by the angle sum property in triangle ABC we can find the value of ∠B and ∠C.

Solution:



It is given that

$$AB = AC$$

$$\therefore \angle C = \angle B \left(\text{Angles opposite to equal sides are also equal}\right)$$

In ΔABC,

$$\angle A + \angle B + \angle C = 180^0 \left(\text{Angle sum property of a triangle}\right)$$

$$90^0 + \angle B + \angle C = 180^0$$

$$90^0 + \angle B + \angle B = 180^0$$

$$2\angle B = 90^0$$

$$\angle B = 45^0$$

$$\therefore \angle B = \angle C = 45^0$$

Q8. Show that the angles of an equilateral triangle are 60° each.

Difficulty Level:

Easy

Known/given:

Triangle ABC is an equilateral triangle.

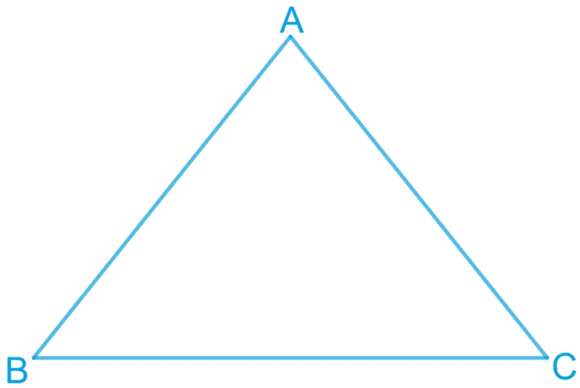
To prove:

The angles of an equilateral triangle are 60° each.

Reasoning:

We can use the property angles opposite to equal sides are equal and then by the angle sum property in triangle ABC we can show the value of each angle is 60 degree.

Solution:



Let us consider that ABC is an equilateral triangle.

Therefore,

$$AB = BC = AC$$
$$AB = AC$$
$$\therefore \angle C = \angle B \text{ (Angles opposite to equal sides of a triangle are equal)}$$

Also,

$$AC = BC$$
$$\therefore \angle B = \angle A \text{ (Angles opposite to equal sides of a triangle are equal)}$$

Therefore, we obtain

$$\angle A = \angle B = \angle C$$

In ΔABC,

$$\angle A + \angle B + \angle C = 180^0$$
$$\angle A + \angle A + \angle A = 180^0$$
$$3\angle A = 180^0$$
$$\angle A = 60^0$$
$$\therefore \angle A = \angle B = \angle C = 60^0$$

Hence, in an equilateral triangle, all interior angles are of measure 60^0.

Triangles

Exercise (7.3)

Q1. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see the given figure). If AD is extended to intersect BC at P, show that

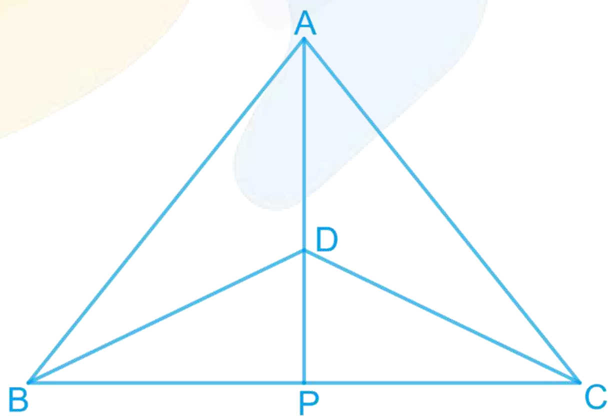
i) $\Delta ABD \cong \Delta ACD$
ii) $\Delta ABP \cong \Delta ACP$
iii) AP bisects $\angle A$ as well as $\angle D$.
iv) AP is the perpendicular bisector of BC.

Difficulty Level:
Medium

Known/given:
ΔABC and ΔDBC are two isosceles triangles.

To prove:
i) $\Delta ABD \cong \Delta ACD$
ii) $\Delta ABP \cong \Delta ACP$
iii) AP bisects $\angle A$ as well as $\angle D$.
iv) AP is the perpendicular bisector of BC.

Reasoning:
We can use Isosceles triangle property to show triangles ABD and ACD by SSS congruency and then we can say corresponding parts of congruent triangles are equal, Similarly ABP and ACP by SAS then with the help of these we can prove other required results.



Solution:
i) In ΔABD and ΔACD, $AB = AC$ (Given)
$BD = CD$ (Given) $AD = AD$ (Common)
$\Delta ABD \cong \Delta ACD$ (By SSS congruence rule)
$\angle BAD = \angle CAD$ (By CPCT)
$\angle BAP = \angle CAP$ .... (1)

ii) In ΔABP and ΔACP, $AB = AC$ (Given)

$\angle BAP = \angle CAP$ [From equation (1)]

$AP = AP$ (Common)

$\therefore \Delta ABP \cong \Delta ACP$ (By SAS congruence rule)

$\therefore BP = CP$ (By CPCT) … (2)

iii) From Equation (1),

$\angle BAP = \angle CAP$

Hence, AP bisects $\angle A$.

In ΔBDP and ΔCDP,

$BD = CD$ (Given)

$DP = DP$ (Common)

$BP = CP$ [From equation (2)]

$\therefore \Delta BDP \cong \Delta CDP$ (By SSS Congruence rule)

$\therefore \angle BDP = \angle CDP$ (By CPCT) … (3)

Hence, AP bisects $\angle D$.

iv) $\Delta BDP \cong \Delta CDP$

$\therefore \angle BPD = \angle CPD$ (By CPCT) …. (4)

$\angle BPD + \angle CPD = 180^0$ (Linear pair angles)

$\angle BPD + \angle BPD = 180^0$

$2\angle BPD = 180^0$ [From Equation (4)]

$\angle BPD = 90^0$ … (5)

From Equations (2) and (5), it can be said that AP is the perpendicular bisector of BC.

Q2. AD is an altitude of an isosceles triangle ABC in which $AB = AC$. Show that

i) AD bisects BC

ii) AD bisects $\angle A$.

Difficulty Level:

Easy

Known/given:

AD is an altitude of an isosceles triangle ABC in which $AB = AC$.

To prove:

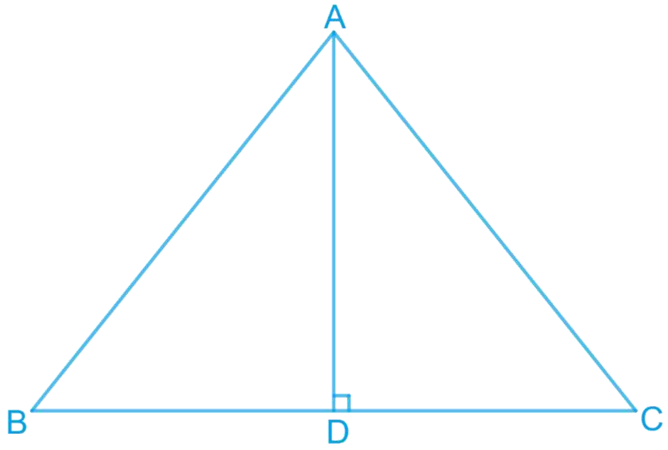
i) AD bisects BC

ii) AD bisects $\angle A$.

Reasoning:

We can show triangle BAD and CAD congruent by using RHS congruency criterion and then we can say corresponding parts of congruent triangle are equal.

Solution:



i) In ΔBAD and ΔCAD,

$\angle ADB = \angle ADC$ (Each 90^0 as AD is an altitude)
$AB = AC$ (Given)
$AD = AD$ (Common)
$\therefore$ $\Delta BAD \cong \Delta CAD$ (By RHS Congruence rule)
$\therefore$ $BD = CD$ (By CPCT)

Hence, AD bisects BC.

ii) Also, by CPCT,
$\angle BAD = \angle CAD$
Hence, AD bisects $\angle A$.

Q3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see the given figure). Show that:

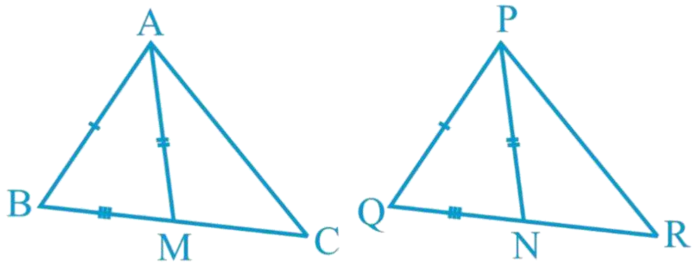
i) $\Delta ABM \cong \Delta PQN$
ii) $\Delta ABC \cong \Delta PQR$

Difficulty Level:
Medium

Known/given:
Sides AB=PQ, BC=QR and AM = PN also AM and PN are medians.

To prove:
i) $\Delta ABM \cong \Delta PQN$
ii) $\Delta ABC \cong \Delta PQR$

Reasoning:
Using the median property, we can show Triangles ABM and PQN are congruent by SSS congruency then we can say corresponding parts of congruent triangles are equal and then Triangles ABC and PQR by SAS congruency.



Solution:

i) In ΔABC, AM is the median to BC.

$$\therefore BM = \frac{1}{2} BC$$

In ΔPQR, PN is the median to QR.

$$\therefore QN = \frac{1}{2} QR$$

However, $BC = QR$

$$\therefore \frac{1}{2} BC = \frac{1}{2} QR$$

$$\therefore BM = QN \quad \ldots (1)$$

In ΔABM and ΔPQN,

$AB = PQ$ (Given)
$BM = QN$ [From Equation (1)]
$AM = PN$ (Given)
$\Delta ABM \cong \Delta PQN$ (By SSS congruence rule)
$\angle ABM = \angle PQN$ (By CPCT)
$\angle ABC = \angle PQR \quad \ldots (2)$

ii) In ΔABC and ΔPQR, $AB = PQ$ (Given)
$\angle ABC = \angle PQR$ [From Equation (2)]
$BC = QR$ (Given)
$\therefore \Delta ABC \cong \Delta PQR$ (By SAS congruence rule)

Q4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

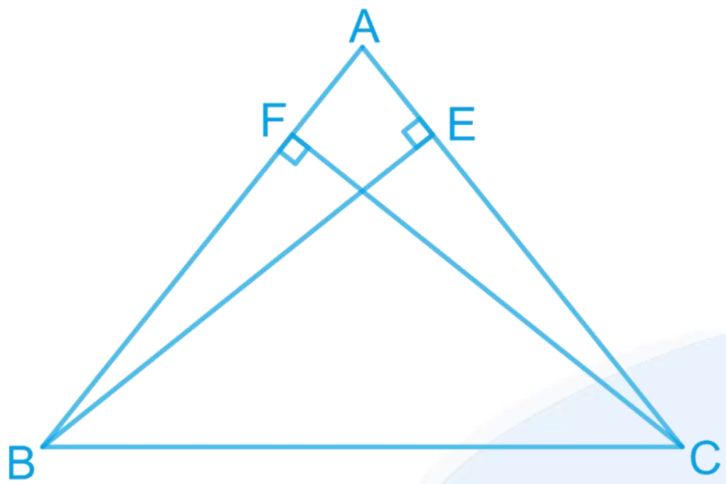
Difficulty Level:
Easy

Known/given:
BE and CF are two equal altitudes of a triangle ABC.

To prove:
Triangle ABC is isosceles by using RHS congruence rule.

Reasoning:
We can show triangles BEC and CFB congruent by using RHS congruency and then we can say corresponding parts of congruent triangles are equal to prove the required result.

Solution:



In ΔBEC and ΔCFB,

$\angle BEC = \angle CFB$ (Each 90^0)
BC = CB (Common)
BE = CF (Given)
$\therefore \Delta BEC \cong \Delta CFB$ (By RHS congruency)
$\therefore \angle BCE = \angle CBF$ (By CPCT)
$\therefore$ AB = AC (Sides opposite to equal angles of a triangle are equal)

Hence, ΔABC is isosceles.

Q5. ABC is an isosceles triangle with AB = AC. Draw AP $\perp$ BC to show that $\angle B = \angle C$.

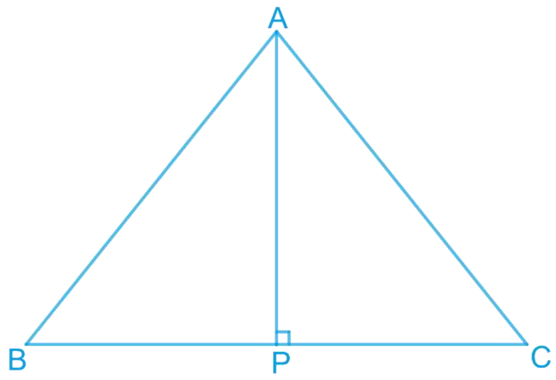
Difficulty Level:
Easy

Known/given:
ABC is an isosceles triangle with AB = AC and AP $\perp$ BC

To prove:
$\angle B = \angle C$

Reasoning:
We can show triangles APB and APC congruent by using RHS congruency and then we can say corresponding parts of congruent triangles are equal to show the required result.

Solution:



In ΔAPB and ΔAPC,

$\angle APB = \angle APC$	(Each 90^0)
$AB = AC$	(Given)
$AP = AP$	(Common)
$\Delta APB = \Delta APC$	(Using RHS congruence rule)
$\angle B = \angle C$	(CPCT)

Triangles

Exercise (7.4)

Q1. Show that in a right-angled triangle, the hypotenuse is the longest side.

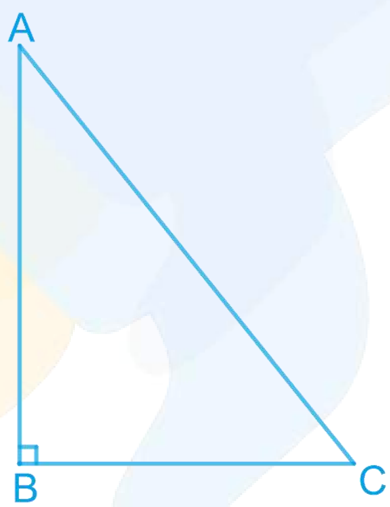
Difficulty Level:
Easy

Known/given:
A right-angled triangle.

To prove:
Hypotenuse is the longest side.

Reasoning:
In a triangle if one angle is of 90 degree then the other two angles have to be acute and we can use the fact that in any triangle, the side opposite to the larger (greater) angle is longer.

Solution:



Let us consider a right-angled triangle ABC, right-angled at B.

In ΔABC,

$$\angle A + \angle B + \angle C = 180^\circ \quad \text{(Angle sum property of a triangle)}$$

$$\angle A + 90^\circ + \angle C = 180^\circ$$

$$\angle A + \angle C = 90^\circ$$

Hence, the other two angles have to be acute (i.e., less than 90°).
$\angle B$ is the largest angle in ΔABC.

$$\angle B > \angle A \text{ and } \angle B > \angle C$$

$$AC > BC \text{ and } AC > AB$$

[In any triangle, the side opposite to the larger (greater) angle is longer.]
Therefore, AC is the largest side in ΔABC.
However, AC is the hypotenuse of ΔABC. Therefore, hypotenuse is the longest side in a right-angled triangle.

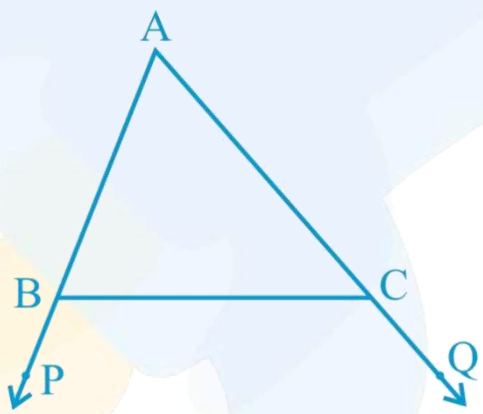
Q2. In the given figure sides AB and AC of ΔABC are extended to points P and Q respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$.

Difficulty Level:
Easy

Known/given:
$\angle PBC < \angle QCB$.

To prove:
$AC > AB$.

Reasoning:
By using linear pair, we can find inequality of interior angles and then we can use the fact that in any triangle, the side opposite to the larger (greater) angle is longer.



Solution:
In the given figure,

$$\angle ABC + \angle PBC = 180^\circ \text{ (Linear pair)}$$
$$\angle ABC = 180^\circ - \angle PBC \quad \ldots (1)$$

Also,

$$\angle ACB + \angle QCB = 180^\circ$$
$$\angle ACB = 180^\circ - \angle QCB \quad \ldots (2)$$

As $\angle PBC < \angle QCB$,

$$180^\circ - \angle PBC > 180^\circ - \angle QCB$$
$$\angle ABC > \angle ACB \text{ [From Equations (1) and (2)]}$$
$$AC > AB \text{ (Side opposite to the larger angle is larger.)}$$

Hence proved, $AC > AB$

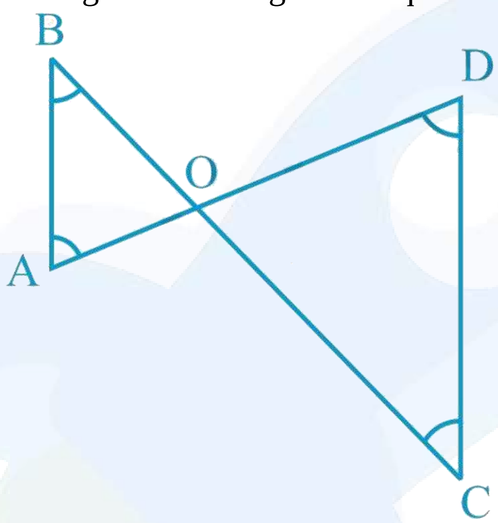
Q3. In the given figure, $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.

Difficulty Level:
Easy

Known/given:
$\angle B < \angle A$ and $\angle C < \angle D$.

To prove:
$AD < BC$

Reasoning:
We can use the fact that In any triangle, the side opposite to the larger (greater) angle is longer We can add both the triangles result to get the required result.



Solution:
In ΔAOB,

$$\angle B < \angle A$$

$$AO < BO \text{ (Side opposite to smaller angle is smaller)} \quad \dots (1)$$

In ΔCOD,

$$\angle C < \angle D$$

$$OD < OC \text{ (Side opposite to smaller angle is smaller)} \quad \dots (2)$$

On adding Equations (1) and (2), we obtain

$$AO + OD < BO + OC$$

$$AD < BC, \text{ proved}$$

Q4. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see the given figure). Show that $\angle A > \angle C$ and $\angle B > \angle D$.

Difficulty Level:
Easy

Known/give:

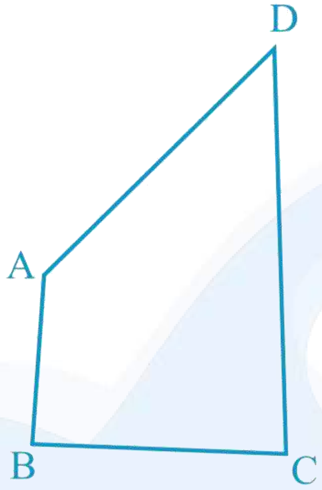
AB and CD are respectively the smallest and longest sides.

To prove:

$\angle A > \angle C$ and $\angle B > \angle D$

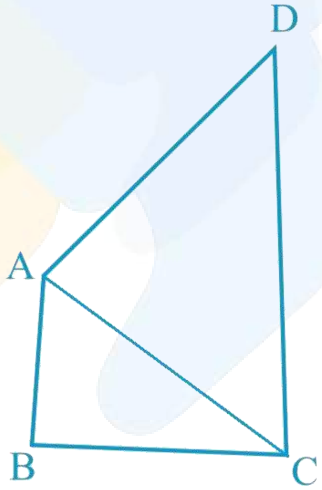
Reasoning:

First of all we can join vertex A to C then triangles ABC and ADC will be formed now we can use the fact that In any triangle, the side opposite to the larger (greater) angle is longer We can add both the triangles result to get the first required result, Similarly we can join vertex B and D and use the fact to get the other required result.



Solution:

Let us join AC.



In ΔABC,

$$AB < BC \ (\text{AB is the smallest side of quadrilateral ABCD})$$

$$\angle 2 < \angle 1 \ (\text{Angle opposite to the smaller side is smaller}) \ \ldots (1)$$

In ΔADC,

$$AD < CD \ (\text{CD is the largest side of quadrilateral ABCD})$$

$$\angle 4 < \angle 3 \ (\text{Angle opposite to the smaller side is smaller}) \ \ldots (2)$$

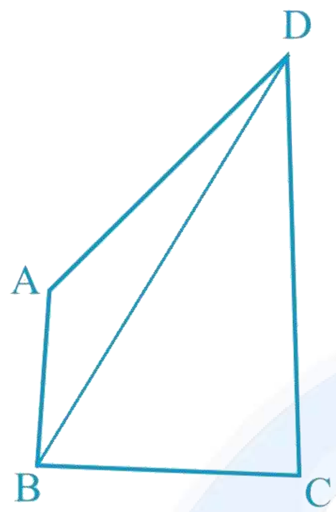
On adding Equations (1) and (2), we obtain

$$\angle 2 + \angle 4 < \angle 1 + \angle 3$$

$$\angle C < \angle A$$

$$\angle A > \angle C$$

Let us join BD.



In ΔABD,

$$AB < AD \ (AB \text{ is the smallest side of quadrilateral } ABCD)$$

$$\angle 8 < \angle 5 \ (\text{Angle opposite to the smaller side is smaller}) \quad ...(3)$$

In ΔBDC,

$$BC < CD \ (CD \text{ is the largest side of quadrilateral } ABCD)$$

$$\angle 7 < \angle 6 \ (\text{Angle opposite to the smaller side is smaller}) \ ... \ (4)$$

On adding Equations (3) and (4), we obtain

$$\angle 8 + \angle 7 < \angle 5 + \angle 6$$

$$\angle D < \angle B$$

$$\angle B > \angle D \ (\text{Hence, proved})$$

Q5. In the given figure, $PR > PQ$ and PS bisects $\angle QPR$. Prove that $\angle PSR > \angle PSQ$.

Difficulty Level:

Easy

Known/given:

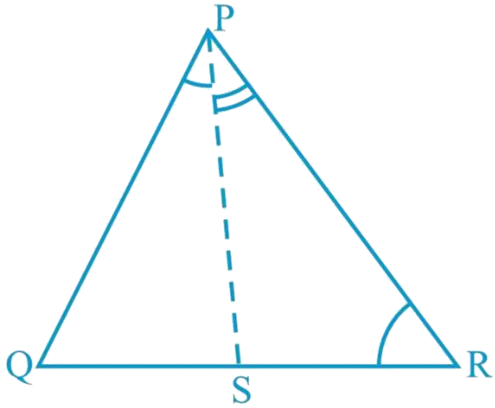
$PR > PQ$ and PS bisects $\angle QPR$

To prove:

$\angle PSR > \angle PSQ$.

Reasoning:

We can use exterior angle sum property to find the required inequality.



Solution:

As PR > PQ,

$\angle PQR > \angle PRQ$ (Angle opposite to larger side is larger) ...(1)

PS is the bisector of $\angle QPR$.

$\angle QPS = \angle RPS$ ...(2)

$\angle PSR$ is the exterior angle of ΔPQS.

$\angle PSR = \angle PQR + \angle QPS$ ...(3)

$\angle PSQ$ is the exterior angle of ΔPRS.

$\angle PSQ = \angle PRQ + \angle RPS$ ...(4)

Adding Equations (1) and (2), we obtain

$\angle PQR + \angle QPS > \angle PRQ + \angle RPS$

$\angle PSR > \angle PSQ$ [Using the values of Equations (3) and (4)]

Q6. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Difficulty Level:

Easy

Known/given:

Line segments drawn from a given point is not on line.

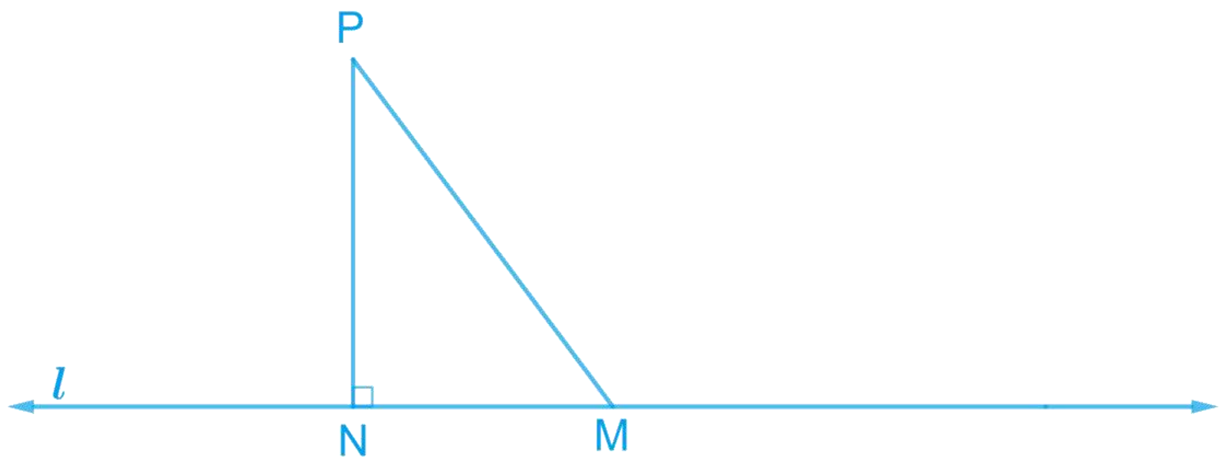
To prove:

The perpendicular line segment is the shortest.

Reasoning:

We know that in a triangle if one angle is 90 degree then other have to be acute.

Solution:



Let us take a line *l* and from point *P* (i.e., not on line *l*), draw two line segments PN and PM. Let PN be perpendicular to line *l* and PM is drawn at some other angle.

In ΔPNM,

$$\angle N = 90^{\circ}$$

$$\angle P + \angle N + \angle M = 180^{\circ} \text{ (Angle sum property of a triangle)}$$

$$\angle P + \angle M = 90^{\circ}$$

Clearly, $\angle M$ is an acute angle.

$$\angle M < \angle N$$

$$PN < PM \text{ (Side opposite to the smaller angle is smaller)}$$

Similarly, by drawing different line segments from P to *l*, it can be proved that PN is smaller in comparison to them. Therefore, it can be observed that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Triangles

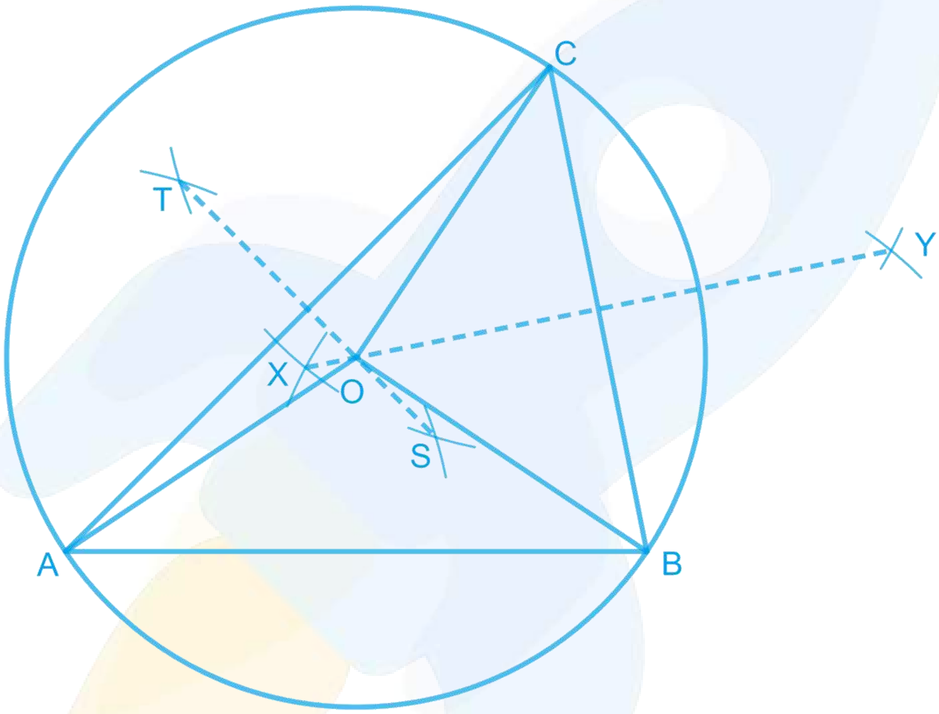
Exercise (7.5)

Q1. ABC is a triangle. Locate a point in the interior of ΔABC which is equidistant from all the vertices of ΔABC.

Solution:

Circumcentre of a triangle is always equidistant from all the vertices of that triangle.

Circumcentre is the point where perpendicular bisectors of all the sides of the triangle meet together.

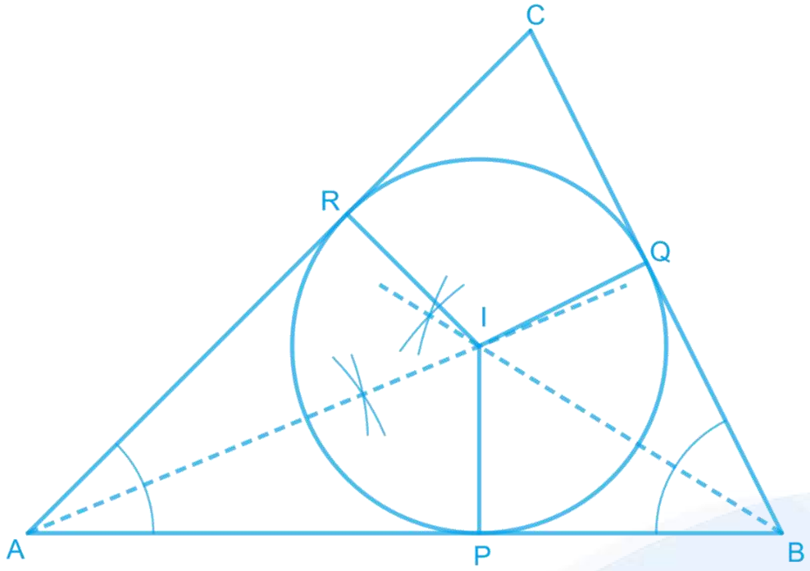


In ΔABC, we can find the circumcentre by drawing the perpendicular bisectors of sides AB, BC, and CA of this triangle. O is the point where these bisectors are meeting together. Therefore, O is the point which is equidistant from all the vertices of ΔABC.

Q2. In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

Solution:

The point which is equidistant from all the sides of a triangle is called the incentre of the triangle. Incentre of a triangle is the intersection point of the angle bisectors of the interior angles of that triangle.



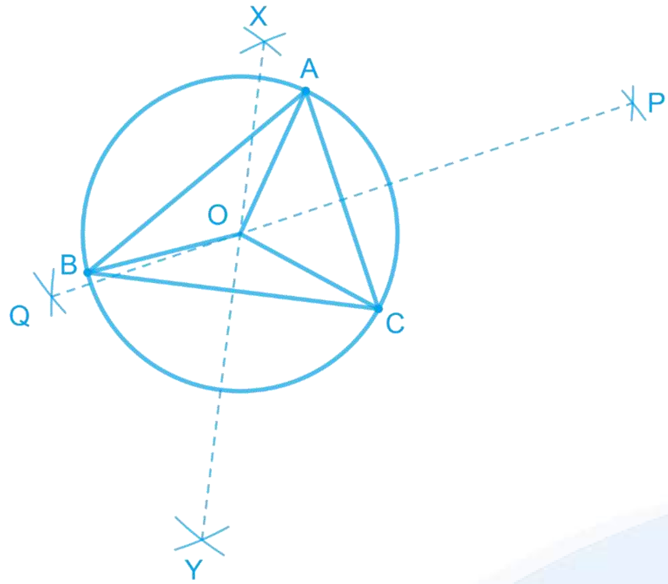
Here, in ΔABC, we can find the incentre of this triangle by drawing the angle bisectors of the interior angles of this triangle. I is the point where these angle bisectors are intersecting each other. Therefore, I is the point equidistant from all the sides of ΔABC.

Q3. In a huge park people are concentrated at three points (see the given figure)

A: where there are different slides and swings for children,
B: near which a man-made lake is situated,
C: which is near to a large parking and exit.

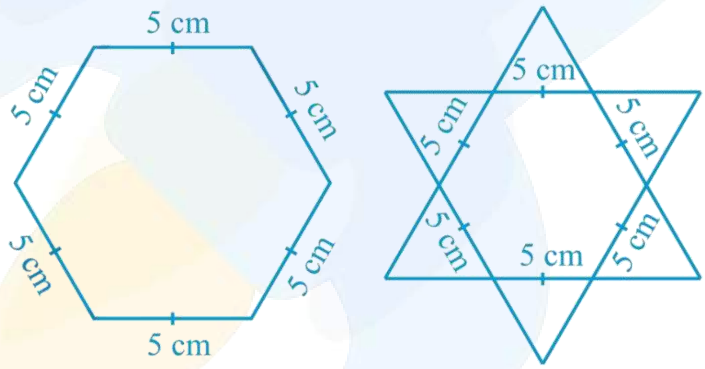
Where should an ice-cream parlour be set up so that maximum number of persons can approach it?
(*Hint:* The parlor should be equidistant from A, B and C)

Solution:
Maximum number of persons can approach the ice-cream parlour if it is equidistant from A, B and C. Now, A, B and C form a triangle. In a triangle, the circumcentre is the only point that is equidistant from its vertices. So, the ice-cream parlour should be set up at the circumcentre O of ΔABC.



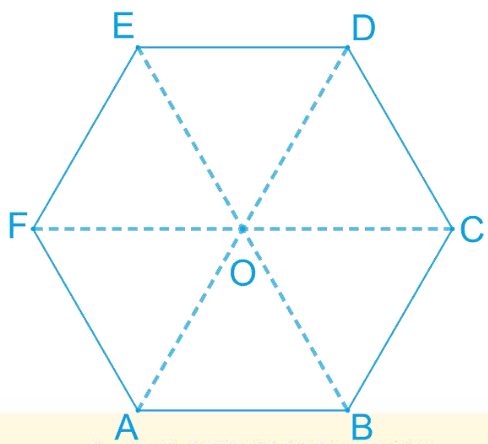
In this situation, maximum number of persons can approach it. We can find circumcentre O of this triangle by drawing perpendicular bisectors of the sides of this triangle.

Q4. Complete the hexagonal and star shaped *rangolies* (see the given figures) by filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?



Solution:

It can be observed that hexagonal-shaped *rangoli* has 6 equilateral triangles of side 5cm in it.



Area of $\Delta OAB = \frac{\sqrt{3}}{4}(side)^2$

$= \frac{\sqrt{3}}{4}(5cm)^2 = \frac{25\sqrt{3}}{4}cm^2$

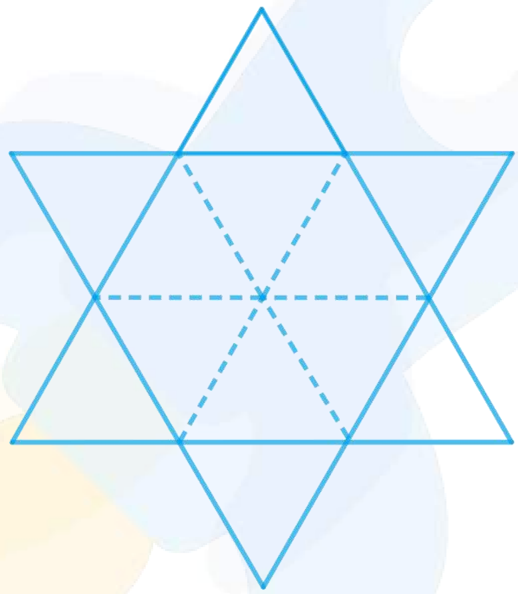
Area of hexagonal-shaped rangoli $= 6 \times \frac{25\sqrt{3}}{4}cm^2 = \frac{75\sqrt{3}}{2}cm^2$

Area of equilateral triangle having its side as $1cm = \frac{\sqrt{3}}{4}(1cm)^2 = \frac{\sqrt{3}}{4}cm^2$

Number of equilateral triangles of 1 cm side that can be filled in this hexagonal-shaped

Rangoli $= \frac{\frac{75\sqrt{3}}{2}cm^2}{\frac{\sqrt{3}}{4}cm^2} = \frac{75\sqrt{3}}{2} \times \frac{4}{\sqrt{3}} = 150$

Star-shaped *rangoli* has 12 equilateral triangles of side 5 cm in it.



Area of star-shaped rangoli $= 12 \times \frac{\sqrt{3}}{4} \times (5cm)^2 = 75\sqrt{3}cm^2$

Number of equilateral triangles of 1 cm side that can be filled in this *star-shaped*

rangoli $= \frac{75\sqrt{3}cm^2}{\frac{\sqrt{3}}{4}cm^2} = 75\sqrt{3} \times \frac{4}{\sqrt{3}} = 300$

Therefore, *star-shaped rangoli* has more equilateral triangles of side 1 cm in it.