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Triangles

Exercise (7.1)

Q1. In quadrilateral ACBD, AC = AD and AB bisects $\angle A$ (See the given figure). Show that $\triangle ABC \cong \triangle ABD$. What can you say about BC and BD?

**Difficulty Level:**
Easy

**Known/given:**
AC = AD and AB bisects $\angle A$

**To prove:**
$\triangle ABC \cong \triangle ABD$ and, what can be said about BC and BD.

**Reasoning:**
We can show two sides and included angle of $\triangle ABC$ are equals to corresponding sides and included angle of $\triangle ABD$, by using SAS congruency criterion both triangles will be congruent and by CPCT, BC and BD will be equal.

**Solution:**
In $\triangle ABC$ and $\triangle ABD$,

- $AC = AD$ (Given)
- $\angle CAB = \angle DAB$ (AB bisects $\angle A$)
- $AB = AB$ (Common)

$\therefore \triangle ABC \cong \triangle ABD$ (By SAS congruence rule)

$\therefore BC = BD$ (By CPCT)

Therefore, BC and BD are of equal lengths.
Q2. ABCD is a quadrilateral in which \( AD = BC \) and \( \angle DAB = \angle CBA \) (See the given figure). Prove that

(i) \( \triangle ABD \cong \triangle BAC \)
(ii) \( BD = AC \)
(iii) \( \angle ABD = \angle BAC \)

**Difficulty Level:**
Easy

**Known/given:**
\( AD = BC \) and \( \angle DAB = \angle CBA \)

**To prove:**
(i) \( \triangle ABD \cong \triangle BAC \)  (ii) \( BD = AC \)  (iii) \( \angle ABD = \angle BAC \)

**Reasoning:**
We can show two sides and included of \( \triangle ABD \) are equals to corresponding sides and included angle of \( \triangle BAC \), by using SAS congruency criterion both triangles will be congruent. Then we can say corresponding parts of congruent triangle will be equal.

**Solution:**
In \( \triangle ABD \) and \( \triangle BAC \),
\begin{align*}
\text{AD} & = \text{BC} \quad \text{(Given)} \\
\angle DAB & = \angle CBA \quad \text{(Given)} \\
\text{AB} & = \text{BA} \quad \text{(Common)}
\end{align*}
\therefore \( \triangle ABD \cong \triangle BAC \) (By SAS congruence rule)
\therefore \( BD = AC \) (By CPCT)
And, \( \angle ABD = \angle BAC \) (By CPCT)
Q3. AD and BC are equal perpendiculars to a line segment AB (See the given figure). Show that CD bisects AB.

**Difficulty Level:**
Easy

**Known/given:**
AD ⊥ AB, BC ⊥ AB and AD = BC

**To prove:**
CD bisects AB or OA = OB

**Reasoning:**
We can show two triangles OBC and OAD congruent by using AAS congruency rule and then we can say corresponding parts of congruent triangles will be equal.

**Solution:**
In ΔBOC and ΔAOD,
∠BOC = ∠AOD (Vertically opposite angles)
∠CBO = ∠DAO (Each 90º)
BC = AD (Given)
∴ ΔBOC ≅ ΔAOD (AAS congruency rule)
∴ BO = AO (By CPCT)
⇒ CD bisects AB.

Q4. l and m are two parallel lines intersected by another pair of parallel lines p and q (see the given figure). Show that ΔABC ≅ ΔCDA.

**Difficulty Level:**
Easy

**Known/given:**
l ∥ m and p ∥ q
To prove:
\( \triangle ABC \cong \triangle CDA. \)

Reasoning:
We can show both the triangles congruent by using ASA congruency criterion

Solution:
In \( \triangle ABC \) and \( \triangle CDA, \)
\[ \angle BAC \text{ and } \angle DCA \text{ (Alternate interior angles, as } p \parallel q \text{)} \]
\[ AC = CA \text{ (Common)} \]
\[ \angle BCA \text{ and } \angle DAC \text{ (Alternate interior angles, as } l \parallel m \text{)} \]
\[ \therefore \triangle ABC \cong \triangle CDA \text{ (By ASA congruence rule)} \]

Q5. Line \( l \) is the bisector of an angle \( \angle A \) and \( B \) is any point on \( l \). \( BP \) and \( BQ \) are perpendiculars from \( B \) to the arms of \( \angle A \) (see the given figure). Show that:

(i) \( \triangle APB \cong \triangle AQB \)
(ii) \( BP = BQ \) or \( B \) is equidistant from the arms of \( \angle A \)

Difficulty Level:
Easy

What is known/given?
\( l \) is the bisector of an angle \( \angle A \) and \( BP \perp AP \) and \( BQ \perp AQ \)

To prove:
\( \triangle APB \cong \triangle AQB \) and \( BP = BQ \) or \( B \) is equidistant from the arms of \( \angle A \)

Reasoning:
We can show two triangles \( \triangle APB \) and \( \triangle AQB \) congruent by using AAS congruency rule and then we can say corresponding parts of congruent triangles will be equal.
Solution:
In $\triangle APB$ and $\triangle AQB$,

$\angle BAP = \angle BAQ$ (l is the angle bisector of $\angle A$)

$\angle APB = \angle AQB$ (Each 90°)

$AB = AB$ (Common)

$\therefore \triangle APB \cong \triangle AQB$ (By AAS congruence rule)

$BP = BQ$ (By CPCT)

Or, it can be said that B is equidistant from the arms of $\angle A$.

Q6. In the given figure, $AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$. Show that $BC = DE$.

Difficulty Level:
Medium

Known/given:
$AC = AE$, $AB = AD$ and $\angle BAD = \angle EAC$.

To prove:
$BC = DE$.

Reasoning:
We can show two triangles BAC and DAE congruent by using SAS congruency rule and then we can say corresponding parts of congruent triangles will be equal. To show both triangles congruent two pair of equal sides are given and add angle DAC on both sides in given pair of angles BAD and angle EAC to find the included angle BAC and DAE.
Solution:
It is given that \( \angle BAD = \angle EAC \)
\[ \angle BAD + \angle DAC = \angle EAC + \angle DAC \]
\[ \angle BAC = \angle DAE \]

In \( \triangle BAC \) and \( \triangle DAE \),
\[ AB = AD \text{ (Given)} \]
\[ \angle BAC = \angle DAE \text{ (Prove above)} \]
\[ AC = AE \text{ (Given)} \]

\[ \therefore \triangle BAC \cong \triangle DAE \text{ (By SAS congruence rule)} \]
\[ \therefore BC = DE \text{ (By CPCT)} \]

Q7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that \( \angle BAD = \angle ABE \) and \( \angle EPA = \angle DPB \) (See the given figure).

Show that
(i) \( \triangle DAP \cong \triangle EBP \)
(ii) \( AD = BE \)

Difficulty Level:
Medium

Known/given
P is its mid-point of AB, \( \angle BAD = \angle ABE \) and \( \angle EPA = \angle DPB \)

To prove:
(i) \( \triangle DAP \cong \triangle EBP \) and (ii) \( AD = BE \).

Reasoning:
We can show two triangles DAP and EBP congruent by using ASA congruency rule and then we can say corresponding parts of congruent triangles will be equal. To show both triangles congruent one pair of equal sides and one pair of equal angles are given and add angle EPD on both sides in given pair of angles EPA and angle DPB to find the other pair of angles APD and BPE.
Solution:
It is given that $\angle EPA = \angle DPB$

$\angle EPA + \angle DPE = \angle DPB + \angle DPE$

$\therefore \angle DPA = \angle EPB$

In $\triangle DAP$ and $\triangle EBP$,

$\angle DAP = \angle EPB$ (Given)

$AP = BP$ (P is mid-point of AB)

$\angle DPA = \angle EPB$ (From above)

$\therefore \triangle DAP \cong \triangle EBP$ (ASA congruence rule)

$\therefore AD = BE$ (By CPCT)

Q8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see the given figure). Show that:

(i) $\triangle AMC \cong \triangle BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\triangle DBC \cong \triangle ACB$

(iv) $CM = \frac{1}{2} AB$

Difficulty Level:
Medium

Known/given:
M is the mid-point of hypotenuse AB, $\angle C = 90^\circ$ and $DM = CM$

To prove:
(i) $\triangle AMC \cong \triangle BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\triangle DBC \cong \triangle ACB$

(iv) $CM = \frac{1}{2} AB$

Reasoning:
We can show two triangles AMC and BMD congruent by using SAS congruency rule and then we can say corresponding parts of congruent triangles will be equal means angle ACM will equal to angle BDM, which are alternate interiors angle and can conclude DB is parallel to AC. Now it will help to find angle DBC by co-interior angles. Similarly, triangles DBC and ACB will be congruent by using SAS criterion and CM will be half of AB by using M mid-point.
Solution:

(i) In $\triangle AMC$ and $\triangle BMD$,
- $AM = BM$ (M is the mid-point of $AB$)
- $\angle AMC = \angle BMD$ (Vertically opposite angles)
- $CM = DM$ (Given)

$\therefore \triangle AMC \cong \triangle BMD$ (By SAS congruence rule)

$\therefore AC = BD$ (By CPCT)

And, $\angle ACM = \angle BDM$ (By CPCT)

(ii) $\angle ACM = \angle BDM$

However, $\angle ACM$ and $\angle BDM$ are alternate interior angles.
Since alternate angles are equal,
It can be said that $DB \parallel AC$

$\angle DBC + \angle ACB = 180^\circ$ (Co-interior angles)
$\angle DBC + 90^\circ = 180^\circ$
$\therefore \angle DBC = 90^\circ$

(iii) In $\triangle DBC$ and $\triangle ACB$,
- $DB = AC$ (Already proved)
- $\angle DBC = \angle ACB$ (Each $90^\circ$)
- $BC = CB$ (Common)

$\therefore \triangle DBC \cong \triangle ACB$ (SAS congruence rule)

(iv) $\triangle DBC \cong \triangle ACB$

$AB = DC$ (By CPCT)

$AB = 2 \times CM$

$\therefore CM = \frac{1}{2} AB$
Triangles

Exercise (7.2)

Q1. In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Join A to O. Show that:
(i) OB = OC
(ii) AO bisects ∠A

Difficulty Level:
Easy

Known/given:
Triangle ABC is isosceles in which AB = AC also OB and OC are bisectors of angle B and angle C

To prove:
(i) OB = OC (ii) AO bisects ∠A

Reasoning:
As OB and OC are bisectors of angle B and angle C means half of angle B will be equal to half of angle C which will help us to conclude OB is equal to OC. Now we can show two triangles OAB and OAC congruent by using SSS congruency rule and then we can say corresponding parts of congruent triangles will be equal.

Solution:
(i) It is given that in triangle ABC, AB = AC
∠ACB = ∠ABC (Angles opposite to equal sides of a triangle are equal)
\[ \frac{1}{2} \angle ACB = \frac{1}{2} \angle ABC \]
\[ \therefore OB = OC \text{ (Sides opposite to equal angles of a triangle are also equal)} \]

(ii) In ΔOAB and ΔOAC,
AO = AO (Common)
AB = AC (Given)
OB = OC (Proved above)
Therefore, \( \triangle OAB \cong \triangle OAC \) (By SSS congruence rule)
\[ \angle BAO = \angle CAO \text{ (CPCT)} \]
\[ \therefore AO \text{ bisects } \angle A. \]

Q2. In \( \triangle ABC \), \( AD \) is the perpendicular bisector of \( BC \) (see the given figure).
Show that \( \triangle ABC \) is an isosceles triangle in which \( AB = AC \).

**Difficulty Level:**
Easy

**Known/given:**
\( AD \) is perpendicular bisector of \( BC \) means \( \angle ADB = \angle ADC = 90^\circ \) and \( BD = DC \)

**To prove:**
\( \triangle ABC \) is an isosceles triangle in which \( AB = AC \).

**Reasoning:**
We can show two triangles \( \triangle ADB \) and \( \triangle ADC \) congruent by using SAS congruency rule and then we can say corresponding parts of congruent triangles will be equal.

![Diagram of \( \triangle ABC \) with \( AD \) as the perpendicular bisector]

**Solution:**
In \( \triangle ADC \) and \( \triangle ADB \),
\[ AD = AD \text{ (Common)} \]
\[ \angle ADC = \angle ADB \text{ (Each } 90^\circ) \]
\[ CD = BD \text{ (AD is the perpendicular bisector of } BC) \]
\[ \therefore \triangle ADC \cong \triangle ADB \text{ (By SAS congruence rule)} \]
\[ \therefore AB = AC \text{ (By CPCT)} \]
Therefore, \( \triangle ABC \) is an isosceles triangle in which \( AB = AC \).
Q3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see the given figure). Show that these altitudes are equal.

Difficulty Level:
Easy

Known/given:
Sides \( AB = AC \), \( BE \perp AC \) and \( CF \perp AB \)

To prove:
Altitudes BE and CF are equal or \( BE = CF \)

Reasoning:
We can show two triangles ABE and ACF congruent by using AAS congruency rule and then we can say corresponding parts of congruent triangles will be equal.

Solution:
In \( \triangle AEB \) and \( \triangle AFC \),
\( \angle AEB \) and \( \angle AFC (Each \ 90^\circ) \)
\( \angle A = \angle A \) (Common angle)
\( AB = AC \) (Given)
\( \therefore \triangle AEB \cong \triangle AFC \) (By AAS congruence rule)
\( \therefore BE = CF \) (By CPCT)

Q4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see the given figure). Show that
(i) \( \triangle ABE \cong \triangle ACF \)
(ii) \( AB = AC \), i.e., ABC is an isosceles triangle.

Difficulty Level:
Easy

Known/given:
Altitudes, \( BE = CF \), \( BE \perp AC \) and \( CF \perp AB \)
To prove:
(i) \( \triangle ABE \cong \triangle ACF \)
(ii) \( AB = AC \), i.e., \( ABC \) is an isosceles triangle.

Reasoning:
We can show two triangles \( ABE \) and \( ACF \) congruent by using AAS congruency rule and then we can say corresponding parts of congruent triangles will be equal.

Solution:
(i) In \( \triangle ABE \) and \( \triangle ACF \),
\[ \angle AEB = \angle AFC \text{ (Each 90°)} \]
\[ \angle A = \angle A \text{ (Common angle)} \]
\[ BE = CF \text{ (Given)} \]
\[ \therefore \triangle ABE \cong \triangle ACF \text{ (By AAS congruence rule)} \]

(ii) It has already been proved that
\[ \triangle ABE \cong \triangle ACF \]
\[ \therefore AB = AC \text{ (By CPCT)} \]

Q5. \( ABC \) and \( DBC \) are two isosceles triangles on the same base \( BC \) (see the given figure). Show that \( \angle ABD = \angle ACD \).

Difficulty Level:
Easy

Known/given:
\( ABC \) and \( DBC \) are two isosceles triangles.

To prove:
\( \angle ABD = \angle ACD \)

Reasoning:
First of all, we can join point A and D then we can show two triangles \( ADB \) and \( ADC \) congruent by using SSS congruency rule after that we can say corresponding parts of congruent triangles will be equal.
Let us join AD.

In \( \triangle ABD \) and \( \triangle ACD \),
- \( AB = AC \) (Given)
- \( BD = CD \) (Given)
- \( AD = AD \) (Common side)

\[ \therefore \triangle ABD \cong \triangle ACD \] (By SSS congruence rule)

\[ \therefore \angle BCD \] is a right angle.

**Q6.** \( \triangle ABC \) is an isosceles triangle in which \( AB = AC \). Side BA is produced to D such that \( AD = AB \) (see the given figure). Show that \( \angle BCD \) is a right angle.

**Difficulty Level:**
Easy

**Known/given:**
Sides \( AB = AC \) and \( AD = AB \)

**To prove:**
\( \angle BCD \) is a right angle.
Reasoning:
We can use the property angles opposite to equal sides are equal and then by the angle sum property in triangle BCD we can show the required result.

Solution:
In ΔABC,

\[ AB = AC \text{ (Given)} \]
\[ \therefore \angle ACB = \angle ABC \text{ (Angles opposite to equal sides of a triangle are also equal)} \]

In ΔACD,

\[ AC = AD \]
\[ \therefore \angle ADC = \angle ACD \text{ (Angles opposite to equal sides of a triangle are also equal)} \]

In ΔBCD,

\[ \angle ABC + \angle BCD + \angle ADC = 180^\circ \text{ (Angle sum property of a triangle)} \]
\[ \angle ACB + \angle ACB + \angle ACD + \angle ACD = 180^\circ \]
\[ 2(\angle ACB + \angle ACD) = 180^\circ \]
\[ 2(\angle BCD) = 180^\circ \]
\[ \therefore \angle BCD = 90^\circ \]

Q7. ABC is a right-angled triangle in which \( \angle A = 90^\circ \) and AB = AC. Find \( \angle B \) and \( \angle C \).

Difficulty Level:
Easy

Known/given:
ABC is right-angled triangle and sides AB=AC

To Find:
Value of \( \angle B \) and \( \angle C \).
Reasoning:
We can use the property angles opposite to equal sides are equal and then by the angle sum property in triangle ABC we can find the value of $\angle B$ and $\angle C$.

Solution:

It is given that $AB = AC$

$\therefore \angle C = \angle B$ (Angles opposite to equal sides are also equal)

In $\triangle ABC$,

$\angle A + \angle B + \angle C = 180^0$ (Angle sum property of a triangle)

$90^0 + \angle B + \angle C = 180^0$

$90^0 + \angle B + \angle B = 180^0$

$2\angle B = 90^0$

$\angle B = 45^0$

$\therefore \angle B = \angle C = 45^0$

Q8. Show that the angles of an equilateral triangle are $60^0$ each.

Difficulty Level:
Easy

Known/given:
Triangle ABC is an equilateral triangle.

To prove:
The angles of an equilateral triangle are $60^0$ each.

Reasoning:
We can use the property angles opposite to equal sides are equal and then by the angle sum property in triangle ABC we can show the value of each angle is 60 degree.
Solution:

Let us consider that ABC is an equilateral triangle.

Therefore,

\[ AB = BC = AC \]
\[ AB = AC \]
\[ \therefore \angle C = \angle B \ (\text{Angles opposite to equal sides of a triangle are equal}) \]

Also,

\[ AC = BC \]
\[ \therefore \angle B = \angle A \ (\text{Angles opposite to equal sides of a triangle are equal}) \]

Therefore, we obtain

\[ \angle A = \angle B = \angle C \]

In \( \triangle ABC \),

\[ \angle A + \angle B + \angle C = 180^\circ \]
\[ \angle A + \angle A + \angle A = 180^\circ \]
\[ 3\angle A = 180^\circ \]
\[ \angle A = 60^\circ \]
\[ \therefore \angle A = \angle B = \angle C = 60^\circ \]

Hence, in an equilateral triangle, all interior angles are of measure 60°.
Triangles

Exercise (7.3)

Q1. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see the given figure). If AD is extended to intersect BC at P, show that
i) ΔABD ≅ ΔACD
ii) ΔABP ≅ ΔACP
iii) AP bisects ∠A as well as ∠D.
iv) AP is the perpendicular bisector of BC.

Difficulty Level:
Medium

Known/given:
ΔABC and ΔDBC are two isosceles triangles.

To prove:
i) ΔABD ≅ ΔACD
ii) ΔABP ≅ ΔACP
iii) AP bisects ∠A as well as ∠D.
iv) AP is the perpendicular bisector of BC.

Reasoning:
We can use Isosceles triangle property to show triangles ABD and ACD by SSS congruency and then we can say corresponding parts of congruent triangles are equal, Similarly ABP and ACP by SAS then with the help of these we can prove other required results.

Solution:
i) In ΔABD and ΔACD, AB = AC (Given)
   BD = CD (Given) AD = AD (Common)
   ΔABD ≅ ΔACD (By SSS congruence rule)
   ∠BAD = ∠CAD (By CPCT)
   ∠BAP = ∠CAP .... (1)
ii) In $\triangle ABP$ and $\triangle ACP$, $AB = AC$ (Given)
$\angle BAP = \angle CAP$ [From equation (1)]
$AP = AP$ (Common)
$\therefore \triangle ABP \cong \triangle ACP$ (By SAS congruence rule)
$\therefore BP = CP$ (By CPCT) … (2)

iii) From Equation (1),
$\angle BAP = \angle CAP$
Hence, AP bisects $\angle A$.

In $\triangle BDP$ and $\triangle CDP$,
$BD = CD$ (Given)
$DP = DP$ (Common)
$BP = CP$ [From equation (2)]
$\therefore \triangle BDP \cong \triangle CDP$ (By SSS Congruence rule)
$\therefore \angle BDP = \angle CDP$ (By CPCT) … (3)
Hence, AP bisects $\angle D$.

iv) $\triangle BDP \cong \triangle CDP$
$\therefore \angle BPD = \angle CPD$ (By CPCT) …. (4)
$\angle BPD + \angle CPD = 180^\circ$ (Linear pair angles)
$\angle BPD + \angle BPD = 180^\circ$
$2 \angle BPD = 180^\circ$ [From Equation (4)]
$\angle BPD = 90^\circ$ … (5)

From Equations (2) and (5), it can be said that AP is the perpendicular bisector of BC.

Q2. AD is an altitude of an isosceles triangle ABC in which $AB = AC$. Show that
i) AD bisects BC
ii) AD bisects $\angle A$.

Difficulty Level:
Easy

Known/given:
AD is an altitude of an isosceles triangle ABC in which $AB = AC$.

To prove:
i) AD bisects BC
ii) AD bisects $\angle A$.

Reasoning:
We can show triangle BAD and CAD congruent by using RHS congruency criterion and then we can say corresponding parts of congruent triangle are equal.
Solution:

i) In $\triangle BAD$ and $\triangle CAD$,

\[ \angle ADB = \angle ADC \text{ (Each } 90^0 \text{ as } AD \text{ is an altitude)} \]
\[ AB = AC \text{ (Given)} \]
\[ AD = AD \text{ (Common)} \]
\[ \therefore \triangle BAD \cong \triangle CAD \text{ (By RHS Congruence rule)} \]
\[ \therefore BD = CD \text{ (By CPCT)} \]

Hence, AD bisects BC.

ii) Also, by CPCT,

\[ \angle BAD = \angle CAD \]

Hence, AD bisects \angle A.

Q3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of $\triangle PQR$ (see the given figure). Show that:

i) $\triangle ABM \cong \triangle PQN$

ii) $\triangle ABC \cong \triangle PQR$

Difficulty Level:
Medium

Known/given:
Sides AB=PQ, BC=QR and AM = PN also AM and PN are medians.

To prove:

i) $\triangle ABM \cong \triangle PQN$

ii) $\triangle ABC \cong \triangle PQR$

Reasoning:
Using the median property, we can show Triangles ABM and PQN are congruent by SSS congruency then we can say corresponding parts of congruent triangles are equal and then Triangles ABC and PQR by SAS congruency.
Solution:

i) In $\triangle ABC$, AM is the median to BC.
\[ \therefore BM = \frac{1}{2} BC \]

In $\triangle PQR$, PN is the median to QR.
\[ \therefore QN = \frac{1}{2} QR \]

However, $BC = QR$
\[ \therefore \frac{1}{2} BC = \frac{1}{2} QR \]
\[ \therefore BM = QN \quad \ldots (1) \]

In $\triangle ABM$ and $\triangle PQN$,
- $AB = PQ$ (Given)
- $BM = QN$ [From Equation (1)]
- $AM = PN$ (Given)

$\triangle ABM \cong \triangle PQN$ (By SSS congruence rule)
\[ \angle ABM = \angle PQN \quad \text{(By CPCT)} \]
\[ \angle ABC = \angle PQR \quad \ldots (2) \]

ii) In $\triangle ABC$ and $\triangle PQR$, $AB = PQ$ (Given)
\[ \angle ABC = \angle PQR \quad \text{(From Equation (2))} \]

$BC = QR$ (Given)
\[ \therefore \triangle ABC \cong \triangle PQR \quad \text{(By SAS congruence rule)} \]

Q4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Difficulty Level:
Easy

Known/given:
BE and CF are two equal altitudes of a triangle ABC.

To prove:
Triangle ABC is isosceles by using RHS congruence rule.
Reasoning:
We can show triangles BEC and CFB congruent by using RHS congruency and then we can say corresponding parts of congruent triangles are equal to prove the required result.

Solution:

In $\triangle BEC$ and $\triangle CFB$,

- $\angle BEC = \angle CFB$ (Each $90^\circ$)
- $BC = CB$ (Common)
- $BE = CF$ (Given)

$\therefore \triangle BEC \cong \triangle CFB$ (By RHS congruency)
$\therefore \angle BCE = \angle CBF$ (By CPCT)

$\therefore AB = AC$ (Sides opposite to equal angles of a triangle are equal)

Hence, $\triangle ABC$ is isosceles.

Q5. $ABC$ is an isosceles triangle with $AB = AC$. Draw $AP \perp BC$ to show that $\angle B = \angle C$.

Difficulty Level:
Easy

Known/given:
$ABC$ is an isosceles triangle with $AB = AC$ and $AP \perp BC$

To prove:
$\angle B = \angle C$

Reasoning:
We can show triangles $APB$ and $APC$ congruent by using RHS congruency and then we can say corresponding parts of congruent triangles are equal to show the required result.
Solution:

In \( \triangle APB \) and \( \triangle APC \),

\[ \angle APB = \angle APC \quad \text{(Each 90°)} \]
\[ AB = AC \quad \text{(Given)} \]
\[ AP = AP \quad \text{(Common)} \]
\[ \triangle APB = \triangle APC \quad \text{(Using RHS congruence rule)} \]
\[ \angle B = \angle C \quad \text{(CPCT)} \]
Q1. Show that in a right-angled triangle, the hypotenuse is the longest side.

**Difficulty Level:**
Easy

**Known/given:**
A right-angled triangle.

**To prove:**
Hypotenuse is the longest side.

**Reasoning:**
In a triangle if one angle is of 90 degree then the other two angles have to be acute and we can use the fact that in any triangle, the side opposite to the larger (greater) angle is longer.

**Solution:**

Let us consider a right-angled triangle ABC, right-angled at B.

In \( \triangle ABC \),
\[
\angle A + \angle B + \angle C = 180^\circ \quad \text{(Angle sum property of a triangle)}
\]
\[
\angle A + 90^\circ + \angle C = 180^\circ
\]
\[
\angle A + \angle C = 90^\circ
\]

Hence, the other two angles have to be acute (i.e., less than 90°).
\( \angle B \) is the largest angle in \( \triangle ABC \).
\[
\angle B > \angle A \text{ and } \angle B > \angle C
\]
\[
AC > BC \text{ and } AC > AB
\]
[In any triangle, the side opposite to the larger (greater) angle is longer.] Therefore, AC is the largest side in $\Delta ABC$. However, AC is the hypotenuse of $\Delta ABC$. Therefore, hypotenuse is the longest side in a right-angled triangle.

Q2. In the given figure sides AB and AC of $\Delta ABC$ are extended to points P and Q respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$.

**Difficulty Level:**
Easy

**Known/given:**
$\angle PBC < \angle QCB$.

**To prove:**
$AC > AB$.

**Reasoning:**
By using linear pair, we can find inequality of interior angles and then we can use the fact that in any triangle, the side opposite to the larger (greater) angle is longer.

**Solution:**
In the given figure,
\[
\angle ABC + \angle PBC = 180^\circ \quad \text{(Linear pair)}
\]
\[
\angle ABC = 180^\circ - \angle PBC \quad \text{... (1)}
\]

Also,
\[
\angle ACB + \angle QCB = 180^\circ
\]
\[
\angle ACB = 180^\circ - \angle QCB \quad \text{... (2)}
\]

As $\angle PBC < \angle QCB$,
\[
180^\circ - \angle PBC > 180^\circ - \angle QCB
\]
\[
\angle ABC > \angle ACB \quad \text{[From Equations (1) and (2)]}
\]
\[
AC > AB \quad \text{(Side opposite to the larger angle is larger.)}
\]
Hence proved, $AC > AB$.
Q3. In the given figure, $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$.

**Difficulty Level:**
Easy

**Known/given:**
$\angle B < \angle A$ and $\angle C < \angle D$.

**To prove:**
$AD < BC$

**Reasoning:**
We can use the fact that in any triangle, the side opposite to the larger (greater) angle is longer. We can add both the triangles result to get the required result.

**Solution:**
In $\triangle AOB$,

$\angle B < \angle A$

$AO < BO$ (Side opposite to smaller angle is smaller) ... (1)

In $\triangle COD$,

$\angle C < \angle D$

$OD < OC$ (Side opposite to smaller angle is smaller) ... (2)

On adding Equations (1) and (2), we obtain

$AO + OD < BO + OC$

$AD < BC$, proved

Q4. AB and CD are respectively the smallest and longest sides of a quadrilateral $ABCD$ (see the given figure). Show that $\angle A > \angle C$ and $\angle B > \angle D$.

**Difficulty Level:**
Easy
**Known/give:**
AB and CD are respectively the smallest and longest sides.

**To prove:**
\[ \angle A > \angle C \text{ and } \angle B > \angle D \]

**Reasoning:**
First of all we can join vertex A to C then triangles ABC and ADC will be formed now we can use the fact that in any triangle, the side opposite to the larger (greater) angle is longer. We can add both the triangles results to get the first required result. Similarly we can join vertex B and D and use the fact to get the other required result.

**Solution:**
Let us join AC.

In \( \triangle ABC \),
\[ AB < BC \quad (AB \text{ is the smallest side of quadrilateral } ABCD) \]
\[ \angle 2 < \angle 1 \quad (\text{Angle opposite to the smaller side is smaller}) \quad \ldots \quad (1) \]

In \( \triangle ADC \),
\[ AD < CD \quad (CD \text{ is the largest side of quadrilateral } ABCD) \]
\[ \angle 4 < \angle 3 \quad (\text{Angle opposite to the smaller side is smaller}) \quad \ldots \quad (2) \]
On adding Equations (1) and (2), we obtain
\[ \angle 2 + \angle 4 < \angle 1 + \angle 3 \]
\[ \angle C < \angle A \]
\[ \angle A > \angle C \]

Let us join BD.

In \( \triangle ABD \),
\[ AB < AD \quad (AB \text{ is the smallest side of quadrilateral } ABCD) \]
\[ \angle 8 < \angle 5 \quad (\text{Angle opposite to the smaller side is smaller}) \quad ...(3) \]

In \( \triangle BDC \),
\[ BC < CD \quad (CD \text{ is the largest side of quadrilateral } ABCD) \]
\[ \angle 7 < \angle 6 \quad (\text{Angle opposite to the smaller side is smaller}) \quad ...(4) \]

On adding Equations (3) and (4), we obtain
\[ \angle 8 + \angle 7 < \angle 5 + \angle 6 \]
\[ \angle D < \angle B \]
\[ \angle B > \angle D \quad (\text{Hence, proved}) \]

Q5. In the given figure, PR > PQ and PS bisects \( \angle QPR \). Prove that
\[ \angle PSR > \angle PSQ. \]

**Difficulty Level:**
Easy

**Known/given:**
PR > PQ and PS bisects \( \angle QPR \)

**To prove:**
\[ \angle PSR > \angle PSQ. \]
Reasoning:
We can use exterior angle sum property to find the required inequality.

Solution:
As \( PR > PQ \),

\[ \angle PQR > \angle PRQ \quad \text{(Angle opposite to larger side is larger)} \quad \text{...(1)} \]

PS is the bisector of \( \angle QPR \).

\[ \angle QPS = \angle RPS \quad \text{...(2)} \]

\( \angle PSR \) is the exterior angle of \( \triangle PQS \).

\[ \angle PSR = \angle PQR + \angle QPS \quad \text{...(3)} \]

\( \angle PSQ \) is the exterior angle of \( \triangle PRS \).

\[ \angle PSQ = \angle PRQ + \angle RPS \quad \text{...(4)} \]

Adding Equations (1) and (2), we obtain

\[ \angle PQR + \angle QPS > \angle PRQ + \angle RPS \]

\[ \angle PSR > \angle PSQ \quad \text{[Using the values of Equations (3) and (4)]} \]

Q6. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Difficulty Level:
Easy

Known/given:
Line segments drawn from a given point is not on line.

To prove:
The perpendicular line segment is the shortest.

Reasoning:
We know that in a triangle if one angle is 90 degree then other have to be acute.
Let us take a line $l$ and from point $P$ (i.e., not on line $l$), draw two line segments $PN$ and $PM$. Let $PN$ be perpendicular to line $l$ and $PM$ is drawn at some other angle.

In $\triangle PNM$,

$\angle N = 90^\circ$

$\angle P + \angle N + \angle M = 180^\circ$ (Angle sum property of a triangle)

$\angle P + \angle M = 90^\circ$

Clearly, $\angle M$ is an acute angle.

$\angle M < \angle N$

$PN < PM$ (Side opposite to the smaller angle is smaller)

Similarly, by drawing different line segments from $P$ to $l$, it can be proved that $PN$ is smaller in comparison to them. Therefore, it can be observed that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.
**Triangles**

**Exercise (7.5)**

**Q1.** ABC is a triangle. Locate a point in the interior of ΔABC which is equidistant from all the vertices of ΔABC.

**Solution:**
Circumcentre of a triangle is always equidistant from all the vertices of that triangle.

Circumcentre is the point where perpendicular bisectors of all the sides of the triangle meet together.

In ΔABC, we can find the circumcentre by drawing the perpendicular bisectors of sides AB, BC, and CA of this triangle. O is the point where these bisectors are meeting together. Therefore, O is the point which is equidistant from all the vertices of ΔABC.

**Q2.** In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

**Solution:**
The point which is equidistant from all the sides of a triangle is called the incentre of the triangle. Incentre of a triangle is the intersection point of the angle bisectors of the interior angles of that triangle.
Here, in \( \triangle ABC \), we can find the incentre of this triangle by drawing the angle bisectors of the interior angles of this triangle. \( I \) is the point where these angle bisectors are intersecting each other. Therefore, \( I \) is the point equidistant from all the sides of \( \triangle ABC \).

**Q3.** In a huge park people are concentrated at three points (see the given figure)

A: where there are different slides and swings for children,
B: near which a man-made lake is situated,
C: which is near to a large parking and exit.

Where should an ice-cream parlour be set up so that maximum number of persons can approach it?
(*Hint:* The parlor should be equidistant from A, B and C)

**Solution:**
Maximum number of persons can approach the ice-cream parlour if it is equidistant from A, B and C. Now, A, B and C form a triangle. In a triangle, the circumcentre is the only point that is equidistant from its vertices. So, the ice-cream parlour should be set up at the circumcentre \( O \) of \( \triangle ABC \).
In this situation, maximum number of persons can approach it. We can find circumcentre \( O \) of this triangle by drawing perpendicular bisectors of the sides of this triangle.

**Q4.** Complete the hexagonal and star shaped **rangolis** (see the given figures) by filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?

**Solution:**
It can be observed that hexagonal-shaped **rangoli** has 6 equilateral triangles of side 5 cm in it.
Area of $\Delta OAB = \frac{\sqrt{3}}{4} (\text{side})^2$

$$= \frac{\sqrt{3}}{4} (5\text{cm})^2 = \frac{25\sqrt{3}}{4} \text{cm}^2$$

Area of hexagonal-shaped rangoli = $6 \times \frac{25\sqrt{3}}{4} \text{cm}^2 = \frac{75\sqrt{3}}{2} \text{cm}^2$

Area of equilateral triangle having its side as $1\text{cm} = \frac{\sqrt{3}}{4} (1\text{cm})^2 = \frac{\sqrt{3}}{4} \text{cm}^2$

Number of equilateral triangle of 1 cm side that can be filled in this hexagonal-shaped rangoli

$$= \frac{75\sqrt{3}}{2} \times \frac{4}{\sqrt{3}} = 150$$

Star-shaped rangoli has 12 equilateral triangles of side 5 cm in it.

Area of star-shaped rangoli = $12 \times \frac{\sqrt{3}}{4} \times (5\text{cm})^2 = 75\sqrt{3}\text{cm}^2$

Number of equilateral triangles of 1 cm side that can be filled in this star-shaped rangoli

$$= \frac{75\sqrt{3}\text{cm}^2}{\frac{\sqrt{3}}{4} \text{cm}^2} = 75\sqrt{3} \times \frac{4}{\sqrt{3}} = 300$$

Therefore, star-shaped rangoli has more equilateral triangles of side 1 cm in it.
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