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## Quadrilaterals

### Exercise (8.1)

**Q1.** The angles of quadrilateral are in the ratio 3:5:9:13. Find all the angles of the quadrilateral.

**Difficulty Level:**

Easy

**What is known/given?**

The angles of quadrilateral are in the ratio 3:5:9:13.

**What is unknown?**

All the angles of the quadrilateral.

**Reasoning:**

Sum of angles in any quadrilateral is 360 degree.

**Solution:**

Let the common ratio between the angles be  $x$ .

Therefore, the angles will be  $3x$ ,  $5x$ ,  $9x$ , and  $13x$  respectively.

As the sum of all interior angles of a quadrilateral is  $360^\circ$ ,

$$\therefore 3x + 5x + 9x + 13x = 360^\circ$$

$$30x = 360^\circ$$

$$x = 12^\circ$$

Hence, the angles are

$$3x = 3 \times 12 = 36^\circ$$

$$5x = 5 \times 12 = 60^\circ$$

$$9x = 9 \times 12 = 108^\circ$$

$$13x = 13 \times 12 = 156^\circ$$

**Q2.** If the diagonals of a parallelogram are equal, then show that it is a rectangle.

**Difficulty Level:**

Easy

**What is known/given?**

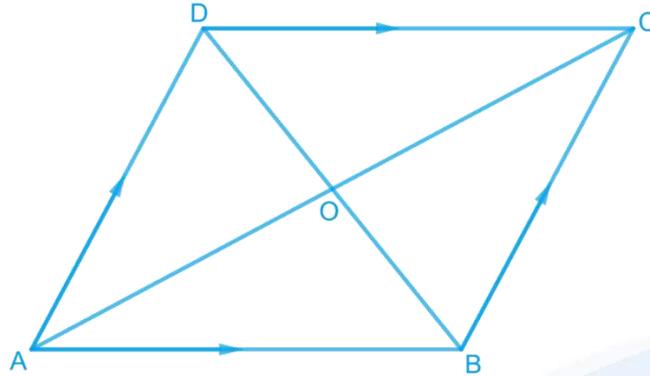
The diagonals of a parallelogram are equal.

**What is unknown?**

How we can show that it is a rectangle.

**Reasoning:**

To show that given parallelogram is a rectangle, we have to prove that one of its interior angles is  $90^\circ$  and this can be done by showing two triangles congruent.

**Solution:**

Let ABCD be a parallelogram. To show that ABCD is a rectangle, we have to prove that one of its interior angles is  $90^\circ$ .

In  $\triangle ABC$  and  $\triangle DCB$ ,

$AB = DC$  (Opposite sides of a parallelogram are equal)

$BC = BC$  (Common)

$AC = DB$  (Given)

$\therefore \triangle ABC \cong \triangle DCB$  (By SSS Congruence rule)

$\Rightarrow \angle ABC = \angle DCB$

It is known that the sum of the measures of angles on the same side of transversal is  $180^\circ$ .

$$\angle ABC + \angle DCB = 180^\circ \text{ (} AB \parallel CD \text{)}$$

$$\Rightarrow \angle ABC + \angle ABC = 180^\circ$$

$$\Rightarrow 2\angle ABC = 180^\circ$$

$$\Rightarrow \angle ABC = 90^\circ$$

Since ABCD is a parallelogram and one of its interior angles is  $90^\circ$ , ABCD is a rectangle.

**Q3.** Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

**Difficulty Level:**

Easy

**What is known/given?**

The diagonals of a quadrilateral bisect each other at right angles.

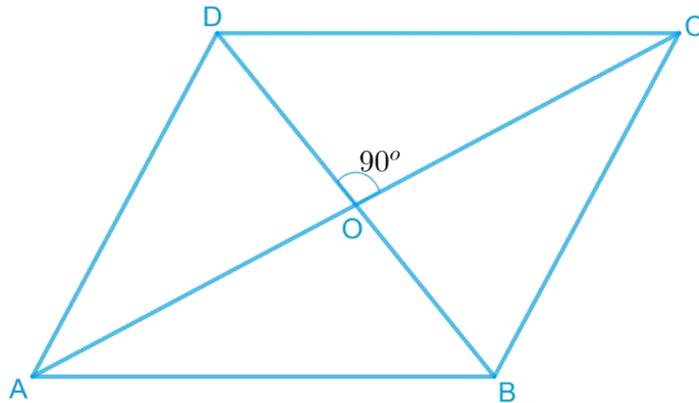
**What is unknown?**

How we can show that it is a rhombus.

**Reasoning:**

To show that given quadrilateral is a rhombus, we have to show it is a parallelogram and all the sides are equal.

**Solution:**



Let ABCD be a quadrilateral, whose diagonals AC and BD bisect each other at right angle i.e.  $OA = OC$ ,  $OB = OD$ , and  $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90^\circ$ .

To prove ABCD a rhombus,

We have to prove ABCD is a parallelogram and all the sides of ABCD are equal.

In  $\triangle AOD$  and  $\triangle COD$ ,

$$OA = OC \text{ (Diagonals bisect each other)}$$

$$\angle AOD = \angle COD \text{ (Given)}$$

$$OD = OD \text{ (Common)}$$

$$\therefore \triangle AOD \cong \triangle COD \text{ (By SAS congruence rule)}$$

$$\therefore AD = CD \text{ ..... (1)}$$

Similarly, it can be proved that

$$AD = AB \text{ and } CD = BC \text{ ..... (2)}$$

From Equations (1) and (2),

$$AB = BC = CD = AD$$

Since opposite sides of quadrilateral ABCD are equal, it can be said that ABCD is a parallelogram. Since all sides of a parallelogram ABCD are equal, it can be said that ABCD is a rhombus.

**Q4.** Show that the diagonals of a square are equal and bisect each other at right angles.

**Difficulty Level:**

Medium

**What is known/given?**

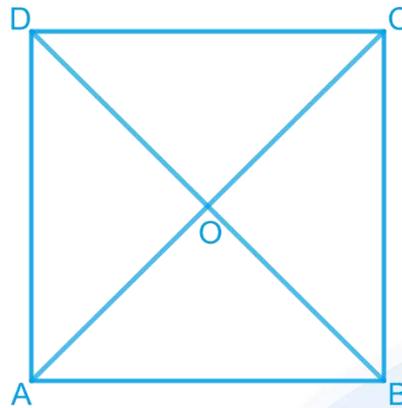
Given quadrilateral is a square.

**What is unknown?**

How we can show that diagonals of a square are equal and bisect each other at right angles.

**Reasoning:**

By showing two triangles congruent consist diagonals and then we can say corresponding parts of congruent triangles are equal.

**Solution:**

Let ABCD be a square.

Let the diagonals AC and BD intersect each other at a point O.

To prove that the diagonals of a square are equal and bisect each other at right angles, we have to prove,

$AC = BD$ ,  $OA = OC$ ,  $OB = OD$ , and  $\angle AOB = 90^\circ$

In  $\triangle ABC$  and  $\triangle DCB$ ,

$AB = DC$  (Sides of a square are equal to each other)

$\angle ABC = \angle DCB$  (All interior angles are of  $90^\circ$ )

$BC = CB$  (Common side)

$\therefore \triangle ABC \cong \triangle DCB$  (By SAS congruency)

$\therefore AC = DB$  (By CPCT)

Hence, the diagonals of a square are equal in length.

In  $\triangle AOB$  and  $\triangle COD$ ,

$\angle AOB = \angle COD$  (Vertically opposite angles)

$\angle ABO = \angle CDO$  (Alternate interior angles)

$AB = CD$  (Sides of a square are always equal)

$\therefore \triangle AOB \cong \triangle COD$  (By AAS congruence rule)

$\therefore AO = CO$  and  $OB = OD$  (By CPCT)

Hence, the diagonals of a square bisect each other.

In  $\triangle AOB$  and  $\triangle COB$ ,

As we had proved that diagonals bisect each other, therefore,

$$AO = CO$$

$$AB = CB \text{ (Sides of a square are equal)}$$

$$BO = BO \text{ (Common)}$$

$$\therefore \triangle AOB \cong \triangle COB \text{ (By SSS congruency)}$$

$$\therefore \angle AOB = \angle COB \text{ (By CPCT)}$$

However,

$$\angle AOB + \angle COB = 180^\circ$$

$$2\angle AOB = 180^\circ$$

$$\angle AOB = 90^\circ$$

Hence, the diagonals of a square bisect each other at right angles.

**Q5.** Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

**Difficulty Level:**

Medium

**What is known/given?**

The diagonals of a quadrilateral are equal and bisect each other at right angle.

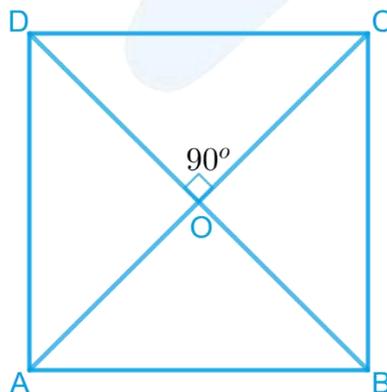
**What is unknown?**

How we can show that it is a square.

**Reasoning:**

We have to show that given quadrilateral is a parallelogram in which all sides are equal and one of its interior angles is  $90^\circ$ .

**Solution:**



Let us consider a quadrilateral ABCD in which the diagonals AC and BD intersect each other at O.

It is given that the diagonals of ABCD are equal and bisect each other at right angles.

Therefore,  $AC = BD$ ,  $OA = OC$ ,  $OB = OD$ , and  $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90^\circ$

To prove ABCD is a square

We have to prove that ABCD is a parallelogram in which  $AB = BC = CD = AD$ , and one of its interior angles is  $90^\circ$ .

In  $\triangle AOB$  and  $\triangle COD$ ,

$$AO = CO \text{ (Diagonals bisect each other)}$$

$$OB = OD \text{ (Diagonals bisect each other)}$$

$$\angle AOB = \angle COD \text{ (Vertically opposite angles)}$$

$$\therefore \triangle AOB \cong \triangle COD \text{ (SAS congruence rule)}$$

$$\therefore AB = CD \text{ (By CPCT) } \dots\dots\dots (1)$$

And,  $\angle OAB = \angle OCD$  (By CPCT)

However, these are alternate interior angles for line AB and CD and alternate interior angles are equal to each other only when the two lines are parallel.

$$AB \parallel CD \quad \dots (2)$$

From Equations (1) and (2), we obtain ABCD is a parallelogram.

In  $\triangle AOD$  and  $\triangle COD$ ,

$$AO = CO \text{ (Diagonals bisect each other)}$$

$$\angle AOD = \angle COD \text{ (Given that each is } 90^\circ)$$

$$OD = OD \text{ (Common)}$$

$$\therefore \triangle AOD \cong \triangle COD \text{ (SAS congruence rule)}$$

$$\therefore AD = DC \dots\dots\dots (3)$$

However,  $AD = BC$  and  $AB = CD$  (Opposite sides of parallelogram ABCD)  
 $AB = BC = CD = DA$

Therefore, all the sides of quadrilateral ABCD are equal to each other.

In  $\triangle ADC$  and  $\triangle BCD$ ,

$$AD = BC \text{ (Already proved)}$$

$$AC = BD \text{ (Given)}$$

$$DC = CD \text{ (Common)}$$

$$\therefore \triangle ADC \cong \triangle BCD \text{ (SSS Congruence rule)}$$

$$\therefore \angle ADC = \angle BCD \text{ (By CPCT)}$$

However,  $\angle ADC + \angle BCD = 180^\circ$  (Co-interior angles)

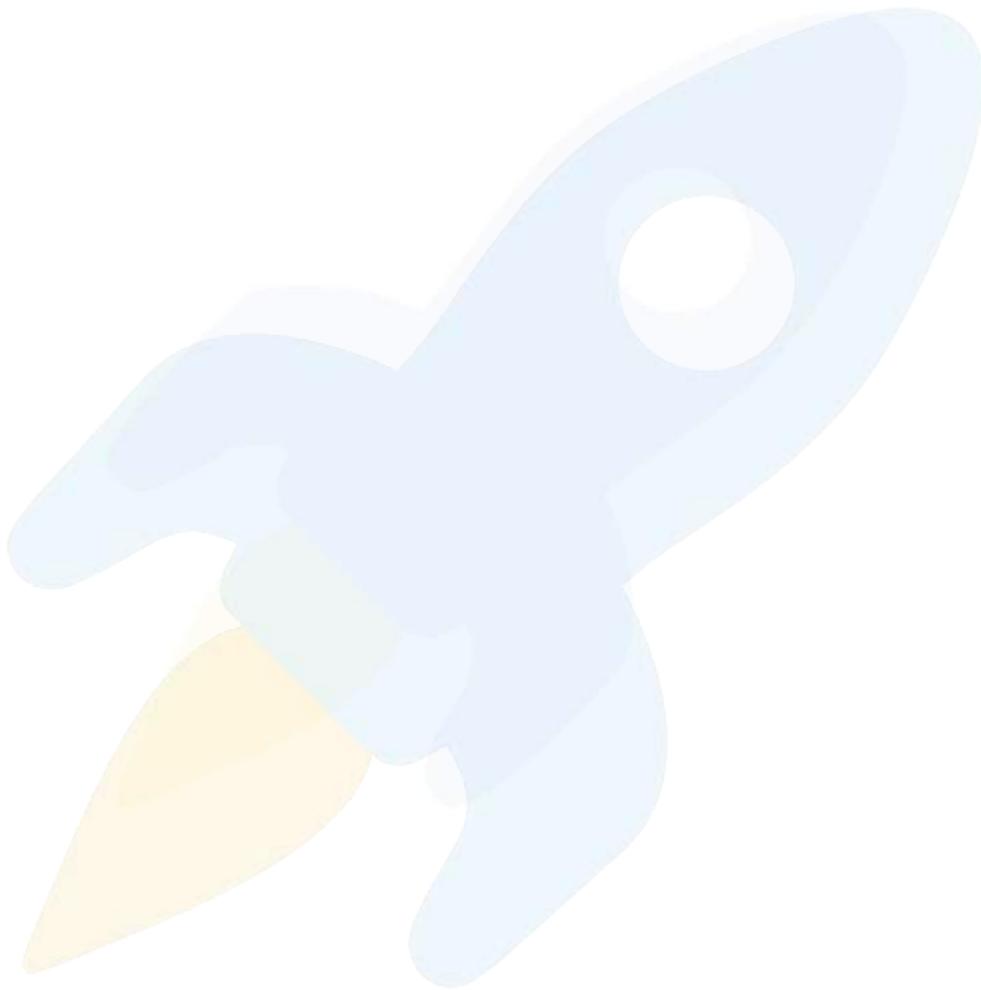
$$\angle ADC + \angle ADC = 180^\circ$$

$$2\angle ADC = 180^\circ$$

$$\therefore \angle ADC = 90^\circ$$

One of the interior angles of quadrilateral ABCD is a right angle.  
Thus, we have obtained that ABCD is a parallelogram,  $AB = BC = CD = AD$  and one of its interior angles is  $90^\circ$ .

Therefore, ABCD is a square.



**Q6.** Diagonal AC of a parallelogram ABCD bisects  $\angle A$  (see the given figure).

Show that

- (i) It bisects  $\angle C$  also,
- (ii) ABCD is a rhombus.

**Difficulty Level:**

Medium

**What is known/given?**

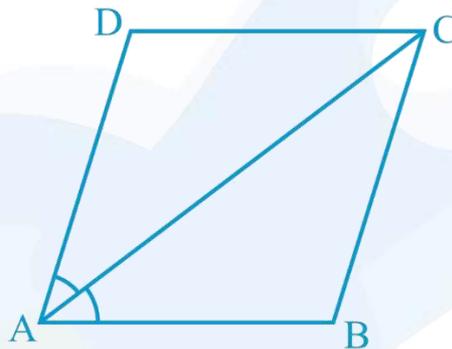
The diagonal AC of a parallelogram ABCD bisects  $\angle A$ .

**What is unknown?**

How we can show that (i) It bisects  $\angle C$  also, (ii) ABCD is a rhombus.

**Reasoning:**

We can use alternate interior angles property to show diagonal AC bisects angle C also, by showing all sides equal it can be said rhombus.



**Solution:**

(i) ABCD is a parallelogram.

$$\angle DAC = \angle BCA \text{ (Alternate interior angles)} \dots\dots\dots (1)$$

$$\text{And, } \angle BAC = \angle DCA \text{ (Alternate interior angles)} \dots\dots\dots (2)$$

However, it is given that AC bisects  $\angle A$ .

$$\angle DAC = \angle BAC \dots\dots\dots (3)$$

From Equations (1), (2), and (3), we obtain

$$\angle DAC = \angle BCA = \angle BAC = \angle DCA \dots\dots\dots (4)$$

$$\angle DCA = \angle BCA$$

Hence, AC bisects  $\angle C$ .

(ii) From Equation (4), we obtain

$$\angle DAC = \angle DCA$$

$$DA = DC \text{ (Side opposite to equal angles are equal)}$$

However,  $DA = BC$  and  $AB = CD$  (Opposite sides of a parallelogram)

$$AB = BC = CD = DA$$

Hence, ABCD is a rhombus.

**Q7.** ABCD is a rhombus. Show that diagonal AC bisects  $\angle A$  as well as  $\angle C$  and diagonal BD bisects  $\angle B$  as well as  $\angle D$ .

**Difficulty Level:**

Medium

**What is known/given?**

ABCD is a rhombus.

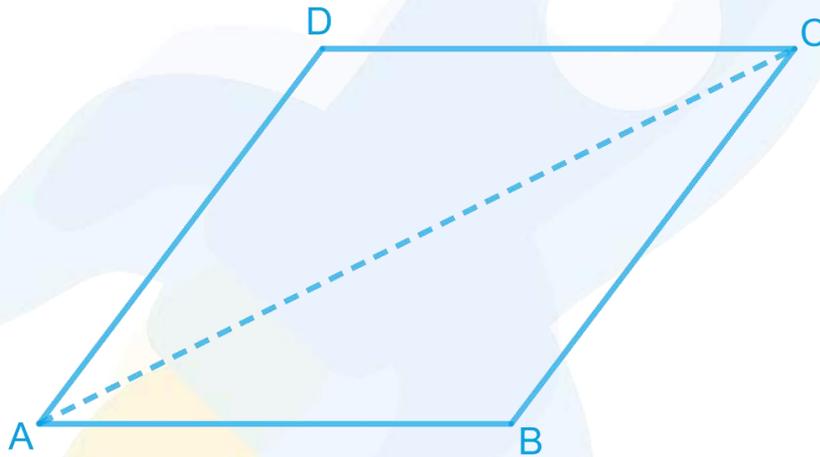
**What is unknown?**

How we can show that diagonal AC bisects  $\angle A$  as well as  $\angle C$  and diagonal BD bisects  $\angle B$  as well as  $\angle D$ .

**Reasoning:**

We can use alternate interior angles property to show diagonal AC bisects angles A and C, similarly diagonal BD bisects angles B and D.

**Solution:**



Let us join AC.

In  $\triangle ABC$ ,

$BC = AB$  (Sides of a rhombus are equal to each other)

$\angle BAC = \angle BCA$  (Angles opposite to equal sides of a triangle are equal)

However,  $\angle BAC = \angle DCA$  (Alternate interior angles for parallel lines AB and CD)

$\angle BCA = \angle DCA$

Therefore, AC bisects  $\angle C$ .

Also,  $\angle BCA = \angle DAC$  (Alternate interior angles for  $\parallel$  lines BC and DA)

$\angle BAC = \angle DAC$

Therefore, AC bisects  $\angle A$  as well as  $\angle C$ .

Similarly, it can be proved that BD bisects  $\angle B$  and  $\angle D$  as well.

**Q8.** ABCD is a rectangle in which diagonal AC bisects  $\angle A$  as well as  $\angle C$ .

Show that:

- (i) ABCD is a square
- (ii) Diagonal BD bisects  $\angle B$  as well as  $\angle D$ .

**Difficulty Level:**

Medium

**What is known/given?**

ABCD is a rectangle in which diagonal AC bisects  $\angle A$  as well as  $\angle C$ .

**What is unknown?**

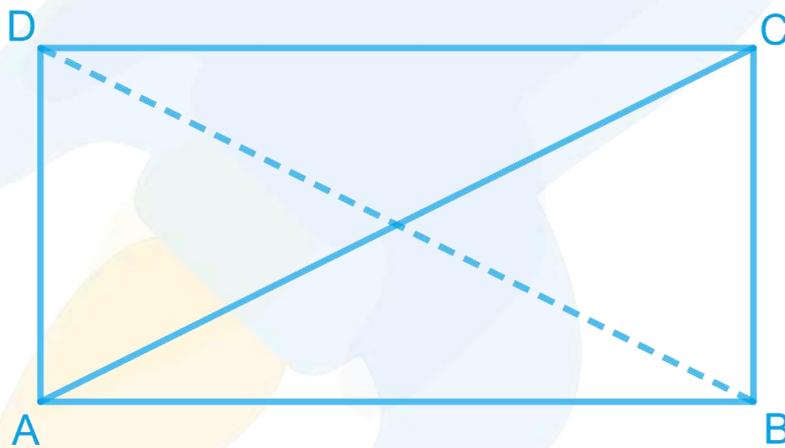
How we can show that (i) ABCD is a square

(ii) Diagonal BD bisects  $\angle B$  as well as  $\angle D$ .

**Reasoning:**

We can use angle bisector property and isosceles triangle property to show given rectangle as square and alternate interior angles property to show BD bisects angles B and D.

**Solution:**



(i) It is given that ABCD is a rectangle.

$$\angle A = \angle C$$

$$\Rightarrow \frac{1}{2} \angle A = \frac{1}{2} \angle C$$

$$\Rightarrow \angle DAC = \angle DCA \quad (\text{AC bisects } \angle A \text{ and } \angle C)$$

CD = DA (Sides opposite to equal angles are also equal)

However, DA = BC and AB = CD (Opposite sides of a rectangle are equal)

$$AB = BC = CD = DA$$

ABCD is a rectangle and all the sides are equal.

Hence, ABCD is a square.

(ii) Let us join BD.

In  $\triangle BCD$ ,

$BC = CD$  (Sides of a square are equal to each other)

$\angle CDB = \angle CBD$  (Angles opposite to equal sides are equal)

However,  $\angle CDB = \angle ABD$  (Alternate interior angles for  $AB \parallel CD$ )

$\angle CBD = \angle ABD$

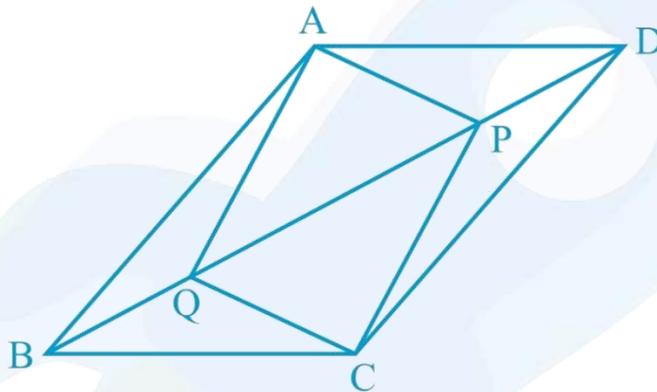
BD bisects  $\angle B$ .

Also,  $\angle CBD = \angle ADB$  (Alternate interior angles for  $BC \parallel AD$ )

$\angle CDB = \angle ABD$

BD bisects  $\angle D$  and  $\angle B$ .

**Q9.** In parallelogram ABCD, two points P and Q are taken on diagonal BD such that  $DP = BQ$  (see the given figure). Show that:



- (i)  $\triangle APD \cong \triangle CQB$
- (ii)  $AP = CQ$
- (iii)  $\triangle AQB \cong \triangle CPD$
- (iv)  $AQ = CP$
- (v) APCQ is a parallelogram

**Difficulty Level:**

Medium

**What is known/given?**

ABCD is a parallelogram and  $DP = BQ$

**What is unknown?**

How we can show that

- (i)  $\triangle APD \cong \triangle CQB$
- (ii)  $AP = CQ$
- (iii)  $\triangle AQB \cong \triangle CPD$
- (iv)  $AQ = CP$
- (v) APCQ is a parallelogram

**Reasoning:**

We can use alternate interior angles and parallelogram property for triangles congruence criterion to show triangles congruent then we can say corresponding parts of congruent triangles will be equal.

**Solution:**

(i) In  $\triangle APD$  and  $\triangle CQB$ ,

$$\angle ADP = \angle CBQ \text{ (Alternate interior angles for } BC \parallel AD)$$

$$AD = CB \text{ (Opposite sides of parallelogram } ABCD)$$

$$DP = BQ \text{ (Given)}$$

$$\therefore \triangle APD \cong \triangle CQB \text{ (Using SAS congruence rule)}$$

(ii) As we had observed that  $\triangle APD \cong \triangle CQB$ ,

$$\therefore AP = CQ \text{ (CPCT)}$$

(iii) In  $\triangle AQB$  and  $\triangle CPD$ ,

$$\angle ABQ = \angle CDP \text{ (Alternate interior angles for } AB \parallel CD)$$

$$AB = CD \text{ (Opposite sides of parallelogram } ABCD)$$

$$BQ = DP \text{ (Given)}$$

$$\therefore \triangle AQB \cong \triangle CPD \text{ (Using SAS congruence rule)}$$

(iv) As we had observed that  $\triangle AQB \cong \triangle CPD$ ,

$$\therefore AQ = CP \text{ (CPCT)}$$

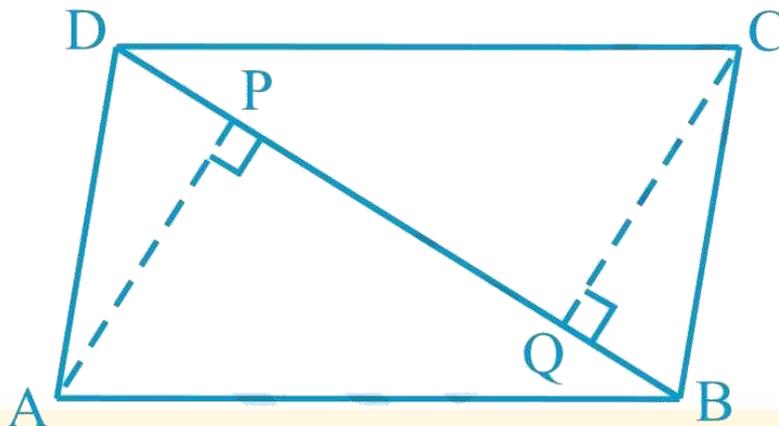
(v) From the result obtained in (ii) and (iv),

$$AQ = CP \text{ and}$$

$$AP = CQ$$

Since opposite sides in quadrilateral  $APCQ$  are equal to each other,  $APCQ$  is a parallelogram.

**Q10.**  $ABCD$  is a parallelogram and  $AP$  and  $CQ$  are perpendiculars from vertices  $A$  and  $C$  on diagonal  $BD$  (See the given figure). Show that



- (i)  $\triangle APB \cong \triangle CQD$   
 (ii)  $AP = CQ$

**Difficulty Level:**

Medium

**What is known/given?**

ABCD is a parallelogram and  $AP \perp DB, CQ \perp DB$

**What is unknown?**

How we can show that (i)  $\triangle APB \cong \triangle CQD$

(ii)  $AP = CQ$

**Reasoning:**

We can use alternate interior angles and parallelogram property for triangles congruence criterion to show triangles congruent then we can say corresponding parts of congruent triangles will be equal.

**Solution:**

(i) In  $\triangle APB$  and  $\triangle CQD$ ,

$$\angle APB = \angle CQD \text{ (Each } 90^\circ\text{)}$$

$$AB = CD \text{ (Opposite sides of parallelogram ABCD)}$$

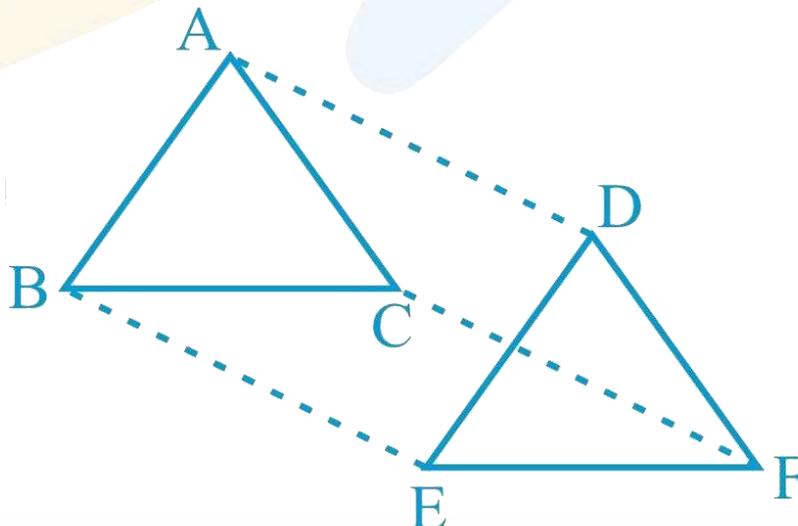
$$\angle ABP = \angle CDQ \text{ (Alternate interior angles for } AB \parallel CD\text{)}$$

$$\therefore \triangle APB \cong \triangle CQD \text{ (By AAS congruency)}$$

(ii) By using the above result

$$\triangle APB \cong \triangle CQD, \text{ we obtain } AP = CQ \text{ (By CPCT)}$$

**Q11.** In  $\triangle ABC$  and  $\triangle DEF$ ,  $AB = DE$ ,  $AB \parallel DE$ ,  $BC = EF$  and  $BC \parallel EF$ . Vertices A, B and C are joined to vertices D, E and F respectively (see the given figure). Show that



- (i) Quadrilateral ABED is a parallelogram
- (ii) Quadrilateral BEFC is a parallelogram
- (iii)  $AD \parallel CF$  and  $AD = CF$
- (iv) Quadrilateral ACFD is a parallelogram
- (v)  $AC = DF$
- (vi)  $\triangle ABC \cong \triangle DEF$ .

### Difficulty Level:

Medium

### What is known/given?

In  $\triangle ABC$  and  $\triangle DEF$ ,  $AB = DE$ ,  $AB \parallel DE$ ,  $BC = EF$  and  $BC \parallel EF$ .

### What is unknown?

- How we can show that
- (i) Quadrilateral ABED is a parallelogram
  - (ii) Quadrilateral BEFC is a parallelogram
  - (iii)  $AD \parallel CF$  and  $AD = CF$
  - (iv) Quadrilateral ACFD is a parallelogram
  - (v)  $AC = DF$
  - (vi)  $\triangle ABC \cong \triangle DEF$ .

### Reasoning:

We can use the fact that in a quadrilateral if one pair of opposite sides are parallel and equal to each other then it will be a parallelogram and converse is also true. Also, by using suitable congruence criterion we can show triangles congruent then we can say corresponding parts of congruent triangles will be equal.

### Solution:

- (i) It is given that  $AB = DE$  and  $AB \parallel DE$ .

If one pair of opposite sides of a quadrilateral are equal and parallel to each other, then it will be a parallelogram.

Therefore, quadrilateral ABED is a parallelogram.

- (ii) Again,  $BC = EF$  and  $BC \parallel EF$

Therefore, quadrilateral BCFE is a parallelogram.

- (iii) As we had observed that ABED and BEFC are parallelograms, therefore

$$AD = BE \text{ and } AD \parallel BE$$

(Opposite sides of a parallelogram are equal and parallel)

$$\text{And, } BE = CF \text{ and } BE \parallel CF$$

(Opposite sides of a parallelogram are equal and parallel)

$$\therefore AD = CF \text{ and } AD \parallel CF$$

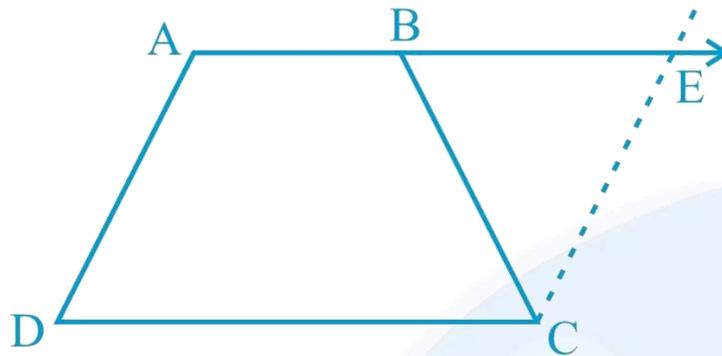
- (iv) As we had observed that one pair of opposite sides ( $AD$  and  $CF$ ) of quadrilateral ACFD are equal and parallel to each other, therefore, it is a parallelogram.

- (v) As ACFD is a parallelogram, therefore, the pair of opposite sides will be equal and parallel to each other.

$$\therefore AC \parallel DF \text{ and } AC = DF$$

- (vi)  $\triangle ABC$  and  $\triangle DEF$ ,  
 $AB = DE$  (Given)  
 $BC = EF$  (Given)  
 $AC = DF$  (ACFD is a parallelogram)  
 $\therefore \triangle ABC \cong \triangle DEF$  (By SSS congruence rule)

**Q12.** ABCD is a trapezium in which  $AB \parallel CD$  and  $AD = BC$  (see the given figure).  
 Show that



- (i)  $\angle A = \angle B$   
 (ii)  $\angle C = \angle D$   
 (iii)  $\triangle ABC \cong \triangle BAD$   
 (iv) diagonal  $AC =$  diagonal  $BD$   
 [Hint: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

**Difficulty Level:**

Medium

**What is known/given?**

ABCD is a trapezium in which  $AB \parallel CD$  and  $AD = BC$ .

**What is unknown?**

- How we can show that (i)  $\angle A = \angle B$   
 (ii)  $\angle C = \angle D$   
 (iii)  $\triangle ABC \cong \triangle BAD$   
 (iv) diagonal  $AC =$  diagonal  $BD$

**Reasoning:**

Consider parallel lines AD and CE, AE is the transversal line for them then sum of co-interior angles will be 180 degree also angles CBE and CBA are linear pairs gives sum 180, angles CEB equals to CBE because opposite to equal sides in triangle BCE. Using these observations angle A is equal to angle B. Then AB is parallel to CD will help to show angle C is equal to angle D. Also, by using suitable congruence criterion we can show triangles congruent then we can say corresponding parts of congruent triangles will be equal.

**Solution:**

Let us extend AB. Then, draw a line through C, which is parallel to AD intersecting AB at point E. It is clear that AECD is a parallelogram.

(i)  $AD = CE$  (Opposite sides of parallelogram AECD)

However,  $AD = BC$  (Given)

Therefore,  $BC = CE$

$\angle CEB = \angle CBE$  (Angle opposite to equal sides are also equal)

Consider parallel lines AD and CE. AE is the transversal line for them.

$\angle A + \angle CEB = 180^\circ$  (Angles on the same side of transversal)

$\angle A + \angle CBE = 180^\circ$  (Using the relation  $\angle CEB = \angle CBE$ ) ... (1)

However,  $\angle B + \angle CBE = 180^\circ$  (Linear pair angles) ... (2)

From Equations (1) and (2), we obtain

$\angle A = \angle B$

(ii)  $AB \parallel CD$

$\angle A + \angle D = 180^\circ$  (Angles on the same side of the transversal)

Also,  $\angle C + \angle B = 180^\circ$  (Angles on the same side of the transversal)

$\therefore \angle A + \angle D = \angle C + \angle B$

However,  $\angle A = \angle B$  [Using the result obtained in (i)]

$\therefore \angle C = \angle D$

(iii) In  $\triangle ABC$  and  $\triangle BAD$ ,

$AB = BA$  (Common side)

$BC = AD$  (Given)

$\angle B = \angle A$  (Proved before)

$\therefore \triangle ABC \cong \triangle BAD$  (SAS congruence rule)

(iv) We had observed that,

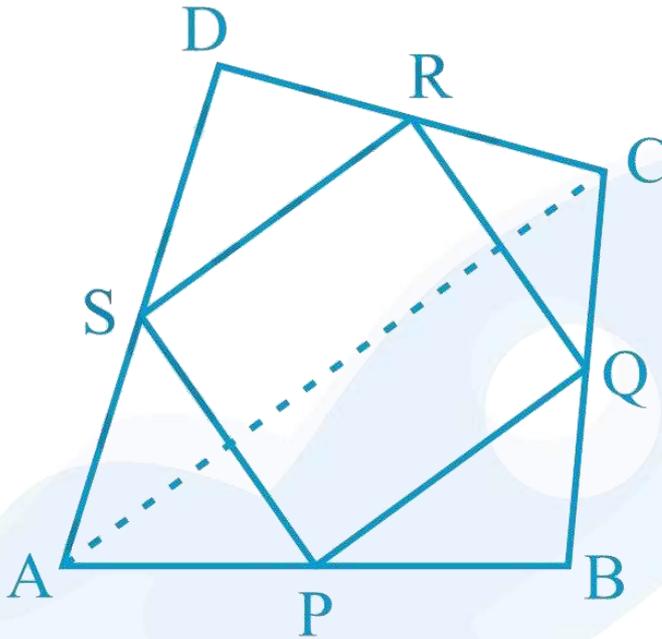
$\triangle ABC \cong \triangle BAD$

$\therefore AC = BD$  (By CPCT)

## Quadrilaterals

### Exercise (8.2)

**Q1.** ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see the given figure). AC is a diagonal. Show that:



- i)  $SR \parallel AC$  and  $SR = \frac{1}{2} AC$
- ii)  $PQ = SR$
- iii) PQRS is a parallelogram.

**Difficulty Level:**  
Medium

#### What is known/given?

ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA. AC is a diagonal.

#### What is unknown?

How we can show that (i)  $SR \parallel AC$  and  $SR = \frac{1}{2} AC$

- (ii)  $PQ = SR$
- (iii) PQRS is a parallelogram.

#### Reasoning:

In a triangle, the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it. Also, if one pair of opposite sides of quadrilateral is parallel and equal to each other then it is a parallelogram.

**Solution:**

- i) In  $\triangle ADC$ , S and R are the mid-points of sides AD and CD respectively. In a triangle, the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it.

$$\therefore SR \parallel AC \text{ and } SR = \frac{1}{2} AC \quad \dots (1)$$

- ii) In  $\triangle ABC$ , P and Q are mid-points of sides AB and BC respectively. Therefore, by using mid-point theorem,

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \quad \dots (2)$$

Using Equations (1) and (2), we obtain  $PQ \parallel SR$  and  $PQ = SR \quad \dots (3)$

$$\therefore PQ = SR$$

- iii) From Equation (3), we obtained  $PQ \parallel SR$  and  $PQ = SR$

Clearly, one pair of opposite sides of quadrilateral PQRS is parallel and equal. Hence, PQRS is a parallelogram.

**Q2.** ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

**Difficulty Level:**

Medium

**What is known/given?**

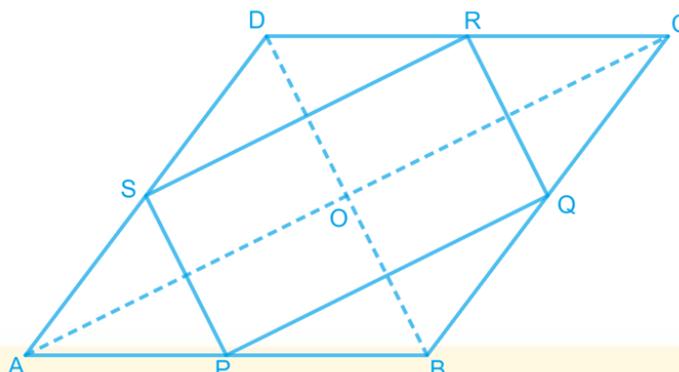
ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.

**What is unknown?**

How we can show that quadrilateral PQRS is a rectangle.

**Reasoning:**

In a triangle, the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it. Also if one pair of opposite sides of quadrilateral is parallel and equal to each other then it is a parallelogram. By showing one of the angle in parallelogram as 90 degree then we can say it is rectangle.

**Solution:**


In  $\triangle ABC$ , P and Q are the mid-points of sides AB and BC respectively.

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \text{ (Using mid-point theorem) ... (1)}$$

In  $\triangle ADC$ ,

R and S are the mid-points of CD and AD respectively.

$$\therefore RS \parallel AC \text{ and } RS = \frac{1}{2} AC \text{ (Using mid-point theorem) ... (2)}$$

From Equations (1) and (2), we obtain

$$PQ \parallel RS \text{ and } PQ = RS$$

Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, it is a parallelogram.

Let the diagonals of rhombus ABCD intersect each other at point O.

In quadrilateral OMQN,

$$MQ \parallel ON \text{ (} \because PQ \parallel AC \text{)}$$

$$QN \parallel OM \text{ (} \because QR \parallel BD \text{)}$$

Therefore, OMQN is a parallelogram.

$$\therefore \angle MQN = \angle NOM$$

$$\therefore \angle PQR = \angle NOM$$

However,  $\angle NOM = 90^\circ$  (Diagonals of a rhombus are perpendicular to each other)

$$\therefore \angle PQR = 90^\circ$$

Clearly, PQRS is a parallelogram having one of its interior angles as  $90^\circ$ .

Hence, PQRS is a rectangle.

**Q3.** ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

#### Difficulty Level:

Medium

#### What is known/given?

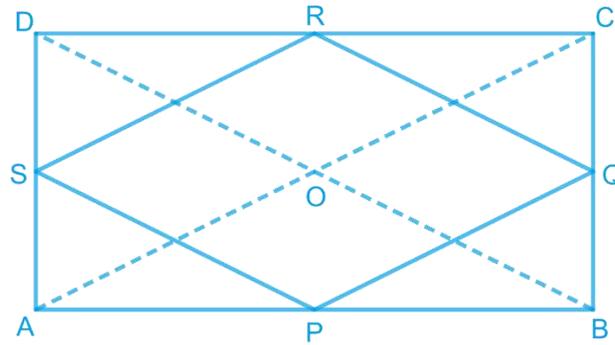
ABCD is a rectangle and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.

#### What is unknown?

How we can show that quadrilateral PQRS is a rhombus.

#### Reasoning:

In a triangle, the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it. Also if one pair of opposite sides of quadrilateral is parallel and equal to each other then it is a parallelogram. By showing all four sides equal in parallelogram we can say it is rhombus.

**Solution:**


Let us join AC and BD.

In  $\triangle ABC$ ,

P and Q are the mid-points of AB and BC respectively.

$$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \text{ (Mid-point theorem) ... (1)}$$

Similarly, in  $\triangle ADC$ ,

$$SR \parallel AC \text{ and } SR = \frac{1}{2} AC \text{ (Mid-point theorem) ... (2)}$$

Clearly,  $PQ \parallel SR$  and  $PQ = SR$

Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, it is a parallelogram.

$$\therefore PS \parallel QR \text{ and } PS = QR \text{ (Opposite sides of parallelogram) ... (3)}$$

In  $\triangle BCD$ , Q and R are the mid-points of side BC and CD respectively.

$$\therefore QR \parallel BD \text{ and } QR = \frac{1}{2} BD \text{ (Mid-point theorem) ... (4)}$$

However, the diagonals of a rectangle are equal.

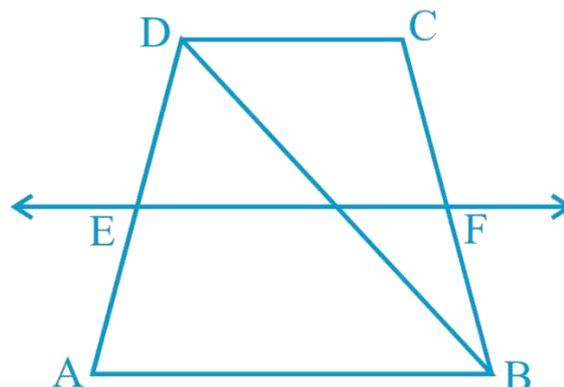
$$\therefore AC = BD \text{ ... (5)}$$

By using Equations (1), (2), (3), (4), and (5), we obtain

$$PQ = QR = SR = PS$$

Therefore, PQRS is a rhombus.

**Q4.** ABCD is a trapezium in which  $AB \parallel DC$ , BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see the given figure). Show that F is the mid-point of BC.



**Difficulty Level:**  
Medium

**What is known/given?**

ABCD is a trapezium in which  $AB \parallel DC$ , BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F.

**What is unknown?**

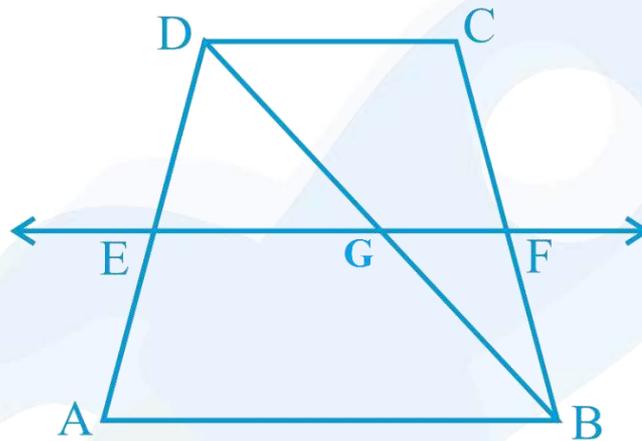
How we can show that F is the mid-point of BC.

**Reasoning:**

By converse of mid-point theorem, we know that a line drawn through the mid-point of any side of a triangle and parallel to another side, bisects the third side.

**Solution:**

Let EF intersect DB at G.



By converse of mid-point theorem, we know that a line drawn through the mid-point of any side of a triangle and parallel to another side, bisects the third side.

In  $\triangle ABD$ ,

$EF \parallel AB$  and E is the mid-point of AD.

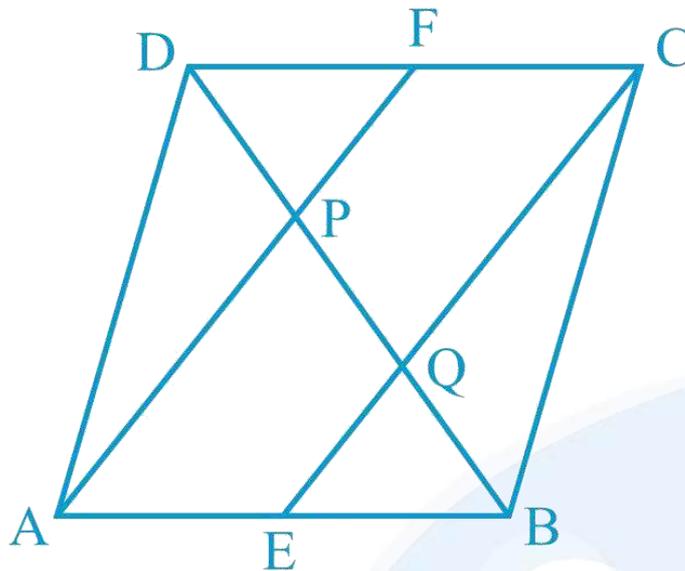
Therefore, G will be the mid-point of DB.

As  $EF \parallel AB$  and  $AB \parallel CD$ ,

$\therefore EF \parallel CD$  (Two lines parallel to the same line are parallel to each other)

In  $\triangle BCD$ ,  $GF \parallel CD$  and G is the mid-point of line BD. Therefore, by using converse of mid-point theorem, F is the mid-point of BC.

**Q5.** In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see the given figure). Show that the line segments AF and EC trisect the diagonal BD.



**Difficulty Level:**  
 Medium

**What is known/given?**

In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively.

**What is unknown?**

How we can show that the line segments AF and EC trisect the diagonal BD.

**Reasoning:**

In a quadrilateral if one pair of opposite sides is parallel and equal to each other. Then it is a parallelogram. Also, by converse of mid-point theorem, we know that a line drawn through the mid-point of any side of a triangle and parallel to another side, bisects the third side.

**Solution:**

ABCD is a parallelogram.

$$AB \parallel CD$$

And hence,  $AE \parallel FC$

Again,  $AB = CD$  (Opposite sides of parallelogram ABCD)

$$\frac{1}{2}AB = \frac{1}{2}CD$$

$AE = FC$  (E and F are mid-points of side AB and CD)

In quadrilateral AECF, one pair of opposite sides (AE and CF) is parallel and equal to each other. Therefore, AECF is a parallelogram.

$\therefore AF \parallel EC$  (Opposite sides of a parallelogram)

In  $\triangle DQC$ , F is the mid-point of side DC and  $FP \parallel CQ$  (as  $AF \parallel EC$ ). Therefore, by using the converse of mid-point theorem, it can be said that P is the mid-point of DQ.

$$\therefore DP = PQ \dots (1)$$

Similarly, in  $\triangle APB$ , E is the mid-point of side AB and  $EQ \parallel AP$  (as  $AF \parallel EC$ ). Therefore, by using the converse of mid-point theorem, it can be said that Q is the mid-point of PB.

$$\therefore PQ = QB \dots (2)$$

From Equations (1) and (2),  
 $DP = PQ = BQ$

Hence, the line segments AF and EC trisect the diagonal BD.

**Q6.** Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

### Difficulty Level:

Medium

### What is known/given?

ABCD is a quadrilateral in which P, Q, R, and S are the mid-points of sides AB, BC, CD, and DA respectively.

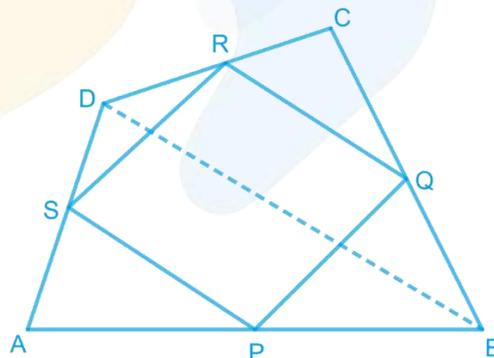
### What is unknown?

How we can show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

### Reasoning:

In a triangle, the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it. In a quadrilateral if one pair of opposite sides is parallel and equal to each other then it is a parallelogram.

### Solution:



Let ABCD is a quadrilateral in which P, Q, R, and S are the mid-points of sides AB, BC, CD, and DA respectively. Join PQ, QR, RS, SP, and BD.

In  $\triangle ABD$ , S and P are the mid-points of AD and AB respectively. Therefore, by using mid-point theorem, it can be said that

$$SP \parallel BD \text{ and } SP = \frac{1}{2} BD \dots (1)$$

Similarly, in  $\triangle BCD$ ,

$$QR \parallel BD \text{ and } QR = \frac{1}{2} BD \quad \dots (2)$$

From Equations (1) and (2), we obtain

$$SP \parallel QR \text{ and } SP = QR$$

In quadrilateral SPQR, one pair of opposite sides is equal and parallel to each other. Therefore, SPQR is a parallelogram.

We know that diagonals of a parallelogram bisect each other.

Hence, PR and QS bisect each other.

**Q7.** ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC

(ii)  $MD \perp AC$

(iii)  $CM = MA = \frac{1}{2} AB$

**Difficulty Level:**

Hard

**What is known/given?**

ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D.

**What is unknown?**

How we can show that (i) D is the mid-point of AC

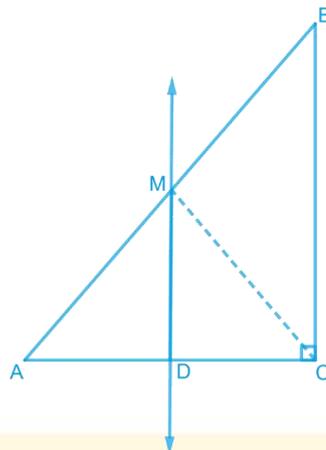
(ii)  $MD \perp AC$

(iii)  $CM = MA = \frac{1}{2} AB$

**Reasoning:**

By converse of mid-point theorem, we know that a line drawn through the mid-point of any side of a triangle and parallel to another side, bisects the third side. To show MD perpendicular to AC we know that sum of interior angles is 180. Also, to get  $CM = MA =$  half of AB we can show two triangles congruent.

**Solution:**



(i) In  $\triangle ABC$ ,

It is given that M is the mid-point of AB and  $MD \parallel BC$ .

Therefore, D is the mid-point of AC. (Converse of mid-point theorem)

(ii) As  $DM \parallel CB$  and AC is a transversal line for them, therefore,

$$\angle MDC + \angle DCB = 180^\circ \text{ (Co-interior angles)}$$

$$\angle MDC + 90^\circ = 180^\circ$$

$$\angle MDC = 90^\circ$$

$$\therefore MD \perp AC$$

(iii) Join MC.

In  $\triangle AMD$  and  $\triangle CMD$ ,

$$AD = CD \text{ (D is the mid-point of side AC)}$$

$$\angle ADM = \angle CDM \text{ (Each } 90^\circ \text{)}$$

$$DM = DM \text{ (Common)}$$

$$\therefore \triangle AMD \cong \triangle CMD \text{ (By SAS congruence rule)}$$

Therefore,  $AM = CM$  (By CPCT)

However,  $AM = \frac{1}{2} AB$  (M is the mid-point of AB)

Therefore, it can be said that

$$CM = AM = \frac{1}{2} AB$$

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