Areas of Parallelograms and Triangles

Exercise - 9.1 (Page 155 of Grade 9 NCERT Textbook)

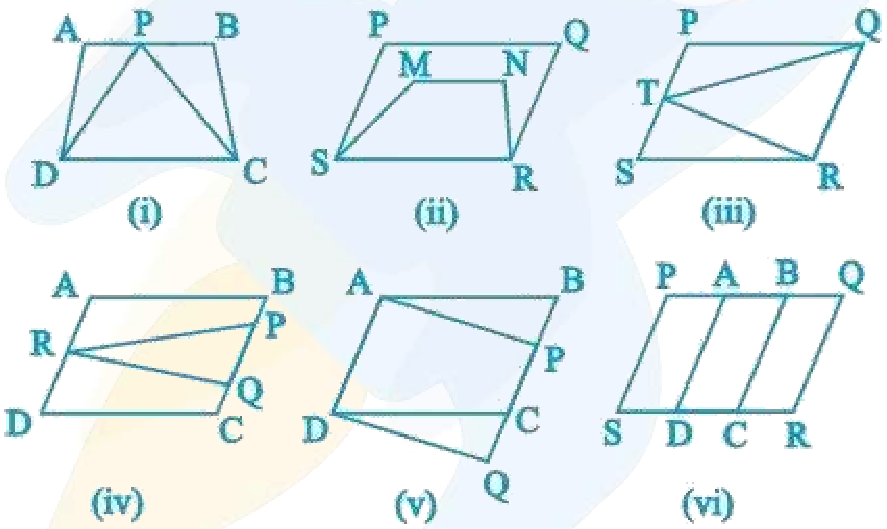
Q1. Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.

Difficulty Level:
Easy

What is known/given?
Quadrilaterals consist two or more than two figures.

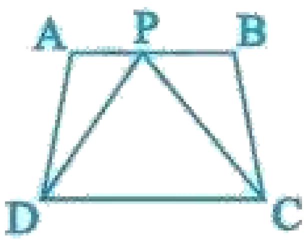
What is unknown?
Which figures having same base and are they lie between parallel lines or not.

Reasoning:
Same base means length of base must be equal of both figures and touch the line parallel to base.



Solution:

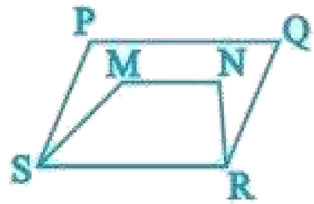
(i)



Yes, Figure (i) lie on the same base and between the same set of parallel lines.

It can be observed that trapezium ABCD and triangle PCD have a common base CD and these are lying between the same set of parallel lines AB and CD.

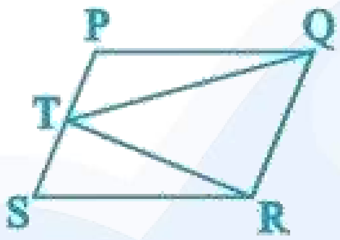
(ii)



No, Figure (ii) does not satisfy the condition of having a common base and lying between the same set of parallel lines.

It can be observed that parallelogram PQRS and trapezium MNRS have a common base RS. However, their vertices, (i.e., opposite to the common base) P, Q of parallelogram and M, N of trapezium, are not lying on the same line. Hence parallelogram PQRS and trapezium MNRS do not lie between the same set of parallel lines.

(iii)



Yes, Figure (iii) lie on the same base and between the same set of parallel lines.

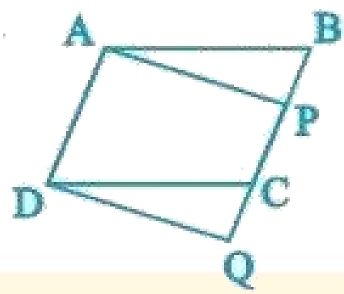
It can be observed that parallelogram PQRS and triangle TQR have a common base QR and they are lying between the same set of parallel lines PS and QR.

(iv)

No, Figure (iv) does not satisfy the condition of having a common base and lying between the same set of parallel lines.

It can be observed that parallelogram ABCD and triangle PQR are lying between the same set of parallel lines AD and BC. However, these do not have any common base.

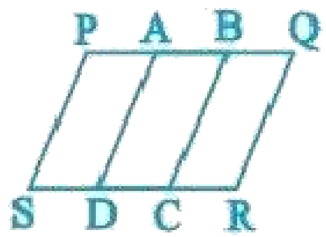
(v)



Yes, Figure (v) lie on the same base and between the same set of parallel lines.

It can be observed that parallelogram ABCD and parallelogram APQD have a common base AD and these are lying between the same set of parallel lines AD and BQ.

(vi)



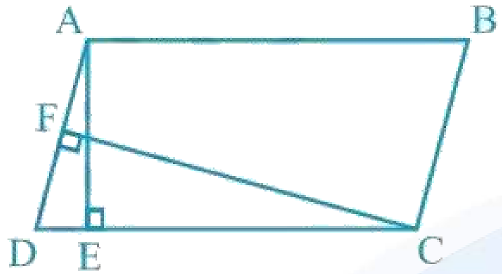
No, Figure (vi) does not satisfy the condition of having a common base and lying between the same set of parallel lines.

It can be observed that parallelogram PBCS and PQRS are lying on the same base PS. However, these do not lie between the same set of parallel lines.

Areas of Parallelograms and Triangles

Exercise 9.2 (Page 159 of Grade 9 NCERT Textbook)

Q1. In the given figure, ABCD is parallelogram, $AE \perp DC$ and $CF \perp AD$. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.



Difficulty Level:
Easy

What is known/given?
ABCD is parallelogram, $AE \perp DC$ and $CF \perp AD$. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.

What is unknown?
Length of AD.

Reasoning:
Area of a parallelogram ABCD = $CD \times AE = AD \times CF$.

Solution
Given:

AB = 16 cm
AE = 8 cm
CF = 10 cm

In parallelogram ABCD, CD = AB = 16 cm
[Opposite sides of a parallelogram are equal]

We know that
Area of a parallelogram = Base × Corresponding altitude
Therefore, Area of a parallelogram ABCD = $CD \times AE = AD \times CF$
Here CD and AD act as base and $AE \perp CD$, $CF \perp AD$ are the corresponding altitudes.

$$\therefore AD = \frac{CD \times AE}{CF}$$

Substituting the values for CD , AE and CF, we get

$$AD = \frac{16 \times 8}{10} \text{ cm} = 12.8 \text{cm}$$

Thus, the length of AD is 12.8 cm.

Q2. If E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD show that ar (EFGH) = $\frac{1}{2}$ ar (ABCD)

Difficulty Level:

Easy

What is known/given?

E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD.

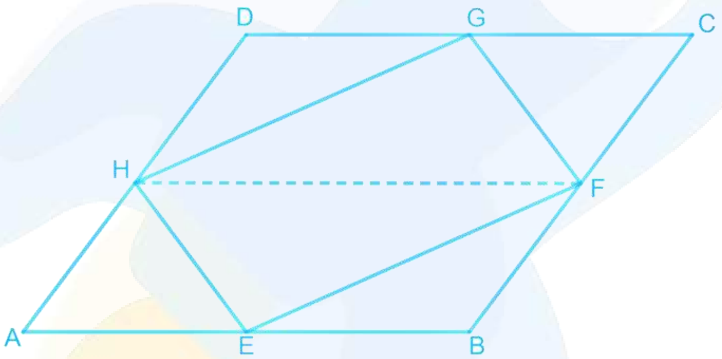
What is unknown?

How we can show that ar (EFGH) = $\frac{1}{2}$ ar (ABCD).

Reasoning:

If a triangle and parallelogram are on same base and between same parallel lines, then area of triangle will be half of area of parallelogram. We can observe in figure that triangle GHF and parallelogram DHFC are on same base and between parallel lines, Similarly, triangle EHF and Parallelogram AHFB are on same base and between parallel lines. By using these results, we can find area and add them to get the required result.

Solution



Let us join HF.

In parallelogram ABCD,

AD = BC and AD || BC (Opposite sides of a parallelogram are equal and parallel)

AB = CD (Opposite sides of a parallelogram are equal)

$\Rightarrow \frac{1}{2}$ AD = $\frac{1}{2}$ BC and AH || BF

⇒ AH = BF and AH || BF (H and F are the mid-points of AD and BC)

Therefore, ABFH is a parallelogram.

Since ΔHEF and parallelogram ABFH are on the same base HF and between the same parallel lines AB and HF,

$$\therefore \text{Area } (\Delta HEF) = \frac{1}{2} \text{Area } (ABFH) \qquad \ldots(1)$$

Similarly, it can be proved that

$$\text{Area } (\Delta HGF) = \frac{1}{2} \text{Area } (HDCF) \qquad \ldots(2)$$

On adding Equations (1) and (2), we obtain

$$\text{Area }(\Delta \text{HEF}) + \text{Area }(\Delta \text{HGF}) = \frac{1}{2}\text{ Area }(\text{ABFH}) + \frac{1}{2}\text{ Area }(\text{HDCF})$$

From the figure, we can see that :

$$\text{Area }(\Delta \text{HEF}) + \text{Area }(\Delta \text{HGF}) = \text{Area (EFGH) and --------(3)}$$

$$\frac{1}{2}\text{ Area }(\text{ABFH}) + \frac{1}{2}\text{ Area }(\text{HDCF}) = \frac{1}{2}\text{Area}(\text{ABCD}) \text{ --------(4)}$$

From (3)& (4) we have,

$$\text{Area }(\text{EFGH}) = \frac{1}{2}\text{Area}(\text{ABCD})$$

Q3. P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).

Difficulty Level:

Easy

What is known/given?

P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD.

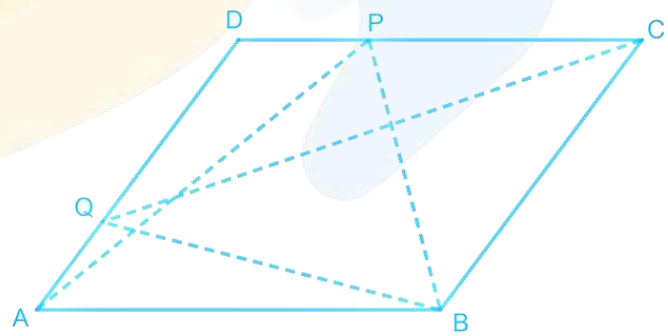
What is unknown?

How we can show that ar (APB) = ar (BQC).

Reasoning:

If a triangle and parallelogram are on same base and between same parallel lines, then area of triangle will be half of area of parallelogram. So, we can find parallelogram area by two ways and equate them to get the required result.

Solution



It can be observed that ΔBQC and parallelogram ABCD lie on the same base BC and these are between the same set of parallel lines AD and BC.

$$\therefore \text{ Area }(\Delta \text{BQC}) = \frac{1}{2}\text{ Area }(\text{ABCD}) \qquad \ldots(1)$$

Similarly, ΔAPB and parallelogram ABCD lie on the same base AB and between the same set of parallel lines AB and DC.

$$\therefore \text{Area } (\Delta APB) = \frac{1}{2} \text{ Area } (ABCD) \qquad \ldots(2)$$

From Equations (1) and (2), we obtain

$$\text{Area } (\Delta BQC) = \text{Area} (\Delta APB)$$

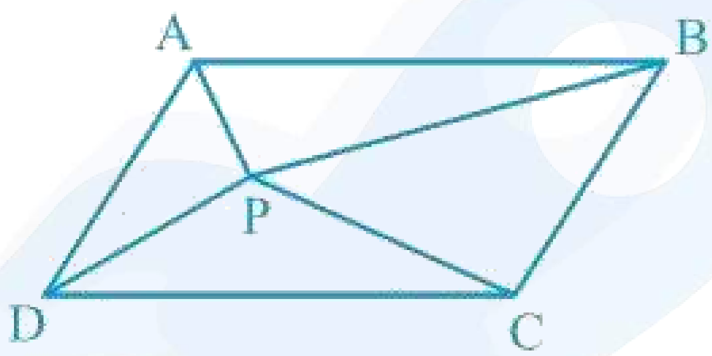
Q4. In the given figure, P is a point in the interior of a parallelogram ABCD. Show that

i) $\text{ar } (APB) + \text{ar } (PCD) = \frac{1}{2} \text{ ar } (ABCD)$

ii) $\text{ar } (APD) + \text{ar } (PBC) = \text{ar } (APB) + \text{ar } (PCD)$

[Hint: Through. P, draw a line parallel to AB]

iii)



Difficulty Level:

Medium

What is known/given?

P is a point in the interior of a parallelogram ABCD.

What is unknown?

How we can show that

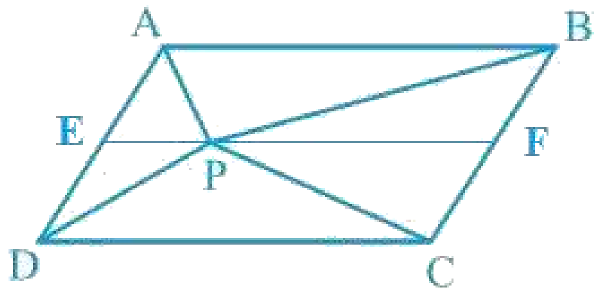
(i) $\text{ar } (APB) + \text{ar } (PCD) = \frac{1}{2} \text{ ar } (ABCD)$

(ii) $\text{ar } (APD) + \text{ar } (PBC) = \text{ar } (APB) + \text{ar } (PCD)$

Reasoning:

Draw a line parallel to AB and CD passes through point P. Now we can observe the figure, if a triangle and parallelogram are on same base and between same parallel lines then area of triangle will be half of area of parallelogram so we can find area of triangles APB and PCD to get the required result. Similarly, we can draw a line parallel to AD and BC to get the required result.

Solution



(i) Let us draw a line segment EF, passing through the point P and parallel to line segment AB in parallelogram ABCD,

$$AB \parallel EF \text{ (By construction)} \quad \dots (1)$$

We know that, ABCD is a parallelogram.

$$\therefore AD \parallel BC \quad \text{(Opposite sides of a parallelogram are parallel)}$$

$$\Rightarrow AE \parallel BF \quad \dots (2)$$

From Equations (1) and (2), we obtain

$$AB \parallel EF \text{ and } AE \parallel BF$$

Therefore, quadrilateral ABFE is a parallelogram.

Similarly, it can be deduced that quadrilateral EFCD is a parallelogram.

It can be observed that ΔAPB and parallelogram ABFE are lying on the same base AB and between the same set of parallel lines AB and EF.

$$\therefore \text{Area}(\Delta APB) = \frac{1}{2}\text{Area}(ABFE) \quad \dots (3)$$

Similarly, for ΔPCD and parallelogram EFCD,

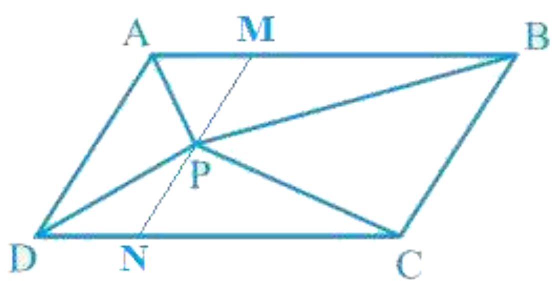
$$\text{Area}(\Delta PCD) = \frac{1}{2}\text{Area}(EFCD) \quad \dots (4)$$

Adding Equations (3) and (4), we obtain

$$\text{Area}(\Delta APB) + \text{Area}(\Delta PCD) = \frac{1}{2}\left[\text{Area}(ABFE) + \text{Area}(EFCD)\right]$$

$$\text{Area}(\Delta APB) + \text{Area}(\Delta PCD) = \frac{1}{2}\text{Area}(ABCD) \quad \dots (5)$$

(ii)



Let us draw a line segment MN, passing through point P and parallel to line segment AD.

In parallelogram ABCD,

MN || AD (By construction) ... (6)

We know that, ABCD is a parallelogram.

$\therefore$ AB || DC (Opposite sides of a parallelogram are parallel)

$\Rightarrow$ AM || DN ... (7)

From Equations (6) and (7), we obtain

MN || AD and AM || DN

Therefore, quadrilateral AMND is a parallelogram.

It can be observed that ΔAPD and parallelogram AMND are lying on the same base AD and between the same parallel lines AD and MN.

$$\therefore \text{Area }(\Delta \text{APD}) = \frac{1}{2}\text{ Area }(\text{AMND}) \quad \text{... (8)}$$

Similarly, for ΔPCB and parallelogram MNCB,

$$\text{Area }(\Delta \text{PCB}) = \frac{1}{2}\text{ Area }(\text{MNCB}) \quad \text{... (9)}$$

Adding Equations (8) and (9), we obtain,

$$\text{Area }(\Delta \text{APD}) + \text{Area }(\Delta \text{PCB}) = \frac{1}{2}\left[\text{Area }(\text{AMND}) + \text{Area }(\text{MNCB})\right]$$

$$\text{Area }(\Delta \text{APD}) + \text{Area }(\Delta \text{PCB}) = \frac{1}{2}\text{ Area }(\text{ABCD}) \quad \text{... (10)}$$

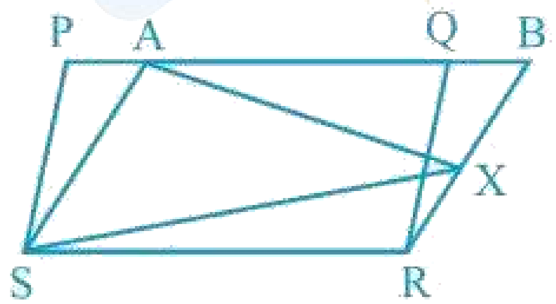
On comparing Equations (5) and (10), we obtain

$$\text{Area }(\Delta \text{APD}) + \text{Area }(\Delta \text{PBC}) = \text{Area }(\Delta \text{APB}) + \text{Area }(\Delta \text{PCD})$$

Q5. In the given figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that

i) $\text{ar }(\text{PQRS}) = \text{ar }(\text{ABRS})$

ii) $\text{ar }(\Delta AXS) = \frac{1}{2}\text{ ar }(\text{PQRS})$



Difficulty Level:

Medium

What is known/given?

PQRS and ABRS are parallelograms and X is any point on side BR.

What is unknown?

How we can show that $\text{ar}\left(\text{PQRS}\right)=\text{ar}\left(\text{ABRS}\right)$ and $\text{ar}\left(\Delta AXS\right)=\frac{1}{2}\text{ar}\left(\text{PQRS}\right)$

Reasoning:

If a triangle and parallelogram are on same base and between same parallel lines, then area of triangle will be half of area of parallelogram or if two parallelograms are on same and between two parallels lines then their area will be equal. By using these theorems, we can show the required result.

Solution

(i) It can be observed from the figure that parallelogram PQRS and ABRS lie on the same base SR and also, they lie between the same parallel lines SR and PB.

According to Theorem 9.1: **'Parallelograms on the same base and between the same parallels are equal in area.'**

$$\therefore \text{Area}\left(\text{PQRS}\right)=\text{Area}\left(\text{ABRS}\right) \quad \ldots(1)$$

(ii) Consider ΔAXS and parallelogram ABRS.

As both lie on the same base AS and between the same parallel lines AS and BR,

$$\therefore \text{Area}\left(\Delta\,\text{AXS}\right)=\frac{1}{2}\text{Area}\left(\text{ABRS}\right) \quad \ldots(2)$$

From Equations (1) and (2), we obtain

$$\text{Area}\left(\Delta\,\text{AXS}\right)=\frac{1}{2}\text{Area}\left(\text{PQRS}\right)$$

Q6. A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

Difficulty Level:

Medium

What is known/given?

A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q.

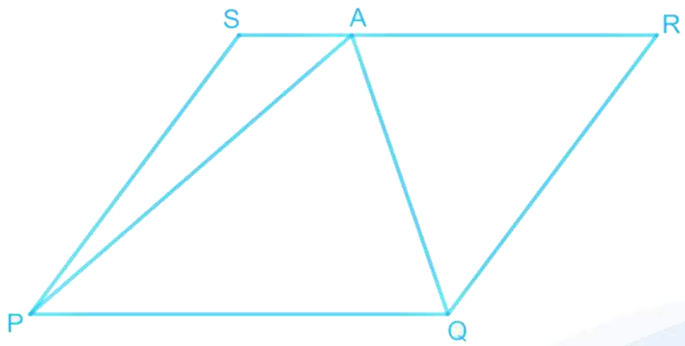
What is unknown?

In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

Reasoning:

By drawing the figure according to question, we can observe no of parts and shapes of these parts. Also, if a triangle and parallelogram are on same base and between same parallel lines then area of triangle will be half of area of parallelogram.

Solution



From the figure, it can be observed that point A divides the field into three parts. These parts are triangular in shape − ΔPSA, ΔPAQ, and ΔQRA

From the figure, we can observe that:

$$\text{Area of } \Delta PSA + \text{Area of } \Delta PAQ + \text{Area of } \Delta QRA = \text{Area of parallelogram } PQRS \quad \ldots (1)$$

We know that if a parallelogram and a triangle are on the same base and between the same set of parallel lines, then the area of the triangle is half the area of the parallelogram.

$$\therefore \text{Area}\left(\Delta PAQ\right) = \frac{1}{2}\text{Area}\left(PQRS\right) \quad \ldots (2)$$

From Equations (1) and (2), we obtain

$$\text{Area}\left(\Delta PSA\right) + \text{Area}\left(\Delta QRA\right) = \frac{1}{2}\text{Area}\left(PQRS\right) \quad \ldots (3)$$

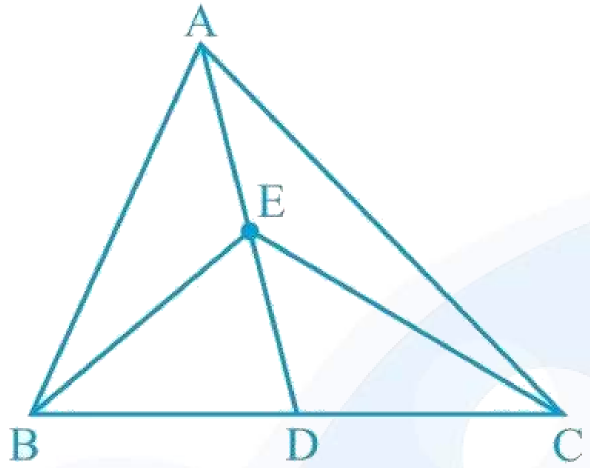
Clearly, it can be observed that the farmer must sow wheat in triangular part PAQ and pulses in other two triangular parts PSA and QRA or wheat in triangular parts PSA and QRA and pulses in triangular parts PAQ.

In this way, the farmer can sow wheat and pulses in equal portions of the field separately.

Areas of Parallelograms and Triangles

Exercise 9.3 (Page 162 of Grade 9 NCERT Textbook)

Q1. In the given figure, E is any point on median AD of a ΔABC. Show that ar (ABE) = ar (ACE)



Difficulty Level:
Medium

What is known/given?
E is any point on median AD of a ΔABC.

What is unknown?
How we can show that ar (ABE) = ar (ACE)

Reasoning:
We know that median divides a triangle in two triangles of equal areas. AD is median for triangle ABC and ED is median of triangle EBC. Using these facts, we can show the required result.

Solution
AD is the median of ΔABC. Therefore, it will divide ΔABC into two triangles of equal areas.

$$\therefore \text{ Area } (\Delta \text{ABD}) = \text{Area } (\Delta \text{ACD}) \qquad \dots (1)$$

ED is the median of ΔEBC.

$$\therefore \text{ Area } (\Delta \text{EBD}) = \text{Area } (\Delta \text{ECD}) \qquad \dots (2)$$

On subtracting Equation (2) from Equation (1), we obtain

$$\text{Area } (\Delta \text{ABD}) - \text{Area} (\text{EBD}) = \text{Area } (\Delta \text{ACD}) - \text{Area } (\Delta \text{ECD})$$

$$\text{Area } (\Delta \text{ABE}) = \text{Area } (\Delta \text{ACE})$$

Q2. In a triangle ABC, E is the mid-point of median AD.

Show that $\text{ar}(\Delta\,\text{BED}) = \frac{1}{4}\,\text{ar}(\Delta\text{ABC})$.

Difficulty Level:

Medium

What is known/given?

E is the mid-point on median AD of a $\Delta\,\text{ABC}$.

What is unknown?

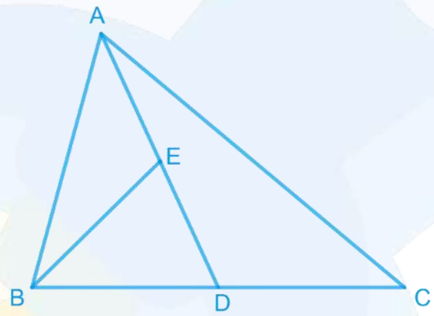
How we can show that, $\text{ar}(\Delta\,\text{BED}) = \frac{1}{4}\,\text{ar}(\Delta\,\text{ABC})$

Reasoning:

We know that median divides a triangle in two triangles of equal areas. AD is median for triangle ABC and BE is median of triangle ABD. Using these facts, we can show the required result.

Solution

Given: A $\Delta\,\text{ABC}$, AD is the median and E is the mid-point of median AD.



To prove: $\text{ar}(\Delta\,\text{BED}) = \frac{1}{4}\,\text{ar}(\Delta\,\text{ABC})$

Proof: In ΔABC, AD is the median.

$$\therefore\ \text{ar}(\Delta\text{ABD}) = \text{ar}(\Delta\text{ADC})$$

$$[\because \text{Median divides a } \Delta \text{ into two } \Delta\text{s of equal area}]$$

$$\text{ar}(\Delta\text{ABD}) = \frac{1}{2}\,\text{ar}(\text{ABC}) \qquad \ldots\text{(i)}$$

In $\Delta\,\text{ABD}$, BE is the median,

$$\text{ar}(\Delta\text{BED}) = \text{ar}(\Delta\text{BAE})$$

$$\therefore\ \text{ar}(\Delta\text{BED}) = \frac{1}{2}\,\text{ar}(\Delta\text{ABD})$$

$$\text{ar}(\Delta\text{BED}) = \frac{1}{2}\left[\frac{1}{2}\,\text{ar}(\Delta\text{ABC})\right] \qquad \text{---------- From (i)}$$

$$\therefore\ \text{ar}(\Delta\text{BED}) = \frac{1}{4}\,\text{ar}(\Delta\text{ABC})$$

Q3. Show that the diagonals of a parallelogram divide it into four triangles of equal area.

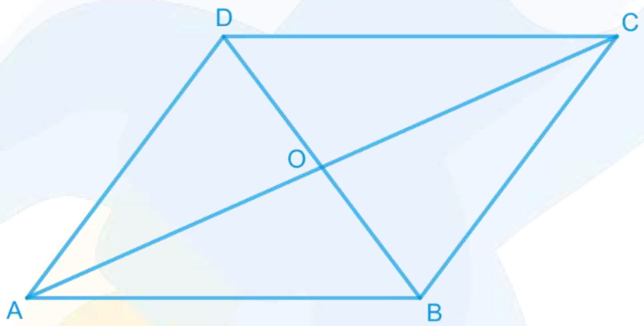
Difficulty Level:
Medium

What is known/given?
Parallelogram and its diagonals.

What is unknown?
How we can show that the diagonals of a parallelogram divide it into four triangles of equal area.

Reasoning:
We know that diagonals of a parallelogram bisect each other it means we will get mid- point for both the diagonals through which medians will be formed. Also, median divides the triangle in two triangles of equal areas. By using these observations, we can show the required result.

Solution



We know that diagonals of parallelogram bisect each other.

Therefore, O is the mid-point of AC and BD.

BO is the median in ΔABC. Therefore, it will divide it into two triangles of equal areas.

$$\therefore \text{Area}\left(\Delta\text{AOB}\right) = \text{Area}\left(\Delta\text{BOC}\right) \qquad \ldots (1)$$

In ΔBCD, CO is the median.

$$\therefore \text{Area}\left(\Delta\text{BOC}\right) = \text{Area}\left(\Delta\text{COD}\right) \qquad \ldots (2)$$

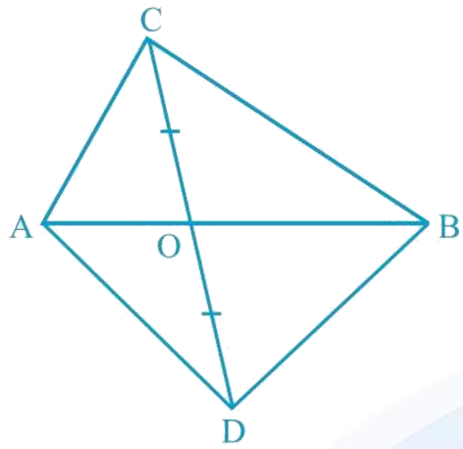
Similarly, Area $(\Delta\text{COD}) = \text{Area}\ (\Delta\text{AOD}) \qquad \ldots (3)$

From Equations (1), (2), and (3), we obtain

$$\text{Area}\left(\Delta\text{AOB}\right) = \text{Area}\left(\Delta\text{BOC}\right) = \text{Area}\left(\Delta\text{COD}\right) = \text{Area}\left(\Delta\text{AOD}\right)$$

Therefore, it is evident that the diagonals of a parallelogram divide it into four triangles of equal area.

Q4. In the given figure, ABC and ABD are two triangles on the same base AB. If line-segment CD is bisected by AB at O, show that ar (ABC) = ar (ABD).



Difficulty Level:

Medium

What is known/given?

In the given figure, ABC and ABD are two triangles on the same base AB. Line-segment CD is bisected by AB at O.

What is unknown?

How we can show that ar (ABC) = ar (ABD).

Reasoning:

CD is bisected by AB at O means O is the mid-point of CD. AO and BO are medians of triangles ADC and BDC. Also, median divides the triangle in two triangles of equal areas. By using these observations, we can show the required result.

Solution

Consider ΔACD.

Line-segment CD is bisected by AB at O. Therefore, AO is the median of ΔACD.

$$\therefore \text{Area}\left(\Delta \text{ACO}\right) = \text{Area}\left(\Delta \text{ADO}\right) \qquad \ldots(1)$$

Considering ΔBCD, BO is the median.

$$\therefore \text{Area}\left(\Delta \text{BCO}\right) = \text{Area}\left(\Delta \text{BDO}\right) \qquad \ldots(2)$$

Adding Equations (1) and (2), we obtain

$$\text{Area}\left(\Delta \text{ACO}\right) + \text{Area}\left(\Delta \text{BCO}\right) = \text{Area}\left(\Delta \text{ADO}\right) + \text{Area}\left(\Delta \text{BDO}\right)$$

$$\Rightarrow \text{Area}\left(\Delta \text{ABC}\right) = \text{Area}\left(\Delta \text{ABD}\right)$$

Q5. D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC. Show that:

(i) BDEF is a parallelogram.

(ii) $\text{ar}(\text{DEF}) = \frac{1}{4}\text{ar}(\text{ABC})$

(iii) $\text{ar}(\text{BDEF}) = \frac{1}{2}\text{ar}(\text{ABC})$

Difficulty Level:

Medium

What is known/given?

D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC

What is unknown?

How we can show that

(i) BDEF is a parallelogram.

(ii) $\text{ar}(\text{DEF}) = \frac{1}{4}\text{ar}(\text{ABC})$

(iii) $\text{ar}(\text{BDEF}) = \frac{1}{2}\text{ar}(\text{ABC})$

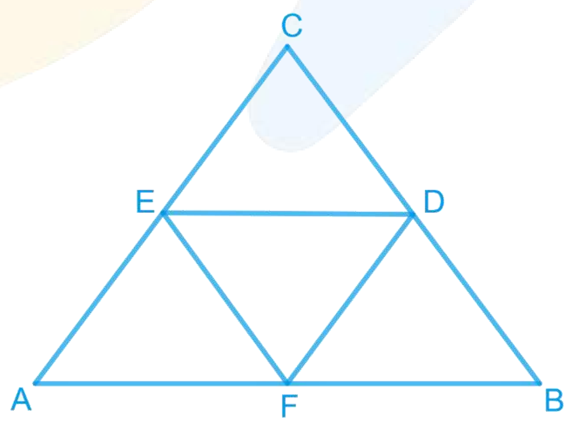
Reasoning:

Line joining the mid-points of two sides of a triangle is parallel to the third and half of its length. If one pair of opposite side in quadrilateral is parallel and equal to each other then it is a parallelogram. Diagonals divides the parallelogram in two triangles of equal areas.

Solution

(i) F is the mid-point of AB and E is the mid-point of AC.

$$\therefore \text{FE} \parallel \text{BC and FE} = \frac{1}{2}\text{ BC}$$



Line joining the mid-points of two sides of a triangle is parallel to the third and half of its length.

$$\therefore \text{FE} \parallel \text{BD} \;[\text{BD is the part of BC}]$$

Also, D is the mid-point of BC.

$$\therefore BD = \frac{1}{2} BC$$

$$FE = BD$$

Now $FE \parallel BD$ and FE = BD

Since one pair of opposite side in quadrilateral BDEF is parallel and equal to each other

Therefore, BDEF is a parallelogram.

(ii) BDEF is a parallelogram.

$$\therefore ar(\Delta BDF) = ar(\Delta DEF) \quad \ldots\ldots\ldots.(i)$$

[The diagonal DF of the parallelogram BDEF divides it in two triangles BDF and DEF of equal area]

Similarly, DCEF is also parallelogram.

$$\therefore ar\ (\Delta DEF) = ar\ (\Delta DEC) \quad \ldots\ldots\ldots(ii)$$

Also, AEDF is a parallelogram.

$$\therefore ar\ (\Delta AFE) = ar\ (\Delta DEF) \quad \ldots\ldots\ldots(iii)$$

From eq. (i), (ii) and (iii),

$$ar\ (\Delta DEF) = ar\ (\Delta BDF) = ar\ (\Delta DEC) = ar\ (\Delta AFE) \quad \ldots\ldots\ldots(iv)$$

Now,

$$ar\ (\Delta ABC) = ar\ (\Delta DEF) + ar\ (\Delta BDF) + ar\ (\Delta DEC) + ar(\Delta AFE) \quad \ldots\ldots\ldots(v)$$

$$ar\ (\Delta ABC) = ar\ (\Delta DEF) + ar\ (\Delta DEF) + ar\ (\Delta DEF) + ar\ (\Delta DEF)$$

$\left[\text{Using (iv) \& (v)}\right]$

$$ar\ (\Delta ABC) = 4 \times ar\ (\Delta DEF)$$

$$ar\ (\Delta DEF) = \frac{1}{4}\ ar\ (\Delta ABC)$$

(iii)

$$ar\ (\parallel gm\ BDEF) = ar\ (\Delta BDF) + ar\ (\Delta DEF) = ar(\Delta DEF) + ar\ (\Delta DEF) \quad \left[\text{Using (iv)}\right]$$

$$ar\ (\parallel gm\ BDEF) = 2\ ar\ (\Delta DEF)$$

$$ar\ (\parallel gm\ BDEF) = 2 \times \frac{1}{4}\ ar\ (\Delta ABC)$$

$$ar\ (\parallel gm\ BDEF) = \frac{1}{2}\ ar\ (\Delta ABC)$$

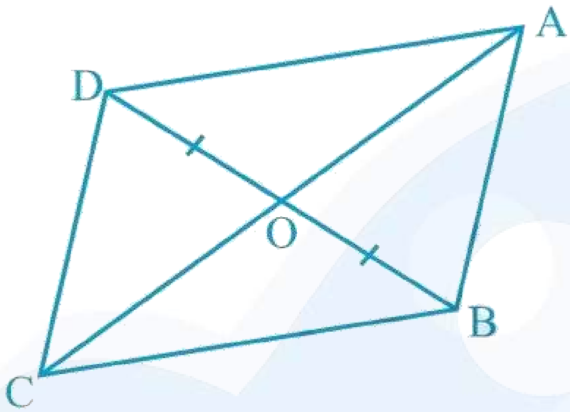
Q6. In the given figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB =OD. If AB = CD, then show that:

(i) ar (DOC) = ar (AOB)

(ii) ar (DCB) = ar (ACB)

(iii) DA || CB or ABCD is a parallelogram.

(iv) [Hint: From D and B, draw perpendiculars to AC.]



Difficulty Level:
Medium

What is known/given?
Diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD.
Also, AB = CD.

What is unknown?
How we can show that

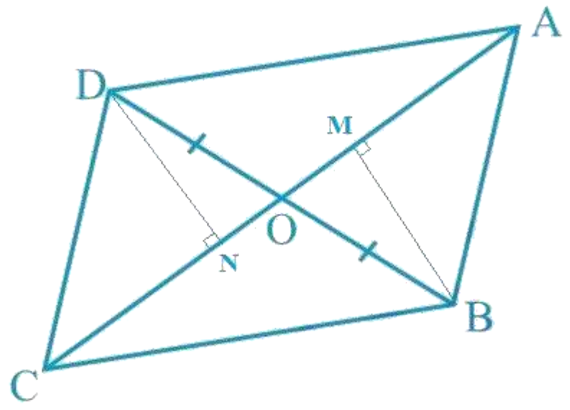
(i) ar (DOC) = ar (AOB)

(ii) ar (DCB) = ar (ACB)

(iii) DA || CB or ABCD is a parallelogram.

Reasoning:
We can draw perpendicular from vertices B and D on diagonal AC which will help us to make triangles congruent and we know that congruent triangles are always equal in areas. If two triangles have the same base and equal areas, then these will lie between the same parallels. If in a quadrilateral one pair of opposite sides is equal and the other pair of opposite sides is parallel, then it will be a parallelogram.

Solution



Let us draw $DN \perp AC$ and $BM \perp AC$.

(i) In ΔDON and ΔBOM,

$\angle DNO = \angle BMO = 90°$ (By construction)

$\angle DON = \angle BOM$ (Vertically opposite angles are equal)

$OD = OB$ (Given)

By AAS congruence rule,

$\Delta DON \cong \Delta BOM$

$DN = BM \quad ... (1)$

We know that congruent triangles have equal areas.

$\text{Area}\,(\Delta DON) = \text{Area}\,(\Delta BOM) \quad ...(2)$

In ΔDNC and ΔBMA,

$\angle DNC = \angle BMA = 90°$ (By construction of perpendicular bisectors)

$CD = AB$ (given)

$DN = BM$ [Using Equation (1)]

NOTE: RHS Congruence rule illustrates that, if the hypotenuse and one side of right-angled triangle are equal to the corresponding hypotenuse and one side of another right-angled triangle; then both the right-angled triangle are said to be congruent.

$\Delta DNC \cong \Delta BMA$ (RHS congruence rule)

$\text{Area}\,(\Delta DNC) = \text{Area}\,(\Delta BMA) \quad ... (3)$

On adding Equations (2) and (3), we obtain

$\text{Area}\,(\Delta DON) + \text{Area}\,(\Delta DNC) = \text{Area}\,(\Delta BOM) + \text{Area}\,(\Delta BMA)$

Therefore, $\text{Area}\,(\Delta DOC) = \text{Area}\,(\Delta AOB)$

(ii) We obtained,

$$\text{Area}(\Delta DOC) = \text{Area}(\Delta AOB)$$

$$\therefore \text{Area}(\Delta DOC) + \text{Area}(\Delta OCB) = \text{Area}(\Delta AOB) + \text{Area}(\Delta OCB)$$

$$\left[\text{Adding Area}(\Delta OCB) \text{ to both sides}\right]$$

$$\therefore \text{Area}(\Delta DCB) = \text{Area}(\Delta ACB)$$

(iii) We obtained,

$$\text{Area}(\Delta DCB) = \text{Area}(\Delta ACB)$$

If two triangles have the same base and equal areas, then these will lie between the same parallels.

$$\therefore DA \,||\, CB \qquad \ldots (4)$$

In quadrilateral ABCD, one pair of opposite sides is equal (AB = CD) and the other pair of opposite sides is parallel (DA || CB).

Therefore, ABCD is a parallelogram.

Q7. D and E are points on sides AB and AC respectively of ΔABC such that ar (DBC) = ar (EBC). Prove that DE || BC.

Difficulty Level:
Medium

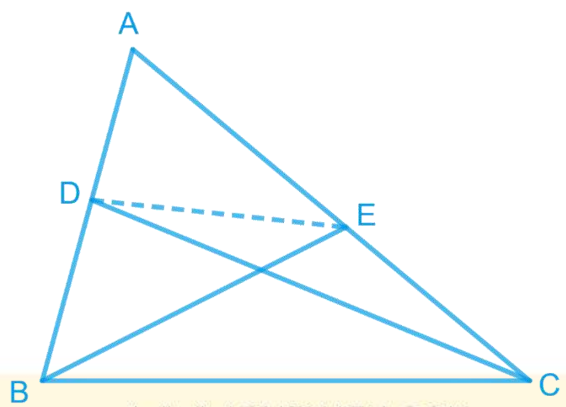
What is known/given?
D and E are points on sides AB and AC respectively of ΔABC such that

$$\text{ar}(DBC) = \text{ar}(EBC).$$

What is unknown?
How we can prove that DE || BC.

Reasoning:
If two triangles are on common base and have equal areas, then they will lie between the same parallel lines.

Solution



Since ΔEBC and ΔDBC are lying on a common base BC and also have equal areas, ΔEBC and ΔDBC will lie between the same parallel lines.

$\therefore$ DE || BC

Q8. XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that ar (ABE) = ar (ACF)

Difficulty Level:

Medium

What is known/given?

XY is a line parallel to side BC of a triangle ABC. BE || AC and CF || AB meet XY at E and F respectively.

What is unknown?

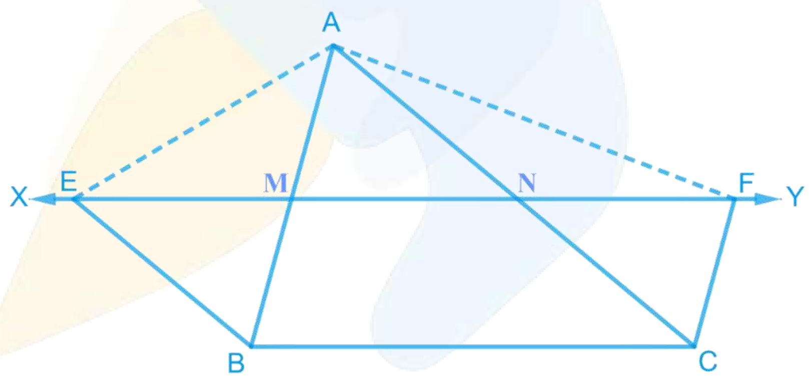
How we can show that ar (ABE) = ar (ACF).

Reasoning:

If a triangle and parallelogram are on same base and between same parallel lines, then area of triangle will be half of area of parallelogram. Also, if two quadrilaterals are on same base and between same pair of parallel lines then both will have equal area. By using these theorems, we can show the required result.

Solution

Let's draw points M and N, intersected by line XY on sides AB and AC respectively.



It is given that, XY || BC and BE || AC

Hence, EN || BC and BE || CN

Therefore, BCNE is a parallelogram.

It is given that, XY || BC and FC || AB

Hence, MF || BC and FC || MB

Therefore, BCFM is a parallelogram.

Parallelograms BCNE and BCFM are on the same base BC and between the same parallels BC and EF.

According to Theorem 9.1: **Parallelograms on the same base and between the same parallels are equal in area**.

$$\therefore \text{Area}\left(\text{BCNE}\right) = \text{Area}\left(\text{BCFM}\right) \qquad \ldots (1)$$

Consider parallelogram BCNE and ΔAEB

These lie on the same base BE and are between the same parallels BE and AC.

$$\therefore \text{Area}\left(\Delta\text{ABE}\right) = \frac{1}{2}\text{Area}\left(\text{BCNE}\right) \qquad \ldots (2)$$

Also, parallelogram ΔBCFM and ΔACF are on the same base CF and between the same parallels CF and AB.

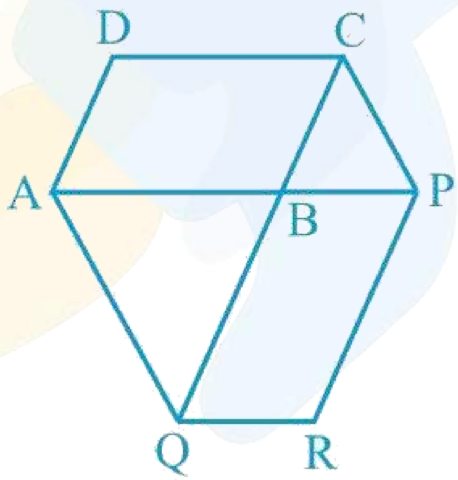
$$\therefore \text{Area}\left(\Delta\text{ACF}\right) = \frac{1}{2}\text{Area}\left(\text{BCFM}\right) \qquad \ldots (3)$$

From Equations (1), (2), and (3), we obtain

$$\text{Area} \;\; \text{Area}\left(\Delta\text{ABE}\right) = \text{Area}\left(\Delta\text{ACF}\right)$$

Q9. The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see the following figure). Show that ar (ABCD) = ar (PBQR).

[Hint: Join AC and PQ. Now compare area (ACQ) and area (APQ)]



Difficulty Level:
Hard.

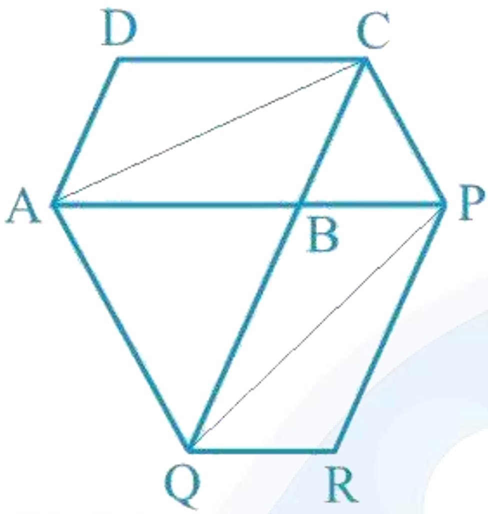
What is known/given?
The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed.

What is unknown?
How we can show that ar (ABCD) = ar (PBQR).

Reasoning:

First of all, we can join AC and PQ. Now we can use theorem for triangles ACQ and AQP if two triangles are on same base and between same pair of parallel lines then both will have equal area. Now we can subtract common area of triangle ABQ from both sides. Now we can compare the result with half of which we have to show.

Solution:



Let us join AC and PQ.

ΔACQ and ΔAQP are on the same base AQ and between the same parallels AQ and CP.

According to Theorem 9.2: **Two triangles on the same base (or equal bases) and between the same parallels are equal in area**.

$$\text{Area }(\Delta ACQ) = \text{Area }(\Delta APQ)$$

$$\text{Area }(\Delta ACQ) - \text{Area }(\Delta ABQ) = \text{Area }(\Delta APQ) - \text{Area }(\Delta ABQ)$$

Subtracting Area (ΔABQ) on both the sides.

$$\text{Area }(\Delta ABC) = \text{Area }(\Delta QBP) \qquad \dots (1)$$

Since AC and PQ are diagonals of parallelograms ABCD and PBQR respectively,

$$\text{Area }(\Delta ABC) = \frac{1}{2}\text{ Area }(ABCD) \qquad \dots (2)$$

$$\text{Area }(\Delta QBP) = \frac{1}{2}\text{ Area }(PBQR) \qquad \dots (3)$$

From Equations (1), (2), and (3), we obtain

$$\frac{1}{2}\text{ Area }(ABCD) = \frac{1}{2}\text{ Area }(PBQR)$$

$$\text{Area }(ABCD) = \text{Area }(PBQR)$$

Q10. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar (AOD) = ar (BOC).

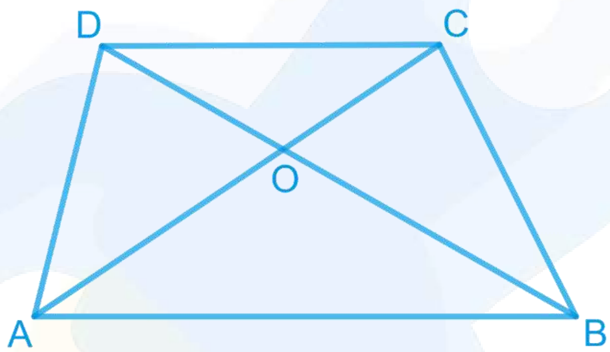
Difficulty Level:
Hard

What is known/given?
Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O.

What is unknown?
How we can prove that ar (AOD) = ar (BOC).

Reasoning:
We can use theorem for triangles DAC and DBC, if two triangles are on same base and between same pair of parallel lines then both will have equal area. Now we can subtract common area of triangle DOC from both sides to get the required result.

Solution:



It can be observed that ΔDAC and ΔDBC lie on the same base DC and between the same parallels AB and CD.

According to Theorem 9.2: **Two triangles on the same base (or equal bases) and between the same parallels are equal in area**.

$$\text{Area }(\Delta DAC) = \text{Area }(\Delta DBC)$$

$$\text{Area }(\Delta DAC) - \text{Area }(\Delta DOC) = \text{Area }(\Delta DBC) - \text{Area }(\Delta DOC)$$

Subtracting $\text{Area }(\Delta DOC)$ *from both sides*

$$\text{Area }(\Delta AOD) = \text{Area }(\Delta BOC)$$

Q11. In the given figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that

(i) ar (ACB) = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)

Difficulty Level:

Medium

What is known/given?

ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F.

What is unknown?

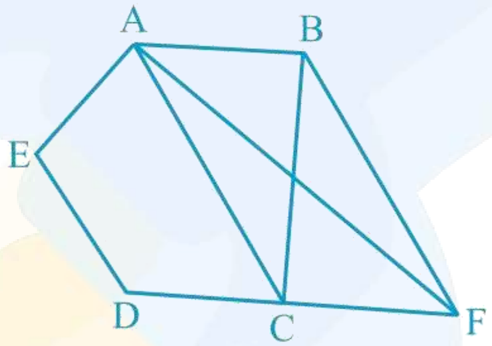
How we can show that

(i) ar (ACB) = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)

Reasoning:

We can use theorem for triangles ACB and ACF, if two triangles are on same base and between same pair of parallel lines then both will have equal area. Now we can add area of quadrilateral ACDE on both sides to get the second part required result.



Solution

(i) ΔACB and ΔACF lie on the same base AC and are between The same parallels AC and BF.

According to Theorem 9.2: **Two triangles on the same base (or equal bases) and between the same parallels are equal in area**.

$$\text{Area}\ (\Delta ACB) = \text{Area}\ (\Delta ACF)$$

(ii) It can be observed that

$$\text{Area}\ (\Delta ACB) = \text{Area}\ (\Delta ACF)$$

$$\text{Area}\ (\Delta ACB) + \text{Area}\ (ACDE) = \text{Area}\ (\Delta ACF) + \text{Area}\ (ACDE)$$

Adding Area $(ACDE)$ *on both the sides.*

$$\text{Area}\ (ABCDE) = \text{Area}\ (AEDF)$$

Q12. A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Difficulty Level:

Medium

What is known/given?

Itwaari has a plot of land of the shape of a quadrilateral **and** some portion of his plot Gram Panchayat use to construct Health Centre. **H**e should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot.

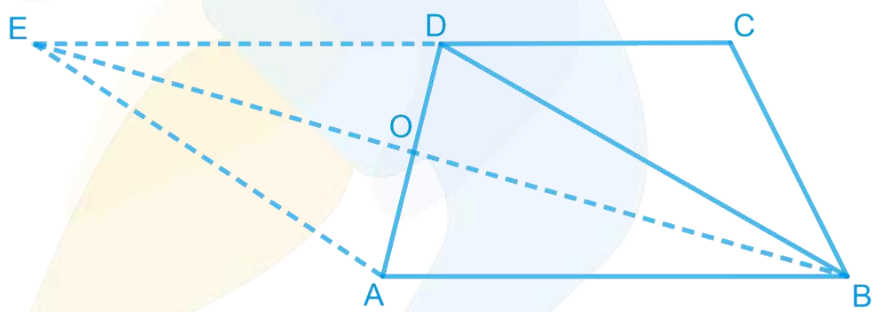
What is unknown?

How this proposal will be implemented.

Reasoning:

Join diagonal BD and draw a line parallel to BD through point A.Let it meet the extended side CD of ABCD at point E.Join BE and AD. Let them intersect each other at O. We can use theorem for triangles DEB and DAB, if two triangles are on same base and between same pair of parallel lines then both will have equal area. Now we can subtract area of triangle DOB from both sides to get the required result.

Solution



Let quadrilateral ABCD be the original shape of the field.

The proposal may be implemented as follows.

Join diagonal BD and draw a line parallel to BD through point A.

Let it meet the extended side CD of ABCD at point E.

Join BE and AD. Let them intersect each other at O.

Then, portion ΔAOB can be cut from the original field so that the new shape of the field will be ΔBCE. (See figure).

We have to prove that the area of ΔAOB (portion that was cut so as to construct Health Centre) is equal to the area of ΔDEO (portion added to the field so as to make the area of the new field so formed equal to the area of the original field).

It can be observed that ΔDEB and ΔDAB lie on the same base BD and are between the same parallels BD and AE.

According to Theorem 9.2**: Two triangles on the same base (or equal bases) and between the same parallels are equal in area**.

$$\text{Area}\left(\Delta DEB\right) = \text{Area}\left(\Delta DAB\right)$$
$$\text{Area}\left(\Delta DEB\right) - \text{Area}\left(\Delta DOB\right) = \text{Area}\left(\Delta DAB\right) - \text{Area}\left(\Delta DOB\right)$$
$$\text{Area}\left(\Delta DEO\right) = \text{Area}\left(\Delta AOB\right)$$

Q13. ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY).
[Hint: Join CX.]

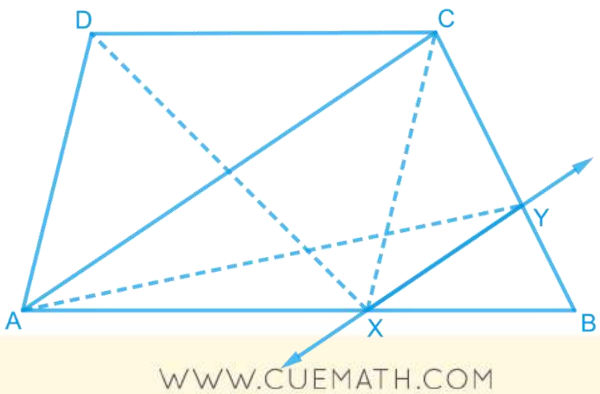
Difficulty Level:
Medium

What is known/given?
ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y.

What is unknown?
How we can prove that ar (ADX) = ar (ACY).

Reasoning:
We can use theorem for triangles ADX and ACX, if two triangles are on same base and between same pair of parallel lines then both will have equal area. Similarly for triangles we can use for triangles ACX and ACY. Now by comparing both the results, we will get the required result.

Solution:



It can be observed that ΔADX and ΔACX lie on the same base AX and are between the same parallels AB and DC.
According to Theorem 9.2: **Two triangles on the same base (or equal bases) and between the same parallels are equal in area**.

$$\text{Area}\left(\Delta ADX\right) = \text{Area}\left(\Delta ACX\right) \quad \dots (1)$$

ΔACY and ΔACX lie on the same base AC and are between the same parallels AC and XY.

According to Theorem 9.2: **Two triangles on the same base (or equal bases) and between the same parallels are equal in area**.

$$\text{Area}\left(\Delta ACY\right) = \text{Area}\left(\Delta ACX\right) \quad \dots (2)$$

From Equations (1) and (2), we obtain

$$\text{Area}\left(\Delta ADX\right) = \text{Area}\left(\Delta ACY\right)$$

Q14. In the given figure, AP || BQ || CR. Prove that ar (AQC) = ar (PBR).

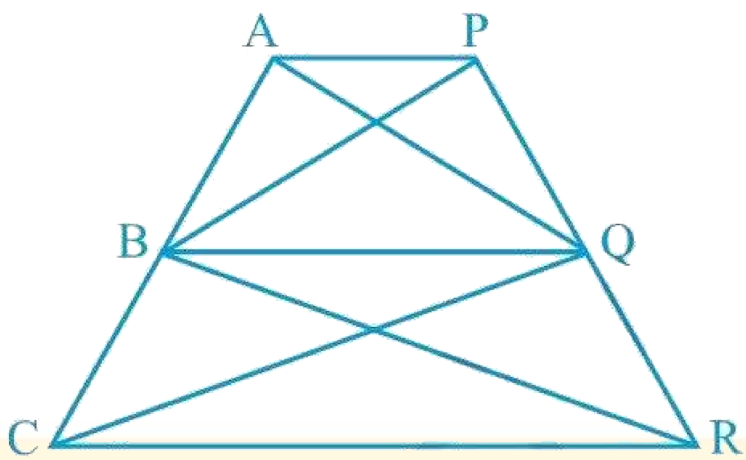
Difficulty Level:
Medium

What is known/given?
AP || BQ || CR.

What is unknown?
How we can prove that ar (AQC) = ar (PBR).

Reasoning:
We can use theorem for triangles ABQ and PBQ, if two triangles are on same base and between same pair of parallel lines then both will have equal area. Similarly for triangles we can use for triangles BCQ and BRQ. Now by adding both the results, we will get the required result.

Solution:



Since ΔABQ and ΔPBQ lie on the same base BQ and are between the same parallels AP and BQ,

According to Theorem 9.2: **Two triangles on the same base (or equal bases) and between the same parallels are equal in area**.

$$\therefore \text{Area}(\Delta ABQ) = \text{Area}(\Delta PBQ) \qquad ...(1)$$

Again, ΔBCQ and ΔBRQ lie on the same base BQ and are between the same parallels BQ and CR.

$$\therefore \text{Area}(\Delta BCQ) = \text{Area}(\Delta BRQ) \qquad ...(2)$$

On adding Equations (1) and (2), we obtain

$$\text{Area}(\Delta ABQ) + \text{Area}(\Delta BCQ) = \text{Area}(\Delta PBQ) + \text{Area}(\Delta BRQ)$$
$$\therefore \text{Area}(\Delta AQC) = \text{Area}(\Delta PBR)$$

Q15. Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.

Difficulty Level:

Medium

What is known/given?

Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC).

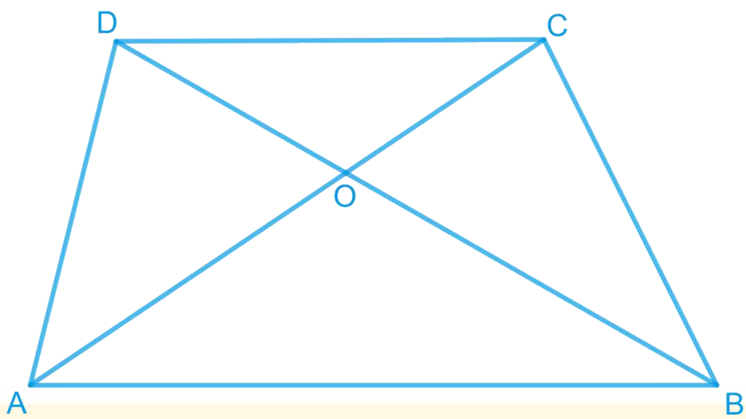
What is unknown?

How we can prove that ABCD is a trapezium.

Reasoning:

We can add area of triangle AOB on both sides in the given equal area's triangle. Now we will get two new triangles of equal areas with common base. Now we can use theorem that if Two triangles having the same base (or equal bases) and equal areas lie between the same parallels.

Solution



It is given that

$$\text{Area}\ (\Delta AOD) = \text{Area}\ (\Delta BOC)$$

$$\text{Area}\ (\Delta AOD) + \text{Area}\ (\Delta AOB) = \text{Area}\ (\Delta BOC) + \text{Area}\ (\Delta AOB)$$

Adding Area (ΔAOB) *on both the sides*

$$\text{Area}\ (\Delta ADB) = \text{Area}\ (\Delta ACB)$$

We know that triangles on the same base having areas equal to each other lie between the same parallels.

Therefore, these triangles, ΔADB and ΔACB, are lying between the same parallels.
i.e., AB || CD

According to Theorem 9.3: **Two triangles having the same base (or equal bases) and equal areas lie between the same parallels**.

Therefore, ABCD is a trapezium.

Q16. In the given figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.

Difficulty Level:
Medium

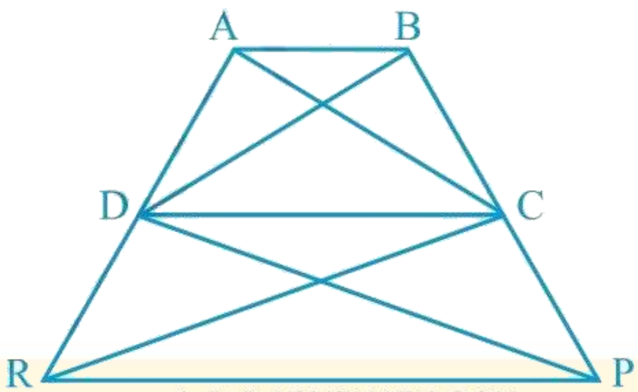
What is known/given?
ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC).

What is unknown?
How we can show that quadrilaterals ABCD and DCPR are trapeziums.

Reasoning:
We know that if Two triangles having the same base (or equal bases) and equal areas lie between the same parallels.

Solution:
It is given that

$$\text{Area}\ (\Delta DRC) = \text{Area}\ (\Delta DPC)$$



As ΔDRC and ΔDPC lie on the same base DC and have equal areas, therefore, they must lie between the same parallel lines.

According to Theorem 9.3: **Two triangles having the same base (or equal bases) and equal areas lie between the same parallels**.

$$\therefore \quad DC \parallel RP$$

Therefore, DCPR is a trapezium.

It is also given that

$$\text{Area}\left(\Delta BDP\right) = \text{Area}\left(\Delta ARC\right)$$

$$\text{Area}\left(\Delta BDP\right) - \text{Area}\left(\Delta DPC\right) = \text{Area}\left(\Delta ARC\right) - \text{Area}\left(\Delta DRC\right)$$

$$\text{Area}\left(\Delta BDC\right) = \text{Area}\left(\Delta ADC\right)$$

Since ΔBDC and ΔADC are on the same base CD and have equal areas, they must lie between the same parallel lines.

According to Theorem 9.3: **Two triangles having the same base (or equal bases) and equal areas lie between the same parallels**.

$$\therefore AB \parallel CD$$

Therefore, ABCD is a trapezium.

Areas of Parallelograms and Triangles

Exercise- 9.4 (Page 164 of Grade 9 NCERT Textbook)

Q1. Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

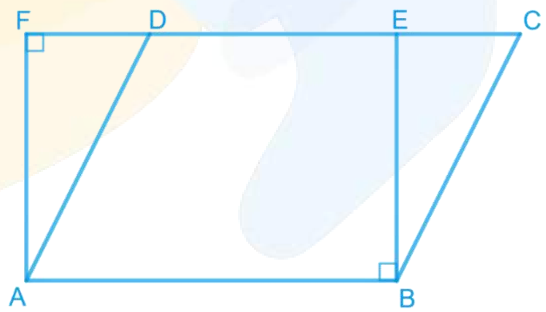
Difficulty Level:
Medium

What is known/given?
Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas.

What is unknown?
How we can show that the perimeter of the parallelogram is greater than that of the rectangle.

Reasoning:
When we compare the sides of rectangle and parallelogram which are on same base, we can see two sides of parallelogram are opposite to 90 degree which shows that these two are longer than rectangle two sides and as we find the perimeter of both quadrilateral, we will get the required result.

Solution:
As the parallelogram and the rectangle have the same base and equal area, therefore, these will also lie between the same parallels.
Consider the parallelogram ABCD and rectangle ABEF as follows.



Here, it can be observed that parallelogram ABCD and rectangle ABEF are between the same parallels AB and CF.

We know that opposite sides of a parallelogram or a rectangle are of equal lengths.
Therefore,

$$AB = EF \ (\text{For rectangle})$$
$$AB = CD \ (\text{For parallelogram})$$
$$\therefore CD = EF$$
$$\therefore AB + CD = AB + EF \qquad ...(1)$$

Of all the line segments that can be drawn to a given line from a point not lying on it, the perpendicular line segment is the shortest.

$$\therefore AF < AD$$

And similarly, $BE < BC$

$$\therefore AF + BE < AD + BC \quad ...(2)$$

From Equations (1) and (2), we obtain

$$AB + EF + AF + BE < AD + BC + AB + CD$$

Perimeter of rectangle ABEF < Perimeter of parallelogram ABCD.

Q2. In the following figure, D and E are two points on BC such that $BD = DE = EC$. Show that $\text{ar}(ABD) = \text{ar}(ADE) = \text{ar}(AEC)$.

Difficulty Level:

Medium

What is known/given?

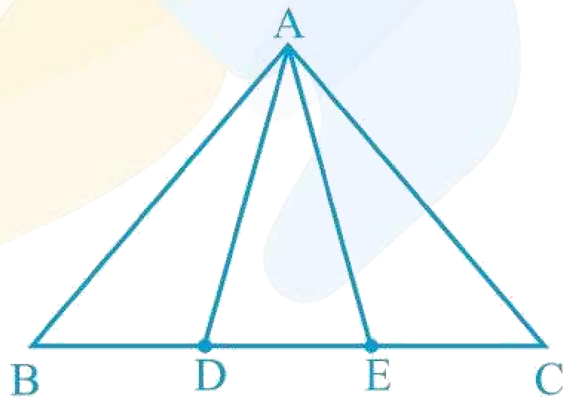
D and E are two points on BC such that $BD = DE = EC$.

What is unknown?

How we can show that $\text{ar}(ABD) = \text{ar}(ADE) = \text{ar}(AEC)$

Reasoning:

First of all, we can draw altitude which will help to find areas of all three triangles. As bases of all three triangles are equal so we can replace the value and we will find the required result.

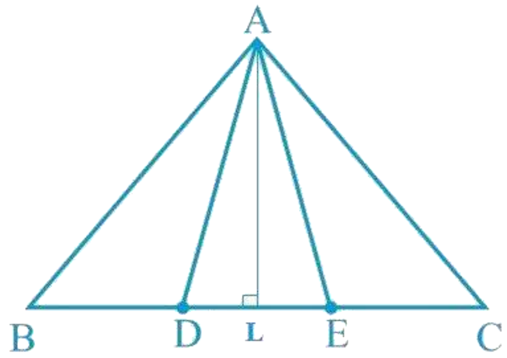


Can you now answer the question that you have left in the 'Introduction' of this chapter, whether the field of Budhia has been actually divided into three parts of equal area?

[*Remark*: Note that by taking $BD = DE = EC$, the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into *n* equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide ΔABC into *n* triangles of equal areas.]

Solution:

Let us draw a line segment $AL \perp BC$.



We know that,

$$\text{Area of a triangle} = \frac{1}{2} \times \text{Base} \times \text{Altitude}$$

$$\text{Area}\left(\Delta ADE\right) = \frac{1}{2} \times DE \times AL$$

$$\text{Area}\left(\Delta ABD\right) = \frac{1}{2} \times BD \times AL$$

$$\text{Area}\left(\Delta AEC\right) = \frac{1}{2} \times EC \times AL$$

It is given that $DE = BD = EC$

$$\frac{1}{2} \times DE \times AL = \frac{1}{2} \times BD \times AL = \frac{1}{2} \times EC \times AL$$

$$\text{Area}\left(\Delta ADE\right) = \text{Area}\left(\Delta ABD\right) = \text{Area}\left(\Delta AEC\right)$$

It can be observed that *Budhia* has divided her field into 3 equal parts.

Q3. In the following figure, ABCD, DCFE and ABFE are parallelograms. Show that $\text{ar}\left(\Delta ADE\right) = \text{ar}\left(\Delta BCF\right)$.

Difficulty Level:

Medium

What is known/given?

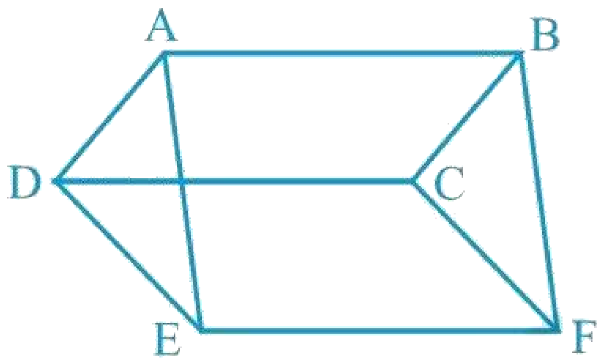
ABCD, DCFE and ABFE are parallelograms.

What is unknown?

How we can show that $\text{ar}\left(\Delta ADE\right) = \text{ar}\left(\Delta BCF\right)$.

Reasoning:

We can see that sides of triangles ADE and BCF are also the opposite sides of the given parallelogram. Now we can show both the triangles congruent by SSS congruency.
We know that congruent triangles have equal areas.



Solution 3:

It is given that ABCD is a parallelogram. We know that opposite sides of a parallelogram are equal.

$$\therefore AD = BC \ldots (1)$$

Similarly, for parallelograms DCFE and ABFE, it can be proved that

$$DE = CF \ldots (2)$$

$$\text{And, } EA = FB \ldots (3)$$

In ΔADE and ΔBCF,

$$AD = BC \ [\text{Using equation } (1)]$$

$$DE = CF \ [\text{Using equation } (2)]$$

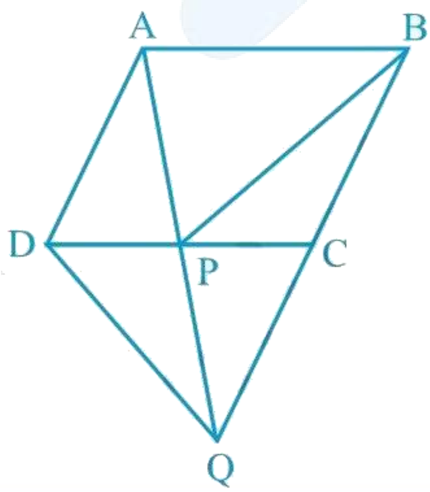
$$EA = FB \ [\text{Using equation } (3)]$$

$$\therefore \Delta ADE \cong \Delta BCF \ (\text{SSS congruence rule})$$

The SSS rule states that: If three sides of one triangle are equal to three sides of another triangle, then the triangles are congruent.

$$\therefore \text{Area } (\Delta ADE) = \text{Area } (\Delta BCF)$$

Q4. In the following figure, ABCD is parallelogram and BC is produced to a point Q such that $AD = CQ$. If AQ intersect DC at P, show that $ar(\Delta BPC) = ar(\Delta DPQ)$.



[*Hint:* Join AC.]

Difficulty Level:
Medium

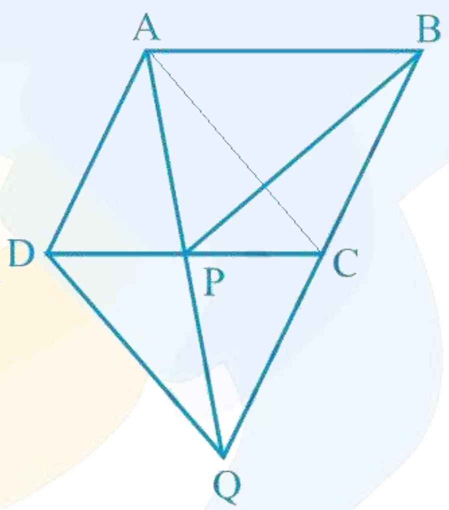
What is known/given?
ABCD is parallelogram and BC is produced to a point Q such that AD = CQ.
AQ intersect DC at P.

What is unknown?
How we can show that $ar(\Delta BPC) = ar(\Delta DPQ)$.

Reasoning:
First of all, join AC. Now we can see that triangles APC and BCP are on same base and between parallel lines so areas will be equal. Similarly, triangles DQC and ACQ areas are equal and now subtract common area of triangle QPC from both sides. Compare the result with first pair of triangles areas to get the required result.

Solution:

It is given that ABCD is a parallelogram.
$AD \parallel BC$ and $AB \parallel DC$ (Opposite sides of a parallelogram are parallel to each other)
Join point A to point C.



Consider ΔAPC and ΔBPC
ΔAPC and ΔBPC are lying on the same base PC and between the same parallels PC and AB.

According to Theorem 9.2: **Two triangles on the same base (or equal bases) and between the same parallels are equal in area**.

Therefore,

$$\text{Area } (\Delta APC) = \text{Area } (\Delta BPC) \quad ...(1)$$

In quadrilateral ACDQ, it is given that AD = CQ

Since ABCD is a parallelogram,
$AD \parallel BC$ (Opposite sides of a parallelogram are parallel)
CQ is a line segment which is obtained when line segment BC is produced.

$$\therefore AD \parallel CQ$$

We have,
$AD = CQ$ and $AD \parallel CQ$

Hence, ACQD is a parallelogram.

Consider ΔDCQ and ΔACQ

These are on the same base CQ and between the same parallels CQ and AD.

Therefore,

$$\text{Area }(\Delta DCQ) = \text{Area }(\Delta ACQ)$$

$$\therefore \text{Area }(\Delta DCQ) - \text{Area }(\Delta PQC) = \text{Area }(\Delta ACQ) - \text{Area }(\Delta PQC)$$

Subtracting Area (ΔPQC) on both the sides.

$$\therefore \text{Area }(\Delta DPQ) = \text{Area }(\Delta APC) \quad ...(2)$$

From equations (1) and (2), we obtain
$\text{Area }(\Delta BPC) = \text{Area}(\Delta DPQ)$

Q5. In the following figure, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that

(i) $\text{ar }(BDE) = \frac{1}{4} \text{ar}(ABC)$

(ii) $\text{ar }(BDE) = \frac{1}{2} \text{ar}(BAE)$

(iii) $\text{ar}(ABC) = 2 \text{ ar }(BEC)$

(iv) $\text{ar }(BFE) = \text{ar }(AFD)$

(v) $\text{ar }(BFE) = 2 \text{ ar }(FED)$

(vi) $\text{ar }(FED) = \frac{1}{8} \text{ar}(AFC)$

Difficulty Level:
Hard

What is known/given?
ABC and BDE are two equilateral triangles such that D is the mid-point of BC. AE intersects BC at F.

What is unknown?

How we can show that

(i) $\text{ar}(BDE) = \frac{1}{4}\text{ar}(ABC)$

(ii) $\text{ar}(BDE) = \frac{1}{2}\text{ar}(BAE)$

(iii) $\text{ar}(ABC) = 2\,\text{ar}(BEC)$

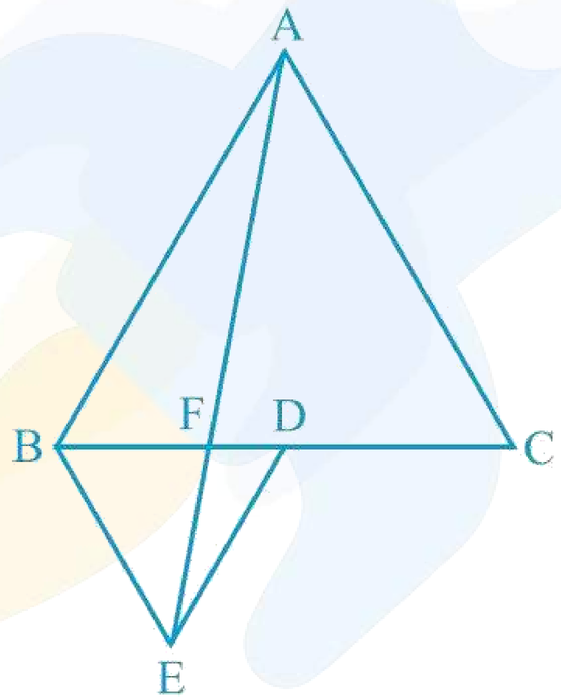
(iv) $\text{ar}(BFE) = \text{ar}(AFD)$

(v) $\text{ar}(BFE) = 2\,\text{ar}(FED)$

(vi) $\text{ar}(FED) = \frac{1}{8}\text{ar}(AFC)$

Reasoning:

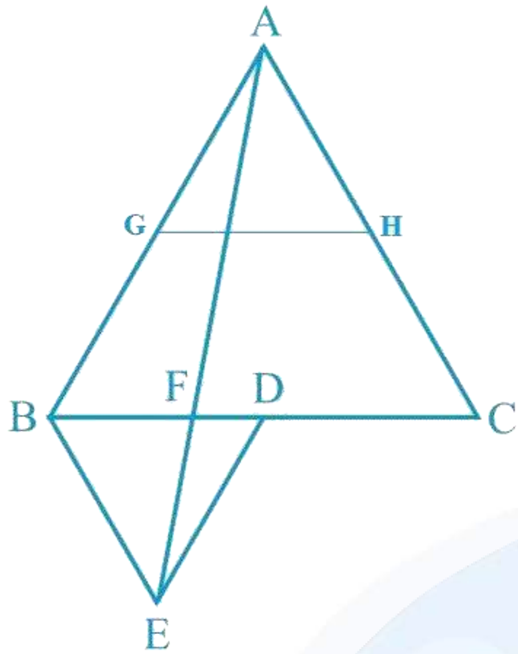
For the first part let G and H be the mid-points of side AB and AC and now we can use mid-point theorem. For remaining parts join vertices E to C and A to D. Now we use the facts that median divides a triangle area in to two triangles of equal areas, If two triangles are on same base and between same pair of parallel lines then areas will be equal of both the triangles.



[Hint: Join EC and AD. Show that $BE \parallel AC$ and $DE \parallel AB$ etc.]

Solution:

(i) Let G and H be the mid-points of side AB and AC respectively.
Line segment GH is joining the mid-points and is parallel to third side.
Therefore, GH will be half of the length of BC (mid-point theorem).



$\therefore GH = \frac{1}{2} BC$ and $GH \parallel BD$

$\therefore GH = BD = DC$ and $GH \parallel BD$ $\left(\text{D is the mid-point of BC}\right)$

Similarly,

- GD = HC = HA
- HD = AG = BG

Therefore, clearly ΔABC is divided into 4 equal equilateral triangles viz $\Delta BGD, \Delta AGH, \Delta DHC$ and ΔGHD

In other words, $\Delta BGD = \frac{1}{4}\ \Delta ABC$

Now consider ΔBDG and ΔBDE

BD = BD (Common base)

As both triangles are equilateral triangle, we can say

BG = BE

DG = DE

Therefore, $\Delta BDG \cong \Delta BDE$ [By SSS congruency]

The SSS rule states that: If three sides of one triangle are equal to three sides of another triangle, then the triangles are congruent.

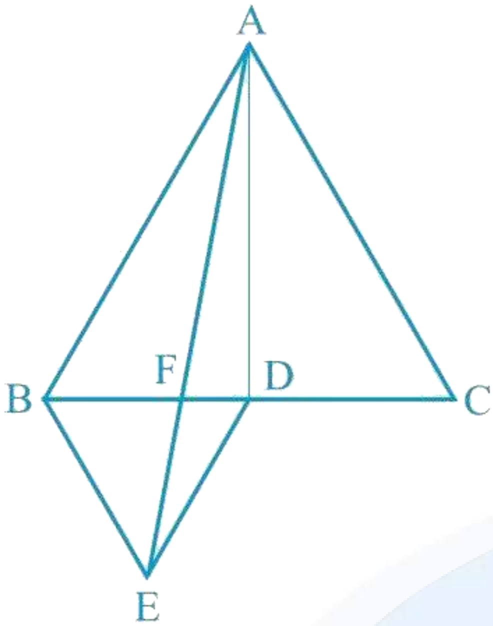
Thus,

$$\text{area } (\Delta BDG) = \text{area } (\Delta BDE)$$

$$\text{ar } (\Delta BDE) = \frac{1}{4} \text{ ar}(\Delta ABC)$$

Hence proved

(ii)



$$\text{Area }(\Delta BDE) = \text{Area }(\Delta AED)$$

$$\left(\textit{Common base } DE \textit{ and } DE \parallel AB\right)$$

$$\text{Area }(\Delta BDE) - \text{Area }(\Delta FED) = \text{Area }(\Delta AED) - \text{Area }(\Delta FED)$$

$$\text{Area }(\Delta BEF) = \text{Area }(\Delta AFD) \;\ldots\; (1)$$

Now

$$\text{Area }(\Delta ABD) = \text{Area }(\Delta ABF) + \text{Area }(\Delta AFD)$$

$$\text{Area }(\Delta ABD) = \text{Area }(\Delta ABF) + \text{Area }(\Delta BEF) \quad [\text{From equation }(1)]$$

$$\text{Area }(\Delta ABD) = \text{Area }(\Delta ABE) \qquad \ldots (2)$$

AD is the median in ΔABC.

$$ar(\Delta ABD) = \frac{1}{2} ar(\Delta ABC)$$

$$= \frac{4}{2} ar(\Delta BDE) \qquad \text{(As proved earlier)}$$

$$ar(\Delta ABD) = 2\, ar(\Delta BDE) \qquad (3)$$

From (2) and (3), we obtain,

$$2\, ar(\Delta BDE) = ar(\Delta ABE)$$

$$ar(BDE) = \frac{1}{2} ar(BAE)$$

(iii)

$$\text{ar}(\Delta ABE) = \text{ar}(\Delta BEC) \qquad (\textit{Common base } BE \textit{ and } BE \parallel AC)$$

$$\text{ar}(\Delta ABF) + \text{ar}(\Delta BEF) = \text{ar}(\Delta BEC)$$

Using equation (1), we obtain,

$$\text{ar}(\Delta ABF) + \text{ar}(\Delta AFD) = \text{ar}(\Delta BEC)$$

$$\text{ar}(\Delta ABD) = \text{ar}(\Delta BEC)$$

$$\frac{1}{2}\text{ar}(\Delta ABC) = \text{ar}(\Delta BEC)$$

$$\text{ar}(\Delta ABC) = 2\,\text{ar}(\Delta BEC)$$

(iv) It is seen that ΔBDE and ar ΔAED lie on the same base (DE) and between the parallels DE and AB.

$$\text{ar}(\Delta BDE) = \text{ar}(\Delta AED)$$

$$\text{ar}(\Delta BDE) - \text{ar}(\Delta FED) = \text{ar}(\Delta AED) - \text{ar}(\Delta FED)$$

Subtracting $\text{ar}(\Delta FED)$ *on both the sides*

$$\text{ar}(\Delta BFE) = \text{ar}(\Delta AFD)$$

(v) Let h be the height of vertex E, corresponding to the side BD in ΔBDE.
Let H be the height of vertex A, corresponding to the side BC in ΔABC.

In (i), it was shown that $\text{ar}(BDE) = \frac{1}{4}\text{ar}(ABC)$

In (iv), it was shown that $\text{ar}(\Delta BFE) = \text{ar}(\Delta AFD)$.

$$\therefore \ \text{ar}(\Delta BFE) = \text{ar}(\Delta AFD)$$
$$= 2\,\text{ar}(\Delta FED)$$

Hence proved.

(vi)

$$\text{ar}(\Delta AFC) = \text{ar}(\Delta AFD) + \text{ar}(\Delta ADC)$$

$$= 2\,\text{ar}(\Delta FED) + \frac{1}{2}\text{ar}(\Delta ABC) \qquad [\textit{using } (\text{v})]$$

$$= 2\,\text{ar}(\Delta FED) + \frac{1}{2}[4 \times \text{ar}(\Delta BDE)] \qquad [\textit{Using result of part } (\text{i})]$$

$$= 2\,\text{ar}(\Delta FED) + 2\,\text{ar}(\Delta BDE)$$

$$= 2\,\text{ar}(\Delta FED) + 2\,\text{ar}(\Delta AED)$$

$[\Delta BDE \text{ and } \Delta AED \textit{ are on the same base and between same parallels}]$

$$= 2\,\text{ar}(\Delta FED) + 2[\text{ar}(\Delta AFD) + \text{ar}(\Delta FED)]$$

$$= 2\,\text{ar}(\Delta FED) + 2\,\text{ar}(\Delta AFD) + 2\,\text{ar}(\Delta FED) \qquad [\textit{Using } (\text{viii})]$$

$$= 4\,\text{ar}(\Delta FED) + 4\,\text{ar}(\Delta FED)$$

$$\Rightarrow \text{ar}(\Delta AFC) = 8\,\text{ar}(\Delta FED)$$

$$\Rightarrow \text{ar}(\Delta FED) = \frac{1}{8}\,\text{ar}(\Delta AFC)$$

Q6. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that; $\text{ar}(\Delta APB) \times \text{ar}(\Delta CPD) = \text{ar}(\Delta APD) \times \text{ar}(\Delta BPC)$

[Hint: From A and C, draw perpendiculars to BD]

Difficulty Level:

Medium

What is known/given?

A quadrilateral ABCD, in which diagonals AC and BD intersect each other at point P.

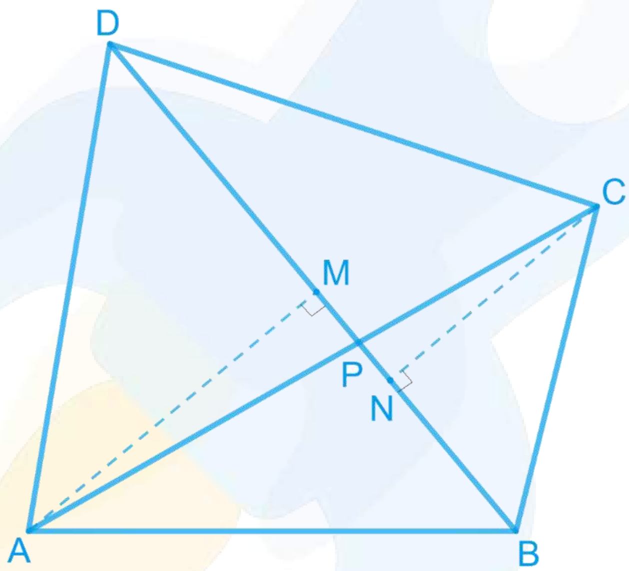
What is unknown?

To Prove:

$$\text{ar}(\Delta APB) \times \text{ar}(\Delta CPD) = \text{ar}(\Delta APD) \times \text{ar}(\Delta BPC)$$

Reasoning:

From A, draw $AM \perp BD$ and from C, draw $CN \perp BD$. Now we can find area of triangles to get the required result.



Solution:

Proof:

$$\text{ar}(\Delta ABP) = \frac{1}{2} \times PB \times AM \quad \text{................(i)}$$

$$\text{ar}(\Delta APD) = \frac{1}{2} \times PD \times AM \quad \text{..................(ii)}$$

Dividing eq. (ii) by (i), we get,

$$\frac{\text{ar}(\Delta APD)}{\text{ar}(\Delta ABP)} = \frac{\frac{1}{2} \times PD \times AM}{\frac{1}{2} \times PB \times AM}$$

$$\Rightarrow \frac{\text{ar}(\Delta APD)}{\text{ar}(\Delta ABP)} = \frac{PD}{PB} \quad \text{.......................(iii)}$$

Similarly,

$$\frac{ar\ (\Delta CDP)}{ar\ (\Delta BPC)} = \frac{PD}{PB} \qquad \text{...............(iv)}$$

From eq. (iii) and (iv), we get

$$\frac{ar\ (\Delta APD)}{ar\ (\Delta ABP)} = \frac{ar\ (\Delta CDP)}{ar\ (\Delta BPC)}$$

$$\Rightarrow ar\ (\Delta APD) \times ar\ (\Delta BPC) = ar\ (\Delta ABP) \times ar\ (\Delta CDP)$$

Hence proved.

Q7. P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that

(i) $ar\ (PRQ) = \frac{1}{2}\ ar\ (ARC)$

(ii) $ar\ (RQC) = \frac{3}{8}\ ar\ (ABC)$

(ii) $ar\ (PBQ) = ar\ (ARC)$

Difficulty Level:

Hard

What is known/given?

P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP.

What is unknown?

How we can show that

(i) $ar\ (PRQ) = \frac{1}{2}\ ar(ARC)$

(ii) $ar\ (RQC) = \frac{3}{8}\ ar(ABC)$

(ii) $ar\ (PBQ) = ar(ARC)$

Reasoning:

Median divides the triangle into two triangles of equal area.

Solution:

(i)

PC is the median of ΔABC.

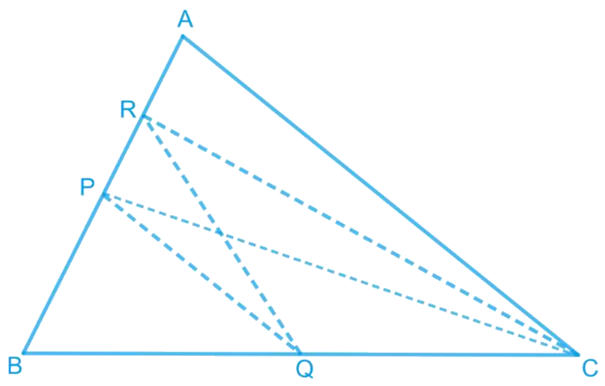
$\therefore ar\left(\Delta BPC\right) = ar\left(\Delta APC\right)$ ....(i)

RC is the median Δ of APC.

$\therefore ar\ \left(\Delta ARC\right) = \frac{1}{2} ar\left(\Delta APC\right)$..........(ii)

[Median divides the triangle into two triangles of equal area]

PQ is the median of ΔBPC.



$$\therefore \text{ar}\left(\Delta PQC\right)=\frac{1}{2}\text{ar}\left(\Delta BPC\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

From eq. (i) and (iii), we get,

$$\text{ar}\left(\Delta PQC\right)=\frac{1}{2}\text{ar}\left(\Delta APC\right) \qquad \ldots\ldots\ldots\ldots \text{(iv)}$$

From eq. (ii) and (iv), we get,

$$\text{ar}\left(\Delta PQC\right)=\text{ar}\left(\Delta ARC\right) \qquad \ldots\ldots\ldots\ldots \left(\text{v}\right)$$

We are given that P and Q are the mid-points of AB and BC respectively.

$$\therefore PQ \parallel AC \text{ and } PQ=\frac{1}{2}AC$$

$$\Rightarrow \text{ar}\left(\Delta APQ\right)=\text{ar}\left(\Delta PQC\right) \qquad \ldots\ldots\ldots\ldots \left(\text{vi}\right) \qquad \left[\begin{array}{l}\textit{triangles between same}\\ \textit{parallel are equal in area}\end{array}\right]$$

From eq. (v) and (vi), we get

$$\text{ar}\left(\Delta APQ\right)=\text{ar}\left(\Delta ARC\right) \qquad \ldots\ldots\ldots\ldots \left(\text{vii}\right)$$

R is the mid-point of AP. Therefore, RQ is the median of ΔAPQ.

$$\therefore \text{ar}\left(\Delta PRQ\right)=\frac{1}{2}\text{ar}\left(\Delta APQ\right) \qquad \ldots\ldots\ldots\ldots \text{(viii)}$$

From (vii) and (viii), we get,

$$\therefore \text{ar}\left(\Delta PRQ\right)=\frac{1}{2}\text{ar}\left(\Delta ARC\right)$$

(ii) PQ is the median of ΔBPC.

$$\therefore \text{ar}\left(\Delta PQC\right)=\frac{1}{2}\text{ar}\left(\Delta BPC\right)=\frac{1}{2}\times\frac{1}{2}\text{ar}\left(\Delta ABC\right) \qquad \ldots\ldots\ldots\ldots \text{(ix)}$$

Also,

$$\text{ar}\left(\Delta PRC\right)=\frac{1}{2}\text{ar}\left(\Delta APC\right) \qquad \left[\textit{Using } \text{(iv)}\right]$$

$$\Rightarrow \text{ar}\left(\Delta PRC\right)=\frac{1}{2}\times\frac{1}{2}\text{ar}\left(\Delta ABC\right)$$

$$=\frac{1}{4}\text{ar}\left(\Delta ABC\right) \qquad \ldots\ldots\ldots\ldots \text{(x)}$$

Adding eq. (ix) and (x), we get,

$$\text{ar}(\Delta PQC) + \text{ar}(\Delta PRC) = \left(\frac{1}{4} + \frac{1}{4}\right)\text{ar}(\Delta ABC)$$

$$\Rightarrow \text{ar}(\text{quad.}PQCR) = \frac{1}{2}\text{ar}(\Delta ABC) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (\text{xi})$$

Subtracting ar (ΔPRQ) from the both sides,

$$\text{ar (quad. PQCR)} - \text{ar}(\Delta PRQ) = \frac{1}{2}\text{ar}(\Delta ABC) - \text{ar}(\Delta PRQ)$$

$$\Rightarrow \text{ar}(\Delta RQC) = \frac{1}{2}\text{ar}(\Delta ABC) - \frac{1}{2}\text{ar}(\Delta ARC) \qquad [\textit{Using result } (\text{i})]$$

$$\Rightarrow \text{ar}(\Delta ARC) = \frac{1}{2}\text{ar}(\Delta ABC) - \frac{1}{2}\times\frac{1}{2}\text{ar}(\Delta APC)$$

$$\Rightarrow \text{ar}(\Delta RQC) = \frac{1}{2}\text{ar}(\Delta ABC) - \frac{1}{4}\text{ar}(\Delta APC)$$

$$\Rightarrow \text{ar}(\Delta RQC) = \frac{1}{2}\text{ar}(\Delta ABC) - \frac{1}{4}\times\frac{1}{2}\text{ar}(\Delta ABC) \qquad [\text{PC } \textit{is median of } \Delta ABC]$$

$$\Rightarrow \text{ar}(\Delta RQC) = \frac{1}{2}\text{ar}(\Delta ABC) - \frac{1}{8}\text{ar}(\Delta ABC)$$

$$\Rightarrow \text{ar}(\Delta RQC) = \left(\frac{1}{2} - \frac{1}{8}\right)\times\text{ar}(\Delta ABC)$$

$$\Rightarrow \text{ar}(\Delta RQC) = \frac{3}{8}\times\text{ar}(\Delta ABC)$$

(iii)

$$\text{ar}(\Delta PRQ) = \frac{1}{2}\text{ar}(\Delta ARC) \qquad [\text{Using result (i)}]$$

$$\Rightarrow 2\text{ar}(\Delta PRQ) = \text{ar}(\Delta ARC) \qquad \ldots\ldots\ldots\ldots \quad (\text{xii})$$

$$\text{ar}(\Delta PRQ) = \frac{1}{2}\text{ar}(\Delta APQ) \qquad [\text{RQ is the medium of } \Delta APQ] \qquad \ldots\ldots\ldots\ldots\ldots \quad (\text{xiii})$$

$$\text{But ar}(\Delta APQ) = \text{ar}(\Delta PQC) \qquad [\text{Using reason of eq. (vi)}] \qquad \ldots\ldots\ldots\ldots \quad (\text{xiv})$$

From eq. (xiii) and (xiv), we get,

$$\text{ar}(\Delta PRQ) = \frac{1}{2}\text{ar}(\Delta PRQ) \qquad \ldots\ldots\ldots\ldots \quad (\text{xv})$$

But,

$$\text{ar}(\Delta BPQ) = \text{ar}(\Delta PQC) \qquad [\text{PQ is the median of } \Delta BPC] \ldots\ldots\ldots\ldots \quad (\text{xvi})$$

From eq. (xv) and (xvi), we get,

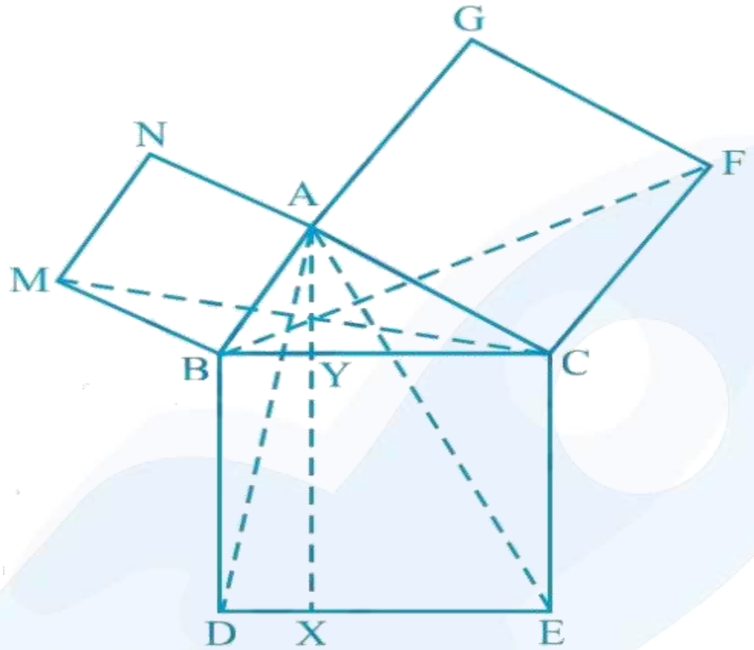
$$\text{ar}(\Delta PRQ) = \frac{1}{2}\text{ar}(\Delta BPQ)\ldots\ldots\ldots\ldots\ldots\ldots(\text{xvii})$$

Now from (xii) and (xvii), we get,

$$2 \times \frac{1}{2} \text{ar}(\Delta BPQ) = \text{ar}(ARC)$$

$$\Rightarrow \text{ar}(\Delta BPQ) = \text{ar}(\Delta ARC)$$

Q8. In the following figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment $AX \perp DE$ meets BC at Y.



Show that:

(i) $\Delta MBC \cong \Delta ABD$

(ii) $\text{ar}(BYXD) = 2\text{ar}(MBC)$

(iii) $\text{ar}(BYXD) = \text{ar}(ABMN)$

(iv) $\Delta FCB \cong \Delta ACE$

(v) $\text{ar}(CYXE) = 2\text{ar}(FCB)$

(vi) $\text{ar}(CYXE) = \text{ar}(ACFG)$

(vii) $\text{ar}(BCED) = \text{ar}(ABMN) + \text{ar}(ACFG)$

Note: Result (vii) is the famous *Theorem of Pythagoras.* You shall learn a simpler proof of this theorem in class X.

Difficulty Level:

Hard

What is known/given?

ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment $AX \perp DE$ meets BC at Y.

What is unknown?

How we can show that

(i) $\Delta MBC \cong \Delta ABD$

(ii) $ar(BYXD) = 2ar(MBC)$

(iii) $ar(BYXD) = ar(ABMN)$

(iv) $\Delta FCB \cong \Delta ACE$

(v) $ar(CYXE) = 2ar(FCB)$

(vi) $ar(CYXE) = ar(ACFG)$

(vii) $ar(BCED) = ar(ABMN) + ar(ACFG)$

Reasoning:

We can use suitable congruency rule to show the triangles congruent. Also, we can use some theorem or properties like if triangle and parallelogram are on the same base and between the same parallels then triangle area will be half of parallelogram area.

Solution:

(i) We know that each angle of a square is 90°.

Hence,

$$\angle ABM = \angle DBC = 90^o$$
$$\therefore \angle ABM + \angle ABC = \angle DBC + \angle ABC$$
$$\therefore \angle MBC = \angle ABD$$

In ΔMBC and ΔABD,

$$\begin{aligned} \angle MBC &= \angle ABD && (\textit{Proved above}) \\ MB &= AB && (\textit{Sides of square } ABMN) \\ BC &= BD && (\textit{Sides of square } BCED) \\ \therefore \Delta MBC &\cong \Delta ABD && (SAS \textit{ congruence rule}) \end{aligned}$$

(ii) We have

$$\Delta MBC \cong \Delta ABD$$
$$\therefore ar(\Delta MBC) = ar(\Delta ABD) \quad \ldots (1)$$

It is given that $AX \perp DE$ and $BD \perp DE$ (Adjacent sides of square BDEC)

$\therefore BD \| AX$ (Two lines perpendicular to same line are parallel to each other)

ΔABD and parallelogram BYXD are on the same base BD and between the same parallels BD and AX.

Area ($\Delta BYXD$) = 2 Area (ΔMBC) [Using equation (1)] ... (2)

(iii) ΔMBC and parallelogram ABMN are lying on the same base MB and between same parallels MB and NC.

$$2\text{ ar}(\Delta MBC) = \text{ar}(ABMN)$$

$$\text{ar}(\Delta BYXD) = \text{ar}(ABMN) \qquad [\text{Using equation } (2)] \ldots (3)$$

(iv) We know that each angle of a square is 90^0.

$$\therefore \angle FCA = \angle BCE = 90^o$$

$$\therefore \angle FCA + \angle ACB = \angle BCE + \angle ACB$$

Adding $\angle ACB$ on both the sides

$$\therefore \angle FCB = \angle ACE$$

In ΔFCB and ΔACE,

$FC = AC$ (*Sides of square* ACFG)
$CB = CE$ (*Sides of square* BCED)
$\Delta FCB \cong \Delta ACE$ (SAS *congruence rule*)

SAS Congruence Rule – If two sides and the included angle of one triangle are equal to two sides and included angle of another triangle, then the triangles are congruent.

(v) It is given that $AX \perp DE$ and $CE \perp DE$ (Adjacent sides of square BDEC)

Hence, $CE \parallel AX$ (Two lines perpendicular to the same line are parallel to each other)

Consider ΔACE and parallelogram CYXE

ΔACE and parallelogram CYXE are on the same base CE and between the same parallels CE and AX.

$$\therefore \text{ar}(\Delta CYXE) = 2\text{ ar}(\Delta ACE) \qquad \ldots\ldots\ldots\ldots\ldots (4)$$

We had proved that

$$\therefore \Delta FCB \cong \Delta ACE$$

$$ar(\Delta FCB) \cong \text{ar}(\Delta ACE) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

On comparing equations (4) and (5),
we obtain

$$\text{ar}(CYXE) = 2\text{ ar}(\Delta FCB) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

(vi) Consider ΔFCB and parallelogram ACFG

ΔFCB and parallelogram ACFG are lying on the same base CF and between the same parallels CF and BG.

$$\therefore \text{ar}(\text{ACFG}) = 2\ \text{ar}(\Delta\,\text{FCB})$$

$$\therefore \text{ar}(\text{ACFG}) = \text{ar}(\text{CYXE}) \qquad [\text{Using equation } (6)] \ \ldots (7)$$

(vii) From the figure, it is evident that

$$\text{ar}(\Delta B\text{CED}) = \text{ar}(\Delta B\text{YXD}) + \text{ar}(\text{CYXE})$$

$$\therefore \text{ar}(\Delta B\text{CED}) = \text{ar}(\text{ABMN}) + \text{ar}(\text{ACFG}) \qquad \left[\textit{Using equations } (3) \textit{ and } (7)\right]$$